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Alternative investments role in institutions' strategic allocation

Tail risk in portfolio optimization

Master's thesis
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Abstract.

Recently there has been a growing demand for alternative assets. Low yield and expected return environment in traditional assets and appealing performance of the alternative investments have been the key drivers for that development. While the different risk-return structures of alternative investments have been under broad interest among academics, the scarce data have driven the practitioners to utilize traditional portfolio construction methods with alternative investments.

This study examines whether alternative assets enhance the efficiency of multi-asset portfolios when issues related to scarce and smoothed data have been taken into account and whether we can construct more efficient portfolios when we take the higher moments of returns into account in the optimization. We examine these questions from the institutional investor point of view, which have the most favorable characteristics to emphasize alternative investments in their strategic allocation. The studied asset classes are Private Equity, Private Credit, Infrastructure, Real Estates, Natural Resources, and Hedge Funds. The alternative assets data are aggregated private market fund quarterly returns from 2008 to 2021, which we desmooth to monthly returns. For traditional asset classes, we utilize monthly index returns.

The results confirm that alternative assets enhance the portfolio efficiency despite the data adjustments and chosen optimization objective. The optimal allocations towards alternative investments decreased when we took the higher moments into account, which implies that while alternative investments still enhance the portfolio efficiency, they do not look as attractive as the traditional mean-variance optimization suggests. When examining portfolio construction with backtests from an ex-ante perspective, we found that mean-variance optimization offered more efficient results than mean-expected shortfall optimization due to higher moments estimation complexity. We also found indications that CRRA objective function could work as an alternative for mean-expected shortfall optimization if we want to take the higher moments into account in optimization.

Key words: Alternative assets, Alternative investments, Strategic allocation, Expected Shortfall, Portfolio optimization, Institutional investors

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Tiivistelmä

Vaihtoehtoisten sijoitusten kysyntä on kasvanut merkittävästi viimeisen kymmenen vuoden aikana ja erityisesti mielenkiinto epälikvidejä vaihtoehtoisia sijoituksia kohtaan on lähtenyt räjähdysmäiseen kasvuun Covid-19 kriisin jälkeen. Matalan tuotto-odotuksen ympäristö perinteisissä omaisuuslajeissa sekä vaihtoehtoisten sijoitusten houkutteleva performanssi nähdään tärkeimpinä ajureina tässä kehityksessä. Vaikka vaihtoehtoisten sijoitusten poikkeavat riski-tuottorakenteet ovat olleet akateemikoiden laajan mielenkiinnon kohteena, niukka ja heikkolaatuinen tuottodata vaihtoehtoisissa sijoituksissa on ohjannut rahoitusalan toimijoita hyödyntämään perinteisiä portfolion rakentamismenetelmiä.

Tämä tutkielma tarkastelee parantaako vaihtoehtoiset sijoitukset yhdistelmäportfolioiden tehokkuutta, kun dataan liittyvät ongelmat pyritään korjaamaan, ja voimmeko rakentaa tehokkaampia portfolioita ottamalla optimoinnissa huomioon tuottojen korkeammat momentit. Tarkastelemme näitä kysymyksiä institutionaalisten sijoittajien näkökulmasta, koska näiden sijoittajien ominaisuudet ovat suotuisimpia vaihtoehtoisten sijoitusten painottamiseen strategisissa allokaatioissa. Tutkittavia vaihtoehtoisia omaisuusluokkia ovat Private Equity, Private Credit, infrastruktuuri, kiinteistöt, luonnonvarat ja Hedge rahastot. Vaihtoehtoisten sijoitusten data on indekseiksi aggregoitua kvartaalittaista listaamattoman markkinan tuottodataa, joka tutkimuksessa avataan kuukausituotoiksi. Perinteisissä omaisuuslajeissa hyödynnämme laajoja markkinoita kuvaavia kuukausittaisia indeksituottoja. Tutkimuksessa tarkkaillaan tuottoja vuosilta 2008-2021.

Tutkielmassa saadut havainnot vahvistavat aiemmissa tutkimuksissa saatuja tuloksia vaihtoehtoisten sijoitusten kyvystä parantaa portfolioiden tehokkuutta, huolimatta datan korjauksista tai optimoinnin tavoitteista. Vaihtoehtoisten sijoitusten osuus väheni optimoiduissa portfolioissa, kun otimme tuottojen korkeammat momentit huomioon optimoinneissa. Tämä viittaa siihen, että vaikka vaihtoehtoiset sijoitukset edelleen parantavat salkkujen tehokkuutta, ne eivät näytä niin houkuttelevilta kuin perinteisessä tuotto-varianssi optimoinnissa. Kun portfolion rakentamista tutkittiin simuloinnilla ex-ante näkökulmasta, havaitsimme että tuotto-varianssi optimointi tarjoaa tehokkaampia tuloksia kuin tuotto-odotettu alijäämä optimointi, mikä todennäköisesti viittaa korkeampien momenttien estimoinnin haasteellisuuteen. Löysimme myös viitteitä CRRA tavoitefunktion käyttökelpoisuudesta tuotto-odotettu alijäämä optimoinnin sijaan, kun haluamme ottaa korkeammat momentit optimoinnissa huomioon.

Avainsanat: Vaihtoehtoiset sijoitukset, Strateginen allokaatio, Odotettu alijäämä, Portfolion optimointi, Institutionaaliset sijoittajat

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1 INTRODUCTION

1.1 Background

Due to the current market situation where valuations are higher, and yield curves are lower than long-run averages, institutional investors have started looking for alternatives to increase their portfolios' expected returns. Alternative investments such as Private Equity (PE), Private Credit (PC), Real-estates (RE), Hedge funds (HF), natural resources (NR), and infrastructure (IF) have faced growing demand. Assets under management (AuM) in the alternative investments industry have grown from 3.1 trillion dollars in 2008 to 12.5 trillion dollars in 2021, which gives a 12% Compounded annual growth rate (CAGR) (Preqin 2021). Also, portfolios' allocations to alternative investments have risen significantly. Global pension funds' allocations to alternative investments have increased from 5% in 1996 to 26% in 2019 (BlackRock 2020). Net flow to alternative investments has been arguably strong. Reasons for this kind of rise in allocation have been argued to be at least; significantly lower interest rates and expected returns, alternatives' additional return premiums, the need created by market uncertainties for increased diversification, and the fact that the number of listed investments have not significantly increased from the beginning of the millennium while investable assets are (The World Bank, 2019). In recent years, exceptional liquidity in the market due to the quantitative easing followed by the aftermath of the COVID-19 crisis has further accelerated the shift in the institutions' strategic allocation from traditional asset classes to alternative ones. Alternative investments are now the second-largest megatrend in the financial markets after the ESG investing.

The trendiness of the asset class after every major crisis raises an interesting question. Does it require extreme market conditions for investors to find this appealing-sounding asset class?[1] Maybe. But does it give investors a simple answer to navigate through financial markets in the coming decade? Probably not. Like every other asset class, alternative assets have their pros and cons. However, investors often fall into a pitfall when they try to enhance their portfolios by adding alternative investments. Compared to conventional asset classes, risk and return structures of alternative investments are often misunderstood. Alternative investments typically have low volatility due to reporting biases. Even if the volatility is low after adjusting the reporting biases, it might be obtained at the

[1] This asset class seems to have a pattern with dot-com bubble, financial crisis, and covid-19 crisis.

cost of negative skewness and higher kurtosis (Amin and Kat 2003). If an investor is not restricted with quadratic preferences, these risks also need to be taken into account. When we use alternative assets in a portfolio, we should also have alternative tools to assess the risk and return of the portfolio.

Alternative investment vehicles are not new inventions in the financial markets. The earliest examples of private equity investments are from the 19th century's industrial revolution. The first alternative investment to infrastructure can be traced back as long as 1862, when individuals invested in the transcontinental railroad in the United States (Duran 2013). Nevertheless, the proliferation of alternative investments did not begin until after the millennium. The dot-com bubble crash reminded investors how the financial markets could be fragile. In 2002, as again in 2021, all traditional asset classes had single-digit expected returns. At the same time, David Swensen introduced Yale Endowment fund's success with alternative investments in his book *Pioneering Portfolio Management*. The Yale Endowment fund, which had around 60% of its allocation in alternative assets at the time, had survived surprisingly well from the crash and had an even more lucrative outlook ahead. This market environment led to the boom of an alternative investment era, and the period between 2001-2007 can be seen as the golden age of alternative assets. Eventually, in 2008 alternative assets disappointed investors when most sub-asset classes crushed down with the market (Ilmanen 2011). Since then, the rallying of the asset class has been substantial, but still, alternative investments are a fairly new part of the institutions' strategic allocation. According to the Prequin survey[2], 24% of institutional investors still do not have alternative investments in their portfolios.

In traditional mean-variance portfolio construction, one has expected return, volatility, and correlation as an input. Optimizing a portfolio is quite straightforward with conventional asset classes, which have relatively efficient markets and abundant data. However, with alternative assets, one usually needs to consider also illiquidity, opaque valuations, scarcity of data, and rigidity of implementation (Chan 2021). This challenge investor to evaluate the risks of the portfolio from new perspectives. For example, illiquid assets usually look better than their liquid counterparts even if the economic factors behind these different asset classes are largely the same. This is due to smoothed data that fools the conventional risk indicator, namely volatility. However, if we use alternative risk indicators which highlight the worst possible outcomes (e.g. Value at Risk and Expected Shortfall), we perceive that investors with illiquid assets probably suffer more profound and

[2]Preqin surveyed 231 institutional investors in June 2021

sustained outcomes. Due to the stale pricing and smoothing bias of illiquid assets, brief tail events, such as the Covid-19 drawdown, do not fully reveal these risks to investors, but when the stress period is more extended, these risks will probably be materialized worst than expected (Group 2021). Nevertheless, when we understand the additional risk factors behind alternative investments, we can manage and deploy them to returns (Ilmanen 2011, 239-240).

1.2 Purpose of the study and research questions

Most part of the portfolio's return and risk contribution comes from strategic allocation (Hoernemann, Junkans and Zarate 2005; Ibbotson and Kaplan 2000). However, instead of this being considered a truism of finance theory, it should be considered more about explaining the behavior of an average investor. Meaning that tactical asset allocation and security selection can add value and have their time and place[3].(Swensen 2009) Nevertheless, the well-diversified portfolio drives rational investors to the starting point where strategic asset allocation serves most opportunities, reduces the degree of unreliable factors, and grounds the decision-making framework on the stable foundation of long-term policy actions. Having strategic asset allocation in the center of investment philosophy enables long-term investment success for institutions' portfolio management. (Swensen 2009, 50-55.)

Since Markowitz's paper in the Journal of finance in 1952, the Modern Portfolio Theory (MPT) has been one of the most studied and challenged finance literature topics. At the same time, its simplicity has made it the number one tool for practitioners to construct portfolios and strategic allocations. Post Modern Portfolio Theory (PMPT) suggests many improved models optimize portfolios, but its results have remained controversial enough to keep MPT as a leading tool in practice (Malkiel 2019, 185-224). However, when we talk about alternative investments, the MPT and its mean-variance framework become arguably too restrictive and straightforward, not least because it assumes that investors have a quadratic utility function and are only interested in the first two moments of the return distribution[4] . It is studied by many that alternative investments return distributions differs significantly from the normal distribution which makes traditional

[3]The more proper way to analyze the added value of tactical asset allocation and security selection would be to analyze attribution instead of examining contribution.

[4]It has been shown that investors are willing to have lower expected returns and higher volatility in exchange for lower kurtosis and higher skewness.(Dittmar 2002; Harvey and Siddique 2000; Mitton and Vorkink 2007; Barberis and Huang 2008.)

MPT optimizations inefficient (Fung and Hsieh 2000, 2001; Brooks and Kat 2002; Popova, Popova, Morton and Yau 2006; Agarwal and Naik 2004; Jondeau and Rockinger 2006). Also, alternative investments' data usually have stale pricing and appraisal smoothing biases, which must be considered before constructing a portfolio (Cumming, Helge Haß and Schweizer 2014).

Institutional investors are usually characterized by infinite investment horizons and future liabilities to cover with investment returns. Institutions' infinite investment horizon supports the risk appetite. Contrarian, investment objectives usually emphasize capital conservation and hedging against liabilities. When we also consider a large absolute proportion of an asset that institutions have, enabling them to invest in illiquid assets, it is easy to see that alternative investments are particularly interesting opportunities in institutions' strategic allocation. Large initial investments and long investment horizons enable institutions to carry risks and harvest premiums that are usually beyond individual investors' reach. This study examines institutions' portfolio construction from the traditional mean-variance framework and post-modern portfolio theory's unconventional tail risk perspective. Two different risk perspectives enable us to assess portfolios risk properties from a broader range and examine how these properties affect the allocation obtained from optimization problems. The mean-variance framework is used as a starting point to see how alternative investments affect the portfolio's performance when added to the optimization problem with traditional assets. After that, we examine how optimal allocations will change when we use optimization methods that control the tail risk. The two research questions are:

1. Can we obtain more efficient portfolios by adding alternative investments with traditional asset classes into the investment universe?

2. Can we obtain more efficient portfolios by optimizing return to expected shortfall instead of variance?

Efficiency is defined in this study as a risk-adjusted return, and consequently, the *efficient portfolio* is the highest expected return portfolio in chosen risk level. In this study, the risk is defined and examined as variance and expected shortfall, focusing on the Sharpe ratio and Return on Expected Shortfall metrics. This definition makes us focus on the efficient frontiers, whether we examine them from a variance or expected shortfall point of view. Utilizing two efficiency metrics allows us to examine whether we obtain more efficient portfolios in one perspective at the expense of another efficiency perspective.

There are two reasons why we use the mean-variance framework in this study, even after earlier stated drawbacks; First, it represents the standard industry approach. Its simplicity makes it the most used tool, and this way, it works as a great benchmark. Secondly, its premise about normally distributed returns makes it an interesting method to be compared with the expected shortfall optimization method, which can deviate from this assumption. This way, we can quantify how much the estimated distributions affect the allocations and the performance of these optimal portfolios. The expected shortfall measure has brought to the optimization problem of this study because this way, we can take the higher moments into the heart of optimization. Higher moments are engaging in this study since they illustrate the tail risks that alternative investments are usually prone to. If we optimize risk measure that is affected by return distributions higher moments, as the expected shortfall is, then we can manage the whole risk spectrum and make more efficient portfolios. In addition, tail risk measures like expected shortfall and Value at Risk are essential for institutions. Even with a long investment horizon, institutions still need to ensure that they are not endangering to fulfill their recurring liabilities. The third reason for examining portfolios from the expected shortfall point of view is the increasing regulatory standards. In 2016 Basel Committee added Expected Shortfall to complement Value at Risk measure to calculate financial institutions capital requirements for the market risk.

This study tries to deep dive into alternative investments as a heterogeneous asset class. The goal is to understand better the characteristics of different alternative investments sub-asset classes and portfolio construction techniques from different risk perspectives. These insights are helpful when building a strategic asset allocation for institutions and help us understand the role of alternative investments in institutions' portfolios.

1.3 Scope and limitations

The scope of this study is to evaluate alternative investments' suitability into institutions' allocation in the long horizon and limit tactical asset allocation outside of the study. The studied assets class and investor profile under examination provide a sound basis for this scope. The illiquidity of alternative investments makes it unreasonable to allocate this asset class tactically due to slow operational execution and costs. The institutions' long-term investment horizon also rationalizes them to focus on long-term asset-liability management and minimize the dependence on uncertain short-term factors.

The idea is to view the allocation problem from a policy maker's point of view, where the institutions' investment policy should guide the portfolio management for decades. That is also why we focus on rather static allocations. There are at least two arguments to justify this decision. First, even though alternatives as an asset class are a heterogeneous group, most are characterized by illiquidity. It is unreasonable to expect that investors could dynamically optimize this asset class's allocation very frequently in practice. For the same reason, we assume that the investor would rebalance its portfolio once a year, even though more frequent rebalance might be possible. The second reason is that alternative asset risk factors and returns premiums are seen at least conceptually static and are studied to be nearly so also in practice (Ilmanen 2011). This enables us to focus on the asset class's long-term return and risk structure and helps us cope with scarce data since we do not have to consider changes in distribution when examining individual sub-asset classes.

This study groups alternative investments as a Preqin classifies them. The sub-asset classes are; Private Equity (PE), Private Credit (PC), Hedge funds (HF), Real-Estate (RE), Infrastructure (IF), and Natural Resources (NR) (Preqin 2021). These assets can be invested directly (e.g., real-estates, timber, investments in private companies) through funds or fund of funds. The data for alternative investments is Preqin quarterly index data from 2008 to 2021, which consists of all reported vintages. Indices are aggregated from reported funds' money-weighted rate of returns (IRR). The data is free from survivorship bias but suffers from back-fill, selection, and liquidation bias which we look over in the methodology chapter. In general, alternative investment data suffers from appraisal smoothing and stale pricing. The data will be desmoothed, and potential lags are fixed. For traditional asset classes, we use index data from Bloomberg, and it covers government bonds, investment grade and high yield corporate bonds separately and developed and emerging market equities separately. The study is conducted in U.S. dollars and the investment universe is global.

Efficiency in research questions is defined as a risk-adjusted return. The efficiency metrics, which this study will mainly focus on, are Sharpe and Return on Expected Shortfall (RoES). These indicators describe portfolios' performance from a different risk point of view. Since mean-variance optimization optimizes Sharpe ratio and on the other hand mean-expected shortfall optimization optimizes RoES ratio, it can be expected that depending on the optimization model on review, the simulated performance should show better results with the pairing efficiency metric. That is why it is vital to examine these metrics together and cross the optimization method. This way, we will get a more comprehensive picture of portfolios' performance, which helps us evaluate alternative investments' potential benefits in strategic allocation. In addition to the performance indicators, we are interested in how the allocations will change when we change the optimization method. When we take the higher moments into account, the allocation to alternative investments will probably decrease compared to mean-variance optimization. Also, the sub-asset classes allocation is expected to change. Since this study is carried out from an institutional investor perspective, it raises for review of the institutions' liquidity needs and expected shortfall objectives. These factors are taken into account when conclusions of optimal allocations are made. The results offer a robust view of why we should allocate at least some proportion of our assets to alternative investments. The scope of this study is motivated by the fact that the current practice in the market is focused on handling alternative investments the same way as the traditional ones.

1.4 Structure of the thesis

This thesis can be divided into four parts: Literature review, theoretical foundation, empirical research, and conclusion. First, we explore earlier studies for characteristics of alternative investments and institutional investors. Here the main point of interest is how alternative investments' product structures and risk characteristics differ from traditional assets. We also get insight into which purpose investors usually link different sub-asset classes. In this chapter, we also identify the most common characteristics of an institutional investor and use the Yale endowment fund as a case study.

After the literature review, we go through the essential theoretical background. The reader of this thesis is assumed to be familiar with the most fundamental theoretical framework of finance, but still, we iterate the most critical theories: Modern Portfolio Theory (MPT) and Post Modern Portfolio Theory (PMPT). Modern Portfolio Theory is the foundation of portfolio optimization, and Post Modern Portfolio Theory brings important tools to deal with asymmetric returns and tail risk. Here we also introduce the basic optimization process in the mean-variance framework and the mean-expected shortfall framework. We introduce the higher-order moment estimation and how these have brought to the optimization problem. We also introduce an objective function utilized in the empirical simulations as an interesting reference portfolio. After that, we go through previously utilized models for portfolio optimization with alternative investments. The idea is to highlight the pros and cons of models utilized in previous studies. This way, we can lay the groundwork for optimization models utilized in empirical research.

The thesis continues with the methodology chapter, demonstrating how the empirical study is conducted. Here we first recall the research objectives and explain the research strategy, which contains a comprehensive review of data collection and analysis, together with the descriptive statistics. This chapter also shows how portfolio optimization routines and out-of-sample simulations have been carried out.

In the end, results are analyzed and reported. Here the main interest revolves around the changes in allocation and efficiency metrics. We go through the validity and reliability of this study, and to establish the robustness, backtests are discussed. Finally, we bring the literature review and the empirical study together and shortly conclude the findings.

2 LITERATURE REVIEW

Alternative investments are typically characterized by an inflation hedge, a weak correlation between other assets, alternative risk premiums, lock-up periods, and illiquidity. At the same time, institutional investors are usually characterized by an infinite investment horizon, liabilities to cover with portfolio returns, and significant investment assets. Institutions' infinite investment horizon supports the risk appetite, while investment strategies usually emphasize capital conservation and hedging against inflation. This starting point gives institutions the ability to utilize alternative investments. However, the question is how this should be done allocation-wise. For example, an infinite investment period could suggest that institutions should harvest liquidity premium by locking up to private equity instead of the public one. Same time capital conservation and yearly cash flow needs suggest that institutions should focus on stable inflation hedged returns like unleveraged real-estates. To find out from which perspective institutions should look at their strategic allocation and whether alternative investments enhance their portfolios, we need to deep dive into alternative investments' heterogeneous characteristics. Before quantitative research, in this chapter, we first evaluate alternative assets and institutional investors from a qualitative perspective.

2.1 Alternative assets

Ilmanen (2011) defines alternative investments as everything other than traditional assets. Fraser-Sampson (2010) challenges the categorization of "Alternative Assets." The problem with the word "Alternative" is that it could be understood as a substitute for something even though they should be recognized as complements for the traditional assets. When we talk about alternative assets, we must know that the term is subject to unconscious prejudice. These prejudices are one potential reason why investors, especially outside the U.S., still allocate little to these assets (Fraser-Sampson 2010). Same time we are dealing with a very heterogeneous group which challenges the rationality of categorizing these assets as one group. So why do we bring these heterogeneous assets together, and why are they called alternatives? One sensible explanation to group these assets is the lack of knowledge that most investors carry over these assets. The adage of famous Warren Buffett states that one should never invest in anything they do not understand. It explains why investors and finance professionals willingly group these assets as "others" to focus on the traditional expertise of publicly traded stocks and bonds. However, alternative investments also have, as a group, recognizable characteristics compared to traditional asset classes. They are typically illiquid, rarely publicly traded, and exhibit higher fees. However, commodities, which are part of the natural resources, can be seen as an exception for the properties mentioned above. Alternative investments are also less scalable, less transparent, and more exposed to information asymmetries. Even though they are, as a group, great diversifies, correlations vary a lot between different sub-asset classes. They also have the potential to enhance returns through alternative risk premiums; illiquidity premia, alternative beta premia, and potential alpha. Alternative investments also have one common drawback: limited historical return data and questionable quality. The historical data problems originate from the fundamental difference in valuation. When traditional asset classes value forms at the market and the main risk comes from the volatility of this value, at the same time, illiquid alternative investments mark-to-markets base on the fund manager's or external service provider's valuations. For example, Private Equity funds have barely any volatility, but the mark-to-market value represents problematic appreciation-based fair value. The difference in asset valuation calls for different risk metrics as well. (Fraser-Sampson 2010; Ilmanen 2011.)

The high costs usually related to alternative investments are due to the active management which they demand. Since alternative investments have less efficient markets and lack an investable benchmark, it is hard to find comprehensive passive products to get exposure purely to alternative betas. From the institutional investor point of view, while one remains passive with the allocation perspective, the underlying fund management is everything else than passive. The value created with alternative investments is due to active investment management. However, the importance of active portfolio management varies between different alternative asset classes. While in real estate and natural resources, the assets themselves drive substantial returns passively, active management plays a critical role in private equity, private credit, hedge funds, and infrastructure. (Swensen 2009)

While categorizing heterogeneous assets is dangerous, it gives us the ability to understand the bigger picture of multi-asset allocation. This study focuses not on individual sub-asset classes but rather on the wider allocation problem that portfolio managers worldwide face. Still, to get there, we need to understand where these different sub-asset classes stand in their own and the minds of market participants. For that reason, we go through each asset class and investors' current general view of them. Table 1 illustrates the general views of asset classes by ranking their properties according to institutional investors' reasons when making an investment decision. Properties are categorized as diversification, high absolute return, inflation hedge, high risk-adjusted return, reliable income stream, and low correlation to other asset classes, and they are scaled from 0-100% according to Preqin interviews (Preqin 2020).

Table 1: Investors' views for alternative assets characteristics

This data has been collected from Preqin. They interviewed almost 400 institutional investors and asked the main reasons for investing in alternative assets. Numbers under each asset class represent how many percent of interviewed investors named characteristics in question to one of their main reasons to invest in that asset class.

