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ILLIQUIDITY EFFECTS ON COMPANY VALUATION

Evidence from the Nordic stock markets

Master´s Thesis
in Accounting and Finance

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1 INTRODUCTION

1.1 Background and motivation for the study

This study focuses on illiquidity of securities and its effects on company valuation. The recent financial crisis and the recession in 1990 showed how important liquidity is and how quickly the markets can tank in the case of materialized liquidity risk (Longstaff 1995). In addition, many shareholders have suffered great losses when the markets are in downturn and at the same time there is liquidity restrictions which forbid selling shares (Kahl, Liu & Longstaff 2003).

A basic method in finance is to value investments by discounting future cash flows with a risk-adjusted discount factor and the resulting value should be the same regardless of the firm. This method is based on an assumption that there are unlimited investor clienteles. (Loderer & Roth 2005.) The price implied from that theory needs unrestrained trading between investors and the arbitrage guarantees that the price reflects the intrinsic value. (Loderer & Roth 2005; Shleifer & Vishny 1997.) However, limited liquidity exposes markets mispricing and due to this, investors may be reluctant to trade (Loderer & Roth 2005). Therefore, the pricing should be adjusted with some risk factor or a liquidity discount multiple.

The liquidity cost comes from the fact that less liquid financial assets have higher execution costs to investors and therefore investors are not willing to pay as much as they would from pure liquid assets which are easy to sell quickly and with a reasonable price level. This refers to limited liquidity. (Amihud & Mendelson 2012.) Higher transaction costs i.e. lower liquidity increases uncertainty of investors due to limited liquidity and therefore investors demand a higher discount for the lack of marketability (Chen, Dyl, Jiang & Juneja. 2015). The liquidity cost is based on two main components: direct trading costs and price-impact costs. First, direct trading costs include commissions for brokerages, exchange fees and taxes which materialized due to the transaction. Second, the price-impact cost is the price effect when selling or buying an asset. The price-impact cost is a premium when you are buying an asset and a discount when selling an illiquid asset. There is a great probability that the price-impact cost is much higher than the direct trading cost due to illiquidity of a security. (Amihud & Mendelson 2012.)

In general, investors are happy to pay for liquidity. While the illiquidity premium is widely approved, there is still strong debate on how to estimate limited liquidity and include it into value. (Damodaran, 2005.) A well-known way to estimate illiquidity is to measure the cost of immediate execution. When an investor wants to sell the asset immediately there is a risk that he/she might get a relatively low price and other investors can take advantage of this i.e. low market liquidity. This can be seen as a tradeoff between time and money. The bid price refers to the first case where the price has a discount due to the immediate selling and the quoted ask price refers to the second case which includes a premium compared to immediate buying. The bid price and the quoted ask price are reflected in the markets and the spread between the bid and ask price can be seen as a measure of illiquidity of a financial asset. (Amihud & Mendelson 1986, Loderer & Roth 2005.)

Damodaran (2005) suggests that there are three ways to incorporate the illiquidity effect on company valuation. First, calculate the value of a liquid asset and then use an illiquidity discount which is regressed from the actual market data (see also Loderer & Roth 2005). Second, calculate the cost of capital for the share and then take into account the asset specific illiquidity and sum the illiquidity premium to the calculated cost of capital. This method seems to be the most used in practice but there is a risk in use of this on valuation purposes. Third way to measure illiquidity is related to relative valuation. In this approach the illiquidity is tried to be estimated through transaction prices of similar illiquid assets, but if this is not possible, then illiquidity is measured by adjusting the discount factor as proposed in the second method. This study is interested in the research of an illiquidity discount method in order to estimate limited liquidity of shares.

Earlier studies have found that the equity value is negatively correlated with the relative bid-ask spread and positively correlated with liquidity characteristics like the number of shareholders, trading volume and the number of market makers trading the share (see e.g. Amihud & Mendelson 1986; Amihud 2002, Loderer & Roth 2005). Moreover, there are evidences that illiquidity can explain differences in expected returns across shares and it has been proposed that over time expected illiquidity has a positive effect on expected stock returns (Amihud, 2002).

The motivation for this study is multidimensional. In corporate finance, liquidity and especially lack of liquidity, has substantial consequences for decision-making and valuation conclusions. Consequences can be divided into two levels. First, liquidity of securities limits possibilities of a company to raise capital with these securities, e.g. stocks and bonds. Second, liquidity of company's assets may restrict decision-making. (Damodaran 2005.) Butler, Grullon and Weston (2002) found that issuance costs of seasoned equity offerings are much higher for firms with less liquid shares and therefore they suggested that firms can lower the cost of capital with better market liquidity of the shares.

One of the most challenging elements of privately-held company valuations is to effectively communicate the value of a non-marketable interest. Especially, the premium paid for marketability or a discount for lack of marketability should be taken into account in valuations and therefore it will be researched in this study. (Zanni, 2014.) Paglia and Harjoto (2010) investigated valuation adjustments for private companies. They tried to match private company transactions with publicly traded counterparts. Their study found that the discount for lack of marketability was averagely 65%-70% and exceeded 80% in some sectors of the economy. In general, limited liquidity can reduce a potential equity value more than 50%. Hence, limited liquidity can be seen as a critical element on corporate finance.

As it is noted earlier, investors pay lower prices or demand higher discounts on illiquid assets. Nowadays an increasing amount of wealth is allocated to asset classes which are typically priced at a discount, such as venture capital, private equity, initial public offerings, commercial real estates and hedge funds. This highlights the fact that it is highly important to understand the illiquidity discounts on these assets. (Chen et al. 2015.) Growing interest in liquidity in asset-pricing has been verified also in empirical literature. However, earlier literature has focused mostly on U.S. data and used trading-volume as a proxy for limited liquidity. (Chan & Faff 2005.) Since the market environment in the Nordic markets differs from the US, it is interesting to research liquidity in asset-pricing concerning the Nordic markets and compare these results to previous studies. The companies in the Nordic markets are typically smaller than in the US and moreover, marketability of shares are more limited, i.e. the stock markets are more efficient in the US than in the Nordics. However, the markets are more efficient globally nowadays than previous decades. Results show that generally there is a negative relation between liquidity and returns when using different proxies for liquidity (Loderer & Roth 2005). However, controversial results have also been found as Chen and Kan (1995, 12) did not find a reliable relation between returns and relative bid-ask spreads.

1.2 Objectives and research limitations of the study

The objective of this study is to investigate the pricing discount for limited liquidity by using directly the current share prices instead of the usually studied relation between historical returns and various liquidity proxies. It is highly important to notice that if the markets value liquidity, it should be reflected in share prices. Therefore, this method is applicable to research limited liquidity.

The research question in this study is following: how much an investor demands discount from perfectly liquid equity value in the case of limited liquidity? The research question is divided into sub-research questions. These sub-research questions define the theoretical framework for this study and highlight the importance of limited liquidity in valuation purposes. The sub-questions are following:

- How to define a fair market equity value and what characteristics should be taken into account in its valuation?
- How to measure limited liquidity in valuation?

By using the above-mentioned method, less data is required and it provides up-to-date information about an illiquidity discount. This kind of information is particularly valuable after the financial crisis and in a market environment like Finland, Sweden and Norway, as a major part of the companies listed on stock exchanges are considered as small cap companies. Moreover, it is possible to use this method in private company valuations with little modifications. There is great probability that the illiquidity discount changes over time and moreover, this approach makes it possible to take it into account, as this approach provides up-to-date market information. In addition, sovereign debt crisis in Europe increased volatility in stock markets like Finland, which had little to do with the actual crisis. These changes affect stock's liquidity and can have permanent effects in stock markets for a long time.

Theoretically, it is highly interesting to investigate limited liquidity with relative bid-ask spreads. Based on previous literature, it has been noted that relative bid-ask spread seems to work as a liquidity proxy due to its negative correlation coefficient between the market capitalizations i.e. the market value of equity and relative bid-ask spreads (see e.g. Loderer & Roth 2005). This implies that larger companies have smaller bid-ask spreads. With smaller bid-ask spread the liquidity is higher and the price-impact costs smaller. However, the bid-ask spread is not the only liquidity proxy and in this study, the average daily value of trading volume will be used as a liquidity proxy.

Since the purpose of this study is to investigate the effects of stock illiquidity on company valuation, the amount of discount for limited liquidity given by investors will be empirically tested. In other words, the purpose is to look at the pricing relevance of illiquidity directly at share prices mainly using methods used by Loderer and Roth (2005). The data will be analyzed from OMX Helsinki, OMX Stockholm and OMX Copenhagen stock exchanges between 1 January 2013 and 30 June 2016 and test whether it is possible to find a statistically and financially significant price-liquidity relation. In addition, the sample is divided into two separate samples to test whether the results are same as for the whole sample.

To perform a regression, there has to be compounded a model which includes regression arguments for each particular firm. These arguments are the expected rate of earnings growth, firm's payout ratio, risk of the share, market value of the firm's equity and share's liquidity. The regression model includes the firm size due to the evidences that risk-adjusted returns are affected by the size of the firm (Loderer & Roth 2005).

Loderer and Roth (2005) state that there is a great advance in the use of this method to research limited liquidity effects on company valuation. According to them, the biggest benefit is that the approach reveals relation between equity value and liquidity. This differs from previous studies which have studied the historical relationships with return-based methods. The main advance of this study, the relation between value and liquidity, is highly relevant since it fluctuates over time and with the approach used by Loderer and Roth, it will be able to take into account. Moreover, Loderer and Roth (2005) clarify that the approach used by them avoids the problem of inadequate time-series data of mean returns.

In addition, this research method is relevant since it has not been studied before in the Nordic markets and the results might be very useful in practice. Therefore, this study tries to fill the research gap in the Nordic markets but also the other research objective is to provide up-to-date information about limited liquidity discounts in the Nordic markets, especially in Finland and in Sweden.

This study has also its research limitations. The limitations are based on availability and reliability of data used in regression models. Because the regression models in this study need input information from the future expected growth rate for a company in question, the only possibility to estimate these growth rates is to use analyst estimates. One should always be aware of the risk of the use of analyst estimates because there is a potential conflict between academic studies and the purpose of analyst estimates. Usually, the analyst's task is to create demand for shares and in the academic research, the objective is to get neutral and unaffected data. However, as long as expected earnings growth rates influence to pricing of assets, these are highly important components in this study and should be taken into account.

The other research limitation concerns the data availability. As mentioned before, a major part of the companies listed on the Nordic exchanges are considered as small cap companies and this can mean that there are no analyst estimates or other needed data available in the databases. However, in this study the data is collected from three different stock exchanges and the total number of constituent firms is almost 600, and at the end of the day, all the needed data is available for 243 companies. At the moment, this kind of research is not possible to do only for the Finnish stock markets due to the lack of data.

1.3 Structure of the study

This section of the study provides background information, motivation, purposes and limitations of the study. In addition, there is a short background section to the test design and data. The next chapters in the study are organized as follows. The second chapter, cost of capital, defines the concept of rate of required return which is a highly important factor in valuation and in this study. The rate of required return can be calculated for assets and liabilities. Therefore, there are two kinds of main components in cost of capital calculations. After defining the cost of assets, the cost of equity and the cost of debt, the weighted average cost of capital (WACC) will be defined, which is used as a discount factor in many valuation approaches.

The third chapter, valuation methods, deals with different types of valuation approaches. The valuation approaches can be divided into two categories, to an income approach and a market approach which differs from each other significantly. This study focuses on the income approach which is relevant regarding the limited liquidity. It is also a more theoretical and technical method to estimate a fair market enterprise value or equity value of the company in question. There are numerous income approaches but in this study, the most relevant is a discounted cash flow (DCF) method. Moreover, the study gives a short insight to a residual earnings valuation model and its close relative, an abnormal earnings growth valuation model.

The fourth chapter, illiquidity and company valuation, first defines limited liquidity in a detail level and then the liquidity proxies are covered, especially the relative bid-ask spread as a proxy for liquidity but also average daily value of trading volume which is the other liquidity proxy in this study. At the end of the chapter, the relation between limited liquidity and company valuation will be studied.