Characteristic	PE	PC	HF	RE	IF	NR
Diversification	60	65	63	68	70	66
Low correlation	15	29	38	30	35	32
High absolute return	55	28	37	26	23	26
High risk-adjusted return	42	37	29	20	28	20
Inflation hedge	4	6	2	30	25	28
Reliable income stream	7	38	0	35	28	8

When we examine this table vertically, we see that diversification is the most important reason any sub-asset class is invested in. This strengthens the narrative that the current market uncertainties support the record inflows to alternative investments. Otherwise, the importance of different characteristics starts to differ when going down the list. However, in this point, it is essential to note that all of these mentioned categories are listed as one of the main reasons to invest in a particular sub-asset class, some more often than others. This said it confirms that investors expect the exact properties from alternative investments stated earlier in this study. When we examine this table horizontally, we can rank different sub-asset classes according to their importance for each characteristic, and this way, we get qualitative differences between assets.

While the diversification row tells that it is the most important for IF and least necessary for PE, the differences are minimal. However, we get more significant differences when we examine the low correlation to other asset classes, which means the diversification in the second-order moment. Again, it is the least important reason for investing in PE, but after that, the rank goes as PC, RE, NR, IF, and for HF, it is the most important reason to invest in. While PE is not seen as a good diversifier, it is seen as an essential asset class from a returns perspective. PE ranks in number one asset class in factors High Absolute Returns and High Risk-Adjusted Returns. We see interesting changes in investors ' expectations from an inflation hedge and cash flow perspective. Here PE and HF shine with their absence, but these are important characteristics for RE and IF investing. Even though reliable income stream and inflation hedge are usually complementing characteristics, PC is not used for inflation hedge at all, but at the same time, it is used as the most important source of reliable income stream. With NR, the opposite is true, and it is easy to see why it is not seen as a source of income stream, but instead an inflation hedge since inflation usually goes fast to raw material prices. However, with PC, it is interesting how this asset class is seen as a reliable income stream but not an inflation hedge. While with PC, one usually gets better terms for the lender money compared to marketable bonds, the companies funded have higher credit risk. At the same time, PC bonds are usually with the floating rate, which should, at least indirectly, hedge against inflation. These views lay an exciting background for alternative assets when we examine each sub-asset class more comprehensively, especially when discussing the descriptive statistics in methodology chapter.

2.1.1 Private Equity

The private market (equity and credit) is one of the most booming alternative investments sub-asset classes right now, not least because of low rates and tightened bank regulations. The former has led to chasing alternative risk premiums, which private investments typically have, and the latter has created a gap in financing (Chan 2021). Also, Ilmanen (2011) defines Private Equity as a return enhancer compared to other alternative assets, while Swensen (2009) points out that its diversifying properties are limited.

As most of the alternative sub-asset classes, also private equity is a heterogeneous class itself. On the other end is venture capital firms which invest in start-ups and early-stage companies usually as a target to take them public, while in other end is buyout firms which, with the help of leverage, turn public companies to private as a target to develop the company and maybe bring it later back to public or sell it to a strategic investor. These different types of private equity firms have usually focused on the specific category, but recently there have been more and more balanced private equity funds that handle the whole spectrum. While venture capital funds are more exposed to fundamental risks of immature businesses, leveraged buyout funds carry the risk involved to heavy leverage. Even though these different categories have quite different risk characteristics, they still carry the same distinguishable structure (Swensen 2009). A private equity fund is formed from a general partner (private equity firm) and limited partners (investors). General partner works as an investment manager whose responsibility is to find attractive opportunities, acquire them and add value by developing them. At the same time, limited partners have first committed money to the fund. When the general partner finds an investment opportunity and makes the capital call, limited partners deploy the money. This structure enables efficient active portfolio management, which reduces capital inefficiencies and agency problems due to the ownership of general partners. However, this raises the limited partner's particular capital commitment type of illiquidity risk, where investors might need to liquidate capital from other investments in the worst possible time in the market to cover called capital. (Ilmanen 2011.)

The main ways to invest in private equity are a direct investment to the firm through private equity fund presented above, funds of funds, publicly-traded private equity firms, and secondary transactions (Kiesel, Zagst and Scherer 2010). This study excludes publicly traded private equity firms from our review. This way, we study components that give us the purest exposure to this market since publicly traded private equity firms have significant public market equity beta and give oversized correlation with the equity market. The return is calculated as a net return to limited partners to represent the actual value added to the investor. We also include fund of funds structure since it gives broad private market exposure, often including funds in the different stages as well as direct investments and secondary transactions. Overall, the fund of funds structure is more diversified but contains an extra layer of costs.

Private equity's historical performance remains controversial despite the large number of papers published around this topic. Ilmanen (2011) results with Cambridge Associates data that private equity earned 12,4% per annum in 1986-2009 while the public equity market earned 9,2% p.a. Also Leitner, Mansour and Naylor (2007) got complimentary results with Thomson's Venture Economics data. Venture capital funds earned 21%, while buyout funds earned 12% from 1986 through 2006. However, most of the returns came in the 90s, and after that, the returns have been more modest (Fraser-Sampson 2010). When we drill down deeper to the returns, some studies conclude that private equity managers, as an average, do not outperform the public market after fees (Phalippou and Gottschalg 2009; Kaplan and Schoar 2005). The fees seem to be the main reason for underperformance since with gross returns, private equity overperforms the public market (S&P 500) by 3% on average. Ilmanen (2011) interprets that PE managers are skillful enough to bring an excess return, but the hurdle after fees is too high to deliver value to limited partners, so fund managers are the only ones collecting the added value. Even though private equity has a high correlation with the equity market, some additional risk factors should provide additional premiums (e.g. illiquidity, size, leverage) compared to the public market. For that reason, the view of the average private equity manager being skillful is at least doubtful.

Since we know this asset class's controversial performance, why does it attract so much interest and cash inflows? One crucial factor is the significant dispersion between a best and worst manager. In the last ten-year time frame, J.P.Morgan Asset Management (2021) studied that the top quartile delivered 24,5% IRR after fees while the bottom quartile delivered only 1,6% IRR. The difference is the largest of all studied asset classes. Global public equity managers have 12,5% and 10,7% corresponding numbers for comparison. At the same time, these top-quartile managers tend to stay as top-quartile managers, indicating performance persistence and skill. More tremendous inefficiencies enable harvest alpha from private markets while a positive circle of success keeps top performers at the top (Ilmanen 2011). It is safe to say that manager selection matters in private equity. Swensen (2009) capsulizes that investors should avoid the private equity market in the absence of truly superior fund selection skills since only top quartile performance is sufficient to compensate for the higher risk and illiquidity.

Nonetheless, other possible reasons for strong cash inflow to this asset class are reporting biases and right-skewed returns, making the investors with lottery-seeking preferences pay overprice. We will study those two factors later in the empirical section. Since there is a lot of dispersion between funds performance and many different data providers that collect the data mainly from general and limited partners and not so much from the public sources, the possibility of selection bias is inevitable, which could explain the controversial results private equity performance.

While in the U.S., the Venture Capital category has historically been a vital part of the private market and socio-economic development of the country, in Europe, not least because of the hostile regulation and tax environment, it has less than 20% of private equity AuM (McKinsey & Company 2021). It can be argued that during this latest trend of private market growth, the U.S. Venture Capital market starts to be overvalued, while in Europe, there still might be possible to find attractive opportunities (Fraser-Sampson 2010). Overall, Venture Capital investments stumble to the same problems as private capital in general. High fees, modest average returns, and usually terrible average risk-adjusted returns challenge the investor to re-evaluate the attractiveness of this strategy. The typical characteristics that are different from other private capital investments are higher operational risk, the exclusivity of the top quartile funds, and the glamour reputation of high-profile start-up success stories. The Venture capital results exhibit wide dispersion of returns. For example, in the Investment Benchmarks report (1985-2005), the VC mean return was 3,2% p.a. while returns ranged between 721% and -100%, resulting in a 51,1% standard deviation. Since the dispersion of returns is highest in this strategy, it emphasizes the importance of manager selection. However, the exclusivity brings its hurdle here, since as an investor, to get the money to the most skillful managers, one has to been already invested earlier funds of this manager. At the same time, most promising success stories want to corporate only with the best VC managers, which makes the success stories naturally flow these funds. This creates a circle of success that diverges more and more from the overall VC market. (Swensen 2009.)

In Europe, the leverage buyout strategy has been the leading private equity investment strategy (Fraser-Sampson 2010). It is the more criticized strategy for the abundant management fees and modest average risk-adjusted returns since, in many cases, it is simply financial structuring more than developing corporate operations. It is also studied that this strategy contains a significant negative relationship between the size of the fund and the performance (Lerner, Schoar and Wang 2008). However, as Swensen (2009) stress, an investor cannot just pick the small funds since those usually contain higher operational risks. Instead, to find successful funds from leveraged buyout firms, Swensen (2009) emphasizes the funds with lower leverage and higher operations development focus. This way, LBO can offer significant risk-adjusted returns. The key to this is still to find skillful PE managers, which is possible only with heavy due diligence. This recipe applies to all private capital investments (Swensen 2009).

2.1.2 Private Credit

Private credit can be defined as higher-risk debt investments in unlisted, typically small and medium-sized companies. PC is often used to finance corporate and real estate transactions, projects, foreign trade, and corporate restructuring. While many LBO private equity funds seek successful businesses with positive cash flows to squeeze a little bit more out of them by developing the company, private credit is usually used to distress and turnaround deals since this allows the creditor to structure propitious loans from an investor point of view (Fraser-Sampson 2010). From a borrower's view, PC is usually desired private capital in family-owned companies since it does not affect the ownership situation as PE does (Kiesel et al. 2010).

In particular, the market exists because post-financial crisis regulation has forced banks to reduce their lending, especially for riskier opportunities. The lack of credit supply created a gap in the market that Private Credit players have grasped. Indeed, the market has grown from 150 billion USD in 2007 to about

1000 billion USD by 2021. Differences to traditional bonds are lack of credit ratings, little information about counterparty, non-existent or poor secondary market, credits that are almost always collateralized, multiply covenants, higher recovery rates, and the effective loan term is usually three to four years. Although the asset class has many more favorable features to the investor than in traditional public bonds, counterparties are still interested in borrowing from private credit investors. Reasons for this are faster loan structuring, a small number of investors, and cost-effectiveness. These features make PC loans more flexible for changes which can be helpful for both lender and borrower. (Kiesel et al. 2010.)

Private credit strategies can be divided into three subcategories depending on their risk level. Namely, these categories are from the high to low risk; opportunistic credit, specialty finance, and covered senior loans. Opportunistic credit comprises, for example, distressed debt for companies that have or have the chance of filing for bankruptcy in the near future and mezzanine investments in debt that subordinate to the primary debt issuance and senior to equity positions. Specialty finance includes, for example, project finance, foreign trade financing, and invoice factoring. Covered senior loans are usually direct loans to small and medium-sized companies, and they are highest on the capital structure so that they will be repaid first if the borrowing company default. (Fraser-Sampson 2010.)

2.1.3 Hedge Funds

Hedge funds, or as Swensen (2009) refer to them as Absolute return funds, are again a very heterogeneous group that is argued not to be asset class on its own. It covers a wide range of investment strategies, which similarities are usually limited to the use of derivatives and arbitrage hunting from the market (Fraser-Sampson 2010). One crucial difference to other alternatives introduced in this study is that hedge funds use mostly liquid assets. However, many times hedge funds are illiquid for the investor. Hedge funds face less regulation than conventional funds with traditional assets, and that way, they are more flexible in using leverage, short selling, and derivatives. Investing in hedge funds is typically investing in the fund managers' skills since the idea is to find inefficiency from a relatively efficient market creatively. The word "hedge" does not mean that the managers hedge all or even systematic risk away but tells more about the standpoint where managers try to isolate the wanted exposure from everything else. In other words, they try to hedge all other risks away from the fund, but the risk exposure they have some view (Ilmanen 2011). To go through all the different hedge fund strategies and their properties, one could write a book about it, which is not the focus of this study. However, namely to go through, there are identified at least the following strategies; long/short, market neutral, Convertible arbitrage, statistical arbitrage, merger arbitrage, fixed income arbitrage, global macro, event-driven, and distressed (Fraser-Sampson 2010).

The freedom often explains the hedge funds' positive performance even after private equity-like fees compared to the average fund manager, which entails a broader spectrum of opportunities and skills acquired to hedge funds with attractive rewards. This would indicate that money moves from not so skillful investors to skillful investors in a systematic and persistent way. It is also argued that hedge funds earn rewards from market completion economic functions and illiquidity premium thanks to investor lock-ups. From a pessimistic point of view, it can be argued that positive results are due to reporting biases and unobserved risks. Since hedge funds reporting is voluntary, most return data sets have some degree of selection, survivorship and backfill biases, making the data overstate returns and understate risks. One way to decrease those biases is to use Fund of funds data. That data, however, suffers from double-layer fees. Also, the compelling risk-adjusted returns are due to chosen risk indicator, volatility. Many hedge fund strategies contain adverse skewness and kurtosis by their nature. These catastrophe insurances like strategies involve, for example, being short in volatility. In a normal market situation, they collect a steady premium from the market, but the value is expected to collapse under market stress. (Ilmanen 2011.)

As Ibbotson, Chen and Zhu (2011) illustrated, survivorship and backfill biases are particularly big problems with hedge funds. In their study, survivorship bias contributed 3,1-5,2 percentage points to yearly hedge fund returns and backfill bias 1,4-3,5 percentage points. Even so, with adjustments for survivorship and backfill biases as well as traditional risk factors, Ibbotson et al. (2011) found that hedge funds, on average, produced 3,0% alpha per annum in the period 1995-2009. However, there are still selection, liquidation, lookback, and lookahead biases, that have not been studied. Also, and more importantly, the risk adjustments do not account for illiquidity or tail risks. Like in private equity, Swensen (2009) highlights that investing successfully in hedge funds demands the ability to find top-quartile managers, which again demands extraordinary resources to do due diligence to identify those managers.

2.1.4 Real-Estates

Real-estates are available to investors through private real estate (direct investment, fund, or fund of funds) and publicly traded real estate investment trusts (listed REITs). The asset class can be divided roughly into two types; commercial and residential real-estates. Typically institutions hold investments in private commercial real estate. Ilmanen (2011) identifies that real-estates are usually added to portfolios, not because of their expected returns but their inflation hedge and diversification properties. This finding is in line with the earlier introduced Preqin survey. Real-estates are one of the three real assets in this study. (Ilmanen 2011.)

Francis and Ibbotson (2009) studied real estate investing compared to traditional asset classes and highlighted its ability to diversify the volatility and enhance the returns through the available leverage. They also point out the problematic smoothing bias in physical real-estate value time series. Indeed, real estate data has its challenges. While REIT data is relatively clean, it correlates more with the equity market than private real estate and, for that reason, has higher volatility. The private real estate return comprises of appreciation and rental yield. While rental cash flows are quite straightforward, calculating the appreciation can be done through appraisal-based or transaction-based values. Appraisal-based indices aggregate real estate values from the previous period[5] and, for that reason, smooths and stales the index values. This gives an erroneous picture of volatility and correlation of the asset class. Transaction-based data tackle these problems but, at the same time, it is very scarce data. (Ilmanen 2011.)

Real-estates can be classified according to their risk and return characteristics. Core and Core Plus categories are the safest, including main properties in primary markets and a low level of leverage. The Plus category differs from the former with the modest value-add approach, wherewith slight improvements in the property can create value. Then we have the value-added category, representing moderate risk/return investments. These real estates need proper development to get them class A buildings, and these type of investments usually requires 50-70% leverage. In the "high yield" end of real estate investments, opportunistic and distressed categories represent the high-risk, high reward investments. While the real-estate investments can be quite heterogeneous as well, the main difference in risk/return structure in real-estates comes from the level of utilized leverage. Data of this study aggregates all those categories to one index, but in practice, the investor can quite

[5] Typically a year.

easily choose a preferred category from the spectrum through private real estate fund, open-end fund, or direct investment to real estate. (Fraser-Sampson 2010.)

Depending on the used appreciation technique, the historical real total return of commercial U.S. real-estates varies between 6% and 7.3% p.a. in different data sets. The former is from Francis and Ibbotson (2009) study and covers the period between 1978-2008. The latter is from NCREIF[6] appraisal based data and covers 1984-2009 (Ilmanen 2011). Real total return for residential U.S. real-estates has greater dispersion depending on reviewed data. Francis and Ibbotson (2009) get a geometric average per annum real return of 1,7% for the 1978-2008 period, while Ilmanen (2011) gets 7% between 1960 and 2008 by combining Davis-Heathcote (2007) HPA data with Davis-Lehnert-Martin (2008) rental yields data. While there is a dispersion in the reported numbers, they leave open a quation about inflation hedge properties. By definition rental contracts are inflation hedged. Also, as Swensen (2009) points out, the real estate market has a strong relationship between market value and replacement cost. Since replacement costs consist largely of labor and material costs that move with inflation, real estate appreciation also enjoys inflation hedge properties. In empirical section we examine the real-estate return correlation with CPI inflation to confirm these properties.

2.1.5 Infrastructure

Fraser-Sampson (2010) defines infrastructure investing as planning, funding, constructing, and operating public sector assets. Infrastructure is a real asset like real estate. However, infrastructure can be divided into two distinctive opportunities; primary and secondary. Primary investing means private investment pools that are used to the long horizon projects, so the investor is not buying anything already built but instead committing to the building projects and that way have the opportunity to earn from the added value of creating something new. Secondary investments instead resemble long-term government bonds as they are already operating real assets with long horizon steady cash flows. The former represents private equity-like investment (higher risk and return characteristics), while the latter is an excellent alternative for government bonds with inflation hedge properties. Even though the asset class is heterogeneous, its investments usually focus on transportation, utilities, communications, and energy. The investments are characterized by quasi-monopolistic position and inelastic demand. Again, investors

[6] National Council of Real Estate Investment Fiduciaries

can invest in infrastructure through private investment pools or publicly traded infrastructure companies (Ilmanen 2011).

Investors need to consider specific risk characteristics when investing in infrastructure, namely regulatory, political, and geopolitical unrest. Most importantly, the market needs to have a low probability of getting governmental intervention in projects and their financing (Fraser-Sampson 2010). At the same time, the government needs to develop, and that way, financing national infrastructure creates investment opportunities for the infrastructure market. For example, it is not by accident that infrastructure investments are booming right now. In addition to the normal need for infrastructure development, the green transition moves tremendous capital to infrastructure. IRENA (2021)[7] approximates that over the period to 2050, there will need to flow 131 trillion USD to energy systems to keep us in the 1,5C pathway[8]. At the same time, Joe Biden's Bipartisan Infrastructure Bill is approved in the U.S., totaling 1,2 trillion USD, where 550 billion USD is new capital to the infrastructure sector (Times 2021). This trend inevitably lays the ground for lucrative return possibilities to this asset class in the future.

As with real-estates, the infrastructure uses the same categories for the constituents for the index used in this study. Namely Core, Core Plus, Value-Added and Opportunistic. The Core represents the low-risk end with secondary investments in stable countries with transparent regulations. These infrastructure operations are characterized by a monopoly position, demonstrated demand, and long-term stable cash flows that are forecastable with a low margin for error. On the other end of the risk spectrum is opportunistic infrastructure investments. These are primary investments that focus on capital growth completely without existing cash flow. (Preqin 2021.)

It is important to notice that the way investors create exposure to this asset class is not irrelevant. For example, many infrastructure ETFs have just exposure to sectors related to infrastructures like utilities and telecommunication. However, that does not mean that these ETFs have any exposure to actual infrastructure projects. At the same time, their correlation with the general equity index in the same market is usually extremely high[9], which spoils the idea of diversification which usually is one of the main reasons to invest in infrastructure in the first place. (Fraser-Sampson 2010.)

[7] International Renewable Energy Agency

[8] Keeping the climate warming under the 1,5°C

[9] For example Fraser-Sampson (2010) calculated correlation of 90% between MSCI European infrastructure index and MSCI European equities index

2.1.6 Natural Resources

Natural resources combine agriculture, energy, commodities, timberland, and water (Preqin 2021). This asset class represents real assets along with earlier introduced real estate and infrastructure, and it is typically characterized by low correlation with other assets and inflation hedging properties (Ilmanen 2011). Natural resources exposure can be created through futures markets or purchasing physical reserves. While one gets easy exposure to commodity price movements through future contracts, it is highly dependent on the spot price movements. As Ilmanen (2011) denotes, future returns consist of purely collateral return, roughly representing a risk-free rate, roll return representing ex-ante risk premium, and spot price change representing the unexpected return. Swensen (2009) highlights the reserves' ability in the energy market to produce high expected returns even without price movements. Same time investors still get exposure to the price movements. Also, investing in timber through private timberland enhances the returns through other activities in addition to changes in timber value. First of all, there is biological growth which silvicultural practices can influence. Biological growth usually contributes the most to the total return of the timberland. At the same time, these are incremental revenue streams for timberland, and an increase in land value is one significant contributor. A steady revenue stream can be gained through ecosystem services. These include carbon sequestration, forest protection, recreational pastimes, and leasing land for alternative energy use like wind power. All this makes timberland investment uncorrelated with the equity market while it is highly correlated with inflation making it a great hedge against it. (Swensen 2009.)

Again, this value creation is due to the complex and inefficient real asset world where expertise can create value compared to the highly efficient commodity futures market. Private acquisition on energy reserves carries the same principles as the private equity fund investment, so it is not trivial which manager runs the operations. This study focuses on physical natural resources that can be invested through private funds, open-end mutual funds, or direct investments in a physical asset.

2.2 Investment vehicles and liquidity

Alternative assets are usually linked to be illiquid investments. While most of the underlying assets of alternative investments are illiquid, different investment vehicles enable investors to invest liquid or at least more liquid ways to alternative investments. However, as Fraser-Sampson (2010) and Swensen (2009) point out, investors should always seek illiquid private investments to get clear exposure to alternative betas and low as possible correlation with equity and bond market. However, liquid publicly traded alternatives can be helpful in situations where illiquidity is a problem for the investor from a regulatory or investment objectives point of view.

Liquid investment vehicles that invest in illiquid alternatives offering liquidity premium to end investors have become trendy products in the asset management industry. Namely, these vehicles are usually alternative investment mutual funds or fund of funds. Nevertheless, as Fraser-Sampson (2010) doubts, it might be hard to keep the fund's liquidity in the market stress situations since the underlying assets are still illiquid. For that reason, mutual funds usually have the right to freeze the redemptions in case of liquidity droughts. However, here is important to stress that even traditional assets that are generally considered liquid have turned out to be everything else under severe market stress (Fraser-Sampson 2010). Herd behavior can make liquidity management assets to be the most illiquid ones in certain market drawdowns. This gives us an interesting perspective on the empirical part of this study, where we look at the tail risks of different asset classes. If the risks of alternative investments are primarily realized under the market stress, as suggested in this study, and at the same time the risks of traditional asset classes are realized both in calm market periods and under the market stress, the question arises as to whether it makes sense for a long-term investor to invest in the public market at all.

Investing in alternative assets ETFs means investing basket of publicly traded stocks that operate on sectors related to alternative assets but are not purely alternative investments. An investor can usually find a single stock from the market which operates either with private equity, private credit, hedge funds, real-estates, infrastructure, or natural resources. However, those usually differ significantly from alternative investments' characteristics, and even though they might have some small correlation with the corresponding alternative asset class, they are mainly driven by stock market beta. (Fraser-Sampson 2010.)

Traditionally liquidity of an asset is seen as a virtue. When an investor gets the sudden need for cash, investments are helpful only when they are liquid. However, as Swensen (2009) points out, liquidity is expensive. Having too much liquidity in one's portfolio is poor investing. It can be argued that the opportunity cost from holding liquid assets is due to weak liability modeling. This is why long-term investors should seek a liquidity premium for assets they will not need in the near future. However, illiquidity can be a problem also when investing money. Especially private equity and credit investors face problematic illiquidity through capital commitments where the capital call can come in an unfavorable moment (Ilmanen 2011). From the fund manager's point of view, illiquidity enables the manager to make long-term decisions necessary for investment success. This longevity should convert higher returns for the limited partners as well. In liquid markets, public companies' managers and investors get feedback constantly. This pressure might push to make decisions that are not constant with the long-term strategy (Swensen 2009). So it is safe to say that liquidity is a double-edged sword. It is highly dependent on the investment objectives of how much liquid assets one should have. Planning liquidity needs accordingly and optimizing the portfolio with that objective in mind should enhance the investment portfolio's performance.

Traditionally in finance, volatility is seen as negative, and liquidity is seen as positive property. To carry illiquid assets, one should earn a liquidity premium. According to that, alternative investments' illiquidity should reduce the utility obtained from their low volatility. However, low volatility is mainly due to low liquidity with alternative investments. This might be one reason why, a little counter-intuitively, there is some evidence that institutional investors are ready to pay extra instead of getting a premium for illiquid assets. The idea is to smooth return volatility away, albeit artificially (Ilmanen, Chandra and McQuinn 2019). This challenges the existence of a liquidity premium.