The fifth chapter starts the empirical part of the study. The fifth chapter includes the research design and the empirical results are presented in the sixth chapter. To be precise, the fifth chapter deals with the data of the study and variables used in the regressions, hypothesis and economic concerns. The sixth chapter presents the empirical results and provides interpretation and discussions about the results. In addition, the results are compared to previous studies and questions arisen from this study are presented. The seventh chapter summarizes the literature and empirical part of thesis.

2 COST OF CAPITAL

2.1 General considerations

This chapter focuses on the cost of capital, its general considerations and the relation between cost of assets, cost of equity and cost of debt. In addition, the tax effects of interest payments are covered. The cost of capital is an important factor in valuation since the value of an asset is theoretically the present value of the net future cash flows of the asset to the owner. The present value is calculated by discounting the future cash flows with a risk-adjusted discount rate. The cost of capital can be defined also as an opportunity rate of return, i.e. the same risk characteristics for an alternative investment or an asset should assure equals required rate of returns for both, a given and an alternative investment.

To be precise, the firm value is a present value of future cash flows which are available for debt and equity holders. This cash flow pattern should be discounted at a cost of capital that reflects the cost of assets in overall, i.e. the risk of these firm’s assets. (Rosenbaum & Pearl 2009, 111.) Therefore, the intrinsic value of a firm reflects the value of assets in the balance sheet and the value of assets should be equal with the sum of debt and equity value, i.e. with the liabilities side of the balance sheet (Holthausen & Zmijewski 2012). The figure 1 illustrates that relation (Damodaran: Fair value accounting 2015).

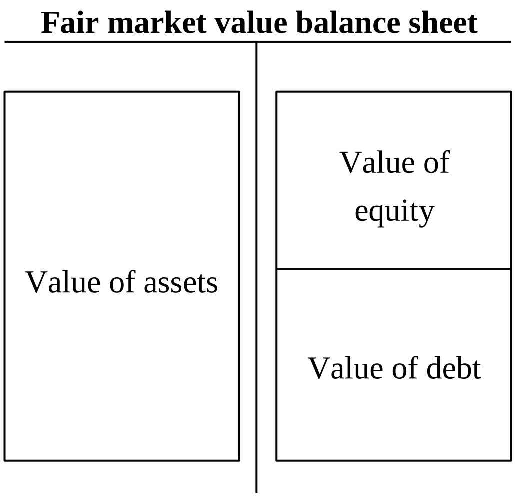

Figure 1 Fair market value balance sheet (Damodaran: Fair value accounting 2015)

Theoretically, the value of debt is the present value of future cash flows from debt financing that is discounted at the cost of debt. Main cash flows from debt financing are interest payments and repayments of face value of the debt. The other part of the liabilities side of the balance sheet is the equity. The equity value is the present value of future cash flows that are available to equity holders, i.e. shareholders. The equity cash flow is discounted at the cost of equity which reflects the risk of equity. The use of the cost of capital will be presented more technically in the valuation section.

However, the main approach in practice is to value the firm with the DCF-model using a free cash flow to firm approach. Therefore, the free cash flows will be discounted with the average after tax return, which the providers of capital require, i.e. with the weighted average cost of capital. (Holthausen & Zmijewski 2012.)

Before taking a closer look on components of the cost of capital, it is important to clarify some general considerations about the topic. First, it is highly needed to define a capital structure used in calculations. The capital structure can be divided into debt, mezzanine and equity financing. In general, the debt consists of all the interest bearing financial liabilities or claims, such as bonds and bank debt. Due to unequal position between debt holders and shareholders, there are different rights for both parties. Rights of debt holders are contractual, such as regular interest payments, repayment of nominal amount of face value of the debt, and if the debt is secured, then the debt holder has a right to get the underlying asset in the case of bankruptcy. Rights for shareholders are in the lower level in comparison with the rights for debt holders. Therefore, shareholders will only get residual cash flows and voting right at the company in question. The residual cash flow will be handled in the valuation section. (Damodaran: Capital structure: the choices and the trade off)

2.2 Cost of assets

At a terminology level, the cost of assets is required expected or average rate of return on assets. Without any outstanding debt, the cost of assets equals with the cost of equity. In a world without frictions, the cost of assets should reflect the risk of the firm's business model and its overall operating risk. This means that it does not matter how the product or service is financed, it will always be the same product or service which will be sold and it generates cash flows to security holders. (Holthausen & Zmijewski 2012.)

Since the balance sheet's both sides are equals, the opportunity cost of the assets is the value of assets times the cost of assets. Thus, the opportunity cost of the debt and equity equals with the opportunity cost of the assets. This can be written as follows

$$k_A = k_D \times \frac{D}{D+E} + k_E \times \frac{E}{D+E}, \tag{1}$$

where A is the value of assets, k_A is the cost of assets, D is the amount of debt, k_D is the cost of debt, E is the amount of equity and k_E is the cost of equity. (Holthausen & Zmijewski 2012; Rosenbaum & Pearl 2009, 159.) Together the amount of debt and the amount of equity makes up the value of liabilities and it is equal to the value of assets.

As proposed above, the cost of assets is only dependent on the risk of the firm's operating activities in the world without frictions. Therefore, one can say that the cost of assets is given for the firm in question and the total value of the company should not depend on the capital structure in the perfect world (Modigliani & Miller 1958; Berk & DeMarzo 2014, 480). However, it can be shown that the leverage increases the firm's cost of equity as the equation for the cost of equity can be written as follows by modifying the equation 1

$$k_E = k_A + (k_A - k_D) \times \frac{D}{E}, \tag{2}$$

where the term D/E is a leverage component. (Modigliani & Miller 1958; Berk & DeMarzo 2014, 489; Holthausen & Zmijewski 2012.) This interprets that the higher leverage increases the cost of equity due to a higher risk. Consequently, this shows that the debt financing is not a cheaper source of financing than equity. (Berk & DeMarzo 2014, 489.)

2.3 Tax effect of interest payments

Since interest payments can be deducted from the taxable income, these tax savings add value of the asset or the investment project. Every interest euro paid saves the amount of the firm's tax rate in tax payments. (Modigliani & Miller 1958.) The value adding tax savings should be taken into account in a valuation process (Berk & DeMarzo 2014, 572). It can happen through a discount rate or by adding a so-called debt tax shield (DTS) to the company value. The DTS is defined as the present value of the future tax savings due to the interest payments. (Inselbag & Kaufold, 1997.) The value of the DTS can be calculated as follows

$$\begin{aligned} PV(DTS) &= PV(\tau_c \times Future\ interest\ payments) \\ &= \tau_c \times PV(Future\ interest\ payments) \\ &= \tau_c \times D, \end{aligned} \tag{3}$$

where τ_c is a corporate tax rate and D is an amount of the firm's permanent debt (Berk & DeMarzo 2014, 513).

In some cases, it is recommended to use an adjusted cost of capital to take into account DTS and in other cases it is more preferrable to calculate the firm value as in the perfect world and add the incremental worth due to tax savings. It depends on the company's financing policy, if the debt target is an absolute dollar or euro etc. amount, the adjusted present value (APV) method is recommended. If the debt target is a specific debt ratio from the fair market value of the company, the adjusted cost of capital method is recommended, i.e. the weighted average cost of capital approach. This is based on the fact that the DTS, therefore, has the same risk as the company's assets. In this case the future tax savings are discounted with the cost of assets. However, in the first case where the amount of debt is an absolute euro amount, it is assumed that the DTS reflects the same risk as the company's debt. In this case the future tax savings are discounted by using the cost of debt. (Inselbag & Kaufold, 1997; Holthausen & Zmijewski 2012.)

The overall cost of capital of the company is needed in the valuation. Since the company's assets are not traded in the markets, the cost of assets of the company is not directly observed even in the markets. Therefore, it is needed to estimate the firm's cost of equity and the cost of debt. After that, by using these costs, one should compute the cost of assets and the weighted average cost of capital which is the cost of assets after taxes and it takes into account the tax effect of interest payments presented above (Berk & DeMarzo 2014, 514).

2.4 Cost of equity

The cost of equity is a required expected or average rate of return on equity. The previous literature presents various methods to estimate the cost of equity. The most popular and well-known method is the capital asset pricing model (CAPM). Moreover, in this study the dividend discount model is shown as a method to estimate the cost of equity. According to Graham and Harvey (2001) the CAPM is most used in practice (73.5%) by CFOs. The dividend discount model is also well-known and it is used by 16% of interviewed CFOs. Other methods in the cost of equity estimations include multifactor models. The arbitrage pricing theory by Ross (1976) is a well-known multifactor model. Multifactor models state that by using multiple portfolios as risk factors, all systematic risk is taken into account. However, different risk factors take into account different components of the systematic risk. (Berk & DeMarzo 2014, 461-462; see also Ross 1976 and Merton 1973.) One-third of CFOs in Graham and Harvey's survey (2001) answered to use multifactor models in the cost of equity calculations. Moreover, 40% of CFOs answered to use a historical average return as a proxy for the cost of equity.

The equation 4 presents the Gordon growth model as follows (Gordon 1962)

$$P_0 = \frac{DIV_0 \times (1+g)}{k_E - g}. \quad (4)$$

All the parameters are known except the required rate of return on equity varies. However, the current share price implicitly defines the cost of equity and the equation can be rearranged as follows to solve the cost of equity

$$k_E = \frac{DIV_0 \times (1+g)}{P_0} + g. \tag{5}$$

Moreover, the cost of equity can be solved from this expression

$$k_E = \frac{DIV_1}{P_0} + g, \tag{6}$$

where DIV_1 is the next year's expected dividend. The expected growth rate of future dividends can be based on historical average payout ratio or analyst estimations.

Sharpe (1964), Lintner (1965) and Mossin (1966) developed capital asset pricing model (CAPM) to depict the reliance between a risk of the share and a required rate of return of the share. (Dimson & Mussavian 1999). CAPM is based on Markowitz's portfolio theory and there are strong assumptions behind the model. These assumptions are also criticized by researchers. (Fama & French 2004.) The assumptions are following: all the investors act as Markowitz's portfolio theory expects and therefore, optimize their portfolios in the sense of the optimal mean-variance relation, also investors have the same subjective or objective probability distributions of assets' returns and relations between them, i.e. investors have a united opinion about the mean, variance and covariance of the return of the asset in question (this implies that all the investors have an equal amount of information in use), there is a same risk-free rate for lending and borrowing, and moreover, there are no transaction costs in the markets (Pilbeam 2010, 182–183).

Sharpe (1964) states that it is possible to avoid a part of the risk of the share by diversifying the whole portfolio broadly and therefore, the total risk of the share is not included into the share value. In the CAPM, the risk of the share is divided into a systematic and an unsystematic risk. It is not possible to avoid the systematic risk by diversifying the portfolio because this part of the risk is common and equal for every market participant. The systematic risk includes general economical phenomena such as trade cycles and political decisions. The unsystematic risk can be avoided by diversifying. The unsystematic risk is the part of the risk which cannot be avoided by diversifying and it is caused by something else than factors that impact the return of the market portfolio, i.e. company specific factors. According to the CAPM, the unsystematic part of the risk does not matter in the defining of the required rate of return since the unsystematic risk is possible to diversify away. The systematic risk is the only source which impacts on the required expected rate of return of the share. An investor will not get any higher expected return for the share by carrying an unsystematic risk. (Niskanen & Niskanen 2007, 186, 189.)

The CAPM can be written as follows

$$E(R_i) = R_f + \beta_i[E(R_M - R_f)], \tag{7}$$

where R_f is the risk-free rate of return, $E(R_M\text{-}R_f)$ is the estimated market risk premium, where $E(R_M)$ is the expected return for the market portfolio and β_i is the beta of the share in question. The beta is calculated by following: $\beta_i = \sigma_i \rho_{im} / \sigma_m$ (Fama & French 2004; Pilbeam 2010, 187). The beta is a function which expresses the variance of the share in question compared to the markets, but also with its covariance with the markets. If the share correlates perfectly with the market portfolio, i.e. $\rho_{im}=1$ and if the standard deviation of the share is the same as the market portfolio's, i.e. $\sigma_i=\sigma_m$, the beta of the share is one and its expected returns is always in line with the return of the market portfolio. However, a share with beta lower than one, helps an investor to stabilize the portfolio. These kinds of shares are called defensive shares. Whereas, if the beta is more than one, a share is called an aggressive share. These kinds of shares increase the volatility of the portfolio and therefore, the expected return is higher but also the risk is larger. (Pilbeam 2010, 187.)

In the estimation of the beta by using a regression model, the expression is written as follows

$$r_{i,t} = \propto_i + \beta_i \times r_{M,t} + \varepsilon_{i,t}, \tag{8}$$

where $r_{i,t}$ is a continuously compounded stock return of a firm i measured from time (t-1) to t, $r_{M,t}$ is a continuously compounded market return measured from time (t-1) to t, α_i and β_i are regression coefficients (β_i is the beta) and $\varepsilon_{i,t}$ is the error term (Fama & French 2004).

The required rate of return can be divided into three components: the risk-free rate which measures the time value of money (i.e. the provision to investors that postpone consumption), the beta of the share measures the amount of the risk and the risk premium measures the market price of the risk (Pilbeam 2010, 188).

The CAPM results a discount rate which takes into account the systematic risk. This discount rate is the required expected rate of return on equity, i.e. the cost of equity and it is highly used in many valuation models. (Niskanen & Niskanen 2007, 189.)

In practice, there are a few important practical remarks in the beta estimation and in the CAPM estimation in general. First, there is no correct risk-free rate and it always depends on the term structure of the cash flows. However, many practitioners do not take this into account and use risk-free rates such as 10-year or 30-year government bond yields. Nowadays 30 years is more used due to very low interest rates globally. Moreover, a historical market risk premium is a backward looking measure and in valuations, the interest is in the expected annualized market risk premium, not in its historical value.

According to Dimson, Marsh and Staunton (2003), the arithmetic mean of historical equity risk premiums may exceed the geometric mean equity risk premium by several percentages due to market volatility. They suggest that investors and managers should adjust historical equity risk premiums downward due to a general decline in the risk and more diversified investment opportunities.

In addition to the above-mentioned, the estimation period affects on the beta. With a longer estimation period, the statistical precision is higher but the question is, what does historical data tell about the future? In practice, a much used estimation period is from two to five years.

Data frequency is a relevant question in the estimation of beta. Should one use daily, weekly or monthly returns? As above-mentioned, the statistical precision increases when the data points increase but what if the share is not traded much, i.e. the share is illiquid? Therefore, the daily data is not relevant for most of the cases, and weekly data is also doubtful in many cases except the most liquid shares. However, there is no correct estimation period or data frequency.

Moreover, it is important to notice that the beta estimations above are performed to traded assets. In the cases where assets or projects are not traded, the beta estimation should be done by finding a company with similar risk characteristics from the capital markets, i.e. an industry peer group company. Due to different capital structures, first it would be better to calculate unlevered beta and then relevering the beta with the capital structure of the company in question. The equations (9) and (10) show unlevering and relevering procedures.

$$\beta_U = \frac{\beta_L}{\left(1 + \frac{D}{E} \times (1 - \tau)\right)} \tag{9}$$

$$\beta_L = \beta_U \times \left(1 + \frac{D}{E} \times (1 - \tau)\right), \tag{10}$$

where β_U is an unlevered beta, β_L is a levered beta from the peer group companies and D/E is the debt-to-equity ratio. (Rosenbaum & Pearl 2009, 130.)

As mentioned before, the CAPM has been heavily criticized in previous literature over the last decades. For example, the CAPM is a single-period equilibrium model and it assumes that asset returns are linearly related to the beta, its only risk factor. Thus, the market beta should cover all the possible risk sources. Moreover, the portfolio of assets should be mean-variance efficient. If this assumption does not hold, the model is not in equilibrium. The CAPM also expects that asset returns have a joint multivariate Gaussian distribution or the utility function of investors is quadratic. (Copeland & Weston 1988, 194-195.)

Due to shortcomings of the CAPM, various alternatives have been proposed. Ross (1976) presented the arbitrage pricing theory (APT) which says the expected return of an asset is a linear function of different macro-economic factors. Moreover, Fama and French (1993) published the three factor model. The three factor model takes into account the size factor, the market capitalization and the book-to-market ratio that are additional determinants of stock returns. Fama and French (1993) presented also a five factor model which identifies five common risk factors in the returns on stocks and bonds. The five factor model has statistically significant explanatory power for average returns on stocks and bonds (Fama & French 1993).

Even if the CAPM has its drawbacks, it is highly used in the markets. Kothari, Shanken and Sloan (1995) showed that the factor models could suffer from hindsight bias. It is also possible that the factors include characteristics that change over time. In addition, it is hard to test the CAPM because it only observes the realized returns, not expected returns.

Jagannathan and Wang (1996) presented the conditional CAPM where the beta and the market risk premium are not constant over time. In other words, in the conditional CAPM, betas and market risk premiums are time-varying. Bali and Engle's (2012) presented a model which produces betas that fluctuate over time, so-called dynamic conditional correlation beta model. This model gives more weight on recent observations whereas an unconditional beta puts equal weight on all observations.

However, Jagannathan and Wang (1996) gave three reasons why the CAPM is still so popular in spite of the lack of empirical support. They proposed that

> *"In a way it [CAPM] reminds us of cartoon characters like Wile E. Coyote who have the ability to come back to original shape after being blown to pieces or hammered out of shape. Maybe the CAPM survives because a) the empirical support for other asset-pricing models is no better, b) the theory behind the CAPM has an intuitive appeal that other models lack, and c) the economic importance of the empirical evidence against the CAPM reported in empirical studies is ambiguous."*

As argued above, the CAPM is an intuitive model to use in practice even though it might have some drawbacks.

2.5 Cost of debt

The cost of debt is a required expected or average rate of return on debt. If the company has different types of debt outstanding, the cost of debt is calculated by taking a weighted average required rate of return on the debt portfolio. The fundamental question of the cost of debt is why the cost of the debt is eventually higher than the risk-free rate, i.e. why it is not as the CAPM proposes that there is one borrowing rate, the risk-free rate? The cost of the debt is a risk-free rate plus a risk premium. The risk premium of a debt based on a credit or a default risk of the borrower, i.e. the borrower cannot make the promised payments on time and/ or in full to a lender. The risk premium is also called as a credit spread. (Berk & DeMarzo 2014, 411-412.) In the following paragraphs, the cost of the debt is defined by using market prices and credit ratings.

The cost of debt can be estimated from market prices, or to be more precise, from bond prices by calculating the yield-to-maturity (YTM) of a bond. The YTM is the promised return of the bond in question during one period (Berk & DeMarzo 2014, 171). The equation can be written as the equation of internal rate of return (IRR) as follows

$$PV = \sum_{t}^{T} \frac{Coupon_t}{(1 + YTM)^t} + \frac{NV_T}{(1 + YTM)^T}, \tag{11}$$

where the period goes from time t to time T, the coupon is the coupon payment in time t and the NV is the nominal value of the bond. (Caks 1977; Berk & DeMarzo 2014, 173-174.) The YTM includes two components, the risk-free rate of return and the promised credit spread. Therefore, the equation 11 can be written as follows

$$PV = \sum_{t}^{T} \frac{Coupon_t}{(1 + R_{F,t} + CS)^t} + \frac{NV_T}{(1 + R_{F,t} + CS)^T}. \tag{12}$$

Since the YTM is based on the present value of the bond, i.e. the market value of the bond, it is needed to adjust the observed price which typically do not include accrued interest. This observed price is called as a quoted price or a clean price. The present value is called as a dirty price of the bond. Therefore, the present value of the bond is the quoted price plus the accrued interest, where the accrued interest is calculated with the equation 13

$$Accrued\ interest = \left(1 - \frac{t}{360}\right) \times coupon, \tag{13}$$

where t is the number of days to the next coupon payment. (Berk & DeMarzo 2014, 179.) Even if the YTM is a highly used proxy for the cost of debt, it still has its drawbacks. The YTM is a good estimate for the cost of debt only in the case where the firm has a low default probability or an outstanding high recovery rate. Moreover, it is good to notice that the YTM is just calculated for one bond of the firm in question and does not represent all the outstanding debt. However, one should always remember that the coupon rate is not a good estimate for the cost of debt. (Berk & DeMarzo 2014, 412.)

In addition to the market price method, the cost of debt can be estimated by using credit ratings from comparable companies or by generating a synthetic credit rating. The main idea in both of these credit rating methods is to estimate the creditworthiness of the firm in question, i.e. the firm's financial risk. Firms with similar financial risk characteristics can be seen as comparable companies in the estimation of the credit spread and the cost of debt for the target firm. To do this, the target company has to have a credit rating or otherwise it is needed to compound a synthetic rating from the comparable companies. In the case of available credit rating for the target company, one should use that credit rating and implement it to an associated credit spread, e.g. provided by Damodaran. (Damodaran 2002, 285-287.) For example, table 1 presents different rating categories and their interpretations.

	S&P	**Moody's**	**Definition**
Investment grade	AAA	Aaa	Prime
	AA	Aa	High grade
	A	A	Upper medium grade
	BBB	Baa	Medium grade
Speculative grade	BB	Ba	Lower medium grade
	B	B	Speculative
	CCC	Caa	Poor standing
	CC	Ca	Highly speculative
	C	C	Lowest quality
	D		In default

Table 1 Credit rating definitions by S&P and Moody's (S&P Global; Moody's)

The above table 1 presents credit rating categories provided by the two most preferred rating agencies in the world, S&P and Moody's. (S&P Global; Moody's.) Based on Damodaran's database, the credit spreads for associated ratings are following

If an interest coverage ratio is			
>	**≤ to**	**Rating is**	**Spread is**
8.50	100,000	AAA/Aaa	0.75%
6.50	8.499	AA/Aa2	1.00%
5.50	6.499	A+/A1	1.10%
4.25	5.499	A/A2	1.25%
3.00	4.2499	A-/A3	1.75%
2.50	2.999	BBB/Baa2	2.25%
2.25	2.499	BB+/Ba1	3.25%
2.00	2.2499	BB/Ba2	4.25%
1.75	1.999	B+/B1	5.50%
1.50	1.7499	B/B2	6.50%
1.25	1.499	B-/B3	7.50%
0.80	1.2499	CCC/Caa	9.00%
0.65	0.799	CC/Ca2	12.00%
0.20	0.6499	C/C2	16.00%
-100,000	0.199	D/D2	20.00%

Table 2 Interest coverage ratios, credit ratings and credit spreads for large, non-financial service companies (Damodaran's database)

Table 2 is based on an interest coverage ratio and is applicable for large, non-financial service companies with the market cap more than five billion dollar. The credit spread range is from 0.75% to 20.00%.

However, the credit rating is not always available for the company in question. Therefore, one should compute a rating, a so-called synthetic rating. First, it is needed to analyze the factors that have impacts on the company's creditworthiness. Based on Berk & DeMarzo (2014, 35-40), the factors that have an effect on the risk profile of the company are e.g. a size of the company, an interest coverage ratio and a leverage or a debt ratio. The interest coverage ratio is a highly used one and it is defined as the earnings before interest and taxes (i.e. EBIT) relative to interest payments (Graham & Harvey, 2001).
Table 3 presents interest coverage ratios and credit ratings for smaller companies based on Damodaran's database.

If an interest coverage ratio is >	≤ **to**	**Rating is**	**Spread is**
12.50	100,000	AAA/Aaa	0.75%
9.50	12.499	AA/Aa2	1.00%
7.50	9.499	A+/A1	1.10%
6.00	7.499	A/A2	1.25%
4.50	5.999	A-/A3	1.75%
4.00	4.499	BBB/Baa2	2.25%
3.50	3.999	BB+/Ba1	3.25%
3.00	3.499	BB/Ba2	4.25%
2.50	2.999	B+/B1	5.50%
2.00	2.499	B/B2	6.50%
1.50	1.999	B-/B3	7.50%
1.25	1.499	CCC/Caa	9.00%
0.80	1.2499	CC/Ca2	12.00%
0.50	0.799	C/C2	16.00%
-100,000	0.499	D/D2	20.00%

Table 3 Interest coverage ratios, credit ratings and credit spreads for small, non-financial service companies (Damodaran's database)

Table 3 is applicable for non-financial service companies with the market cap less than five billion dollars. The credit spread range is from 0.75% to 20.00%.