Assets illiquidity is usually statistically estimated with serial correlation (Cavenaile, Coën and Hübner 2011; Getmansky, Lo and Makarov 2004). However, we must acknowledge that liquidity typically drains in extreme market stress conditions. For that reason, using serial correlation of returns, which mostly incorporate normal market times, is wanting estimator, but it is typically recognized as the best available option for lack of a better one. We will examine asset classes serial correlation in our data sets in the empirical section of this study.

2.3 Institutional investors & Case Yale Endowment

> Goal is not to make money,
> making money is how one
> accomplishes the goal
>
> *Robert van den Meer*

Institutions differ from individuals as an investor a lot. While individuals have preferences that we try to model with utility functions, institutions have objectives that can be modeled with economic factors and liabilities. While individuals' investment horizons and allocations rely on the life-cycle hypothesis, institutions typically have an infinite investment horizon. At the same time, we have seen institutions allocations evolve. While Dutch Pension funds have led the way in increasing equity proportion in pension funds, the Yale Endowment fund has increased the interest towards alternative investments as a major asset class in institutions' portfolios (Van der Meer 2001; Swensen 2009). In this chapter, the idea is to drill down how these differences affect asset allocation and how, on the other hand, the institutions have evolved in their asset allocation policies.

Portfolio construction is about risk/return optimization, and like previously discussed, risk can be defined and observed in many ways. After we have an efficient set of choices, individual investors pick portfolios according to their subjective utility function. However, institutional investors usually have some liabilities to meet in the future that are not constant. For example, they can vary according to inflation, demographic trends, number of participants, maturity of the workforce, or interest rates. These factors add return and liabilities correlation to review and eventually lead to return/liabilities optimization. Since institutions' liabilities can differ a lot from each other, this return/liabilities aspect in portfolio optimization explains the allocation differences that institutions have. However, even though the eventual objectives for institutions' portfolios can vary a lot, a starting point for every institution's portfolio construction is to cover inflation, and on top of that, institutions start to pile up different objectives. Swensen (2009) for example, distinguish endowments and foundations investment goals due to regulatory. While endowment can dampen fluctuations in the market value by stretching the rules with asset preservation or stable allowance distribution, foundations are usually obliged to achieve a minimum payout level to retain their public benefit tax status[10]. This way, they are bound to stretch with the asset preservation.

[10] In the U.S. the payout level is strictly given to 5% of assets, while in Finland the payout level

Swensen (2009) argues that this should lead to more conservative allocations in foundations than in endowments. (Swensen 2009; Van der Meer 2001.)

To succeed, institutions need to do the investment planning from the objective perspective where the only concern is to meet the institution's purpose, whatever future liabilities that hold. However, in practice, institutions have many stakeholders with different incentives. This adds complexity to the institutions' portfolio management and is the most common reason institutions fail to meet their liabilities (Swensen 2009). However, from the risk perspective, Sortino, Satchell and Sortino (2001) argue that every stakeholder is concerned about the expected shortfall being more than the minimum acceptable return (MAR) to meet the liabilities. That is also why we are interested in the expected shortfall in this study since it is a risk metric that everyone agrees to be managed. The rest of the institutions' investment planning lies in the fiduciaries' hands, who have a critical job to decide the allowance level trade-off between current and future beneficiaries (Swensen 2009).

The Van der Meer (2001) introduced the Dutch Triangle to illustrate how dutch pension funds operate. The idea is that, at the strategic asset allocation level, every pension fund should start with comprehensive asset-liability management (ALM) analysis where every stakeholder's risk-return structures are analyzed in relation to their constraints. This way, we get constraints to the actual optimization problem, where the idea is to maximize the ratio of expected return over MAR relative to expected shortfall under the MAR. Van der Meer (2001) argues that the Dutch Triangle differs from traditional pension fund management by bringing the MAR in the center of asset allocation and performance measurement. It represents the benchmark to which all risks are related. The level of MAR represents the objective goal to meet the institution's purpose, while the ALM analysis brings the stakeholders different incentives to the optimization problem as a constraint. This way, the stakeholder's individual risks are taken into account while the maximization objective is based on purely institution objectives.

Another interesting and groundbreaking asset allocation in institutional portfolio management is introduced by Swensen (2009) in the Yale Endowment fund. Swensen (2009) was the first portfolio manager who brought alternative investments into the major role of endowments' asset allocation. While many asset classes he introduced were unique in the institutional portfolio, the share of the alternative assets in the overall portfolio stays unique in the Yale Endowment fund

is defined in income tax law to be "sufficient" and is more evaluated case by case (Swensen 2009; Verohallinto 2021) even today. The figure 1 shows the asset allocation development in the Yale Endowment fund from 2004 to 2020. As we see, traditional asset classes have never exceeded even half of the portfolio during that period[11]. Honor for this exceptional allocation goes to David Swensen, who joined Yale University as a CIO in 1985. Even though there were alternative assets in endowment allocation already in the '80s, in 1989, almost three-quarters of the endowment's assets were still in traditional asset classes. However, in the '90s, Swensen increased allocation to alternative assets drastically. As Swensen (2009) states, committing less than 5% in some particular asset class is not worth of effort since the effect on the overall portfolio is so small, and committing more than 30% is predisposed to overconcentration. So accordingly, the ideal number of asset classes is about half a dozen, as we see in the Yale Endowment fund.

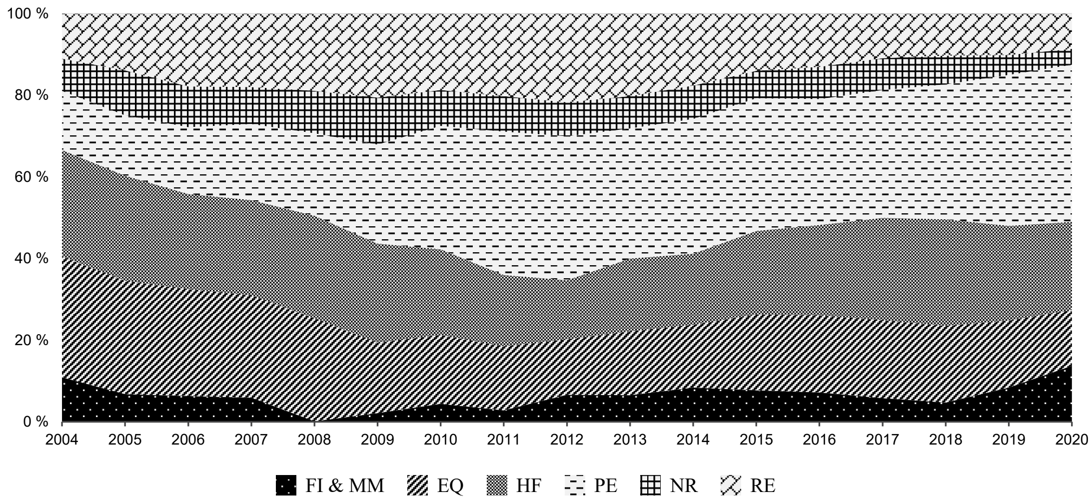

Figure 1: Yale Endowment fund asset allocation

If we compare this, for example, to Finnish Pension funds, the difference is substantial (see Appendix II). The difference can be partly explained by the different additional factors affecting pension fund liabilities like demographic trends, number of participants, and workforce maturity. However, endowment and pension funds also share many similarities. Contributions in a pension fund are more predictable but represent additional cash inflows similarly to gifts in the endowment fund. They increase assets as well as liabilities for the fund. However, the investment objectives of the two institutions are the same, preserving allowances purchasing power in the future and preserving fund's assets. Finnish pension funds' investment performance has been in the recent target of interest in Finland, and

[11]Highest allocation to traditional asset classes is 40,5% in 2004.

it is argued that current investment policy might not be able to meet the future liabilities (Andersen 2021). As previously mentioned, the last ten-year market conditions have also driven these funds to seek alternative assets. At an aggregated level, Finnish pension funds allocation to alternative investments have increased from 14,3% in 2004 to 28,5% in 2020 (Tyoelakevakuuttajat TELA Ry 2021). The Yale Investment Office (2020) motivates the heavy alternative investment allocation to assets' diversifying power and return potential properties. They further ground their heavy allocation to alternative investments by tying it to their infinite investments horizon, which is, according to Yale Investment Office (2020), well suited for illiquid, less efficient markets. Throughout the Swensen (2009) book, the attractiveness of alternative investments is based on exploitable inefficiencies that they offer for a skillful active manager. These inefficiencies are due to the illiquidity of these markets. This way, the tools to manage liquidity risk offer entry to these markets, and we are back to emphasizing the importance of financial planning.

Naturally, the interest arises in how the Yale Endowment fund has performed in history with this unconventional asset allocation. As we see from the figure 2, the endowment has substantially overperformed compared to wide market indices in the observed period. We view the performance in logarithmic scale so we get accurate effects from crisis to performance. Endowment returned 10,6% annually between 2004 and 2021, while in the same period, global MSCI World returned 7,9%, and U.S.-based S&P500 returned 9,6%, respectively. Even though we compare apples to oranges, this gives a sense of how the market-insensitive portfolio have performed compared to the sensitive equity market. Some might even argue that comparing endowment with risky equity-only indices is unfair since the endowment also includes conservative assets. Here is, however, important to remember that during that period, the endowment had 14-40% in Private Equity, which is usually seen riskier investment than public equity. The most interesting remarks from the figure are the timing and size of drawdowns in various crises. The way endowment survived from the dot-com bubble in 2001 is one of the main reasons alternative investments and especially hedge funds gained attention in the 20th century. The endowment was substantially affected by the financial crisis along with the market. However, while MSCI world recorded -40,7% and S&P500 -37% in 2008, the endowment recorded a positive return in the same year. Instead, in 2009 endowment recorded -24,6% while the overall market went already up. The lag was due to the endowment's illiquid assets and yearly reporting. This logarithmic return index illustrates well how the reporting biases affect the risk metrics of illiquid assets. A great example is the Covid-19 crisis in spring 2020, which can not be seen from the Yale endowment return index, even though the dip was severe, since the rise was so fast. The lag with alternative investments is partly due to reporting bias and partly due to market inefficiencies, which slows the market information to transmit asset values, called stale pricing. The infrequent mark to market with alternative assets gives portfolio managers and the investments themselves peace of mind to work towards long-term success and hedge investments from the market fluctuations. This is one of the key factors why Swensen (2009) sees private assets so remarkable in institutions' strategic allocation. However, in portfolio optimization and risk management context, we need to consider these lags to understand the potential insolvency risks.

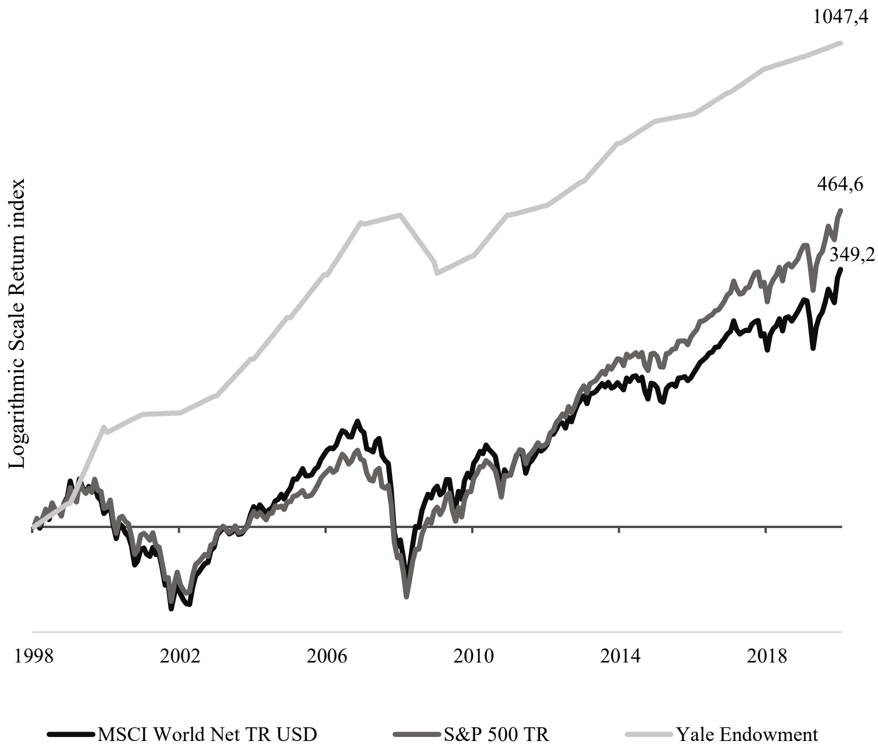

Figure 2: Yale Endowment fund performance

3 THEORETICAL FOUNDATION

3.1 Portfolio construction

The Modern Portfolio Theory (MPT) introduced by Markowitz (1968) demonstrates how to construct and select portfolios based on the expected returns of the investments and desired risk appetite of the investor. It works as a backbone of portfolio construction even today. In addition to the heavy utilization of the modern-day practical finance industry, this model works as an important starting point for new methods and inventions in the field of portfolio optimization (Fabozzi, Focardi and Kolm 2010). It shapes a clear framework that can be utilized when we challenge the premises of MPT. Figure 3 represents the process Fabozzi, Gupta and Markowitz (2002) created for MPT. In MPT, *Risk & Co-movement estimates* mean volatility and correlation estimates. However, when we interpret that part of the process to illustrate risk and co-movements more generally, the process becomes valid for other optimization models as well. This study utilizes this framework in mean-variance optimization and mean-expected shortfall optimization. In the latter, we have Expected Shortfall as a risk measure, and in addition to correlation, we also consider co-skewness and co-kurtosis.

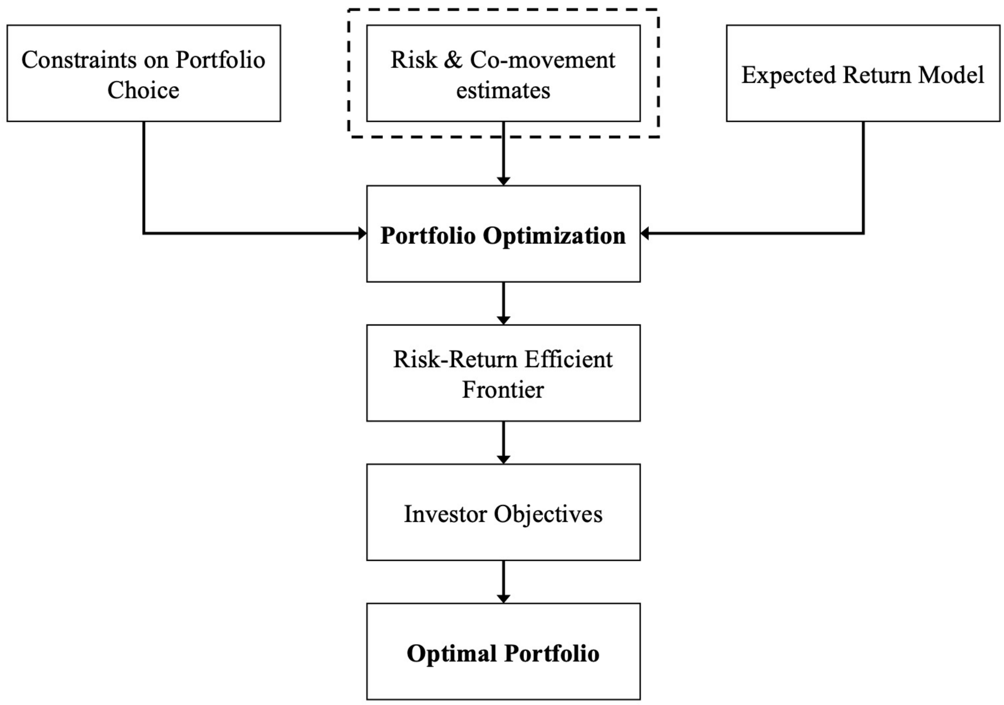

Figure 3: Portfolio optimization process

3.2 Modern Portfolio Theory

Modern Portfolio Theory (MPT) is based on the idea that the investor balances with risk and return. The return is expressed by the ex-ante expected return, and the risk is expressed as the ex-ante expected variance of this return. Markowitz (1968) showed that one gains diversification benefits by constructing a portfolio of not perfectly correlated securities. This benefit is often called the only free lunch in finance (Choueifaty and Coignard 2008). One of the major contributions of the theory is the perception that when adding security to a portfolio, relevant for the risk of this portfolio is the correlation between security and portfolio, not the variance of this security. Markowitz (1968) proved this mean-variance theorem where investors can either minimize variance (i.e., the expected risk on return being constant) or maximize the expected return when variance is constant.

The diversification benefit means that security's risk in the portfolio is lower than holding it standalone. After Markowitz (1968) proved this to be accurate, we needed a new risk definition. Sharpe (1964) introduced the Capital Asset Pricing Model (CAPM) for this need. In this model, the total risk of a security is divided into two, systematic market risk and non-systematic security-specific risk. So the model continued to fine down the diversification benefit introduced by Markowitz (1968). Indeed, Sharpe (1964) showed that diversification could explicitly eliminate the unsystematic risk of securities when adding securities to the portfolio so that the overall risk of the portfolio approaches the systematic risk of the securities. As unsystematic risk can be diversified away, the market does not reimburse this part of the risk to the investor. Since the model assumes homogeneous expectations and efficient markets, the unsystematic risk is not included in the CAPM since investors should not expect any returns from carrying idiosyncratic risk. Therefore, according to the model, investors are only interested in the systematic risk and correlation of securities with the market portfolio. The relationship between systemic risk and expected return of an asset can be presented as the CAPM formula:

$$E(R_i) = r_f + \beta_i(r_m - r_f), \tag{1}$$

where $E(R_i)$ is expected return of an asset i, r_f is the risk-free rate, r_m is the return of a market portfolio, and β_i is a market risk premium multiplier for security i. In other words, β_i describes the systematic risk of security (Brandimarte 2017). Now we can divide risk of an individual asset $\sigma_i = \beta\sigma_m + \sigma_{ie}$ to two, systematic risk ($\beta\sigma_m$) and unsystematic or idiosyncratic risk (σ_{ie}), where σ_m is the market risk.

The idea of diversifying idiosyncratic risk away by forming efficient portfolios is the most important insight of the Modern Portfolio Theory.

We can form an efficient frontier for the best-performing portfolios at each risk level by calculating the expected returns and their variances on all alternative portfolios in the market. The efficient frontier represents the portfolios with the best return-to-risk ratio at each risk level and thus in which every rational investor invests. By minimizing the function of the efficient frontier, we obtain the minimum variance portfolio, which represents the portfolio with the lowest risk on the efficient frontier. According to the mean-variance theorem, all portfolios above the minimum variance portfolio are efficient when moving on the frontier. All well-diversified portfolios belong to the efficient frontier if the capital markets are efficient. (Sharpe 1966.)

Another important portfolio in the model is the tangent portfolio, which, under the assumptions of the Efficient Market Hypothesis (EMH) introduced by Fama (2021), represents the market portfolio if all possible securities have been involved in optimizing the efficient frontier. Thus, the market portfolio includes all risky assets with weights according to their market values (Markowitz 1968). When we add the opportunity to lend and borrow at a risk-free rate, we can form a capital market line (CML). Lending in risk-free rate is practically done by being long in short-term Treasury bills and borrowing by being short in those same Treasury bills (Elton, Gruber, Brown and Goetzmann 2009). The capital market line is formed from various portfolios consisting of a market portfolio and a risk-free rate investment. Assuming that the investor is only interested in the expected return and its variance, every rational investor maximizes their utility by forming a portfolio from the market portfolio and risk-free rate investment according to their risk preferences (Sharpe 1964). The theoretical relationship between efficient frontier, risk-free rate, and market portfolio are illustrated in figure 1. Here P_i represents the market portfolio, R_F the risk-free rate, and the line between them is a Capital Market Line (CML). The line between B and C represents the efficient frontier, and point B represents the minimum variance portfolio.

As the figure illustrates, the investors maximize their utility by selecting a portfolio from CML, assuming that they can invest in the market portfolio and risk-free rate. It implies that the only efficient portfolio in the efficient frontier is the market portfolio. This is consistent with the CAPM since, under CAPM assumptions, investors should hold the same market portfolio, and so the tangency portfolio must be the market portfolio (Brandimarte 2017).

It is essential to point out, that Markowitz (1968) MPT does not assume joint normality of security returns, contrary to popular beliefs. MPT is just consist-

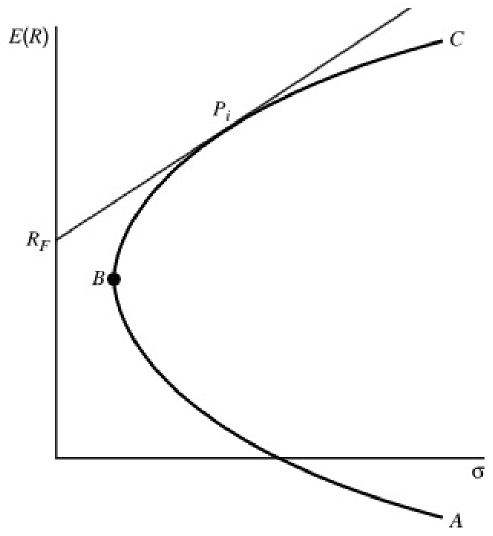

Figure 4: Efficient frontier with riskless lending and borrowing

ent with the assumption, that security returns are jointly normally distributed. However, since the optimization and the efficient frontier focuses only first two moments of the return (mean and variance), it does not tell us anything about risk related to higher moments. For that reason, it is an optimal optimization method for investors having a quadratic utility function. (Fabozzi et al. 2010.)

In modern portfolio theory, the market portfolio is said by definition to include all risky assets. However, practically the difference between the tangent portfolio and theoretical market portfolio is the lacking alternative investments in the former one. In theory, leaving out the alternative investments should lead us to a sub-optimal solution where we have not utilized all the diversification benefits, and our portfolios are susceptible to unsystematic risks which are not rewarded since it is diversifiable. Hence, it is studied that in practice allocating between the risk-free rate and tangent portfolio has not delivered optimal results since more efficient portfolios can be found. The problem stems from the difference between tangent and market portfolios. For example, the MSCI All Country World Index, which is usually used as a tangent portfolio, is actually quite focused on a couple of large companies as well as sectors, and for that reason, the degree of diversification is not ultimately maximal (Choueifaty, Froidure and Reynier 2013.). However, it is often stated that most of the idiosyncratic risk is diversified away only with 15 different securities, and the marginal utility of adding more securities decreases drastically after that. It is important to note that these studies usually focus only on equity portfolios with less diversification benefit available than multi-asset portfolios[12] (Mao 1970; Statman 1987; Elton and Gruber 1977.)

So, at least in theory, adding alternative investments to the portfolio should raise the efficient frontier, assuming that they are not perfectly correlated with other investments. This is demonstrated in Figure 5, which represents a practical view of the efficient frontier. Here the line between B_1 and A_1 represents the efficient frontier with traditional investments. If we include alternative investments in the investment universe, we can move to the upper efficient frontier that the line between B_2 and A_2 represents. The figure also illustrates the well-known problem of tangent (P_1) and market (P_2) portfolios being unequal. If investors cannot invest in the market portfolio due to some restrictions, they will be moving on CML_1 instead of CML_2, which has a higher expected return on every level of risk or lower expected risk on every level of return. In this framework, the next natural question is whether this rise of an efficient frontier is exploitable. In other words, is it remarkable enough to cover costs and other inefficiencies related to alternative investments?

3.3 Mean-Variance optimization

Before optimizing portfolios, we need to review how we generate the inputs we utilize. Key concepts for portfolio management are risk and return. Returns are used in portfolio management instead of prices because they offer a complete picture of the investment opportunities that are not affected by the amount of average investor invests in competitive markets and because they have desirable statistical properties compared to prices (Campbell, Lo and MacKinlay 2012). Return of an investment can be calculated either by simple percentage returns or logarithmic continuously compounded returns. The relation between these two can be shown as:

$$r_t = \ln(1 + R_t) = \ln\left(\frac{P_t + D_t}{P_{t-1}}\right), \tag{2}$$

where r_t is continuously compounded return at time t, R_t is percentage returns

[12] With his theoretical model, Mao (1970) tested the percentage diversification benefits of correlations (ρ) and numbers of different securities and found that if securities $\rho = 0.5$, then 17 securities achieve 90% of the available diversification benefits. If, on the other hand, $\rho = 0.2$, the available diversification benefit is greater, and then 34 securities are required for the corresponding 90% relative benefit.