The above-mentioned methods are focused on promised returns even if investors are usually interested in expected returns. It is important to remember that promised returns will be paid only if the bond does not default until maturity. However, for a company with a low default probability, it is a reasonable assumption to use the promised return as a proxy for the expected return. If the company in question has a high probability of default, it is needed to incorporate the default risk and the recovery rate to the estimation of the cost of debt. It can be calculated with the following equation

$$k_D = (1-p)YTM + p(YTM - L) = YTM - pL, \quad (14)$$

where p is the probability of default and L is an expected loss rate of the bond in the event of default. (Berk & DeMarzo 2014, 412.)

Generally speaking, changes in current bond prices should reflect better the credit risk of the company in question compared with credit ratings. This is based on market's assessment: the bond prices already include credit ratings and other public information and moreover, the bond prices are continuously investigated by the market participants.

2.6 Weighted average cost of capital

As mentioned before, the free cash flows will be discounted with the average after tax return, which the providers of capital require, i.e. with the weighted average cost of capital (Damodaran 2002, 19). As also said, the cost of assets is not observed directly from the markets and therefore, it is needed to estimate the cost of equity and the cost of debt separately. By using these estimated required rate of returns in the cost of capital, and taking into account the capital structure and the fact that interest expenses are tax deductible, it is possible to compound the weighted average cost of capital (WACC). The WACC includes tax savings by measuring the cost of debt after taxes as follows

$$WACC = k_D \times (1 - \tau) \times \frac{D}{D + E} + k_E \times \frac{E}{D + E}, \quad (15)$$

where kD*(1-τ) is the cost of debt after taxes. The WACC can be written as follows also (Holthausen & Zmijewski 2012; Inselbag & Kaufold 1997.)

$$WACC = k_A - k_D \times \tau \times \frac{D}{D + E}. \quad (16)$$

The equation 16 can be applied if the cost of assets is known and the cost of equity is unknown.

3 VALUATION METHODS

3.1 General considerations

The valuation methods can be divided into two different approaches, to an income approach and a market approach. Theoretically, the income approach takes into account all the company specific information in a cash flow projection for an explicit forecasting period. This method is more technical and tries to capture all details of the company in question. (Damodaran 2002, 16-34.) In this study, the income approach covers a discounted cash flow model (DCF), a residual earnings valuation model (RE) and an abnormal earnings growth model (AEG). The first is the preferred model by practice and two other models are accounting-based valuation models.

In addition to the income approach, the market approach is a highly used valuation method in practice also. The market approach is a relative valuation method where a fair value of an asset based on realized prices of assets with similar characteristics. Generally, the market approach covers a trading multiple analysis and a precedent transaction multiple analysis. The idea of the trading multiple analysis is to value a company by using multiples at which publicly listed comparable companies trade in the markets. The idea of the transaction multiples analysis is to value a company by using actual historical multiples paid in mergers and acquisitions (M&A) transactions involving comparable companies. (Damodaran 2002, 16-34; Rosenbaum & Pearl 2009, 11-13, 71-106.) However, this study focuses only on the income approach which is a relevant approach regarding to the limited liquidity.

3.2 Discounted cash flow model

In the DCF-model, the focus is on the firm’s future ability to generate cash which ultimately belongs to the providers of capital. However, cash is needed for pay e.g. salaries, taxes and bills. It has been seen that one of the most common reason for a bankruptcy is a lack of cash flow of the business (Uhrig 2005).

The firm’s activities can be divided into three categories: investments, financing and operations. Investments are assets owned by the company, financing activities of the company define the ways how the assets are financed and operation activities is the uses of firm’s assets. (Nguyen, Cai & McColgan 2016.) Firm activities can be defined also in the terms of financial statements. Investments are presented in the left-hand side of the balance sheet in long-term assets, financing is presented in the right-hand side of the balance sheet in liabilities and equity and operating activities are presented in the income statement. Moreover, the cash flows can be defined by using these three activities, i.e. operating cash flow related to a production and sales, cash flow from investment and cash flow from financing. (Penman 2010, 32-39.)

It is highly important to understand the mechanics of the cash flow statement used in the DCF-model. First, the accounting framework in the cash flow analysis is vital. There are three kinds of firm's activities that lead to the net income in the income statement. Basically, every item above of earnings before interest and taxes (EBIT) is related to the operating activities of the company. However, interest expenses are financing activities and therefore, income taxes and the net income is a mixture of operating and financing activities of the company. (Penman 2010, 36.)

The cash flow analysis starts from an adjustment of EBIT. The EBIT should be adjusted with tax expenses, i.e. with the tax amount without debt financing. This leads to a net operating profit less adjusted taxes (NOPLAT). The NOPLAT is the net income plus after-tax interest expenses. However, the NOPLAT is not a cash flow. To derive a free cash flow to the firm, the NOPLAT needs generally three adjustments: it is needed to add back depreciation and amortization, deduct capital expenditures and change in net working capital. (Damodaran 2002, 533-535.) The net working capital is defined as follows (Berk & DeMarzo 2014, 242)

$$\begin{aligned} Net\ working\ capital &= current\ assets - current\ liabilities \\ &= cash + inventory + receivables - payables. \end{aligned} \tag{17}$$

The free cash flow (FCF) is an important and well-known cash flow figure in the DCF-model valuation. The FCF presents the amount of money that is not tied up in the firm's operating or investment activities and therefore, it is discounted with the WACC. Theoretically, the FCF can be divided to the providers of capital, i.e. security holders. (Berk & DeMarzo 2014, 284.) In conclusion, the equation 18 presents the FCF calculation

$$\begin{aligned} &Free\ cash\ flow \\ &\quad = EBIT \times (1-\tau) + depreciation\ and\ armortization \\ &\quad - capital\ expenditures - change\ in\ net\ working\ capital \end{aligned} \tag{18}$$

Even if the FCF is the most important cash flow figure, others are also used. The residual cash flow (RCF) is derived from FCF by deducting after-tax interest expenses and repayment of debt, i.e. by deducting cash flow outflows to debt holders and adding cash inflows to debt holders. Therefore, the RCF is theoretically available to the shareholders and it can be used in the equity valuation. The RCF has to be discounted with the cost of equity. (Damodaran 2002, 459.)

To derive normalized cash flows reflecting the firm actual businesses, the cash flow analyses need adjustments in some cases. The operating leases can greatly affect to a value conclusion. The operating leases are recorded as operating expenses even if those should be presented as debt financed asset investments in valuation purposes. To adjust the statements, it is needed to calculate the present value of future leasing expenses, activate this amount to the balance sheet and add a corresponding debt amount, depreciate the activated item in the balance sheet according to a relevant depreciation plan and decrease the debt at the same time with the same amount. In addition to that, operation leases should be added back to the EBIT. (Damodaran 1999.)

After all of the above-mentioned steps and other needed cash flow adjustments, the cash flows are ready to valuation purposes. The next step in the DCF-model is to compute forecasts over an explicit forecasting period. In theory, there are two ways to project items in the income statement and in the balance sheet and therefore in the cash flow statement. It can be done by using a T-account method or a percentage-of-sales method. The first means that individual positions are estimated explicitly and the second means that the sales are forecasted first and all the other items are based on the forecasted sales as a percentage of sales, where the percentage ratio can be estimated by using historical average.

As mentioned in the chapter 2.3, the DTS can be taken into account two different ways, by using an adjusted cost of capital (i.e. WACC approach) or by discounting cash flows with the cost of assets and adding the incremental worth created by leverage (i.e. APV approach). The decision depends on the financing policy of the company. If the debt target is in nominal currency amount, the APV approach is recommended. If the debt target is a fixed debt ratio, the WACC approach is recommended. This is based on the fact that the DTS has the same risk as the company's assets since the debt amount fluctuates with the firm value. Thus, the future tax savings are discounted with the cost of assets. However, if the debt amount is a nominal amount, it is assumed that the DTS reflects the same risk as the company's debt since the DTS fluctuates with the amount of debt. In this case the DTS is discounted by using the cost of debt. (Inselbag & Kaufold, 1997; Holthausen & Zmijewski 2012.) The present value of the DTS can be calculated as follows in the APV approach

$$PV(DTS) = \sum_{t=1}^{T} \frac{\tau \times interest\ payments_t}{(1 + k_D)^t}. \tag{19}$$

In the DCF valuation the selection between APV and WACC approaches is crucial. The WACC approach values the company as a levered corporation and the APV approach values the firm as an equity financed corporation and adds all the incremental worth created by leverage. The equation 20 presents the DCF-WACC approach

$$Firm\ value = \sum_{t=1}^{T} \frac{FCF_t}{(1 + WACC)^t}. \tag{20}$$

Thus, the firm value is capitalized future free cash flows where the discount rate is the WACC. (Inselbag & Kaufold, 1997; Damodaran 2002, 537.)

If the future tax savings can be forecasted explicitly, then the APV method can be used. However, it means that the future amounts of debt outstanding should be known. As mentioned, then the future tax savings have the same risk as the debt outstanding and therefore it can be discounted with the cost of debt. Then the firm value can be calculated with the equation 21

$$Firm\ value = \sum_{t=1}^{T} \frac{FCF_t}{(1 + k_A)^t} + PV(DTS), \tag{21}$$

where PV(DTS) is the present value of the debt tax shield as presented in the equation 19 (Inselbag & Kaufold, 1997).

In conclusion, the DCF-model tries to capture all the relevant information of the company in question. The fair market value of the firm (or the enterprise value (EV)) is the sum of each year's discounted free cash flow to the firm and a continuing value which can be defined by using previous year's figures and the Gordon growth model as presented in the equation 1 where the DIV_0 is replaced with the previous year's FCF, k is the cost of capital and g is an assumed growth rate. (Berk & DeMarzo 2014, 285; Penman 2010, 117.) The fair market value of equity is calculated from the firm value by deducting debt and adding cash and cash equivalents (Penman 2010, 123). Moreover, the equity value can be calculated by capitalizing residual cash flows.

3.3 Residual earnings model

The main concept in the residual earnings valuation is to anchor the value on book value of an asset and future forecasted residual earnings. Hence, the anchoring principle is based on that the asset which earns a return equal to its required return on its book value, the asset is worth of its book value. If the forecasted return on book value is higher than the required rate of return, in absolute terms, the asset generates positive residual earnings and the asset value is higher than its book value, i.e. the price-to-book (P/B) ratio is more than 1.0. However, the situation is opposite if the return on book value is less than the required rate of return and then the P/B ratio is less than 1.0. The equity value can be calculated as follows

$$Value\ of\ common\ equity\ (V_0^E) = B_0 + \sum_{t=1}^{T} \frac{RE_t}{{\rho_E}^t}, \tag{22}$$

where B_0 is the current book value of equity on the balance sheet, ρ_E is $(1+k_E)$ and RE is residual earnings for equity. The RE is calculated as follows

$$RE_t = Earn_t - (\rho_E - 1)B_{t-1}, \tag{23}$$

where $Earn_t$ is comprehensive earnings and B_{t-1} is the beginning-of-period book value. (Penman 2010, 153; Easton, Taylor, Shroff & Sougiannis 2002.)

The intrinsic premium over book value of equity is value of common equity minus the current book value of equity and this is equal to the present value of residual earnings. If residual earnings are forecasted only for an explicit forecasting period, e.g. for five years, then it is needed to add the terminal value term to the equation 22. Hence, the equation 22 will be modified as follows

$$V_0^E = B_0 + \sum_{t=1}^{T} \frac{RE_t}{{\rho_E}^t} + \frac{V_T^E - B_T}{\rho_E^T}. \tag{24}$$

In conclusion, by using the residual earnings model in valuation, three things are needed: the current book value, expected residual earnings over an explicit forecasting period and a horizon premium. The horizon premium is the premium of a share's expected value over the book value in time T. (Penman 2010, 153-155.)

However, there is a problem in applying the equation 24 to practice since the present value of equity is a function of value of equity T periods from now. Therefore, the horizon premium in equation 24 is replaced with the Gordon growth model in order to be able to calculate the continuing value for the share in question. The equation 25 expresses this (Penman 2010, 163.)

$$V_0^E = B_0 + \sum_{t=1}^{T} \frac{RE_t}{{\rho_E}^t} + \left(\frac{RE_T \times (1+g)}{\rho_E - (1+g)}\right) / \rho_E^T. \tag{25}$$

The fair market equity value is the current book value of equity sum capitalized expected residual earnings over an explicit forecasting period and the discounted terminal value calculated with the Gordon growth model.

3.4 Abnormal earnings growth model

In the abnormal earnings growth valuation model, the main concept is to calculate capitalized forward earnings and add the extra value created by abnormal cumulative dividend (cum-dividend) earnings growth. Therefore, the principle is that the asset value should be equal to its capitalized earnings if the cum-dividend earnings will grow at a rate equal to the required rate of return. (Penman 2010, 199; Ohlson & Juettner-Nauroth 2005; Jorgensen, Lee & Yoo 2011.)

If the cum-dividend earnings growth rate is higher than the required rate of return, the asset value is higher than its capitalized earnings and vice versa. The cum-dividend earnings can be defined as follows

$$Cumulative\ dividend\ earnings_t = Earnings_t + (\rho - 1) \times dividend_{t-1}, \tag{26}$$

where the ρ is 1 plus the cost of capital. (Penman 2010, 196.)

The earnings that grow at a rate equal to the required rate of return are called normal earnings at time t, i.e.

$$Normal\ earnings_t = \rho Earnings_{t-1}, \tag{27}$$

However, one is not interested in paying for normal earnings. Investors are willing to pay for the cum-dividend earnings growth over these normal earnings, i.e. investors pay for the abnormal earnings growth. The abnormal earnings growth at time t can be calculated as follows (Penman 2010, 197.)

$$\begin{aligned} &Abnormal\ earnings\ growth_t \\ &\quad = Cumulative\ dividend\ earnings_t - Normal\ earnings_t \\ &\quad = [Earnings_t + (\rho - 1)dividend_{t-1}] - \rho Earnings_{t-1}. \end{aligned} \tag{28}$$

In conclusion, if the cum-dividend growth is equal to the required return, the asset value is equal to its capitalized earnings, therefore one is willing to pay only for value-adding growth. Moreover, the cum-dividend growth is a better measure than ex-dividend growth in the analyses of earnings growth since the first mentioned takes into account the reinvestment of dividends. (Penman 2010, 197.) As Loderer and Roth (2005) also found, the payout ratio is irrelevant to valuation purposes. Penman (2010, 197) also states that the payout ratio is irrelevant to valuation since the cum-dividend earnings growth is not depending on dividends.

The abnormal earnings growth (AEG) is used in valuation. The value of equity can be estimated by using following equation

$$V_0^E = \frac{Earn_t}{\rho_E - 1} + \frac{1}{\rho_E - 1}\left[\sum_{t=1}^{T} \frac{AEG_{t+1}}{\rho_E^t}\right], \tag{29}$$

where there are basically two different components, the capitalized forward earnings and extra value for abnormal cum-dividend earnings growth. The equation 29 can be written also as follows (Penman 2010, 199; Penman 2005.)

$$V_0^E = \frac{1}{\rho_E - 1}\left[Earn_t + \sum_{t=1}^{T} \frac{AEG_{t+1}}{\rho_E^t}\right]. \tag{30}$$

Moreover, the value of equity can be calculated by using current earnings instead of forwards earnings. Hence, the valuation equation is following

$$V_0^E + d_0 = \frac{\rho_E}{\rho_E - 1}\left[Earn_0 + \sum_{t=1}^{T} \frac{AEG_t}{\rho_E^t}\right], \tag{31}$$

where d_0 is the current's year dividend. (Penman 2010, 203; Ohlson 2005; Penman 2005.)

The AEG can also be written as a change in the residual earnings, i.e. AEG = ΔRE. The equations 32, 33 and 34 prove this (Penman 2010, 208; Penman 2005)

$$\begin{aligned} AEG_t &= [Earn_t + (\rho_E - 1)d_{t-1}] - \rho_E Earn_{t-1} \\ &= Earn_t - Earn_{t-1} - ((\rho_E - 1)(Earn_{t-1} - d_{t-1}). \end{aligned} \tag{32}$$

Due to the equation of stocks and flows, the book value of equity can be written as follows (Penman 2010, 208; Penman 2005)

$$\begin{aligned} B_{t-1} &= B_{t-2} + Earn_{t-1} - d_{t-1} \\ Earn_{t-1} - d_{t-1} &= B_{t-1} - B_{t-2}. \end{aligned} \tag{33}$$

Therefore, the AEG equation 32 can be written as follows again (Penman 2010, 208)

$$\begin{aligned} AEG_t &= Earn_t - Earn_{t-1} - (\rho_E - 1)(B_{t-1} - B_{t-2}) \\ &= [Earn_t - (\rho_E - 1)B_{t-1}] - [Earn_{t-1} - (\rho_E - 1)B_{t-2}] \\ &= RE_t - RE_{t-1}. \end{aligned} \tag{34}$$

In valuation purposes, the AEG model should be written by using also a continuing value term. Therefore, the model is written as follows (Penman 2010, 210)

$$V_0^E = \frac{1}{\rho_E - 1}\left[Earn_t + \sum_{t=1}^{T} \frac{AEG_{t+1}}{\rho_E^t} + \left(\frac{AEG_T \times (1+g)}{\rho_E - (1+g)}\right) / \rho_E^{T-1}\right], \tag{35}$$

where there are basically three different components, the capitalized forward earnings, extra value for abnormal cum-dividend earnings growth over an explicit forecasting period and the discounted terminal value calculated with the Gordon growth model.

4 ILLIQUIDITY AND COMPANY VALUATION

4.1 Limited liquidity

In the perfect world, the investments should have the same values for every company in the market. The proof of that is the arbitrage. An asset which is sold under the fair market value will be bought by an investor who can sell the asset afterwards with the fair market value and therefore earns risk-free return. However, the arbitrage is not possible if there are no other investors who want to buy the asset even if it would be sold at a fair market price. It means that there is limited tradability, i.e. limited liquidity or illiquidity in the markets. Assets with limited liquidity are trading at a discount in order to be interesting in the markets. Moreover, the transactions costs are related to limited liquidity. (Amihud & Mendelson 2012; see also Loderer & Roth 2005.) An illiquid company typically has a larger bid-ask spread and therefore the buyers and sellers are more difficult to find.

Liquidity has significant effects on valuation of corporate shares and bonds. The limited liquidity increases required returns and thus lowers prices. However, increasing liquidity of a firm’s shares and bonds can decrease the firm’s required rate of return and therefore increase the market value of the firm in question. (Amihud & Mendelson 2012.) Amihud and Mendelson (1986) research paper reveals that financial policy which supports liquidity of a share increases the share value by decreasing the opportunity cost of capital of the company. In general, publicly traded companies have better trading volume on their shares and the liquidity costs remain in a low level compared to private companies. If a company is small, trading is done on an over-the-counter basis and it is a private business, this implies that the possible investor clientele is restricted also. (Damodaran 2005.)

One well-known way to estimate illiquidity of an asset is to measure its immediate execution costs. In that case, an investor faces a tradeoff between time and money: the investor can sell the asset immediately at a lower price or wait to get a more favorable price from the asset. (Amihud & Mendelson 1986.)

Amihud and Mendelson (2012) state briefly the definition of liquidity costs. They argued that the “liquidity costs are the costs associated with executing a transaction in the capital markets.” Moreover, they proposed that these costs can be disentangled into two components: direct trading costs and price-impact costs. First includes brokerage commissions, exchange fees and taxes. Second is a pricing discount or a premium when buying and selling an asset, respectively. These price-impact costs can have a significant effect on company valuation and this price-impact will be researched in this study. (Amihud & Mendelson 2012.)

Limited liquidity has serious consequences on different levels in finance. Damodaran (2005) defined three aspects where illiquidity is a crucial element in finance: in the decision of going public or private, in portfolio management and in corporate finance. First, the decision between going public or staying private and vice versa reflects the tradeoff between control and liquidity of the company. If the company goes public, the opportunity cost of capital, i.e. required rate of return decreases probably and the value of the company increases since the liquidity would be higher. However, the control of a public company is widely divided and the management or the old owners cannot do decisions without other shareholders' approval. Of course, the decision of going private is also a tradeoff between the above-mentioned. There Damodaran still sees that the cost of illiquidity should be lower due to an assumption that the company will be listed in the near future again. (Damodaran 2005.)

Second, investors take into account illiquidity in their investment decisions, performance evaluation and risk management. Third, illiquidity affects also in corporate finance where measures should be adjusted to reflect illiquidity, e.g. the cost of capital needs an illiquidity premium or an illiquidity discount is used in valuation. Moreover, compensation, financing and dividend decisions of the company are considered by taking into account the limited liquidity of the company. Companies with liquid securities can pay more dividend and keep cash and cash equivalents at a minimum since they know that they can raise new capital with low transaction costs to fund shortfalls. (Damodaran 2005.)

4.2 Proxies for illiquidity

There are various illiquidity proxies used in the previous literature. Chen et al. (2015) measure illiquidity by using turnover, bid-ask spread, exchange listing and market capitalization variables. Loderer and Roth (2005) used also bid-ask spread as a main proxy for limited liquidity but they also used an average daily value of trading volume as a proxy for limited liquidity. Amihud and Mendelson (2002) used a ratio of a share's absolute daily return to its daily dollar volume as a proxy for illiquidity of a share. With this approach they try to capture the daily share price change relative to a dollar of trading volume.

Chalmers and Kadlec (1998) suggested that effective spread and amortized spread could work as proxies for illiquidity. Datar, Naik and Radcliffe (1998) proposed that a share turnover or a turnover rate would be a proxy for limited liquidity. Share turnover is defined as a number of shares traded divided by the number of shares outstanding. Brennan, Chordia and Subrahmanyam (1998) suggested trading volume for a liquidity proxy. Moreover, market depth, the number of shareholders, the number of market makers and the price impact of trading have been proposed in the previous literature (Loderer & Roth 2005).

In this study, the bid-ask spread is used as a proxy for limited liquidity due to supporting empirical evidence from previous literature (see e.g. Loderer & Roth 2005; Amihud & Mendelson 1986; Chen et al. 2015). However, it is admitted that there are discussions about the bid-ask spread in the terms of liquidity measure. The previous literature has questioned whether the bid-ask spread measures liquidity correctly or not. (Grossman & Miller 1988; Loderer & Roth 2005.) Moreover, trading volume will be also used as an additional measure of liquidity.

Roll (1984) studied the effective bid-ask spread with the first-order serial covariance of price changes. He found that the implicit trading costs measured by the bid-ask spread were strongly negatively related to firm size. Moreover, Roll (1984) found a sizeable difference of the effective bid-ask spread estimated from daily and weekly data. According to Roll (1984), it implies that the markets were informational inefficiency.

According to Glosten and Harris (1988), the bid-ask spread is a function of trade size. Furthermore, they discussed that the bid-ask spreads are driven by asymmetric information and inventory considerations. The results of their study cannot reject the hypothesis that the bid-ask spreads are due to asymmetric information. Moreover, Coller and Yohn (1997) examined management forecasts and information asymmetry by using the bid-ask spread as a measure of information asymmetry. Their results showed that bid-ask spreads increase the day of and the day after the management forecasts date. However, the results also showed that the bid-ask spread was significantly lower nine days after the announcement than it was prior to the forecast release. This implies that management forecasts reduce information asymmetry in the market and the bid-ask spread includes a component of asymmetric information. It can be also implied that lower information asymmetry increase liquidity of the share. However, in this study asymmetric information is not taken into account in the bid-ask spread since the spread is seen as a function of liquidity of the company.

4.3 Illiquidity in valuation

4.3.1 General considerations

As mentioned before, limited liquidity has direct and indirect costs. The direct costs in valuation will be taken into account in cash flow analyses since they are easy to observe. However, indirect price-impact costs are much harder to take into account in valuation. (Damodaran 2005.)

Damodaran suggested three ways to incorporate the illiquidity effect on company valuation. First, calculate the value of a liquid asset and then use an illiquidity discount which is regressed from the actual market data (see also Loderer & Roth 2005). Second, calculate the cost of capital for the share and then take into account the asset specific illiquidity and sum the illiquidity premium to the calculated cost of capital. This method seems to be the most used in practice but there is a risk in the use of this on valuation purposes. Third way to measure illiquidity is related to relative valuation. In this approach the illiquidity is being tried to estimate through transaction prices of similar illiquid assets, but if this is not possible, then illiquidity is measured by adjusting the discount factor as proposed in the second method. In this study, the illiquidity premium and the illiquidity discount methods are studied further. It is important to avoid to the use of both of these at the same time.