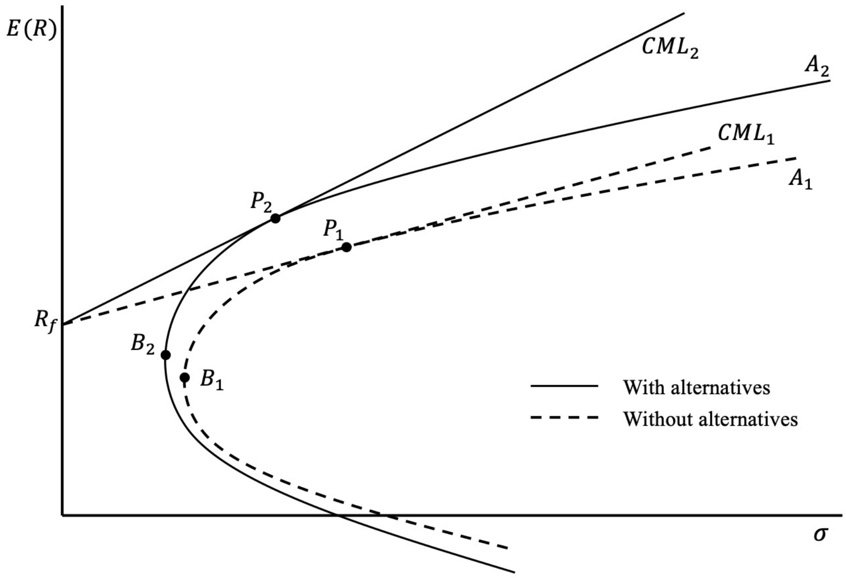

Figure 5: Efficient frontier with alternative assets

at time t, P_t is price of an instrument at time t, and D_t is the cash flow of an instrument in period t. Continuously compounded logarithmic returns have many desired properties compared to percentage returns. They do not have boundaries, are easy to calculate and can be normally distributed in time series aggregation. The continuously compounded returns beneficial time aggregation properties are easy to see when we compare its calculation to percentage returns in Equation 3:

$$\begin{aligned} R_{i,T} &= \prod_{t=1}^{T}(1 + R_{i,t}) - 1 \\ r_{i,T} &= \sum_{t=1}^{T} r_{i,t}, \end{aligned} \tag{3}$$

where T means the aggregated time period. While percentage multi-period return is a product of single-period percentage returns, the logarithmic multi-period return is the sum of single-period logarithmic returns. However, we need to use simple percentage returns in cross-sectional aggregation since portfolio return is a weighted average of percentage returns instead of logarithmic ones. Also, it is important to convert returns back to simple returns at the end of the portfolio optimization process since, at the end of the day, simple returns are the ones that matter for the investor.

In MPT, the risk is defined as a standard deviation from its mean return. It is typically calculated as a variance of returns for computational convenience instead of standard deviation (Brandimarte 2017). This symmetric risk metric for security can be calculated from the return sample as follows:

$$\sigma^2 = \frac{1}{N}\sum_{t=1}^{N}(r_t - \mu)^2, \tag{4}$$

where r_i is the return of security, μ is the sample's mean return, and N is the sample size. When we start to aggregate risks at the portfolio level, covariance comes to an important role. Covariance measures the linear dependence of two assets' returns. In other words, it tells how two individual asset returns move together. It can be calculated from the return sample as follows:

$$\sigma_{ij} = \frac{1}{1-N}\sum_{t=1}^{N}(r_{i,t} - \mu_i)(r_{j,t} - \mu_j). \tag{5}$$

We should use a normalized version from covariance to compare different assets relationships. Correlation is a metric for that purpose since it can get values only between 1 and -1. Correlation can be calculated from Equations 5 and 4 as follows:

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}. \tag{6}$$

Now we get to the actual mean-variance optimization, utilizing Markowitz (1968) findings of Modern Portfolio Theory. As stated in the last section, the diversification benefit stems from the idea that security's risk is lower in the portfolio context than individually. To define the portfolio risk, we are interested in covariances of assets included in the portfolio. For that reason, we need to construct the following variance-covariance (VarCov) matrix:

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1N} \\ \vdots & \ddots & \vdots \\ \sigma_{N1} & \cdots & \sigma_{NN}, \end{bmatrix} \tag{7}$$

where σ_{1N} is the covariance between asset 1 and N. In diagonal, we have variances of an assets since covariance of an asset with itself is variance (Fabozzi et al. 2010). Portfolio return can be calculated in matrix context:

$$\mu_p = \mathbf{w}'\boldsymbol{\mu}, \tag{8}$$

where $\mathbf{w}'$ is transpose from a vector of individual assets weights in portfolio and μ is a vector of expected returns of assets. While portfolio return is calculated as a weighted average of individual asset returns, the portfolio's variance utilizes VarCov matrix in Equation 9:

$$\sigma_p^2 = \mathbf{w}'\mathbf{\Sigma}\mathbf{w}. \tag{9}$$

It follows that the portfolio's variance is lower than the simple weighted average of individual asset variance.

When we move on to the optimization problem, we must notice that we are dealing with expectations. This means that we are interested in constructing optimal portfolio ex-ante. For that reason, inputs to the return vector and VarCov matrix will play a significant role. There are many alternatives to estimate expected returns and their covariances. One can observe historical returns and use sample mean for returns and sample covariance for covariance matrix as we introduced above. However, it can be argued that past performance is not a reliable indicator of future performance. To get more forward-looking prediction power, one can use statistical techniques such as Bayesian and shrinkage estimator or factor models to generate more robust inputs (Fabozzi et al. 2010). However, here we go through the portfolio optimization problem assuming expected returns and their covariances are given and utilize historical samples in the methodology chapter.

In order to optimize asset weights in a mean-variance framework, one can either maximize expected return for target variance or minimize expected variance for target return. We go the optimization problem through with the latter setup. Following MPT, investors have a constrained minimization problem. It is quadratic optimization with equality constraints:

$$\begin{aligned} \min_{w} \quad & \frac{1}{2}\mathbf{w}'\mathbf{\Sigma}\mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}'\boldsymbol{\mu} = \mu_0 \\ & \mathbf{w}'\boldsymbol{\iota} = 1, \end{aligned} \tag{10}$$

where μ_0 is the target return, and ι is a vector of ones. Since this optimization problem can be solved with Lagrange multipliers, we get the Lagrangian function:

$$\mathcal{L} = \frac{1}{2}\mathbf{w}'\mathbf{\Sigma}\mathbf{w} + \lambda(1 - \mathbf{w}'\boldsymbol{\iota}) + \gamma(\mu_0 - \mathbf{w}'\boldsymbol{\mu}), \tag{11}$$

where λ and γ are the Lagrange multipliers. Now we can take first-order conditions from to function, which means partial derivatives with respect to each parameter:

$$
\begin{aligned}
\frac{\delta \mathcal{L}}{\delta \mathrm{w}} &= \boldsymbol{\Sigma}\mathbf{w} - \gamma\boldsymbol{\mu} - \lambda\boldsymbol{\iota} = 0 \\
\frac{\delta \mathcal{L}}{\delta \gamma} &= 1 - \mathbf{w}'\boldsymbol{\iota} = 0 \\
\frac{\delta \mathcal{L}}{\delta \lambda} &= \mu_0 - \mathbf{w}'\boldsymbol{\mu} = 0.
\end{aligned} \tag{12}
$$

Solving first-order conditions with respect to weights $\mathbf{w}$, we get:

$$
\mathbf{w} = \lambda\boldsymbol{\Sigma}^{-\mathbf{1}}\boldsymbol{\iota} + \gamma\boldsymbol{\Sigma}^{-\mathbf{1}}\boldsymbol{\mu}. \tag{13}
$$

Combining this with constraints, we get a linear system for λ and γ:

$$
\begin{aligned}
\mathbf{w}'\boldsymbol{\iota} &= \lambda\boldsymbol{\iota}'\boldsymbol{\Sigma}^{-\mathbf{1}}\boldsymbol{\iota} + \gamma\boldsymbol{\iota}'\boldsymbol{\Sigma}^{-\mathbf{1}}\boldsymbol{\mu} = 1 \\
\mathbf{w}'\boldsymbol{\mu} &= \lambda\boldsymbol{\mu}\boldsymbol{\Sigma}^{-\mathbf{1}}\boldsymbol{\iota} + \gamma\boldsymbol{\mu}'\boldsymbol{\Sigma}^{-\mathbf{1}}\boldsymbol{\mu} = \mu_0,
\end{aligned} \tag{14}
$$

which can be solved using the following matrix form:

$$
\begin{pmatrix} A & B \\ B & C \end{pmatrix} \begin{pmatrix} \lambda \\ \gamma \end{pmatrix} = \begin{pmatrix} 1 \\ \mu_0, \end{pmatrix} \tag{15}
$$

where $A = \boldsymbol{\iota}'\boldsymbol{\Sigma}^{-\mathbf{1}}\boldsymbol{\iota}$, $B = \boldsymbol{\iota}'\boldsymbol{\Sigma}^{-\mathbf{1}}\boldsymbol{\mu}$ and $C = \boldsymbol{\mu}'\boldsymbol{\Sigma}^{-\mathbf{1}}\boldsymbol{\mu}$. Now we can combine equation 9 and equation 15 to get the variance of the portfolio:

$$
\sigma_p^2 = \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} = \frac{A\mu_0^2 - 2B\mu_0 + C}{\Delta}, \tag{16}
$$

where $\Delta = AC - B^2$. To obtain weights for the portfolio at the efficient frontier for chosen return level, we solve Equation 16 with parameter μ_0. To get the global minimum variance portfolio P_i presented in figure 4, we can take derivative respect to μ_0 from equation 16:

$$
\frac{d\sigma_0^2}{d\mu_0} = \frac{2A\mu_0 - 2B}{\Delta} = 0, \tag{17}
$$

and now we get analytical solution for portfolio weights:

$$
w_g = \frac{\boldsymbol{\Sigma}^{-\mathbf{1}}\boldsymbol{\iota}}{\boldsymbol{\iota}'\boldsymbol{\Sigma}^{-\mathbf{1}}\boldsymbol{\iota}}. \tag{18}
$$

The basic mean-variance optimization is very sensitive to expected return inputs. The way expected returns are estimated can greatly impact weights obtained from optimization (Fabozzi et al. 2010). While historical mean returns are poor estimators for future returns, the problem even increases when we are dealing with fat-tailed return distributions since sample mean is best linear unbiased estimator (BLUE) only for normal tailed distributions (Ibragimov 2007).

3.4 Post Modern Portfolio Theory

> Risk, like beauty, is in the eye of the beholder
>
> *Leslie A. Balzer*

In the 1990s, Rom and Ferguson (1994) published a paper called Post-Modern Portfolio Theory comes of age in the *Journal of Investing.* This paper created a concept called Post-Modern Portfolio Theory (PMPT) which recognized the limitations of MPT and focused especially on defining the risk to answer better in real world implications. In a nutshell, the idea of PMPT is to combine the long-identified investor preferences to value upside volatility over downside volatility to the MPT framework (Sortino et al. 2001). However, as the name of their publication refers, this was not a particularly new perspective. Rom and Ferguson (1994) start by pointing out that the fathers of MPT, Markowitz and Sharpe, stated already when creating MPT, that downside risk measures would be more appropriate to define risk than dispersion measures (variance). However, due to the limited computational possibilities in the 1950s, they needed to base their analysis on variance and standard deviation. Rom and Ferguson (1994) saw that the time to implement downside risk into the MPT framework in the 1990s was right and introduced the tools to do that in their paper. They concluded that MPT is a special case of PMPT where applies symmetric returns and quadratic utility function. Since then, the different asymmetric downside risk measures have gained attention in the risk and portfolio management field, expanding the idea of PMPT.

Rom and Ferguson (1994) defined downside risk as tied to investors' objectives. They introduced a concept called Minimum Acceptable Return MAR, representing the boundary between risk and opportunity. Unlike in MPT, where volatility around mean return is seen as a risk, in PMPT, volatility below target return is only seen as a risk. Volatility over target return is seen as a riskless opportunity instead. This approach removes the symmetric risk measure problem, which penalizes overperformance as much as underperformance. Rom and Ferguson (1994) utilized expected returns, standard deviation, and skewness to measure downside risk. They divided downside risk into two components, downside probability, and average downside magnitude. Today's downside risk measures, Value at Risk, and Expected Shortfall can be seen to represent these components. In portfolio management, this risk definition leads investors to optimize a new efficiency metric, the Sortino ratio. Since PMPT, in addition to downside risk, also focuses a lot on investors' objectives, it emphasizes the importance of investment planning, which defines the strategic allocation.

Hadar and Russell (1969) introduced already in 1969 Stochastic Dominance rules, which are grounded on the idea that expected utility theory, which MPT is also based on, is a function of all moments of the probability distribution function, not just the first two. These rules can assess which return distribution dominates with any other distribution and is not affected by the assumed utility function. In other words, it tells the investor's choice despite one's preferences. The first-order stochastic dominance rule states that investors would always prefer distribution A with the lowest return higher or the same as the highest return of distribution B, even though its variance would be much higher. In MPT, that would not be a clear choice since it is blind to the fact that A always does better, and it only focuses on fitting the return/risk ratio to the investor's preferences. The second-order stochastic dominance rule ties the investor objectives to the risk. It states that despite the risk preferences, an investor with MAR objective would always prefer distribution C over A, presented in Figure 6. This is because C has more upside potential and less downside deviation than A when we proportion the risk to the objective MAR. Again, MPT focuses on variance and sees C way riskier than A, and for that reason, the selection would not be apparent without the investor's utility function.

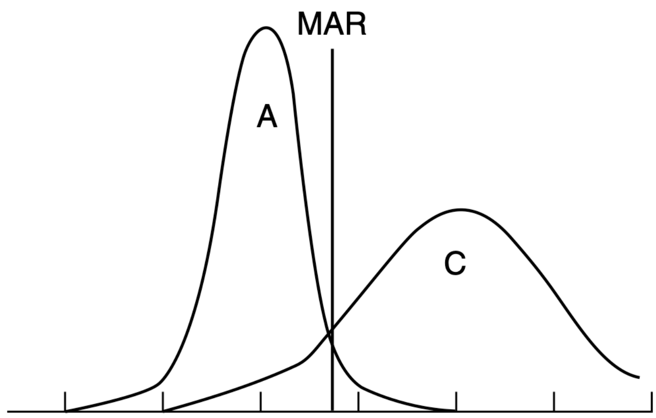

Figure 6: Second order stochastic dominance rule

Since the 1990s, there has been increasing interest in examining risk from a wider perspective than just volatility. It is understood that the mean-variance optimization incorporates only one definition of risk and does not capture the other undesirable properties of return. While investors prefer less volatile returns over more volatile returns, other things being equal, the preferences to variance start to change when we do not have other things equal, which leads to problems of using variance as a measure of risk. In particular, these other things refer to skewness and kurtosis (Sortino et al. 2001). Since returns are not often normally distributed, the mean-variance optimization model does not give desired result (Fabozzi et al. 2010).

In his groundbreaking work for MPT, Markowitz (1968) noted that semi-variance could be used instead of variance to cover asymmetric preferences. However, Rom and Ferguson (1994) introduced a downside risk measure that was a general description of return probability below target, so it differed from semi-variance by its capability to view the whole probability structure. In the case of normal distribution and mean as a target, it did not offer any additional information compared to semi-variance, but the idea was its capability to model risk with asymmetric distributions and different target returns than a mean. The general downside risk measure by Rom and Ferguson (1994) quickly created many metrics that linked in it from different perspectives. Namely, Van der Meer (2001) spoke about shortfall probability, risk capital, and discounted downside risk, while Forsey (2001) talked about upside probability, downside deviation, upside potential, and upside potential ratio. Rom and Ferguson (1997) emphasized the downside frequency and average downside deviation. All of these had in common that they described the return probability distribution and used some target return level as a reference point. However, maybe the most utilized downside risk measure is the Value at Risk (VaR) introduced by JPMorgan in 1994. Value at risk is the quantile measure of the loss distribution (left tail of the return distribution). It thus indicates the critical loss level that will be exceeded during the reference period with a probability of α. Portfolio's Value at Risk can be defined as follows:

$$\mathrm{VaR}_{1-\alpha}(L) = \inf\{l|P(L < l) \geq 1 - \alpha\}, \tag{19}$$

where portfolio loss is $L = -\mu'\mathbf{w}$, and α is the probability that loss exceeds VaR. It can also be defined with cumulative distribution function (CDF). If we have a random profit or loss R over a holding period, and this random variable R has continuous CDF $F(r)$, then for $0 < \alpha < 1$, VaR is defined as the maximum potential loss realized by R over the holding period at $(1-\alpha)$ confidence level and can be written as:

$$\mathrm{VaR}_{\alpha} = -F^{-1}(\alpha). \tag{20}$$

Thus, VaR can be interpreted as the negative of left α-quantile of the distribution. (Wong 2008.)

The Value at Risk became the standard measure for financial market risk since Basel Committee introduced it in 1996 to determine capital requirements for banks (Committee 1996). It is a standard tool for commercial banks, risk management consulting firms, and other financial entities. It gained popularity among downside risk measures since it is easy to understand, applicable, and universal after Basel requirements. However, its simplicity has its downside.

As a Artzner, Delbaen, Eber and Heath (1999) states, VaR is not a coherent risk measure. Risk measure is coherent if it fulfills the four axioms: subadditivity, translation invariance, monotonicity, and positive homogeneity. While VaR satisfies all the other axioms, it lacks subadditivity. This flaw means that VaR does not count the diversification benefit, which is essential in the portfolio optimization framework. VaR has gained many critiques as a risk measure after being selected as the official market risk measure for the banks by Basel Committee in 1996. For example, Danielsson, Embrechts, Goodhart, Keating, Muennich, Renault, Shin et al. (2001) stated that since VaR does not measure the shape or extent of the distribution's tail risk but instead estimates a particular point in the distribution, it is misleading if the returns are not normally distributed. For that reason, VaR models generate imprecise and fluctuating risk forecasts. Return distribution X with light tail probabilities can have the same VaR as return distribution Y with heavy tail probabilities (Tasche 2002).

To overcome all the previously mentioned shortcomings that VaR has, we can utilize Expected Shortfall (ES), sometimes called Conditional Value at Risk, as a risk measure[13]. Expected Shortfall generates a conditional expectation of losses breaching VaR level. It is probability-weighted average returns beyond determined VaR in a given confidence level. Wong (2008) presents ES with confidence level $(1-\alpha)$ as follows:

$$\mathrm{ES}_\alpha(R) = -E(R|R < \mathrm{VaR}) = -\alpha^{-1}\int_{-\infty}^{\mathrm{VaR}} rf(r)dr. \tag{21}$$

where R is the expected negative return below Value at Risk on given confidence α. ES integrates all values of r, weighted by probability density function $f(r)$, from minus infinity to VaR, and therefore accounts for all losses beyond VaR level. (Wong 2008.)

[13]While Expected Shortfall (ES) and Conditional Value at Risk are equivalent, it is important to notice that they are defined differently (Fabozzi et al. 2010). In ES, we fix the target loss priori, while in CVaR, VaR defines it (Brandimarte 2017).

To illustrate the difference between these two metrics, VaR and ES is presented in the figure 7 with a probability density function. While VaR is one point in the distribution, ES integrates the whole area beyond VaR. Even that small VaR does not necessarily mean small ES; a small ES means small VaR, so for this reason too, ES optimization covers the risks more broadly.

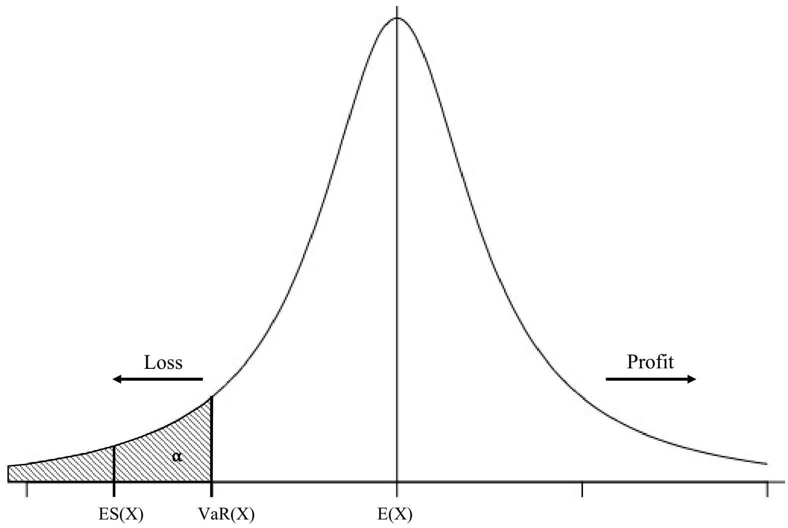

Figure 7: Profit-Loss probability density function with VaR and ES

Expected Shortfall overcomes the following shortcomings of VaR: it accounts for the severity of losses, owns subadditivity properties, is a coherent risk measure, and mitigates the impact of confidence level decision. In statistical terms, to connect VaR to investors' probability of solvency, we need the smallest coherent and law invariant risk measure that dominates VaR. It is shown that ES owns these properties and better statistical properties than the first introduced downside risk measure standard semi-deviation since it is also co-monotonically additive (Tasche 2002).

When investors can invest assets susceptible to infrequent but severe losses (i.e., has fat tales) as some alternative investments, the problems of using VaR as a downside risk measure increase. Using more fat-tailed assets, investors can decrease VaR while downside risk increases in terms of ES. Yamai and Yoshiba (2005) illustrated this problematic feature with the figure 8 of the cumulative distribution function. Rather than minimizing VaR, we can minimize ES due to its ability to say something about the shape of the tail and the structure of maximum loss. At the same time, we can utilize the eligible mathematical properties that ES has. It is important to notice also that while we minimize ES, the VaR is necessarily low as well (Rockafellar, Uryasev et al. 2000).

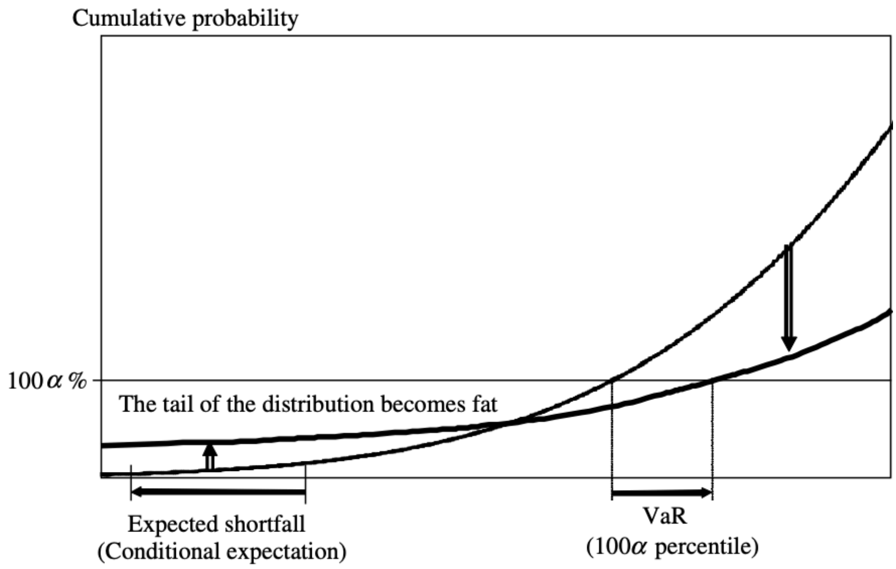

Figure 8: Cumulative distribution

As we see, VaR and ES are downside risk metrics, just as Rom and Ferguson (1994) introduced downside risk. They describe the probability and expected downside, but instead of deciding the MAR level and calculating the probability to fall under that, we first fix the loss probability in these metrics and then get the amount of loss we can expect from holding an asset. However, in practice, α is usually set as 1-5%, so it represents preferences to avoid near-catastrophic events, and for that reason, it is used for capital requirement calculations for institutions. This study tries to use ES to determine institutions' strategic allocation.

However, also ES has its drawbacks. It depends on the chosen confidence level and the underlying return distribution function. In portfolio optimization, it is often criticized to be very sensitive to the chosen confidence level and this way highly disposed to estimation errors (Lim, Shanthikumar and Vahn 2011). It also lacks elicitability properties, which theoretically precludes the backtesting of Expected Shortfall. However, it is shown that ES is jointly elicitable with VaR, and that way, it could be backtested with the Value at Risk measure. The most important property to use this measure in this study is its flexibility to be measured from chosen probability density function, and that way, optimization is not locked to the Gaussian premise. Estimation error also becomes a problem in downside risk measures. Even more extensive than in dispersion risk measures since one uses only a fraction of the data (Fabozzi et al. 2010). Neither VaR nor ES is an exception to this. Yamai and Yoshiba (2005) showed that ES is particularly prone to significant estimation error with fat-tailed distributions, and this is due to more infrequent and larger losses that fluctuate the estimations. One way to tackle this problem is to increase the sample size of the simulations significantly.[14]

One common way to estimate portfolio ES, is to assume that returns are conditionally normally distributed with mean μ and covariance matrix Σ:

$$\mathrm{ES}_w(\alpha) = -\mathbf{w}'\mu + \sqrt{\mathbf{w}'\mathbf{\Sigma}\mathbf{w}}\frac{\phi(z_\alpha)}{\alpha}, \tag{22}$$

where z_α is the α quantile of the standard normal distribution and ϕ is the standard normal density function. However, we can estimate ES by utilizing Cornish-Fisher expansion to account for excess skewness and kurtosis of portfolio returns. As Boudt, Peterson and Croux (2008) demonstrate, the Cornish-Fisher expansion offers a parametric way to correct Gaussian distribution with historical return series' multivariate moments. This asymptotic expansion enables us to estimate the ES without explicitly defining the empirical return distribution function. Instead, we can utilize the empirical return series to estimate higher-order moments to take excess skewness and kurtosis into account. This way, we get the portfolio returns' higher moments to the Expected Shortfall metric and ultimately to the optimization problem. Now ES can be defined:

$$\mathrm{ES}_\alpha(w) = -\mathbf{w}'\mu + \sqrt{m_{(2)}} \times \frac{1}{\alpha}[a_\alpha + b_\alpha k(w) + c_\alpha s(w) + d_\alpha s^2(w)], \tag{23}$$

where a_α, b_α, c_α, and d_α are the chosen loss level α and respectively $s(w)$ and $k(w)$ are the portfolio's skewness and excess kurtosis. We will show in chapter 3.6 how the skewness and kurtosis are estimated in this study.

One possible way to examine ES is always by historical values. However, since tail events are infrequent, it is shown that historical values are bad estimates for tail risk measures. The Cornish-Fisher expansion estimate has been shown to deliver accurate estimates for ES, and for that reason, every ES value shown in the empirical section is calculated with function 23. The more comprehensive definition for this estimator is shown in Appendix I. (Boudt, Carl and Peterson 2013.)

3.5 Mean-Expected Shortfall optimization

As we have now discussed the PMPT and downside risk measures, it is time to derive how we can utilize these risk perspectives in the portfolio optimization same

[14] Yamai and Yoshiba (2005) increased the sample size in simulations from 10000 to 1000000, decreasing the estimation error to third.

way we utilized dispersion measure variance in chapter 3.3. Now the objective is to maximize the expected return subject to the expected shortfall, and we can express this as:

$$\begin{aligned} \max_{w} \quad & \boldsymbol{\mu}'\mathbf{w} \\ \text{s.t.} \quad & \mathrm{ES}_{\alpha}(w) \leq c_0, \end{aligned} \tag{24}$$

where c_0 is constant, expressing the maximum level of risk. Next, we need to define the loss function $f(\mathbf{w}, \mathbf{y})$, where $\mathbf{w}$ is the N-dimensional vector of assets weights in the portfolio and $\mathbf{y}$ is a random vector describing uncertain outcomes. If random values are continuous and $p(\mathbf{y})$ is the probability related to scenario $\mathbf{y}$, we can denote that the cumulative probability gives the probability of loss function exceeding value γ

$$\Psi(\mathbf{w}, \mathbf{y}) = \int_{f(\mathbf{w},\mathbf{y}) \leq \gamma} p(\mathbf{y}) dy, \tag{25}$$

where we get ES by utilizing the definition 21 to following form:

$$\mathrm{ES}_{\alpha}(w) = \alpha^{-1} \int_{f(\mathbf{w},\mathbf{y}) \geq \mathrm{VaR}_{\alpha}(w)} f(\mathbf{w}, \mathbf{y}) p(\mathbf{y}) dy. \tag{26}$$

It can be shown that the function 26 is concave and therefore has a unique minimum. (Fabozzi et al. 2010.) However, the problem mentioned above has VaR in the formula and is complex to utilize in practice. Therefore we follow Rockafellar et al. (2000) method, which simplifies the direct ES optimization in the way that we avoid incorporating VaR into our optimization problem. The idea is to utilize the following auxiliary function in optimization:

$$F_{\alpha}(\mathbf{w}, \xi) = \xi + \alpha^{-1} \int_{f(\mathbf{w},\mathbf{y}) \geq \gamma} (f(\mathbf{w}, \mathbf{y}) - \xi) p(y) dy. \tag{27}$$

This function 27 involves three important properties. First, it is a continuously differentiable convex function that is particularly easy to optimize numerically. Second, VaR is a minimizer of $F_{\alpha}(\mathbf{w}, \xi)$, which means that we get the VaR simultaneously when minimizing ES and third and most importantly, minimum value of $F_{\alpha}(\mathbf{w}, \xi)$ is minimum value of ES_{α}. So the optimization problem simplifies to the following function:

$$\min_{w,\xi} F_{\alpha}(\mathbf{w}, \xi). \tag{28}$$

From here we get optimal weights $\mathbf{w}^*$ for portfolio and the corresponding VaR as ξ^*. However, the probability density function is hard to estimate in practice, so we need to use sampled scenarios. In this case, we can modify the 27 function as:

$$F_\alpha^Y(\mathbf{w},\xi) = \xi + \alpha^{-1}T^{-1}\sum_{i=1}^{T}\max(f(\mathbf{w},\mathbf{y_i}) - \xi, 0), \tag{29}$$

where we have T different scenarios for y_i. When we replace $\max(f(\mathbf{w},\mathbf{y_i}) - \xi, 0)$ with variable z_i we can rewrite the optimization problem with appropriate constraints as:

$$\begin{aligned} \min_{w,\xi} \quad & \xi + \alpha^{-1}T^{-1}\sum_{i=1}^{T} z_i \\ \text{s.t.} \quad & z_i \geq 0, i = 1, ..., T \\ & z_i \geq f(\mathbf{w},\mathbf{y_i}) - \xi i = 1, ..., T. \end{aligned} \tag{30}$$

If $f(\mathbf{w},\mathbf{y})$ is linear, we have a linear optimization problem that can be solved efficiently with standard linear programming techniques. This optimization model can be seen as an extension for the mean-variance model introduced earlier, and it is advantageous when the underlying assets' return distributions have skewness and kurtosis. (Fabozzi et al. 2010.)

3.6 Higher-order portfolio moments

To consider the non-normality of the portfolio assets' returns in the portfolio optimization context, we need to get higher-order moments explicitly under consideration. To do so, we define coskewness and cokurtosis as we defined covariance in chapter 3.3. The coskewness and cokurtosis matrices account for diversification benefit through higher-order terms and are crucial when we aggregate assets' risks in the portfolio context. These higher-order moment tensors can be calculated as follows. First,

$$\phi_{ijk} = E[(r_{i,t} - \mu_i)(r_{j,t} - \mu_j)(r_{k,t} - \mu_k)], \tag{31}$$

where ϕ_{ijk} is coskwness of assets i,j and k. Then,

$$\psi_{ijkl} = E[(r_{i,t} - \mu_i)(r_{j,t} - \mu_j)(r_{k,t} - \mu_k)(r_{l,t} - \mu_l)], \tag{32}$$

where ψ_{ijkl} is cokurtosis of assets i,j,k and l.

We already showed how covariance is constructed in matrix form in equation7. Respectively we can form coskewness Φ and cokurtosis Ψ matrices,

$$\begin{aligned}\Phi &= E[(r-\mu_r)(r-\mu_r)'\otimes(r-\mu_r)'],\\ \Psi &= E[(r-\mu_r)(r-\mu_r)'\otimes(r-\mu_r)'\otimes(r-\mu_r)'],\end{aligned} \tag{33}$$

where $\otimes$ denotes the Kronecker product. Now we can calculate portfolio return moments for the portfolio with weights w like we did with covariance in equation 9:

$$\begin{aligned}m_2(w) &= \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w},\\ m_3(w) &= \mathbf{w}'\boldsymbol{\Phi}(\mathbf{w}\otimes\mathbf{w}),\\ m_4(w) &= \mathbf{w}'\boldsymbol{\Psi}(\mathbf{w}\otimes\mathbf{w}\otimes\mathbf{w}).\end{aligned} \tag{34}$$

We can define portfolio skewness and kurtosis through the portfolio return moments as follows,

$$s(w) = \frac{m_3(w)}{m_2^{\frac{3}{2}}(w)} \tag{35}$$

$$k(w) = \frac{m_4(w)}{m_2^2(w)} - 3. \tag{36}$$

Estimating these higher-order moment tensors is essential when we optimize portfolios in the empirical section since we want the tail risks to be appropriately aggregated to get robust portfolio performance evaluations for different portfolios. (Boudt, Lu and Peeters 2015.)

Another way to utilize estimated moment tensors is to bring them directly to the optimization problem. We introduce the objective function as a third portfolio constructing technique utilized in this study, the Constant Relative Risk Aversion (CRRA) utility function with Taylor Expansion. This technique is one of the standard expected utility maximization frameworks from which the MPT and PMPT originate. The fourth-order Taylor expansion enables us to interpret investors' preferences towards skewness and kurtosis intuitively, making the objective function interesting in an optimization problem with non-normal return distributions. Since we have already defined the higher-order moment tensors, the objective function optimization problem can be defined as:

$$\begin{aligned}\min_{w} \quad & \frac{\gamma}{2}m_2(w) - \frac{\gamma(\gamma+1)}{6}m_3(w) + \frac{\gamma(\gamma+1)(\gamma+2)}{24}m_4(w)\\ \text{s.t.} \quad & \mathbf{w}'\boldsymbol{\iota} = 1,\end{aligned} \tag{37}$$

where γ is the risk aversion parameter and w is the portfolio weights. In this CRRA objective function, investors prefer higher first and third moments and lower second and fourth moments. It is intuitive that investors prefer higher returns and positive skewness (extreme positive returns) and avoid high variance and kurtosis (extreme deviations from mean). (Martellini and Ziemann 2010.)

Investors' preferences towards risk aversion are studied a lot, and the consensus often points out that investors typically have DARA, which means that investors invest more in risky assets in absolute terms when their wealth increases. However, in relative terms, investors usually argued to have either DRRA or CRRA. The first one implies that investors would invest a higher proportion of their wealth to the risky assets when their wealth increases, and the second one implies that investor allocates the same proportion of their wealth to the risky assets when their wealth increases (Levy 1994). While both Relative Risk aversion preferences have good arguments, we can divide the individual and institutional investors' preferences to represent the different RRA preferences. While individuals' consumption is bound to a standard of living, which eventually has some limits, individuals start to reinvest this wealth surplus, increasing the relative proportion of risky assets in their portfolio. At the same time, institutional investors are usually obligated to consume a constant relative proportion of their wealth for their purpose. For example, when endowments' wealth increases, they offer more allowances in absolute terms while keeping the relative outflow constant (Swensen 2009). On these grounds, we choose the objective function to follow DARA and CRRA which function 37 satisfies.

3.7 Previously utilized models, obtained results and critique

When discussing practical portfolio optimization, we often have to answer the question, which we value more, appropriate model specification or improved estimates. Martellini and Ziemann (2010) propose that with low data frequency, investors should depart from correct model specification (e.g., using gaussian premise with non-normally distributed returns) to get lower estimation errors since the utility from latter exceeds the former. Hitaj, Martellini and Zambruno (2011) got the same kind of results. They showed that with sufficiently large sample size and improved estimators, the higher-order optimization outperforms mean-variance optimization consistently from a utility perspective with hedge funds. They used out-of-sample analysis and dynamic allocation (Hitaj et al. 2011). However, Hitaj et al. (2011) study utilized the Constant Absolute Risk Aversion (CARA) utility function. From institutions' preference perspective, a more representative utility function can be argued to be Decreasing Absolute Risk Aversion (DARA).

Allen, Lizieri and Satchell (2020) challenged the mean-variance framework by improving the risk estimate accuracy leading to an increase in out-of-sample portfolio performance from the power utility point of view. They incorporated heavy tails, skewness, volatility clustering, and asymmetric dependencies in financial data using univariate exponential GARCH, generalized Pareto distribution, and skewed-t copula. Their results show a significant increase in the economic value added to the investor portfolio. (Allen et al. 2020.)

Jondeau and Rockinger (2006) studied the opportunity cost between optimization with mean-variance criterion and with the third and fourth moment. To do so, they used Taylor expansion for expected utility. They showed that only with large departure from returns' normality will make the higher moment approximation useful. In other words, the four-moment approximation for expected utility will add value in specific markets or asset classes, but in relatively well-diversified or liquid markets, the mean-variance approximation is close to direct optimization. The study shows an efficient way to optimize portfolios with non-normal return distributions even when the number of assets increases. (Jondeau and Rockinger 2006.)

Jurczenko and Teiletche (2019) optimized cross-asset portfolio in expected shortfall framework. Their risk parity optimization dealt with valuation, illiquidity, asymmetry, and tail risks. They were interested in integrating two separate research paths, deviation from normality in asset allocation and illiquidity on asset moments. To get the contribution of each asset class to the overall expected shortfall, they decompose ES to the first derivative relative to weight. They assumed expected returns to include mispricing to manage valuation risk, which they estimated with assets carry. They used popular Cornish-Fisher (CF) expansion to approximate assets and aggregated portfolio moments to capture asymmetry and tail risk. To incorporate with illiquidity Jurczenko and Teiletche (2019) used moving average properties as Getmansky et al. (2004) has used earlier. This way, they could correct covariance, coskewness, and cokurtosis, which all suffer from stale pricing and smoothed returns. Their study illustrates how major risk contribution to traditional assets comes from volatility, while to alternative investments, it comes from skewness, kurtosis, and illiquidity. Their study shows how taking these risk factors into account in portfolio optimization, one can get an allocation with the same expected Sharpe but higher expected RoES and lower expected drawdown. (Jurczenko and Teiletche 2019.)

The natural extension for the Jurczenko and Teiletche (2019) study is to use out-of-sample analysis to compare the model performance to other optimization models (e.g., minimum variance). However, this analysis is limited due to the low-frequency data that hamper reliable comovement estimation. This problem could be solved by using shrinkage estimators or a factor model. In fact, Martellini and Ziemann (2010) have shown that investors can only enhance their ex-ante utility, compared to a mean-variance analysis, by exploiting these improved risk estimates. Other applicable extensions are the wider asset universe and different investments horizon which this study tries to answer.

Cavenaile et al. (2011) studied the impact of illiquidity and higher moments on performance and optimal allocation with different hedge fund strategies. They conducted portfolio analysis for portfolios fully consisting of different hedge fund strategies and portfolios consisting of equity, bonds, and hedge fund indices. With the latter portfolio analysis, they were able to evaluate the hedge funds' diversification properties. To incorporate illiquidity in the hedge funds risk measures, they unsmoothed the returns with two different methods, the same moving average properties by Getmansky et al. (2004), which also Jurczenko and Teiletche (2019) used in their study, as well as Okunev and White (2003) method where observed returns are alleged to be a weighted average of "true" returns. Even though the technical implementation of these two methods is slightly different, they produced very similar results[15]. They adjusted risk and performance measures to incorporate the higher moments of hedge fund returns. This was done by adjusting the Value at Risk measure for non-normal skewness and kurtosis with Cornish-Fisher expansion. They also used Bell's utility function as a risk measure since it has an intuitive interpretation for the parameters representing higher moments. They used Taylor series expansion as Capocci, Duquenne and Hübner (2006) have previously done to get four first moments directly to the utility function and optimization problem. Since they used two different methods to review illiquidity and non-Gaussian properties in the returns, they offered comprehensive and reliable results. They showed that risk measures that do not consider higher moments or illiquidity can underestimate hedge funds' risks up to 80%. The diversification properties showed that even when taking higher moments and illiquidity into account, the hedge funds offer diversification benefits in a traditional equity-bond portfolio, and that way, hedge funds can be found in optimal portfolios. However, diversification benefits are significantly lower than what mean-variance optimization would suggest. (Cavenaile et al. 2011.)

Cavenaile et al. (2011) study was a great example of what my study tries to achieve. Instead of focusing on different hedge fund strategies, this study will

[15]For example, the smoothing parameter is estimated in the Okunev and White (2003) method with first-order autocorrelation coefficient, while in Getmansky et al. (2004) method, it is estimated with maximum likelihood estimation of moving average process.

bring a broader range of alternative asset classes to the review. Since alternative investments are a more heterogeneous group than different hedge fund strategies, it can be expected that illiquidity and higher moment adjustments will show even more significant changes in performance and diversification results. The results are likely to be parallel with Cavenaile et al. (2011) study; alternative investments offer diversification benefits but not as much as mean-variance optimization from smoothed returns would indicate.

4 METHODOLOGY

4.1 Research strategy

This empirical research aims to determine how alternative assets affect the allocation and performance of a multi-asset portfolio. We also examine how risk perspective affects the output of portfolio optimization. The motivation behind that is to consider better the heterogeneous characteristics that multi-asset portfolios and earlier describer alternative assets have. This way, the study aims to help institutions determine their strategic allocation reflecting their objectives and liabilities.

To improve a portfolio's performance ex-ante, one can improve forecasts or optimization. This study tries to improve performance by improving the optimization to consider return distribution's higher moments. It also introduces new asset classes that should offer diversification benefits to improve performance.

We are particularly interested in the portfolio's efficiency (return/risk ratio) with different investment universes and risk metrics. We compare mean-variance optimization against mean-expected shortfall optimization. The idea is to find out how much different risk perspectives in optimization affect the allocation and efficiency of the portfolio. Mean-variance optimization works as a benchmark since it only takes the first two moments into account. We include skewness and kurtosis characteristics of different asset classes in the optimization model with mean-expected shortfall optimization. In addition to that, we also use CRRA objective function in simulations. This objective function brings higher-order moments to optimization from expected utility theory and works as an alternative for our risk metric approach, working as a great benchmark. The empirical research process goes as follows:

1. Desmoothing and cleaning up the data

2. Descriptive analysis of the clean data

3. Efficient frontier construction with different investment universes and optimization objectives

4. Analyzing the static portfolios' efficiency and allocation

5. Conducting out-of-sample backtests with three optimization objectives

6. Analyzing the historically simulated portfolio's efficiency and allocation

The first step of the process tries to improve the poor-quality data that alternative assets are typically prone to. In this step, the private alternative asset data is disaggregated from quarterly to monthly. The disaggregation method reduces the problematic stale pricing and appraisal smoothing biases. At the same time, it forces us to make some assumptions about the underlying time series factors. We utilize same kind of methods as Getmansky et al. (2004) and Okunev and White (2003) where we assume Markov process or Markov process and random walk depending on the serial correlation obtained from Generalized Least Squares Regression. This step is not trivial for the optimization results, but as mentioned before, it is not the main focus of this study. In the next chapter, we go more comprehensively through the disaggregation process and also how we handle other biases involved in the data.

The descriptive analysis focuses on different asset classes' statistics and risk/return structures. The main focus of this analysis is the correlation of different asset classes and the normality tests of the returns. This step also compares the data to the investors' expectations introduced in chapter 2.1. The efficiency metrics we utilize throughout the study are the Sharpe ratio,

$$SR = \frac{R_p - R_f}{\sigma_p}, \tag{38}$$

and the return on expected shortfall,

$$RoES = \frac{\mathrm{ES}_p - R_f}{\sigma_p}, \tag{39}$$

where R_p is portfolio's mean monthly return, R_f is risk-free rate which is assumed to be zero and σ_p is portfolio's monthly variance. The portfolio ES_p is calculated as represented in equation 23.

In the third and fourth step, two different dimensions, risk and investment universe are examined with two different alternatives in each dimension. Risk is examined from variance and expected shortfall perspective, while the investment universe is examined with and without alternative investments. Mean-variance efficient frontiers are constructed with quadratic programming solver, solving the problem introduced in equation 10. We utilize a linear programming solver for mean-expected shortfall efficient frontiers, solving the problem introduced in equation 30. For these four situations, we utilize historical sample data from the whole period of 2008-2021. The examination includes minimum risk, maximum efficiency, and efficient frontier portfolios. The idea is to comprehensively compare the four different situations from risk, return, and allocation perspectives. This part gives insight into how alternative investment allocations evolve when moving along the risk range.

The fifth and sixth steps' role is to offer robustness to the results obtained from previous steps. Before this, we have only examined results ex-post. However, by conducting backtests (historical simulation), we can examine how things would have looked ex-ante and what results would have followed from decisions based on ex-ante estimations. This procedure can be seen as an out-of-sample analysis, where we examine how different optimization objectives would have performed if they had been utilized in practice. While in-sample analysis examines how portfolios should be constructed to perform well in history, the out-of-sample analysis relies on ex-ante estimates and uses historical data to simulate the results. We also changed the method to differential evolution heuristic optimization to get more simulated data in the fifth step. The backtesting is performed for different maximum efficiency portfolios with three objectives; Sharpe, RoES, and CRRA. We utilize investment universes with and without alternative investments, and the only constraints are fully invested and long-only portfolios. Since we have 12 years of data, we utilize three years for training to get a sufficiently long review period. The rolling window is locked to inception to improve the estimations over time. Rebalancing is done once a year, consistent with the illiquid assets causing fairly static allocations.

With the differential evolution optimization method, the idea is to repeat the objective function to evolute towards the global maximum. Here, the investment constraints are directly imposed by mapping the generated populations into the feasible space, and as the optimization outcome, we get a random variable. This algorithm performs genetic biology-inspired operations to search global optimum and efficiently optimize non-linear optimization problems with many comovement parameters. This way, we get the CRRA objective function also optimized efficiently.

Conclusions for the two research questions, can we obtain more efficient portfolios by adding alternative investments with traditional asset classes into the investment universe and by optimizing return to expected shortfall instead of variance, are formed reflecting the empirical research with the literature review.

4.2 Data & Descriptive statistics

Private alternative investment data is scarce and suffers previously mentioned biases more adversely than public benchmarks. For that reason, alternative investment studies usually focus on publicly listed alternative investments. One distinguishing factor compared to other studies is that we use private return data when available in this study. An essential difference between traditional and alternative assets data in this study is that alternative assets' data are aggregated fund data and include fees. At the same time, this alternative assets' data has backfill, selection, and liquidation bias. These biases compensate the penalty from the net-of-fee return calculation and make the asset class comparison meaningful.

From the six studied alternative investment sub-asset classes, five utilize Preqin private fund data. These are Private Equity (PE), Private Credit (PC), Real-Estate (RE), Infrastructure (IF), and Natural Resources (NR). The Preqin data is quarterly index data, consisting of all reported vintages. Indices are aggregated from reported funds' money-weighted rate of returns (IRR). However, in this study, we view monthly returns, and for that reason, quarterly returns have been disaggregated. This disaggregation enables us to create more data, unsmooth, and fix the lagging returns. The disadvantages of disaggregation are that we need to estimate these time series, increasing the uncertainty of our obtained results' robustness. This study utilizes monthly Hedge Fund Research (HFRI) Fund of Funds Composite index data for hedge funds. HFRI is one of the most comprehensive hedge fund data providers, and the fund of funds composite index ensures the most comprehensive sample possible from a very heterogeneous asset class. However, the fund of funds structure suffers from the double-layered fee structure. Equities are included in the study with two separate components; developed (DM) and emerging (EM) markets. The indices used for equities are MSCI World total return and MSCI Emerging Markets total return indices. Government bonds are studied through Bloomberg US Treasury Total Return index, investment-grade corporate bonds through Bloomberg Global aggregated Corporate total return index and high-yield corporate bonds through Bloomberg Global High Yield Corporate total return index. The money market is left out of the study because of its contradictory role in portfolio optimization in a negative rate and high inflation environment. The study is conducted in U.S. dollars and data is collected from the period 2008-2021.

A summary of the data used in this study is presented in table 2. This table displays the public indicator time series utilized to disaggregate private data. To disaggregate and unsmooth quarterly data to monthly data, we use Litterman (1983) technique for asset classes whose serial correlation coefficient is greater than zero, and Fernandez (1981) method for assets whose serial correlation coefficient is negative. Both methods are regression-based, performing Generalized Least Squares Regression (GLS) to the quarterly values of the private data and the monthly indicator series. The Litterman (1983) technique combines AR(1) Markov process and random walk and assumes that monthly residuals follow the non-stationary process. The process is $u_t = u_{(t-1)} + v_t$, where v is an AR(1) process $v_t = \rho v_{(t-1)} + \epsilon_t$, and where ϵ is White Noise (0,σ_ϵ). Fernandez (1981) method has $\rho = 0$, so it follows a purely random walk. Litterman (1983) showed that his method produced the smallest mean squared errors with time-series which had positive autoregressive parameters. However, distributions with negative autoregressive parameter, Fernandez (1981) and Chow and Lin (1971) methods produced better results. This disaggregation keeps the mean of the original time series but increases the variance of returns. Forming monthly returns adds up to the original quarterly returns, decreasing the stale pricing of the time series.