4.3.2 Illiquidity premium

In valuation, the illiquidity premium is used as an additional component in the WACC. It can also be referred to as a small company risk premium. This leads to higher discount rate and relatively lower expected present value of future cash flows. However, the size of the illiquidity premium is discussed. (Damodaran 2005.)

Damodaran (2005) presents three ways to estimate the illiquidity premium: add a constant illiquidity premium to the discount rate for all illiquid assets, add a firm-specific illiquidity premium or relate the observed illiquidity premium on traded assets to specific characteristics of those assets.

First, by adding a constant illiquidity premium the risk of the size of a company is been reflected. Usually, the cost of equity is assumed to be 3.0-3.5% higher for smaller companies, reflecting the excess returns earned by small cap companies over long estimation periods. Moreover, Ibbotson Associates provides small company risk premiums for different company sizes and many practitioners consider these as illiquidity premiums and add those to the cost of equity. It is should be noted that the smallest companies listed by Ibbotson Associates are much larger than typical private companies. Venture Economics also provides own estimations for illiquidity premiums. They estimated that the returns are 4% higher for venture capital investors than returns on traded shares. (Damodaran 2005.)

Second, by adding a firm-specific illiquidity premium the firm-specific characteristics are taken into account. Hence, the results should reflect better the fair value of the business. In this approach, it is needed to measure how much the company is exposed to liquidity risk, i.e. one should be interested in liquidity betas for individual companies.

The liquidity beta measures how the trading volume varies with the market trading volume over time. (Damodaran 2005.)

Third, companies with a healthier business model and with liquid assets should have lower illiquidity premium than companies with poor business model and illiquid assets. There the discount rate is a subjective view. (Damodaran 2005.)

4.3.3 Illiquidity discount

In the DCF valuation, the illiquidity effects are not taken into account in cash flows and usually the discount rate is calculated for the pure liquid business. There the solution is to apply the illiquidity discount. Thus, the WACC should be calculated without any small company risk premiums, i.e. illiquidity premium in the WACC as presented in the previous sub-chapter. For large publicly traded firms, it is not necessarily needed to use a premium or a discount since the liquidity is high but for small companies, the adjustment is needed. (Damodaran 2005.) The question is how much this illiquidity discount should be and is it always the same amount? This is the main question and research topic in this study.

Damodaran (2005) suggests different approaches to estimate a size for the illiquidity discount in practice: fixed discount for all firms, firm-specific discount, synthetic bid-ask spread and option-based discount. In this study, the fixed discount and firm-specific discount are covered.

The fixed discount or a discount range provided by analysts is used among practitioners in company valuation. Moreover, this method is widely used in textbooks regarding private company valuations but also in court cases. The previous studies have found that the discount range for illiquid shares is 25-35%. (Damodaran 2005.)

The firm-specific discount is supported by previous literature since illiquidity discounts should be different between assets and businesses. Especially, the illiquidity discount should be a function of the company size in the case of a private company. (Damodaran 2005.) The size of this firm-specific discount for a private company will be researched in this study also. Damodaran (2005) states five components that cause illiquidity differences across firms: liquidity of assets owned by the company, financial health and cash flows of the company, possibility of going public in the future, size of the company and control component, i.e. are you buying controlling stake of the company (51% vs. 49%).

4.4 Empirical results of previous studies

Reilly and Rotkowski (2007) have collected empirical results from the studies related to discounts for lack of marketability. They dealt with equity placement studies where shares were unregistered, i.e. restricted, which means that the shares cannot be freely traded on stock exchanges even if the issuer company is publicly traded. Therefore, the company should accept a price discount on their issued restricted shares. The price discount is the difference between the public share price and the same company identical private share price in this context. The collected results are presented in the next paragraph.

First, SEC institutional investor study (1971) showed that the average price discount was 24% for transactions between 1966 and 1969. This study compared public shares to identical private shares in the US. SEC's study also concluded that the price discount was 35% for shares traded on the over-the-counter markets. In Gelman's (1972) study, average and median price discounts were 33% and moreover, 60% of the restricted shares include the price discount more than 30%. Trout (1977) studied the price discounts with regression analysis and he concluded that the average price discount was 34% for restricted share transactions between 1968 and 1972. Based on Moroney's (1973) study, the average price discount for restricted shares was 36% between 1969 and 1972. Maher's (1976) almost identical results support Moroney's study. The study of Hertzel and Smith (1993) proposed that the average price discount for the private placements should be approximately 20%. Moreover, Silber (1991) studied the private placements of public companies between 1981 and 1988 and the results showed the average price discount of 34%. The private placements were studied by Bajaj, Denis, Ferris and Sarin (2001) also and they found the average price discount for unregistered share issues of 28%. Their empirical analyses included transactions between 1990 and 1995. Johnson (1999) studied restricted share issues between 1991 and 1995 and his results proposed the average price discount of 20%. The newest study was Robak's (2007) paper where the results showed the average price discount of 33%.

However, one should remember that this study is not focused on restricted share issues but the liquidity discounts for publicly traded shares. Therefore, the above-mentioned results are not perfectly comparable. Moreover, the estimates resulted from the restricted share issue studies can be damaged by factors which are not related to limited liquidity (Loderer & Roth 2005; see also Hertzel & Smith 1993).

Loderer & Roth (2005) studied the pricing discount across time from the Swiss stock markets but also from the NASDAQ. They found that the median pricing discount in the Swiss stock markets fluctuated between 6% and 21% during 1995 and 2001. At the same time, the median pricing discount in the NASDAQ fluctuated between 25% and 33%. In conclusion, previous literature suggests that average or median discounts are more than

20% in the most of the cases even though discounts could also be much smaller as noted in the Swiss stock markets.

Officer (2007) studied the price of corporate liquidity by looking at acquisition discounts for unlisted targets. He found that stand-alone private companies and subsidiaries of other firms have average acquisition discounts of 15% to 30% relative to acquisition multiples for similar publicly traded targets.

Paglia and Harjoto (2010) investigated valuation adjustments for private companies. They tried to match private company transactions with publicly traded counterparts. Their study found that the discount for lack of marketability was averagely 65%-70% and exceeded 80% in some sectors of the economy. The results are much higher in comparison with restricted stock, IPO and acquisition studies. Therefore, according to them, the results question sizes of discounts typically applied in valuation engagements for privately-held companies.

Zanni (2015) collected results from previous studies concerning private companies and found support to Paglia and Harjoto’s results. Collectively, the previous studies provide evidence that private companies often sell at lower multiples than their public counterparts due to the lack of liquidity or marketability. Zanni (2015) concluded that when valuing a private company, by reference to a similar characteristics public company, the liquidity discount should be taken into account. According to Zanni (2015), the transaction multiples are influenced by subject company size and profitability.

5 RESEARCH DESIGN

5.1 General considerations

A cross-sectional regression approach will be applied as an econometric method in this study. First, a general framework is set for this study, which enables the observation of share prices, i.e. valuation of shares as the present value of constantly growing dividend flow. The equation for this is the same as the Gordon growth model

$$Stock\ price\ (ex - dividend)\ P_0 = \frac{DIV_0 \times (1+g)}{k-g}, \tag{36}$$

where P is the share price, DIV_0 is dividend per share at the moment, g is the expected dividend growth rate and k is the required rate of return (Gordon 1962). The ex-dividend status means that the dividend is distributed to the shareholders of the company in question (Penman 2010, 199). Moreover, it is needed to do an assumption that the payout ratio (π) will be constant in the future. Therefore, the dividend component is replaced with product of earnings-per-share and payout ratio. By doing this assumption, the equation can be rearranged and it is possible to get an expression for a price to earnings (P/E) ratio as following

$$P/E\text{ ratio:}\ \ \frac{P}{EPS_0} = \frac{\pi \times (1+g)}{k-g}, \tag{37}$$

where EPS_0 is the current earnings-per-share of a firm. As the P/E ratio is stated in equation 37, the cross-sectional determinants of P/E ratios should include payout ratios, expected earnings growth rates and required rate of returns. However, equation 37 implicitly assumes the current earnings-per-share which are given. In this study, it is really important to note that in a cross-section for both prices and current earnings-per-share can change. Hence, it is not likely that the functional form of equation 37 applies in a cross-sectional analysis. Therefore, a more general relation must be formed in order to test the research question. (Loderer & Roth 2005.) First, in order to be able to do a cross-sectional comparison, it should be assumed that the risk-free rates and market risk premiums are the same through the sample (Loderer & Roth 2005)[1].

The function (38) models the P/E ratio

[1] This assumption has naturally limits as especially market risk premiums tend to be time-varying but also risk-free rates has been seen volatile in the near past.

$$P/E\,\text{ratio}_i = f(Growth_i, Payout_i, Risk_i, Size_i, Liquidity_i), \tag{38}$$

where *i* refers to the firm in question. Based on regression specifications reported in the return-liquidity literature, the equation 38 is consistent (Loderer & Roth 2005).
This regression model interprets that the P/E ratio can be explained by following factors: growth rate, payout ratio, risk of the firm, firm's size and liquidity. The section 5.1 gives more insight into these factors used in the regression model.

Loderer and Roth (2005) define some expectations related to the cross-sectional regularities. First, they expect that higher expected earnings growth increases share prices and therefore raise P/E ratios. Second, a higher risk usually increases EPS, lowers share prices and net effect from these should be lower than the P/E ratio. Third, greater company size lowers the required rate of return and this raises P/E ratios. Fourth, higher liquidity decreases required rate of return and therefore increases P/E ratios. Last, Loderer and Roth state that the higher payout ratio has an unclear effect on the P/E ratio even though in perfect markets the payout should be irrelevant (see Modigliani & Miller 1961).

5.2 Variables

The dependent variable in this study is the P/E ratio (P/E ratio) and its natural logarithm (ln(P/E)). The P/E ratio is assumed to reflect the company's valuation since it is not plausible that illiquidity can affect a firm's earnings even if it can have a significant effect on company valuation. Thus, the illiquidity effects can be researched with the P/E ratio. The P/E ratio is based on the share price as at 30 June 2016 and 2015 reported earnings. The price and earnings data is downloaded from Thomson Reuters Eikon.

The independent variables in this study are the above-mentioned regression factors. The growth factor in this study is forecasted earnings-per-share growth rates for 2016E and 2017E by analysts (EPSg2016 & EPSg2017). The growth rate estimates are downloaded from Thomson Reuters Eikon as at 24 August 2016.

The payout factor (Payout ratio) in this study is based on reported 2015A dividend and earnings information. All the values are calculated in euros. The data is collected from Thomson Reuters Eikon.

The risk factor in this study is the beta of the company (Beta). The beta is calculated by using total return indices and taking a natural logarithm from monthly returns between 1 January 2013 and 30 June 2016. The market portfolio is the index of the country in question, i.e. in Finland OMX Helsinki, in Sweden OMX Stockholm and in Denmark OMX Copenhagen. All the necessary beta data is downloaded from Thomson Reuters Datastream.

The size factor is the market capitalization of the company (Size). The market capitalization is calculated by using an average share price between 1.1.2016 and 30.6.2016 and number of shares outstanding as at 30.6.2016. All the company values in appendix 1 are in millions of euros. It should be noted that if the company has more than one share class, the most liquid one is used. Moreover, the size of the company is shown as a natural logarithm in the regression model (LNSIZE). The data is downloaded from Thomson Reuters Eikon.

In this study the liquidity factor is approximated by using two different liquidity proxies. At first, a relative bid-ask spread (RELSP) is used and in addition to that, the natural logarithm of an average value of daily trading volume is studied (LNVOLUME). The relative bid-ask is calculated following: (ask price – bid price)/bid price. The relative bid-ask spread is calculated daily basis in this study and the average of these daily estimates is based on 1 January 2013 - 30 June 2016 point estimates. Furthermore, in a first linear regression the relative bid-ask spread is raised to the second power ($RELSP^2$) as Loderer and Roth (2005) did. The data is downloaded from the Thomson Reuters Datastream.

5.3 Sample characteristics

The whole sample, the constituent firms listed on Helsinki, Stockholm and Copenhagen, includes 588 companies, 134 from OMX HEL, 313 from OMX STO and 141 from OMX CPH. However, all the needed data is not available for the whole sample. This refers to unavailable analyst estimates and other market data in the database. Thus, 243 companies are researched at the end of the day. The whole sample includes 66 companies from OMX HEL, 142 from OMX STO and 35 from OMX CPH. The figure 2 illustrates the process of the data collection in this study.

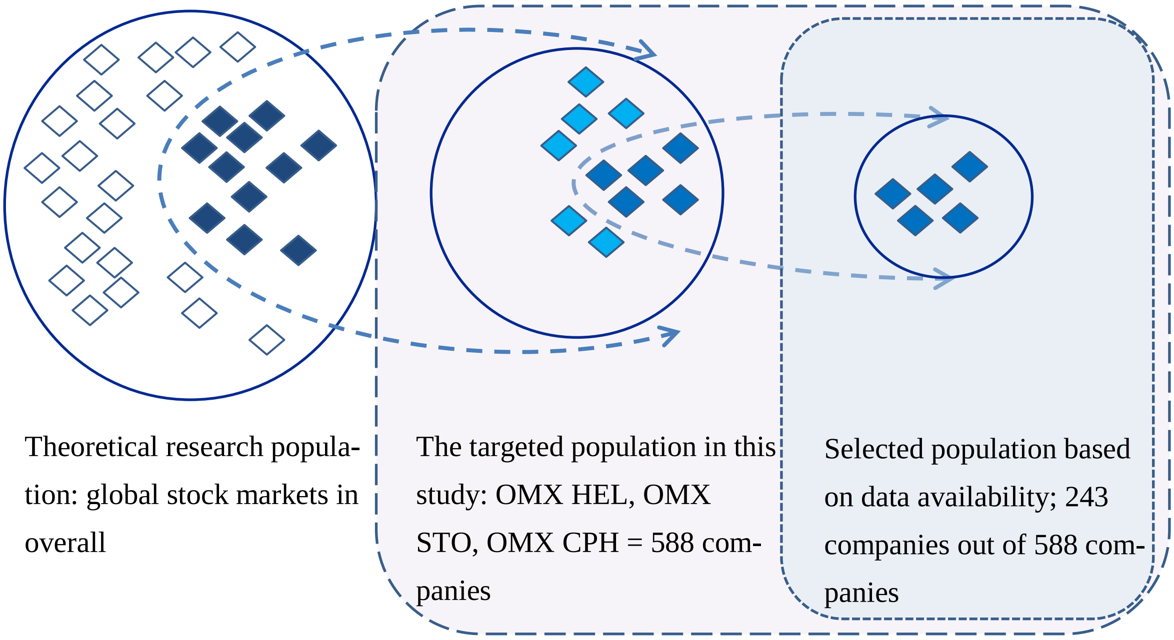

Figure 2 Data collection and availability

Even if 59% of the companies have been eliminated in the data collection process, still 243 companies is a good starting point to perform regression analyses and enables to divide the whole sample into two separate samples to test whether the results are same as for the whole sample.

Table 4 presents descriptive statistics of the whole sample used in this study.

Variables	Sample size	Min	Lower quartile	Median	Average	Upper quartile	Max	Standard deviation
P/E ratio	243	2.93	13.53	18.07	21.33	24.43	171.84	15.65
ln(P/E)	243	1.07	2.61	2.89	2.89	3.20	5.15	0.58
EPSg2016	243	-83.2%	-4.2 %	12.8%	23.9%	37.9%	288.9%	56.0%
EPSg2017	243	-53.6%	4.8 %	11.2%	16.1%	24.3%	184.6%	25.4%
Payout ratio	243	0.0%	34.0 %	49.3%	58.1%	70.6%	352.6%	48.7%
Beta	243	0.02	0.57	0.79	0.81	1.06	1.81	0.36
Size	243	1.6	199.8	799.1	4034.9	2935.7	97841.0	9967.1
LNSIZE	243	14.3	19.1	20.5	20.6	21.8	25.3	1.8
RELSP	243	0.06%	0.13 %	0.48%	0.74%	1.02%	9.20%	1.05%
$RELSP^2$	243	0.00%	0.00 %	0.00%	0.02%	0.01%	0.85%	0.08%
LNVOLUME	243	9.79	13.64	15.52	15.45	17.26	20.53	2.66
RES-SIZE/RELSP	243	-3.107	-0.957	-0.122	0.000	0.675	6.319	1.436
RES-SIZE/LNVOL	243	-3.160	-0.667	-0.022	0.000	0.634	3.254	0.970

Table 4 Descriptive statistics of the sample data

As one can see from the above table 4, the P/E ratio range is from 13.53 to 24.43 (15.65) based on the lower quartile and upper quartile. Earnings-per-share growth rate varies between -4.2% and 37.9% (25.4%-56.0%) based on the lower and upper quartile. It is remarkable that the beta ranges between 0.57 and 1.06 (0.36), however with a significant standard deviation. Based on the sample data, the market capitalization range is from 199.8 million euro to 2,935.7 million euro (9,967.1) based on the lower quartile and upper quartile. However, the standard deviation in market capitalization is almost 10 billion euro. The interest in this study is in the liquidity and its proxies. A liquidity proxy used in this study, the relative bid-ask spread, varies between 0.13% and 1.02% (1.05%) based on the lower quartile and upper quartile. However, the bid-ask spread can be material and the maximum bid-ask spread is more than 9%. It means that the liquidity has a significant impact on markets.

5.4 Cross-sectional regressions and hypotheses

The first regression model R1 tests how the relative bid-ask spread works as a proxy for liquidity. The regression model is following

$$P/E = \alpha_0 + \alpha_1 \times EPSg2016_i + \alpha_2 \times EPSg2017_i + \alpha_3 \times RELSP_i + \alpha_4 \times {RELSP_i}^2 + \varepsilon_i, \quad \text{(R1)}$$

where α_0 is the intercept, α_j is the coeffient and ε_i is the error term for the firm i. The hypothesis for this regression is that the coefficient for RELSP should be negative since lower liquidity should reduce the firm value. The regression model is used by Loderer and Roth (2005) also.

The second regression model the P/E ratio is converted to a natural logarithm and the White heteroscedasticity-consistent standards errors and covariance are applied. Thus, the regression model is unlinear and the $RELSP^2$ is eliminated from the model. The regression model is following

$$\ln(P/E)_i = \alpha_0 + \alpha_1 \times EPSg2016_i + \alpha_2 \times EPSg2017_i + \alpha_3 \times Payout\ ratio_i + \alpha_4 \times Beta + \alpha_5 \times LNSIZE + \alpha_6 \times RELSP_i + \varepsilon_i, \quad \text{(R2)}$$

In this model the focus is to test whether all the factors are statistically significant. Especially, the statistically significance of RELSP is highly interested. As the correlation between the relative bid-ask spread and the market capitalization is -0.232 (not shown), it can be seen that the market capitalization is a measure of both risk and liquidity of a share.

This result means that the larger companies have tighter relative bid-ask spread. Thus, the size of the company can be seen as a proxy for liquidity also. (Loderer & Roth 2005)

As the correlation between the relative bid-ask spread and the market capitalization indicates that the market capitalization measures risk and liquidity of the share, it is needed to separate the possible risk and liquidity effects in the coefficient of LNSIZE in the regression model R2. The unraveling of these two effects is done by regression the natural logarithm of market capitalization on relative bid-ask spread. (Loderer & Roth 2005.) The regression (R3) is used to disentangle the effects

$$LNSIZE_i = \mu_0 + \mu_1 \times RELSP_i + \eta_i, \tag{R3}$$

where i is the firm in question and η is an error term. According to Loderer and Roth (2005), this regression tells which part of the cross-sectional variation in market capitalization is due to the cross-sectional variation in liquidity. Moreover, by construction, the residuals from the regression model R3 are unrelated to liquidity since the relative bid-ask spread works as a proxy for liquidity of the share. Thus, the residuals capture the size effects without liquidity effects. (Loderer & Roth 2005.) The residuals from the model R3 are called RES-SIZE/RELSP and the residuals are used instead of LNSIZE in this study. Hence, the next regression model R4 used in this study is following

$$\begin{aligned} \ln(P/E)_i = \alpha_0 + \alpha_1 \times EPSg2016_i + \alpha_2 \times EPSg2017_i + \alpha_3 \times Payout\ ratio_i \\ + \alpha_4 \times Beta + \alpha_5 \times RES - SIZE/RELSP + \alpha_6 \times RELSP_i + \varepsilon_i, \end{aligned} \tag{R4}$$

However, if all the factors are not statistically significant in the regression model R2, then the insignificant factor is removed from the regression model and the regression is performed again. In the regression model R4, White heteroscedasticity-consistent standards errors and covariance are applied. The main hypothesis in this regression model is that the coefficient of RES-SIZE/RELSP is positive and the coefficient of Beta is negative. A positive coefficient of RES-SIZE/RELSP implies that the equity beta does not capture all the risk of the company and the size can include risk which is not measured by the beta. Thus, the less risky large companies have higher valuation based on the assumptions of a positive RES-SIZE/RELSP coefficient and a negative Beta coefficient. (Loderer & Roth 2005.) The robust regression is applied also for the regression model R4 if needed as Loderer and Roth have done in their paper.

Furthermore, instead of the relative bid-ask spread, the natural logarithm of average value of daily trading volume is used as a proxy for liquidity. Thus, the regressiol model R3 is modified as follows

$$LNSIZE_i = \mu_0 + \mu_1 \times LNVOLUME_i + \eta_i, \tag{R3.2}$$

where the residuals are called RES-SIZE/LNVOL. Hence, the regression model R4 is written as follows

$$\begin{aligned}\ln(P/E)_i = \alpha_0 + \alpha_1 \times EPSg2016_i + \alpha_2 \times EPSg2017_i + \alpha_3 \\ \times Payout\ ratio_i + \alpha_4 \times Beta \\ + \alpha_5 \times RES - SIZE/LNVOL + \alpha_6 \times LNVOLUME_i + \varepsilon_i.\end{aligned} \tag{R4.2}$$

However, if some of the coefficient is statistically insignificant in the regression model R2, it would be removed from the regression model R4.2 as Loderer and Roth (2005) have done.

Moreover, one regression model is used in addition to models used by Loderer and Roth (2005). The fifth regression model R5 is following

$$\begin{aligned}\ln(P/E)_i = \alpha_0 + \alpha_1 \times EPSg2016_i + \alpha_2 \times EPSg2017_i + \alpha_3 \times Beta \\ + \alpha_4 \times LNSIZE + \varepsilon_i,\end{aligned} \tag{R5}$$

and White heteroscedasticity-consistent standards errors and covariance are applied. The hypothesis is that the coefficient of LNSIZE is positive and the coefficient of Beta is negative. The coefficient of LNSIZE is used to derive a pricing discount for perfectly liquid equity value. This regression model is based on the course material of Advanced Valuation at the University of Bern in the spring 2015 by Zeller. There the problem is that the bid-ask spreads are not available for unlisted firms. Hence, the firm size is used as a proxy for bid-ask spreads. According to Zeller’s material, the P/E ratio can be predicted for a given firm named “Company” and for the benchmark firm named “Benchmark” and compute the illiquidity discount as follows

$$\frac{\widehat{P/E}_{Benchmark} - \widehat{P/E}_{Company}}{\widehat{P/E}_{Benchmark}} = 1 - \frac{\widehat{P/E}_{Company}}{\widehat{P/E}_{Benchmark}}$$

$$= 1 - \frac{exp\left[\widehat{\alpha_0} + \sum_{j=1}^{3} \widehat{\alpha_j} \times variable_{Company} + \widehat{\alpha_4} \times LNSIZE_{Company}\right]}{exp\left[\widehat{\alpha_0} + \sum_{j=1}^{3} \widehat{\alpha_j} \times variable_{Benchmark} + \widehat{\alpha_4} \times LNSIZE_{Benchmark}\right]} \quad (39)$$

$$= 1 - \frac{exp\left[\widehat{\alpha_0} + \sum_{j=1}^{3} \widehat{\alpha_j} \times variable_{Company}\right] \times \exp\left[\widehat{\alpha_4} \times LNSIZE_{Company}\right]}{exp\left[\widehat{\alpha_0} + \sum_{j=1}^{3} \widehat{\alpha_j} \times variable_{Benchmark}\right] \times \exp\left[\widehat{\alpha_4} \times LNSIZE_{Benchmark}\right]}.$$

Following it is assumed that the "Company" and the "Benchmark" is assumed to be identical companies except for their size.

$$1 - \frac{\widehat{P/E}_{Company}}{\widehat{P/E}_{Benchmark}} = 1 - \frac{exp\left[\widehat{\alpha_4} \times LNSIZE_{Company}\right]}{exp\left[\widehat{\alpha_4} \times LNSIZE_{Benchmark}\right]}$$

$$= 1 - \left(\frac{exp\left[LNSIZE_{Company}\right]}{exp\left[LNSIZE_{Benchmark}\right]}\right)^{\widehat{\alpha_4}} \quad (40)$$

Hence, the illiquidity discount can be presented as follows

$$Illiquidity\ discount = 1 - \left(\frac{SIZE_{Company}}{SIZE_{Benchmark}}\right)^{\widehat{\alpha_4}}, \quad (41)$$

where $\widehat{\alpha_4}$ is the estimated coefficient of LNSIZE. However, the market capitalization of the company in question is unknown since the market's discount for limited liquidity is not taken into account. The equation for illiquid value of equity is following (Zeller 2015)

$$E_{Illiquid} = E_{Liquid} \times (1 - Illiquidity\ discount). \quad (42)$$

Hence, the equation 41 can be modified as follows (Zeller 2015)

$$Illiquidity\ discount = 1 - \left(\frac{E_{Liquid}}{E_{Benchmark}}\right)^{\frac{\widehat{\alpha_4}}{1-\widehat{\alpha_4}}}, \quad (43)$$

The equation 43 is used to derive an illiquid discount range for perfectly liquid equity value.