The introduced desmoothing is done with the temporal disaggregation algorithms introduced by Sax and Steiner (2013). The method is widely used to convert low-frequency to high-frequency data in economic statistics like Gross Domestic Product calculations. This method generates new high-frequency data consistent with the original low-frequency data. This method aims to bring some of the variance found in the public market to private market data, consistent with the view that many economic factors behind public and private asset returns are the same. Mathematically this GLS process can be presented as,

$$\hat{\beta}(\Sigma) = [X'C'(X\Sigma C')^{-1}CX]^{-1}X'C'(C\Sigma C')^{-1}y_1, \tag{40}$$

where $\hat{\beta}$ is the GLS estimator, CX is the monthly indicator series, y_1 is the quarterly value of original series, X is a $n \times m$ matrix and where m represents the number of indicators. Critical assumption in this method is that the linear relationship between CX and y_1 also holds with X and y. The distribution matrix, D, is the function of covariance matrix:

$$D = \Sigma C'(C\Sigma C')^{-1}, \tag{41}$$

and the covariance matrix is calculated in Fernandez (1981) and Litterman (1983) method as,

$$\Sigma_L(\rho) = \sigma_\epsilon^2[\Delta' H(\rho)' H(\rho)\Delta]^{-1}, \tag{42}$$

Table 2: Utilized data and corresponding indicator series

Indices which have indicator series are converted from quarterly to monthly data. Indices which does not have indicator series are originally monthly data. The data is collected from period 2008-2021. Adjusted R-squared tells how well indicator series explains quarterly index. Indicator series have been selected from a broad universe of available indices based on their coefficient of determination.

Asset class	Index	Indicator series	Adjusted R-squared	Regression coefficient
PE	PrEQin quarterly index	LPX 50	0.61	0.16
PC	PrEQin quarterly index	Bloomberg Global High Yield Corporate TR Index	0.84	0.49
IF	PrEQin quarterly index	S&P Global Infrastructure Index	0.07	0.08
NR	PrEQin quarterly index	S&P GSCI Total Return CME	0.70	0.20
RE	PrEQin quarterly index	FTSE NAREIT	0.14	0.12
HF	HFRI Fund of Funds Composite Index			
EQ - DM	MSCI World TR			
EQ - EM	MSCI Emerging Markets TR			
GOV	Bloomberg US Treasury TR			
IG	Bloomberg Global Agg Corporate TR index			
HY	Bloomberg Global High Yield Corporate TR index			

where Δ denote as $n \times n$ difference matrix, $H(\rho)$ as a $n \times n$ matrix with 1 on its diagonal, $-\rho$ on its subdiagonal and 0 elsewhere. Since in Fernandez (1981) case we had $\rho = 0$, the equation 42 simplifies:

$$\Sigma_L(0) = \sigma_\epsilon^2 \Sigma_D. \tag{43}$$

The autoregressive parameter $\hat{\rho}$ is estimated with maximization of the likelihood of the GLS-regression following Sax and Steiner (2013). Now we have everything estimated for the final estimation of the monthly series,

$$\hat{y} = p + Du_1, \tag{44}$$

where preliminary series is $p = \hat{\beta}X$, D is the distribution matrix from equation 41 and u_1 is a vector containing the differences between quarterly and monthly values. While this method can be argued to desmooth time series quite artificially, it resembles Getmansky et al. (2004) and Okunev and White (2003) methods in their studies. Similar methods are also seen to be utilized in practice when portfolio management needs to give some estimates about illiquid assets' volatility.

Indicator series for each alternative investment time series have been selected based on their R-squared values, and means that selected public indices have the best coefficient of determination to the private index in question. The small regression coefficient and R-squared value with IF index implicates the earlier stated difference between private and public infrastructure investment and reinforces the perception that the only way to get exposure for the alternative betas in infrastructure investing is through private investments rather than listed companies.

After disaggregating the data, we have 156 observations of monthly logarithmic return data per asset class. It represents the period from 2008 to 2021, including market pressures of the financial crisis in fall 2008 and the Covid-19 crisis in march 2020. Between those years, we had the longest bull market in history. Reference period's first and last years are crucial for the utilized data since that way we get versatile return behavior for the studied asset classes and observations for the scarce tail events.

The descriptive statistics and correlation matrix of studied asset classes are represented in table 3. When we examine the statistics, we see that PE offers the highest mean returns while HF has the lowest. The result with PE is not surprising since it is studied that PE typically has 1.3-2.4 equity market beta, depending on the utilized data (Buchner 2020). However, the annual HF return of 1.68% is surprisingly low compared to earlier studies, which have recorded returns between 7.6%-12.3% after fees, survivorship, and backfill bias adjustments (Ilmanen 2011;

Ibbotson et al. 2011). Those studies included periods from 1990 to 2009, so the returns were from different decades than this study. It is good to point out that, for example, Ilmanen (2011) utilized the same HFRI index data as we do in this study, so we can argue that there has been a regime shift in the HF asset class. If we compare these statistics to Preqin's findings of investor expectations introduced in chapter 2, we see that the expectations for high absolute returns with PE align with the historical data, while the HF return expectations do not get supported with the historical data. When we move on to the risk metrics, we see that government bonds represent the lowest risk in standard deviation and expected shortfall as expected. From the other end of the risk spectrum, emerging markets equity has the highest standard deviation and expected shortfall.

Another interesting part of the data is the historical efficiency metrics of different asset classes. In the asset allocation context, Sortino et al. (2001) conclude that downside risk measures will always provide more correct efficiency rankings for assets than the mean-variance framework if return estimates are reliable, which means that we should emphasize RoES more than Sharpe. This statement brings up the critical note that estimating the optimization inputs is not a trivial task, and in many cases, it is even more important than utilizing an appropriate optimization model. However, as stated earlier, this study focuses on assessing an appropriate optimization model and, for that reason, leaves an in-depth analysis of estimation methods for further research. For that reason, we can examine ex-post efficiency rankings of different asset classes to learn something from history. The EM equity has the worst Sharpe ratio while IF has the best. If we look at RoES, EM equity is also worst while PE is the best. The ranking with Sharpe is from most efficient to least: IF, PE, PC, GOV, IG, HY, RE, NR, DM, HF, EM. While with RoES, it is: PE, GOV, IF, PC, IG, NR, HY, DM, RE, HF, EM. If we compare these two rankings, we see quite many differences, which strengthens the view that we should also evaluate the risks above the variance. This gives the first indicators that taking higher moments into account dilutes the attractiveness of alternative investments since government bonds overtake the private credit and infrastructure with RoES and at the same time RE collapsed in rankings. Statistics of alternative investments align with investors' expectations since PE, IF, and PC offer the best risk-adjusted returns from alternative investments with both efficiency metrics.

In panel B, we have asset classes and inflation correlations with each other. With IF, we have most correlations that do not significantly differ from zero. GOV has the most negative correlations, which supports the view of asset class being an excellent risk diversifier. Surprisingly, the best inflation hedge seems to be with HF from a correlation perspective. However, since the mean of inflation has been the same as the HF returns, 1.68% annually, HF seems to have perceived the purchasing power but not offered any real returns during the period. As a second and third in correlation with inflation comes NR and RE, which are expected since materials such as timber and rental yields of real-estates usually follow inflation. If we compare these results with investors' expectations shown in chapter two, the RE and NR are in line with that study. However, the inflation hedge often connected with infrastructure was not found from observed data.

Table 3: Descriptive Statistics and correlations

Panel A describes the summary statistics for each asset class. The stats are calculated from monthly log-returns and comprise 156 observations of each asset. ES is calculated with $\alpha = 0.95$. Sharpe and RoES is calculated with $R_f = 0$. All Jarque Bera statistics have $p < 0.01$ so we can reject the null hypothesis of normally distributed returns. Panel B displays the correlations between the asset classes and inflation. Correlation matrix also include correlation tests and the significance is displayed with *. Correlation test null hypothesis states that there is not linear relationship between the two variables. *) means that the $p > 0.05$, and we can not reject the null hypothesis.

Panel A: Descriptive Statistics

Class	Mean	St. Dev.	ES	Sharpe	RoES	Skewness	Kurtosis	Max	Min	JB stat.
PE	0.0085	0.0167	0.0429	0.5082	0.1703	−1.8901	8.9930	0.0514	−0.0708	324.56
PC	0.0054	0.0188	0.0529	0.2899	0.0672	−2.8783	16.7081	0.0434	−0.1167	1432.70
IF	0.0066	0.0123	0.0353	0.5368	0.1422	−2.8705	15.4881	0.0332	−0.0657	1225.26
NR	0.0020	0.0172	0.0436	0.1188	0.0433	−1.2983	5.7201	0.0312	−0.0722	91.08
RE	0.0024	0.0183	0.0600	0.1352	0.0333	−3.8114	21.4078	0.0214	−0.1141	2572.94
HF	0.0014	0.0163	0.0441	0.0860	0.0245	−1.7641	9.0172	0.0384	−0.0793	314.71
DM[1]	0.0051	0.0492	0.1200	0.1031	0.0380	−0.9104	5.2120	0.1203	−0.2102	52.94
EM[2]	0.0022	0.0652	0.1611	0.0341	0.0110	−0.9135	6.4667	0.1577	−0.3198	99.40
GOV	0.0031	0.0120	0.0212	0.2558	0.1679	0.5231	4.5450	0.0517	−0.0296	22.49
IG	0.0038	0.0204	0.0503	0.1849	0.0574	−1.0079	7.6640	0.0640	−0.0861	167.29
HY	0.0059	0.0330	0.0855	0.1801	0.0416	−1.7004	12.6917	0.1123	−0.1904	684.27
CPI	0.0014	0.0330						0.0100	−0.0182	

Panel B: Correlations

	PE	PC	IF	NR	RE	HF	DM	EM	GOV	IG	HY	CPI
PE	1											
PC	0.81	1										
IF	0.27	0.12*	1									
NR	0.60	0.62	0.38	1								
RE	0.74	0.60	0.34	0.47	1							
HF	0.41	0.37	−0.03*	0.37	0.36	1						
DM[1]	0.76	0.82	0.15*	0.57	0.49	0.12*	1					
EM[2]	0.67	0.80	0.02*	0.54	0.42	0.11*	0.87	1				
GOV	−0.39	−0.32	0.02*	−0.37	−0.20	−0.24	−0.32	−0.24	1			
IG	0.44	0.71	0.03*	0.36	0.23	0.03*	0.68	0.74	0.20	1		
HY	0.66	0.93	−0.05*	0.53	0.41	0.23	0.82	0.84	−0.28	0.79	1	
CPI	0.34	0.33	0.08*	0.48	0.40	0.61	0.14*	0.14*	−0.37	0.02*	0.26	1

[1] Equity - Developed markets.

[2] Equity - Emerging markets.

When we move on to evaluate the normality of log returns, we find out that every other asset class except GOV has negative skewness. This means that all the other assets have long left tails and are prone to severe losses. PE, PC, IF and especially RE have significant negative skewness. PE having such a negative skewness is an interesting finding, since earlier stated in the literature review, Phalippou and Gottschalg (2009) rationalized PE investments, despite their controversial performance, with lottery-seeking preferences and right-skewed returns. One explanation is that they emphasized venture capital investments, and this way, this heterogeneous asset class includes very different return structures, which aggregates to the significant negative skewness, again emphasizing strategy (buyout vs. venture capital) and manager selection in this asset class. From a kurtosis perspective, GOV, NR, DM, and EM are closest to the normal distribution value of 3. PC, IF and RE are the most leptokurtic distributions (have the highest kurtosis), meaning they are more prone to extreme values than other asset classes. After skewness and kurtosis statistics, it is no surprise that according to the Jarque Bera test, we can reject the null hypothesis of normally distributed returns with every asset class.

However, since the Jarque Bera test's chi-squared approximation is very sensitive with small samples, it is important to evaluate the normality also with graphs. The return distributions are shown visually in figure 9. The lighter line illustrates the normal distribution with the given mean and variance, while the darker line shows the empirical distribution.

From these graphs, we can say that the traditional assets are closer to the normal distribution than the alternative ones. We can still evaluate the normality with Henry's line in figure 10, which has 95% confidence bands, and we get the same kind of results as from histograms. Alternative assets tend to depart from the normality, especially in the left (negative) tail of the distribution. GOV and EQ - DM tend to be closest to the normality.

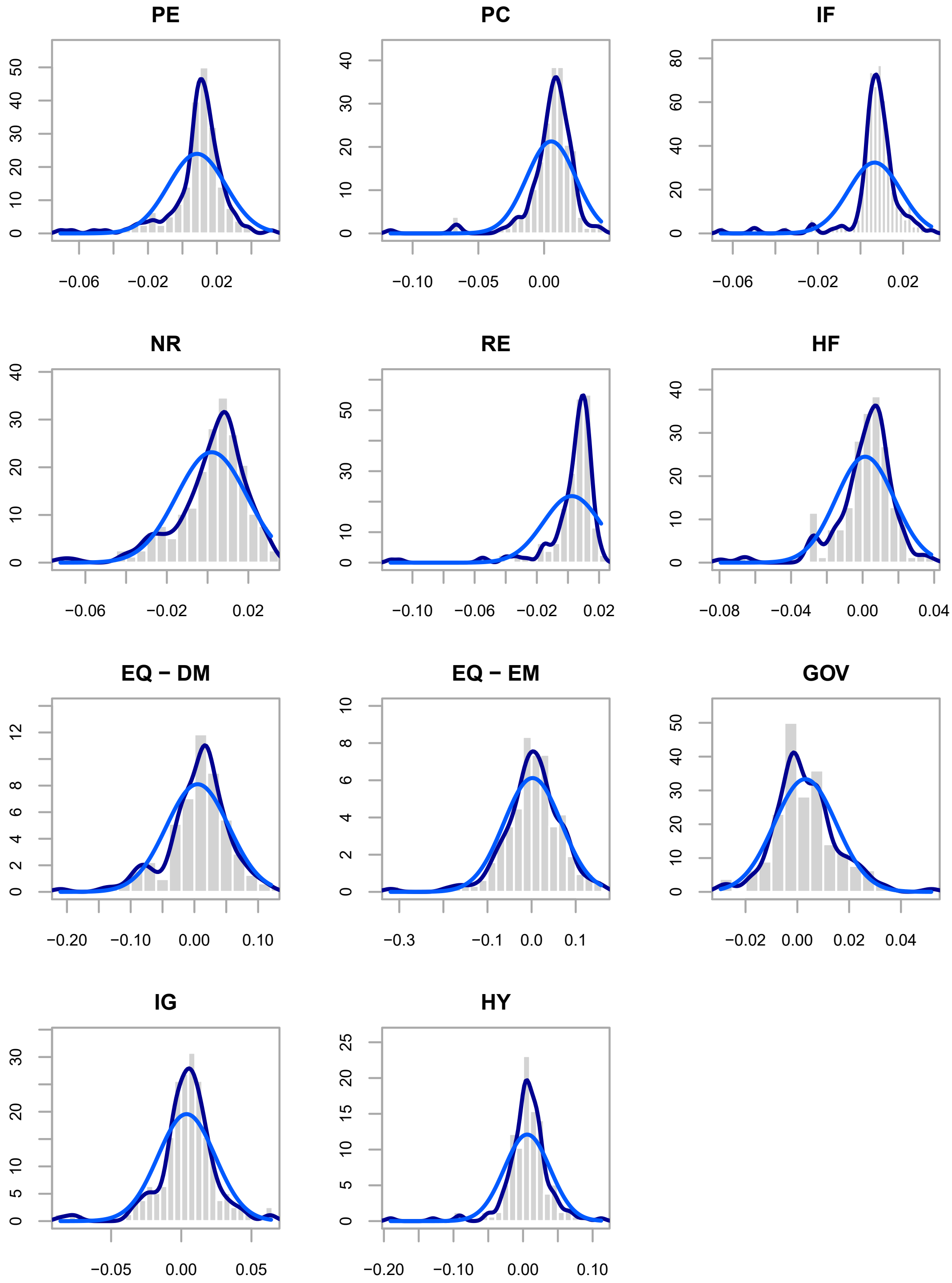

Figure 9: Histograms with distribution function

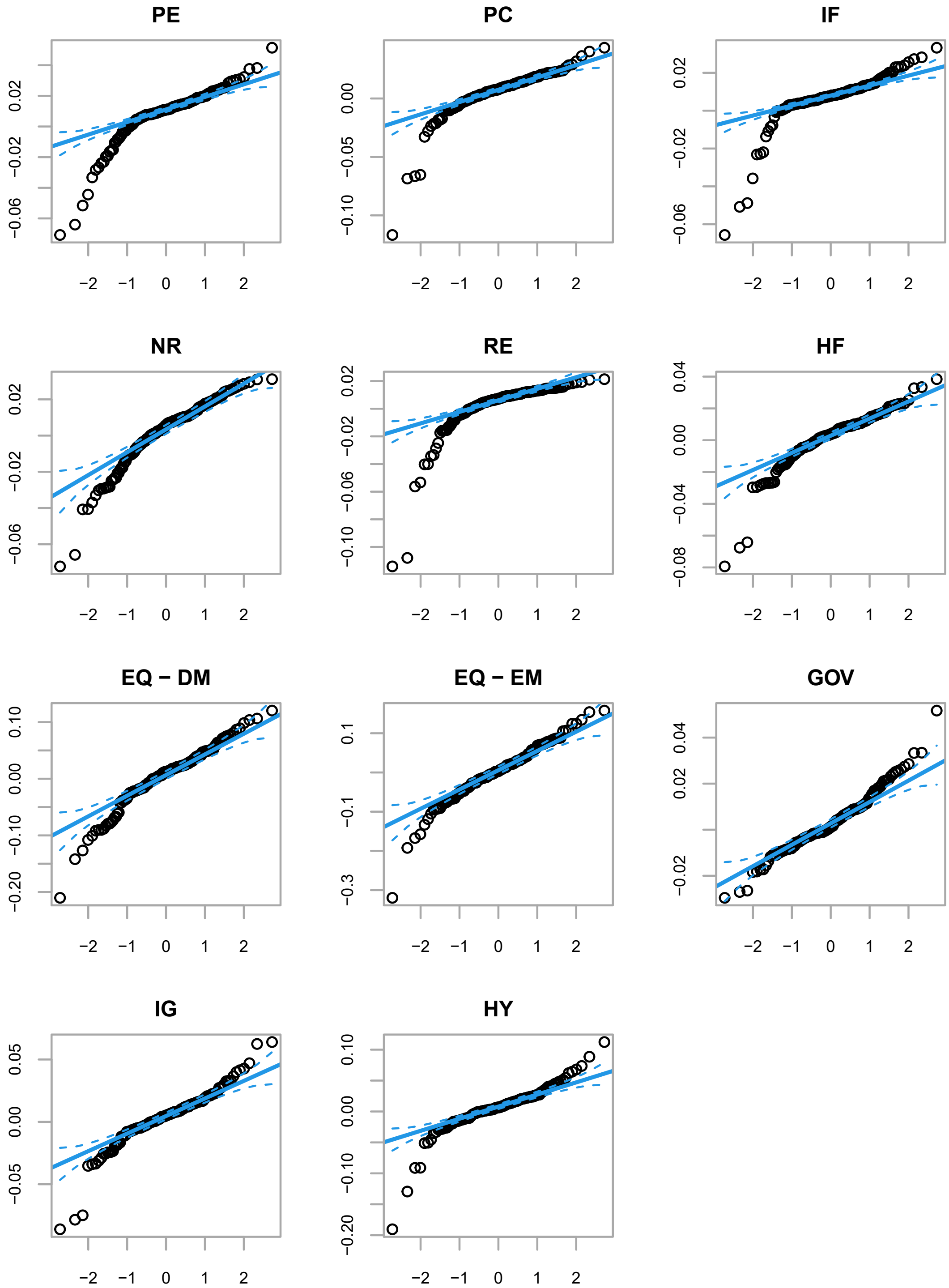

Figure 10: Normality observation with Henry's line

5 RESULTS

We start by optimizing portfolios in the mean-variance (Mean-Var) framework as in chapter 3.3. In practice, we do this with quadratic programming solver and use historical samples as input estimates for mean and variance. We are interested in maximizing the Sharpe ratio and, in the first point, comparing the portfolios' efficiency when we have just traditional assets versus when we include alternative assets to the investment universe as well. The only constraints we have in optimization are the long-only and fully invested portfolio and the objective is to maximize Sharpe ratio. The first and easy way to examine the differences in portfolios' efficiency is to construct efficient frontiers with and without alternative investments. As we see from figure 11, it is clear that the diversification benefit moved the efficient frontier upwards, as we expected in chapter 3. The curve rose especially from the riskier end of the risk spectrum. If we examine the allocation of the efficient frontiers in figure 12, we can notice that this progress is mainly driven by the opportunity to invest in PE. It is also noticeable that the optimization gives quite concentrated portfolios in both cases, with and without alternatives, since efficient portfolios utilizes mainly just PE, IF and GOV with alternatives and HY and GOV without alternatives (see Appendix II). The concentration is due to utilizing a historical sample mean and variance as an estimate for an expected return and variance, making the estimates susceptible to estimation error. In reality, to make efficient ex-ante portfolios, we should utilize more diversification across asset classes. We will examine these ex-ante portfolios later in backtesting.

In number terms, the efficiency improved from 0.366 to 0.789 ex-post Sharpe in maximum SR portfolios and dropped the minimum Standard Deviation from 3.44% to 2.24% per annum in minimum variance portfolios. We have similar results when examining alternative assets' effect on the efficient frontier with ES as a risk metric and RoES as an optimization objective. The maximum RoES improved from 0.225 to 0.385 and the minimum ES from 1.60% to 1.25%. It is interesting to see how the HF are included in the efficient portfolios even though it has the worst efficiency metrics as an individual asset class. However, it happens to have the smallest correlation with the dominating assets, PE, IF, and GOV, and for that reason, it comes to the portfolio to offer pure diversification. This finding is in line with the Preqin study viewed in chapter 2.

The utilized asset classes in efficient frontiers were very similar with mean-variance and mean-expected shortfall (Mean-ES) optimization, but as we see from figure 13, the latter is not as linear with the allocations as the former. The dif-

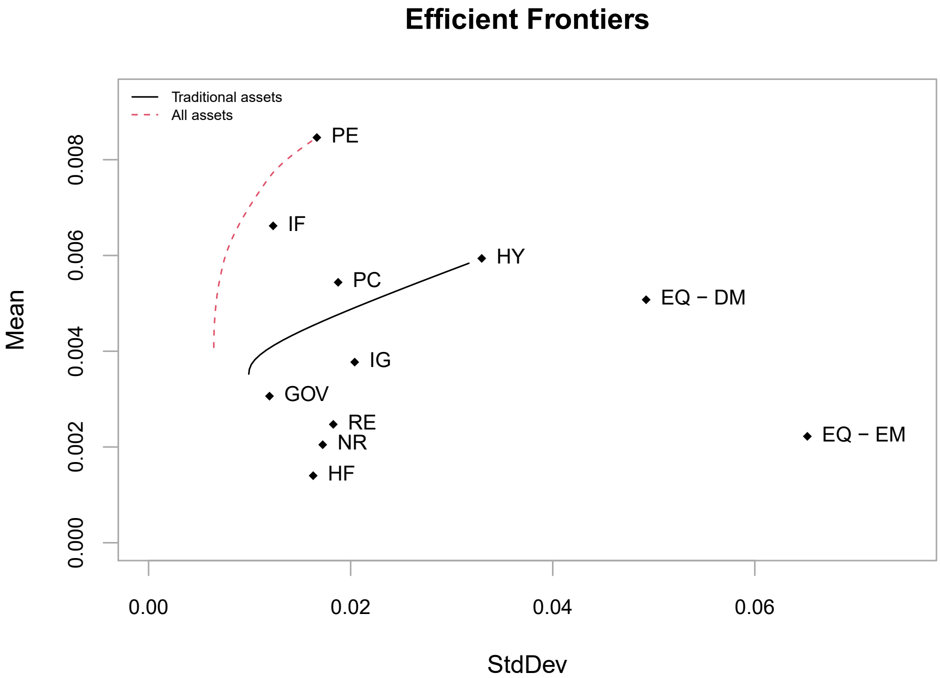

Figure 11: Efficient frontiers in mean-variance space

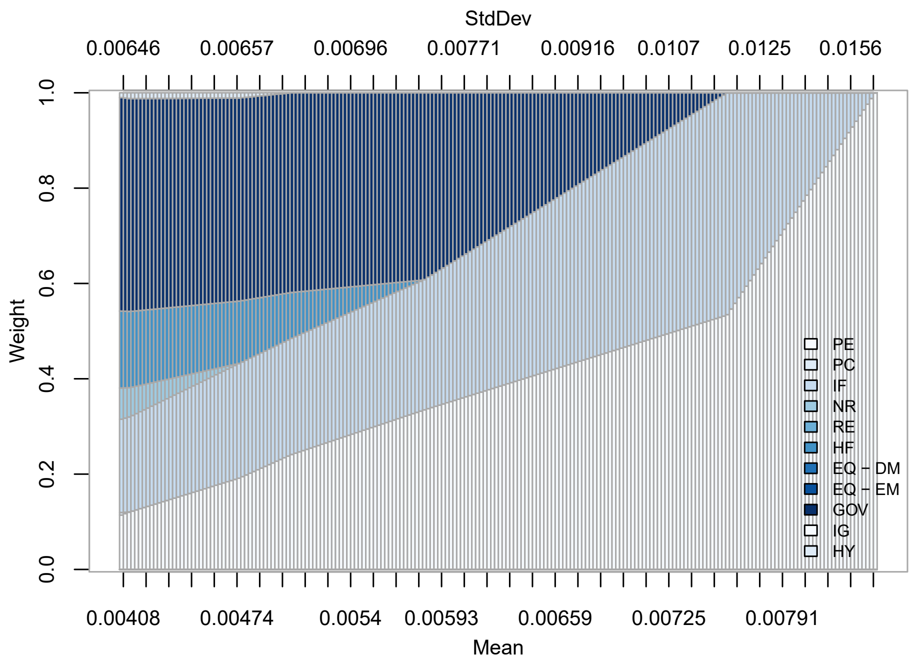

Figure 12: Asset weights in mean-variance efficient portfolios

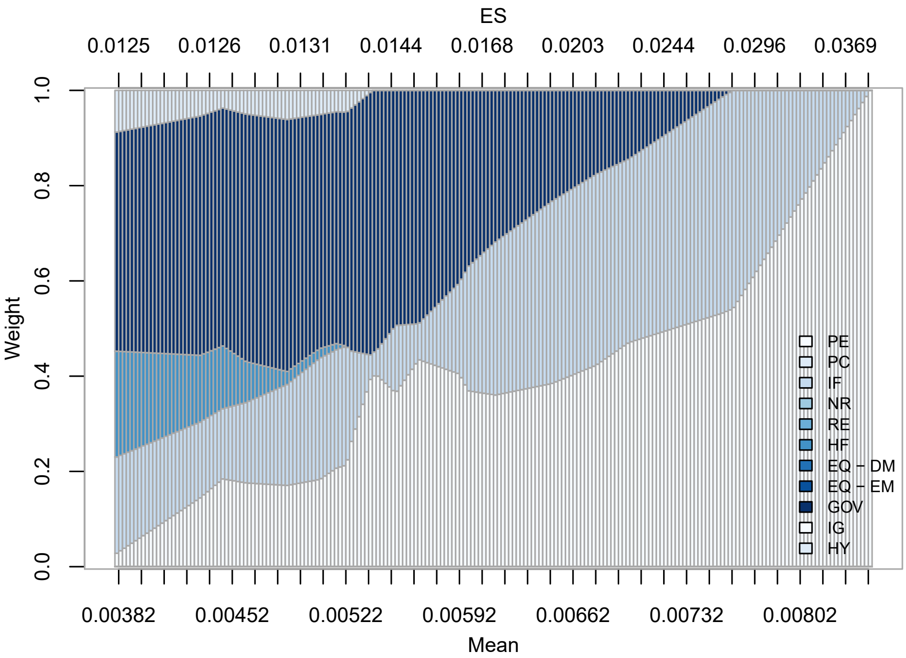

Figure 13: Asset weights in mean-es efficient portfolios

ference in allocation is due to the higher moments taken into account in mean-ES optimization with Cornish-Fisher ES estimation. The improvement in portfolios efficiency compared to mean-variance optimization (or mean-ES optimization with normal distribution assumption) is shown in Figure 14. Even though the improvement is small, it implies that taking higher moments into account improves portfolios performance at least ex-post. The results look very similar to that Krokhmal, Palmquist and Uryasev (2002) obtained from their study. Improvement is because coskewness and cokurtosis offer diversification benefits as well. When we compare how these higher moments affect the allocation towards alternative investments, we see from table 4 that with mean-variance optimization, the average allocation to alternatives is 75% in the efficient frontier, while with mean-expected shortfall optimization, this number decreases to 69%. This implies that alternative investments do not look as attractive when examined from the expected shortfall perspective compared to variance. This finding confirms the initial expectations for alternative investment optimization. However, at the same time, these efficient frontiers confirm that adding alternative investments in the investment universe can significantly improve portfolio efficiency, despite the non-normal return distributions. It is safe to say that alternative investments can increase returns and decrease risks when added to a multi-asset portfolio.

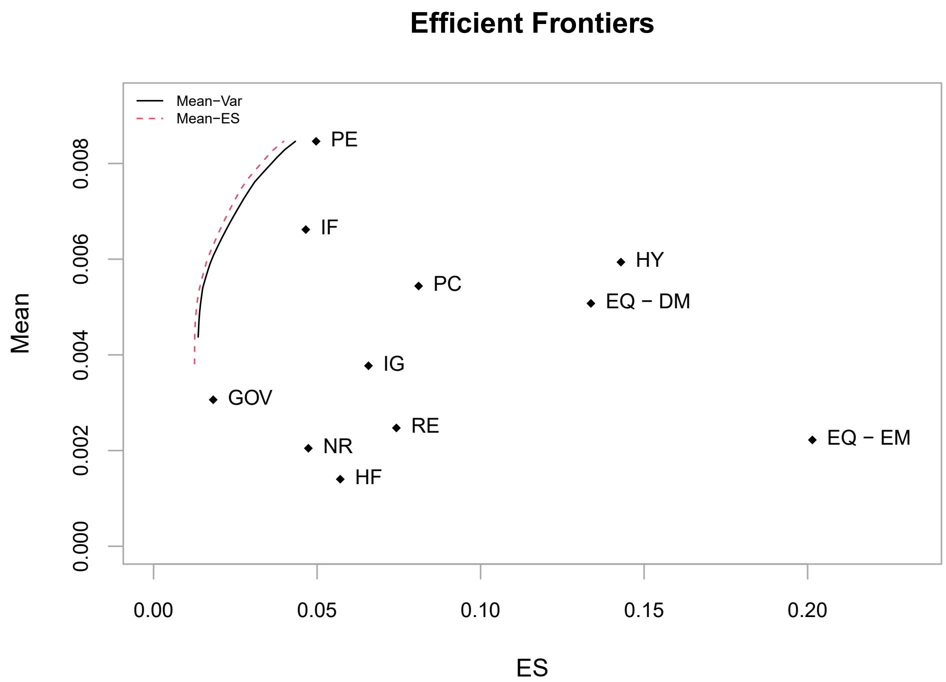

Figure 14: Mean-variance and mean-ES efficient frontiers

While mean-ES optimization revealed that, even though alternatives are improving traditional portfolios, they are not as attractive as typically have seen with mean-variance optimization. At the same time, optimization findings suggest that mean-ES optimization leads to better results regardless of the available assets. However, here is vital to remember that we have examined only ex-post numbers so far, and portfolio construction is all about ex-ante decisions. For that reason, we conduct out-of-sample simulations to put optimizations to test.

The discussed results can be found in table 4 panel A. Next, we move on to the backtests, which results can be found from panel B. Figure 15 summarizes the different portfolios' backtest results. Solid lines are performance for backtested portfolios with alternative investments in the investment universe, and dashed lines are portfolios without alternative investments in the investment universe.

If we jump right to the most interesting results, efficiency metrics, we can easily support our earlier findings of alternative investments' capability to work as efficiency enhancers. No matter which objective we have in optimization, when we add alternative investments into the investment universe, the Sharpe and RoES ratios doubles. However, when we deep dive into different optimization objectives, we see how the traditional mean-variance framework outperforms the other optimization methods, both in absolute returns and efficiency terms. This finding

Table 4: In-sample and out-of-sample results

Returns and standard deviations are annualized, expected shortfall is over one month period. % Alt. stands for the share of alternative assets in the portfolio.

Panel A: In-sample portfolios

		Return%	StdDev%	ES%	SR	RoES	% Alt.
Without alt. assets	Mean-Var[1]	5.67	6.65	4.57	0.85	0.12	0
	Max SR	4.47	3.53	1.94	1.27	0.19	0
	Min Var	4.23	3.44	1.79	1.23	0.19	0
	Mean-ES[1]	5.70	6.74	3.81	0.85	0.15	0
	Max RoES	4.41	3.48	1.63	1.27	0.23	0
	Min ES	4.28	3.45	1.60	1.24	0.22	0
With alt. assets	Mean-Var[1]	7.53	3.19	2.30	2.36	0.30	75.01
	Max SR	7.06	2.58	1.71	2.73	0.34	61.81
	Min Var	4.90	2.24	1.36	2.19	0.32	54.21
	Mean-ES[1]	7.37	3.18	2.01	2.32	0.32	68.97
	Max RoES	6.29	2.47	1.36	2.54	0.39	46.06
	Min ES	4.58	2.33	1.25	1.96	0.31	45.24

Panel B: Out-of-sample portfolios

		Return%	StdDev%	ES%	SR	RoES	% Alt.[2]
Without alt. assets	MaxSR	3.76	2.86	1.45	1.32	0.22	0
	MaxRoES	3.55	3.13	1.47	1.13	0.20	0
	MaxRoES90	3.58	3.05	1.74	1.17	0.17	0
	MaxCRRA	3.67	2.89	1.47	1.27	0.21	0
With alt. assets	MaxSR	6.88	2.00	1.03	3.45	0.56	50.2
	MaxRoES	5.42	2.42	1.04	2.24	0.44	21.7
	MaxRoES90	6.46	2.25	1.47	2.87	0.37	50.8
	MaxCRRA	4.61	1.86	0.81	2.48	0.48	50.6

[1] Efficient frontiers average results.

[2] Average weight during the review period.

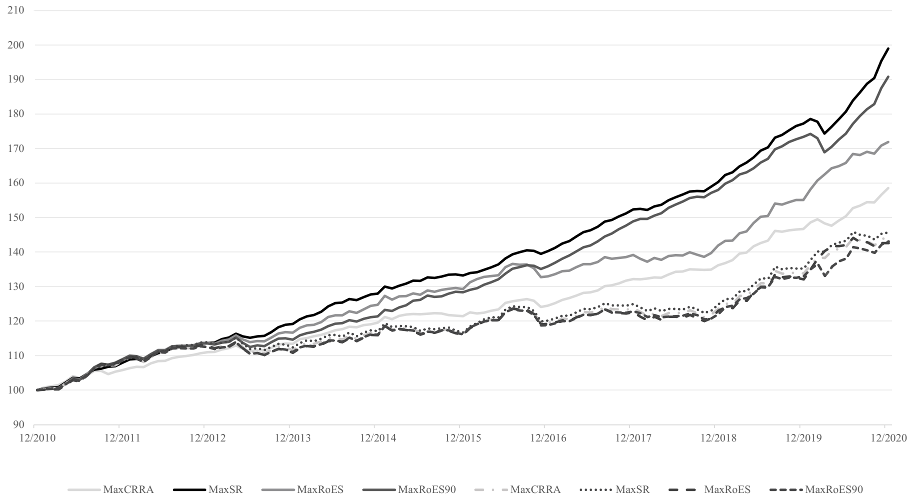

Figure 15: Historical simulations return indices

suggests that while the mean-expected shortfall optimization might look attractive ex-post because we can utilize the actual realized coskewness and cokurtosis numbers, the situation is different when we need to estimate comovements in ex-ante. Since the number of estimated parameters increases rapidly when we move from covariance to the third and fourth moment, the risk for estimation error increases substantially. These out-of-sample backtests suggest that the efficiency gained from considering higher moments does not payout since the increasing estimation error overtakes it. This is consistent with the Martellini and Ziemann (2010) findings.

When we compare ES optimization with different confidence levels ($p = 1 - \alpha$), we recognize the sensitivity of this metric, as earlier stated (Lim et al. 2011). The efficiency and allocation dramatically change when we utilize a 10% loss probability level instead of a 5% level. While ES with 5% α focuses mainly on GOVs over the whole review period, ES with 10% α has significant allocations to PE, IF, and HF. Of course, this allocation change increases the absolute returns, but with efficiency metrics, the portfolio performs better with SR but worse with RoES. When we increase the loss probability level, the optimization is not so sensitive for the fat-tailed distributions, and the weight of the alternative assets in the portfolio increases. At the same time variance of returns decreases. However, when we examined asset classes individually, it is noticeable that alternative investments did not have higher ES than traditional assets. Instead, lower variance and comovement properties attract alternative assets to the portfolio when we allow overall riskier assets with 10% loss probability. While the SR increased and the asset allocation developed closer to levels we obtained from MaxSR optimization (see figures 16 and 17), at the same time, the RoES efficiency metric went in the opposite direction. This means that the higher returns came at the cost of even higher risks. For example, when we examine the drawdown metric in figures 18 and 19, we can see that MaxRoES90 optimizations have more severe drawdowns than MaxSR optimization. This finding suggests that the mean-ES optimization did the opposite of expected results. We argued that mean-ES optimization could fit for institutions that are ready to carry high volatility portfolios if they can at the same time decrease maximum drawdowns through expected shortfall. In our backtests, expected shortfall optimization did not offer that feature, and the mean-variance framework did the best job from that viewpoint.

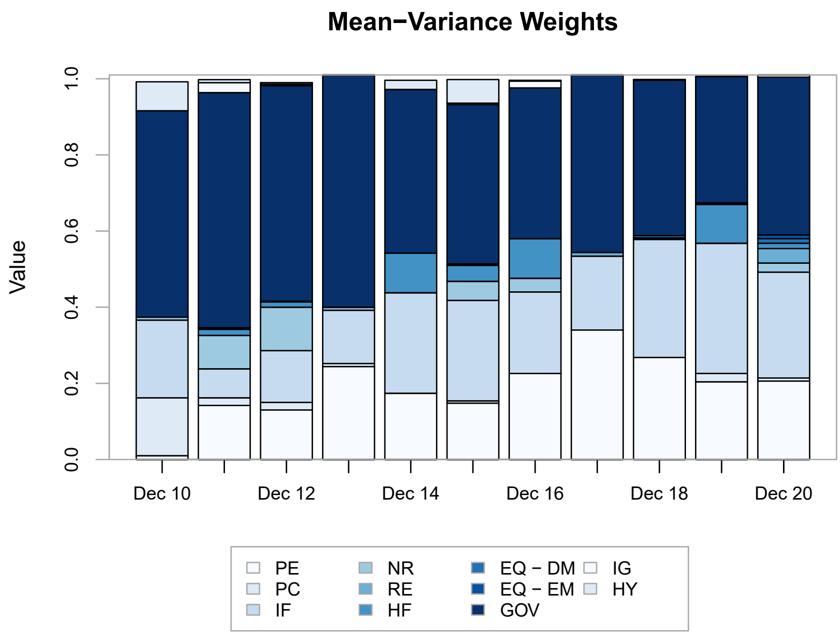

Figure 16: Mean-variance out-of-sample weights

The observations about chosen loss probability level bring the problematic feature of having ES as an objective in optimization. While in the mean-variance framework, one can create an efficient frontier and pick a portfolio according to the risk preferences from that frontier, the subject has one risk level to decide. With ES, the investor needs to pick the initial risk level by choosing the probability of loss α and, after creating an efficient frontier, decide another risk level, preferred expected shortfall from formed efficient frontier. These problems with ES highlight the practicality of variance as a risk metric and assumption of normally distributed returns.

When we finally examine the CRRA objective function out-of-sample perform-

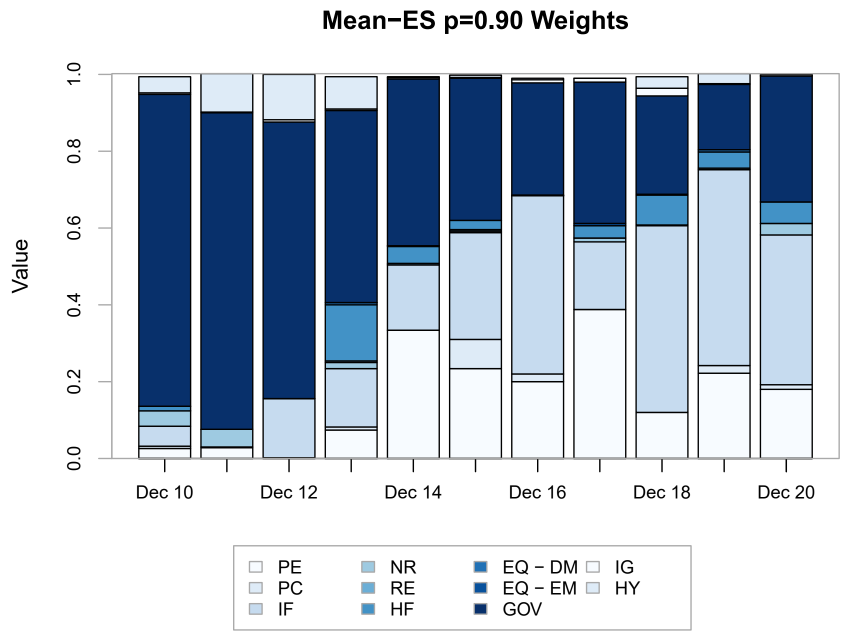

Figure 17: Mean-ES p=0.90 out-of-sample weights

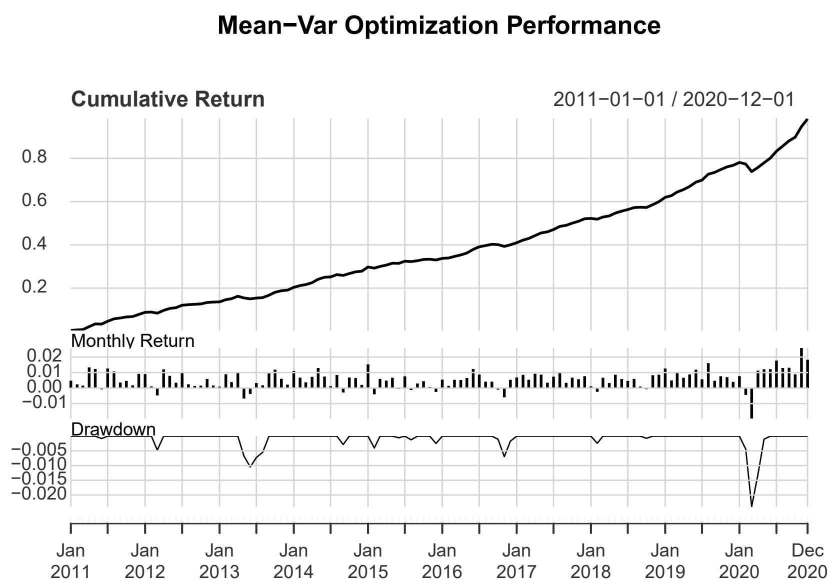

Figure 18: Mean-variance out-of-sample performance

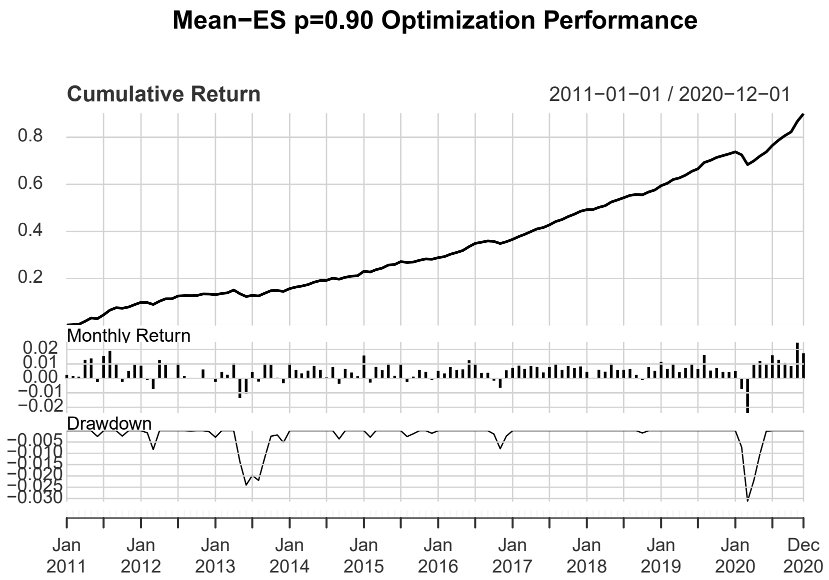

Figure 19: Mean-ES p=0.90 out-of-sample performance

ance, we can notice the smallest drawdown levels in figure 21 but at the same time also the most negligible absolute return. The allocation with alternative investments is most comprehensive with MaxCRRA, it contains significant level HF throughout the review period, and the allocation also stays relatively stable in the review period (see figure 20). It also has the slightest standard deviation and expected shortfall with the alternative investments in the investment universe. However, its modest returns leave it third in SR and second in RoES. Overall, MaxCRRA optimization performance is relatively stable, and it could work as a tool when we try to create small drawdown portfolios. It would be interesting to study how it behaves when we increase the risk level. This study utilized the risk aversion parameter $\gamma = 1$, since we wanted to describe long investment horizon institutions that can bear high levels of risks in their investment portfolios.

Overall, the backtests revealed that the mean-variance framework, in all its simplicity, offers efficient results while the mean-ES framework was unable to deliver the hoped results from maximum drawdown and efficiency perspective. At the same time, CRRA objective function optimization offered promising results to incorporate higher moments effectively in the optimization. For that reason, it could work as an alternative for the mean-variance framework when optimizing multi-asset portfolios with alternative investments.

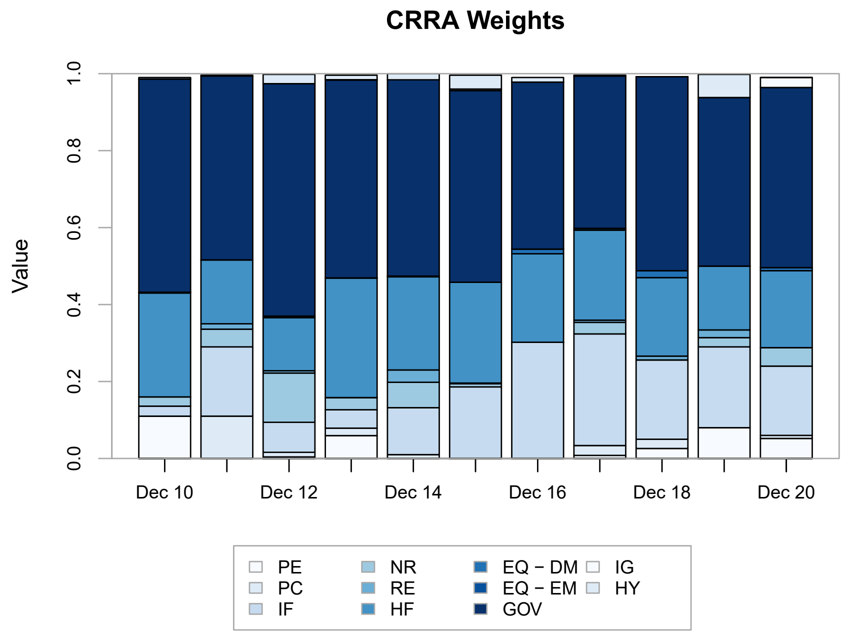

Figure 20: CRRA objective function out-of-sample weights

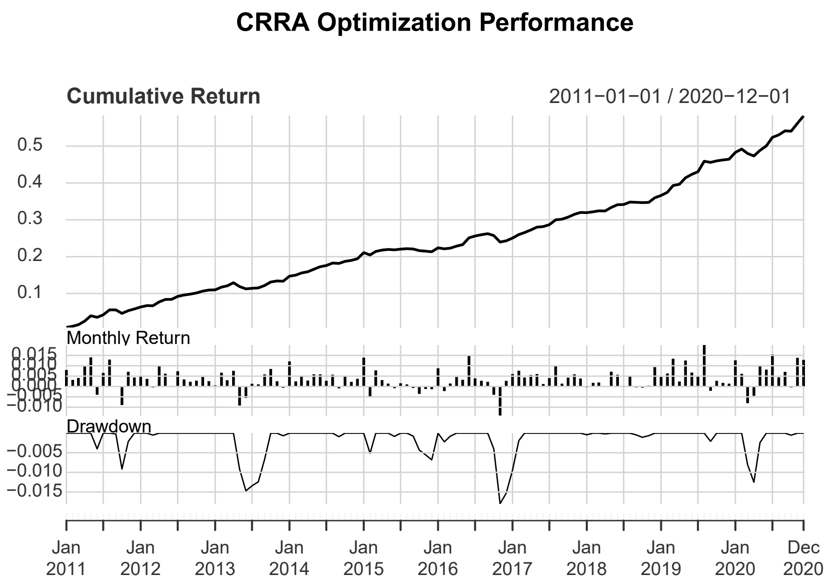

Figure 21: CRRA objective function out-of-sample performance

6 CONCLUSION

The alternative assets appear as a safe haven from the recent markets turmoils created by the Covid-19 crisis and the Russian war against Ukraine. At the same time, the long-term outlook for traditional asset classes' expected returns has uniformly moderated. These uncertainties have further accelerated the long-lived trend of alternative investments getting a share of the investors' wallets. While the shift in strategic allocations has been remarkable, the practice of constructing portfolios has not significantly changed from the traditional MPT mean-variance framework.

The research presented in this thesis examines whether it is naive to think of alternative assets as an answer to increasing portfolios' expected returns and hedge against market turmoils. This research also takes a stand if we should develop how we construct the portfolios when we add new ingredients to the problem. From these standpoints, we got the research questions of this study: Can we obtain more efficient portfolios by adding alternative investments with traditional asset classes into the investment universe, and can we obtain more efficient portfolios by optimizing return to expected shortfall instead of variance?

We examined these questions from an institutional investor perspective because they have particular characteristics that make these research questions most relevant. First, the long investment horizon enables institutional investors to harvest liquidity premium to a greater extent than other investors. The investment horizon reduces the limitations for illiquid assets and enables a greater share of alternative assets in the portfolio. At the same time, we can assume that institutions' large investable assets, in absolute terms, do not create constraints for available assets. However, we need to emphasize that the framework utilized in the empirical section is applicable as a starting point for individual investors as well. With individual investors, we need to consider some extra practicalities to add to the problem. These things are available assets with small investable assets and liquidity concerns on the life cycle hypothesis. So the institutional investor perspective simplifies the study to focus better on the research questions.

As we found in the literature review, different alternative investments were profiled as standalone asset classes for different purposes. At the same time, how exposure was created to different asset classes mattered. All the alternative investments were seen as great diversifiers, but still, there were differences between different asset classes. While PE was characterized as a return enhancer, real assets were utilized for diversification. RE and IF were seen as great cash flow producers and inflation hedges. When we compare these views to the findings we obtained from this study, we can confirm that our data supported the view for PE as a return enhancer, real-assets as diversifiers, and RE as an inflation hedge. However, we also found that HF worked as a great diversifier and the best diversifier for the portfolios we constructed. We did not find the inflation hedge from IF, and the HF performance was inconsistent with the investors' view. When we studied different investment vehicles to get the alternative asset class exposure, the earlier studies emphasized the importance of private market vehicles, particularly for IF, RE, and NR. The reasoning behind those views was that these investments had distinctly different risk and return characteristics compared to publicly traded substitutes. This difference was well demonstrated in the empirical section where we desmoothed the private investment data with the public one. RE and especially IF had the worst coefficient of determination emphasizing the different characteristics of private investments than public ones in these asset classes. We did not find that quality from NR, emphasizing that the private market NR index still holds a significant amount of commodity funds.

In the empirical section, we put the characteristics mentioned above in the portfolio context, and we started to examine whether alternative investments create value in the portfolio and whether the way we optimize portfolios affects that. When we added alternative assets in the investment universe, we saw an increase in portfolio efficiency regardless of our optimization objective. This improvement was confirmed with both efficiency metrics, SR and RoES, and the results were also confirmed in backtests. However, the proportion of alternative investments in efficient portfolios differed depending on the optimization objective. In the mean-variance framework, the proportion of alternative investments was systematically higher than in the mean-expected shortfall framework. This finding supports the view that alternative investments are not so attractive when higher moments are taken into account. At the same time, we saw that alternative investments standalone ES did not significantly differ from traditional assets. This finding suggests that the comovements of higher moments did not offer as good diversification as covariance did. However, as we saw in the out-of-sample backtest portfolios, the chosen confidence level of ES significantly affects the portfolio constituents. For that reason, the introduced findings can not be generalized to mean-ES optimization but instead illustrate the results with a particular confidence levels 95% and 90%. For further research would be interesting to examine portfolio construction comprehensively with a wide range of ES confidence levels.

When we changed the optimization method from mean-variance framework to mean-expected shortfall framework, we improved return on expected shortfall in the ex-post situation. However, the efficiency did not improve from the Sharpe ratio perspective, and even more importantly, in the ex-ante situation with historical simulations, the efficiency performance decreased significantly with both metrics. The possible explanation for poor backtest performance is the increasing estimation error with mean-ES optimization due to the increasing number of estimated parameters. This problem is further accelerated with the less available data in the simulation compared to ex-post examination. These findings suggest that the mean-variance framework should be the preferred optimization method for all its simplicity, even with alternative investments. However, it is essential to state that before any of these optimizations, we improved the smoothed data of alternative investments, which already in itself improves the mean-variance optimization. Indeed it would be a good topic for further research to examine how this data desmoothing affects the obtained portfolios.

We also wanted to examine an alternative way to take higher moments into account in portfolio optimization, and for that reason, we introduced the CRRA objective function in the ex-ante examination. This method produced promising preliminary results from a risk management perspective since it produced the smallest drawdown and other risk metric levels in the review period while maintaining a large allocation to alternative assets. However, since the result highly depends on the chosen risk aversion parameter, this would be an interesting topic for further research with more comprehensive examination with different parameter levels.

Let us summarize the study through research questions. Can we obtain more efficient portfolios by adding alternative investments with traditional asset classes into the investment universe? Yes, we can. Despite the desmoothing of the data and chosen risk perspective, alternative investments enhance portfolios efficiency. Can we obtain more efficient portfolios by optimizing return to expected shortfall instead of variance? Ex-post yes if we value expected shortfall more than variance as a risk metric. Ex-ante no without more comprehensive data and better comovement estimates.

We can consider at least the following if we discuss what these results imply in a broader portfolio management picture. It can be argued that with alternative investments, the heterogeneous nature of the asset classes and dispersion in portfolio managers' performance reduces the rationality of wasting too much effort for portfolio optimization. Instead, investors should focus on manager selection and finding skills from the private market. The allocations obtained from optimization problems should more likely see as a confirmation that there should be alternative investments in the portfolio, but the exact weights are secondary. Since the estimation errors increase with scarce data and higher moments, one might utilize some rule of thumb[16] to obtain as efficient portfolios as with complex optimization models. We need to remember that we never invest for the sake of investing, but instead to achieve some other objectives. Simple as it may sound and still often ignored, determining the goal for one's investments is the most crucial part of investing (Sortino et al. 2001). This again emphasizes the asset and liability management introduced in chapter 2.3. While this study's results do not offer a straightforward solution for a superior way to construct a portfolio, it will hopefully make investors critically re-evaluate their risk perspectives and investment universes for constructing the strategic allocation for their portfolios.

[16] Like the rule of half a dozen asset classes in the Yale Endowment fund (Swensen 2009).

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APPENDIX I

The Cornish-Fisher estimation for ES introduced in chapter 3.4 is derived as Boudt et al. (2013) propose. The second order Cornish-Fisher expansion of the quantile function around the Gaussian quantile function equals:

$$g_{\alpha t} = z_\alpha + \frac{1}{6}(z_\alpha^2 - 1)s_t + \frac{1}{24}(z_\alpha^3 - 3z_\alpha)k_t - \frac{1}{36}(2z_\alpha^3 - 5z_\alpha)s_t^2, \tag{45}$$

with $z_\alpha = \phi^{-1}(\alpha)$. Now the ES for the portfolio in month t is given by:

$$\mathrm{ES}_t(\alpha) = -\mathbf{w}'\mu_{\mathbf{t}} + \sqrt{m_{2t}}d_t(\alpha), \tag{46}$$

where:

$$\begin{aligned} d_t(\alpha) = \frac{1}{\alpha}\{\phi(g_{\alpha t}) + \frac{1}{24}[I^4 - 6I^2 + 3\phi(g_{\alpha t})]k_t + \\ \frac{1}{6}[I^3 - 3I^1]s_t + \frac{1}{72}[I^6 - 15I^4 + 45I^2 - 15\phi(g_{\alpha t})]s_p^2\}, \end{aligned} \tag{47}$$

and where:

$$I^q = \begin{cases} \sum_{i=1}^{\frac{q}{2}} \left(\frac{\prod_{j=1}^{\frac{q}{2}} 2j}{\prod_{j=1}^{i} 2j} \right) g_{\alpha t}^{2i}\phi(g_{\alpha t}) + \left(\prod_{j=1}^{\frac{q}{2}} 2j \right) \phi(g_{\alpha t}) & \text{for } q \text{ is even} \\ \sum_{i=0}^{q^*} \left(\frac{\prod_{j=0}^{q^*} 2j+1}{\prod_{j=0}^{i} 2j+1} \right) g_{\alpha t}^{2i+1}\phi(g_{\alpha t}) + \left(\prod_{j=0}^{q^*} 2j+1 \right) \Phi(g_{\alpha t}) & \text{for } q \text{ is odd} \end{cases} \tag{48}$$

respectively $q^* = (q-1)/2$ and $\phi(\cdot)$ and $\Phi(\cdot)$ are the standard normal density and cumulative distribution functions.

APPENDIX II

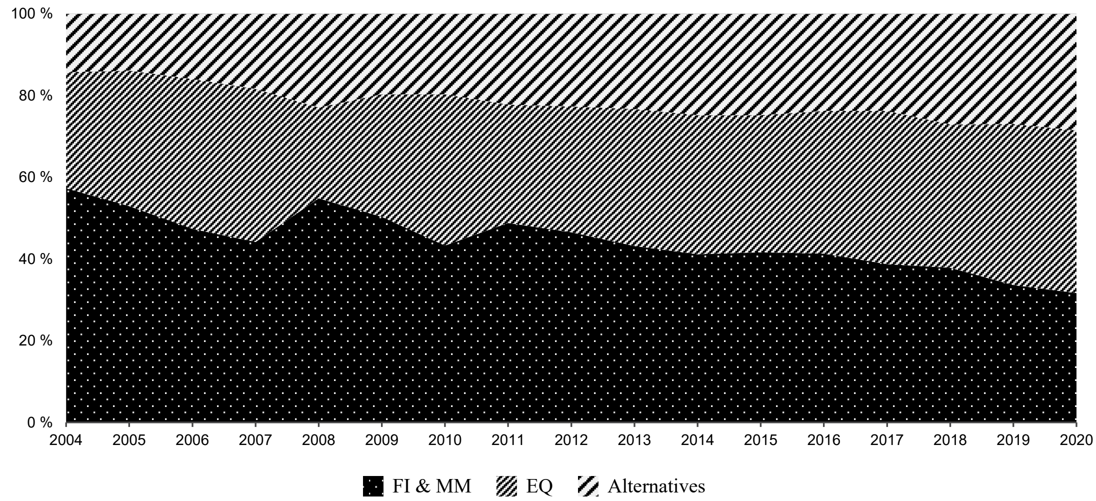

Figure 22: Finnish Pension Funds asset allocation

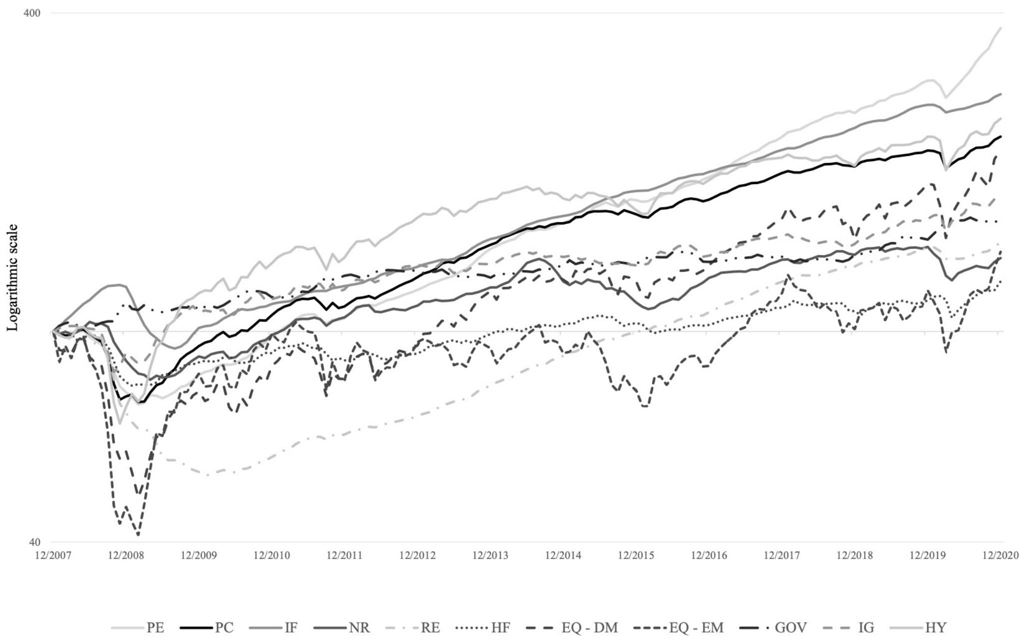

Figure 23: Desmoothed return indices for asset classes

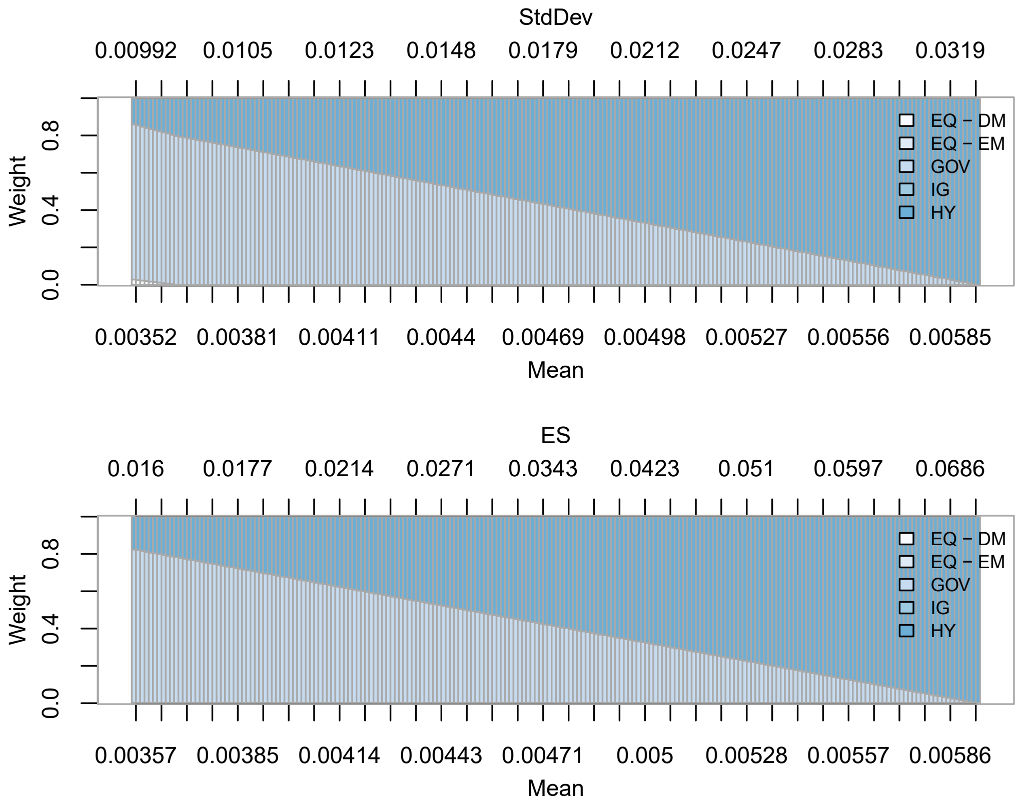

Figure 24: Asset weights in efficient portfolios without alternative investments

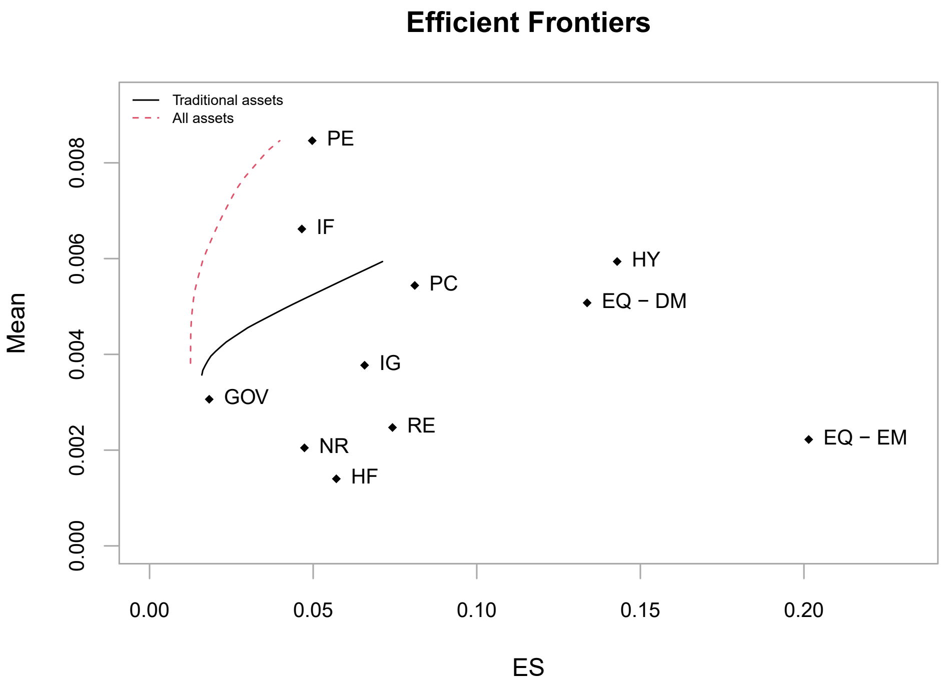

Figure 25: Efficient frontiers in mean-ES space

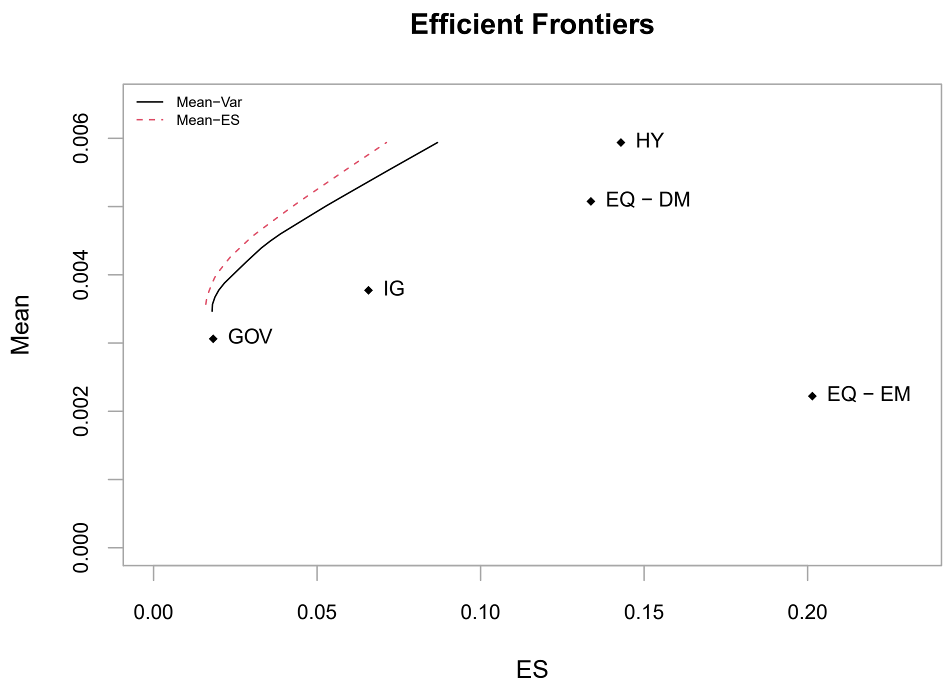

Figure 26: Mean-variance and mean-ES efficient frontiers without alternatives

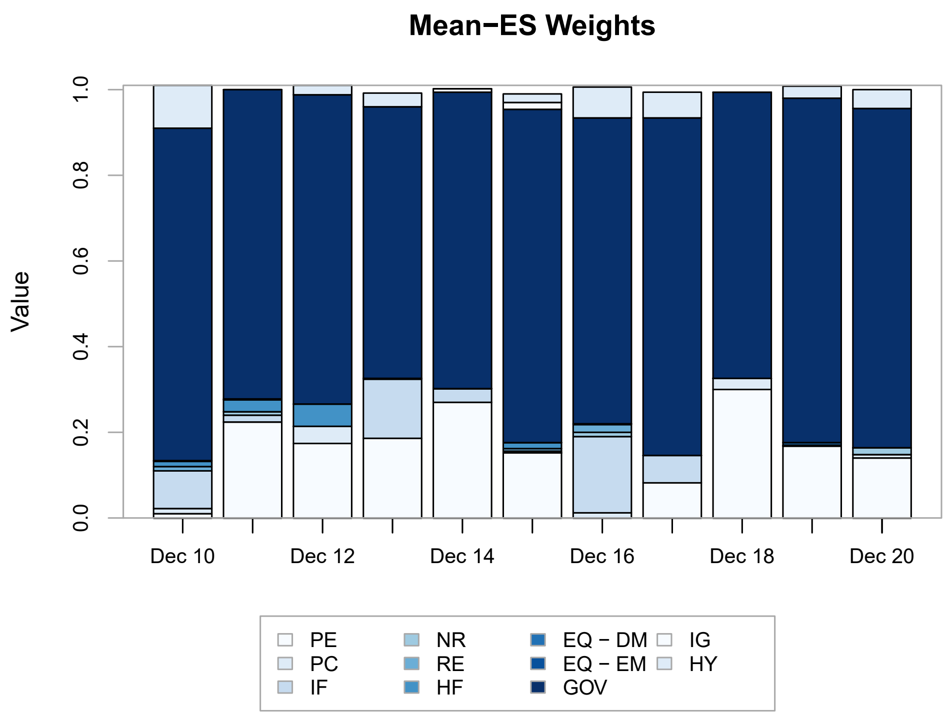

Figure 27: Mean-ES out-of-sample weights

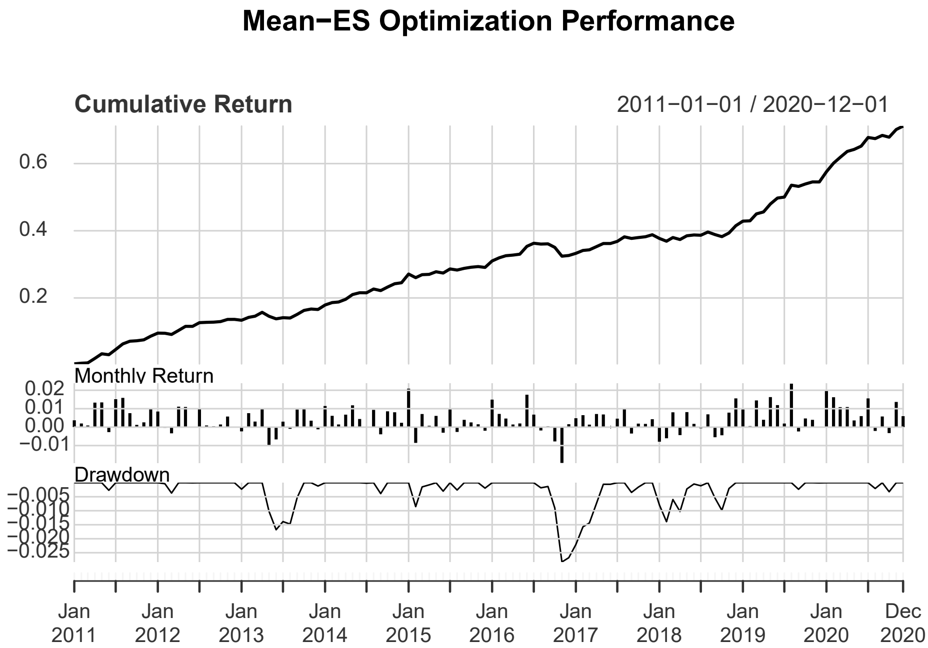

Figure 28: Mean-ES out-of-sample performance