5.5 Econometrics concerns

There are a few possible concerns related to econometric methods in this study. First, it is possible that the used data is not suitable for this study for various reasons, for example, the relative bid-ask spread is not a good liquidity proxy or there is no price-liquidity relation. However, in the terms of econometrics concerns, the biggest issue is the multicollinearity in the sample and it will be investigated. In addition to that, the residuals can be skewed in the regression models. The normality of the residuals will be tested by using different diagnostic analyses.

The White heteroscedasticity-consistent standards errors and covariance are applied due to possible heteroscedasticity of error terms, i.e. the variance is not the same across all observation points. Moreover, a method of robust regression will be applied. In this study, the M-estimation robust regression introduced by Huber (1964) is used. M-estimation is suitable to use if the error distribution is not normal and especially when the errors are heavy-tailed (Fox 2002). In general, the problem with the abnormal residuals is that they are inconsistent with the t- and F-tests (Loderer & Roth 2005). In this study, this procedure is used to compute regression coefficients of the regression models R4 and R4.2.

6 EMPIRICAL RESULTS

This section of the study presents the regression results for all the regression models presented in the chapter 5. Moreover, the economic concerns related to regression models are studied. After that, the results will be interpreted and discussed.

6.1 Regression results

The first regression model R1 tests whether the bid-ask spread works as a proxy for illiquidity at all. Table 5 presents the main observations related to the R1.

Variable	**Regression coefficient**	**t-Statistics**	**p-values**
Intercept	16.8598	12.6888	0.0000
EPSg2016	12.8480	8.7356	0.0000
EPSg2017	21.4818	6.5509	0.0000
RELSP	-394.7084	-2.2817	0.0234
$RELSP^2$	5,122.3930	2.2481	0.0255
Number of observations	243		
F-statistics (p-value)	31.4978 (0.000)		
R-squared	0.3461		
Adjusted R-squared	0.3351		

Table 5 Regression results from R1

All the coefficients in table 5 are statistically significant with the confidence level of 0.95. The F-test measures the significance of regression coefficients as a whole. The null hypothesis is that all independent variables have coefficient of zero, i.e. a linear regression is not suited. A counterhypothesis is that at least one of the coefficients is significantly different from zero, i.e. linear regression model is suited. In this case, the F-statistics is significantly high and the linear regression seems to be able to use. The R-squared and adjusted R-squared are close to each other, 0.35. Therefore, this estimated relation explains 35% of the cross-sectional variation in P/E ratios. Compared this to Loderer & Roth's results (86.5%), this is relative low value but still useful.

Both of the earnings-per-share growth coefficients are positive and significant with the confidence level of 0.95. It is obvious that the higher EPS results higher P/E ratio, i.e. in this case higher valuation. The coefficient for relative bid-ask spread is negative and its squared version has a positive coefficient, and both are significant with the confidence level of 0.95. As Loderer & Roth (2005) estimated that a marginal 1% increase in the bid-ask spread decreases P/E ratio by 12, the corresponding decrease in this study is -3.44 ($-394.7084*0.01+5{,}122.393*0.01^2 \approx -3.44$).

However, the resulted residuals from the regression model R1 are not normally distributed and this can question tests in table 5. The figure 3 represents the residual distribution of R1 where the normal P/E ratio is applied.

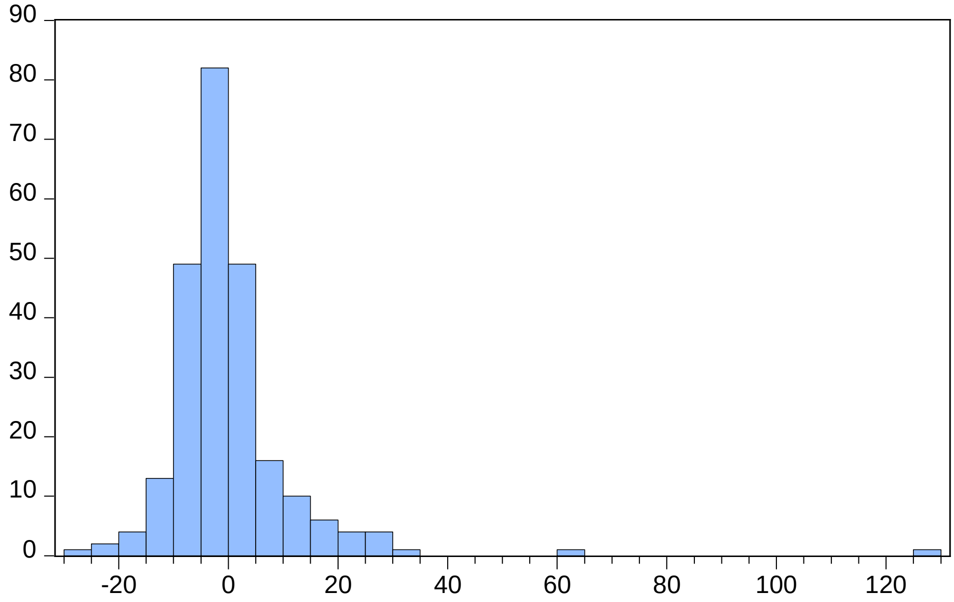

Figure 3 Residuals from the regression R1

Skewness and Kurtosis of the distribution presented in the figure 3 are 4.96 and 47.11, respectively. By taking a natural logarithm from the P/E ratio, the residuals are more normally distributed as shown in the figure 4.

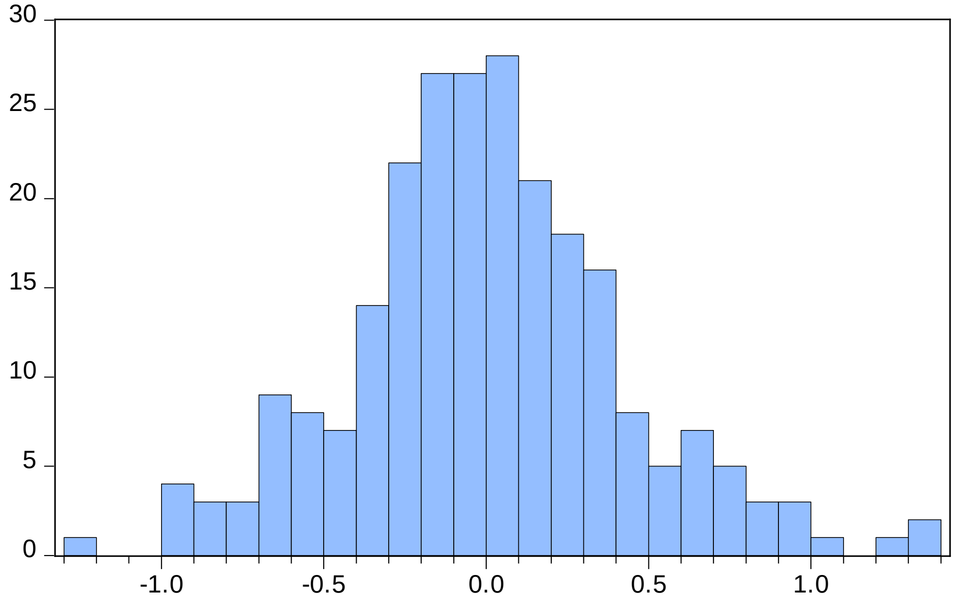

Figure 4 Residuals from the regression R1 with the natural logarithm of P/E ratio

As one can observe, the residuals are much normally distributed with the natural logarithm of P/E ratio. Skewness and Kurtosis of the distribution presented in the figure 4 are 0.24 and 3.68, respectively.

Due to above-mentioned reasons, the second regression model is applied. Table 6 presents results related to R2.

Variable	Regression coefficient	t-Statistics	p-values
Intercept	1.0578	2.3352	0.0204
EPSg2016	0.5152	7.3695	0.0000
EPSg2017	0.8684	5.2066	0.0000
Payout ratio	0.0710	1.0235	0.3071
Beta	-0.1904	-2.5180	0.0125
LNSIZE	0.0803	3.7503	0.0002
RELSP	3.9499	2.3352	0.2223
Number of observations	243		
F-statistics (p-value)	34.2792 (0.000)		
R-squared	0.4657		
Adjusted R-squared	0.4521		

Table 6 Regression results from R2

The regression intercept has a positive and significant coefficient as before. Moreover, coefficients for estimated EPS growth rates remain positive and significant. However, payout ratio's coefficient is close to zero with insignificant value. This is the same observation as Loderer & Roth (2005) had. Moreover, Penman (2010) also states that the payout ratio is irrelevant in valuation purposes. The coefficient of Beta is negative and significant. This implies that the beta is a relevant measure for risk. The coefficient of LNSIZE is significant and a positive value. It can be seen that larger companies have higher valuation due to lower risk (Loderer & Roth 2005).

However, the coefficient of RELSP is insignificantly different from zero. This result is consistent with the study of Loderer and Roth (2005). They suggested two reasons for that: there is no price-liquidity relation or LNSIZE measure both risk and liquidity. The negative correlation between market capitalization and relative bid-ask spread (presented above; -0.232) supports this second reason (Loderer & Roth 2005). In conclusion, the results from R2 interpret that the larger companies have higher valuation than small companies. Since the purpose in this study is to estimate the limited liquidity discount on value, the possible LNSIZE effects should be disentangled, i.e. how much it measures risk and how much liquidity. Therefore, the regression R3 is applied. Table 7 presents results from the R3.

Variable	Regression coefficient	t-Statistics	p-values
Intercept	21.36305	189.2705	0.0000
RELSP	-110.4989	-12.5051	0.0000
Number of observations	243		
F-statistics (p-value)	156.3780 (0.000)		
R-squared	0.3935		
Adjusted R-squared	0.3910		

Table 7 Regression results from R3

Both coefficients are significant and the F-test result is high. The adjusted R-squared is 39.1%. The residuals from this regression are used in the regression model R4 where the liquidity-related size effects are tried to distinguish from the liquidity-unrelated size effects. Table 8 presents the results from the regression model R4.

Variable	Regression coefficient	t-Statistics	p-values
Intercept	2.8137	36.1227	0.0000
EPSg2016	0.5403	9.1606	0.0000
EPSg2017	0.8926	5.5537	0.0000
Beta	-0.1972	-2.6304	0.0091
RES-SIZE/RELSP	0.0835	3.9421	0.0001
RELSP	-5.388	-1.9775	0.0491
Number of observations	243		
F-statistics (p-value)	40.4874 (0.000)		
R-squared	0.4629		
Adjusted R-squared	0.4515		

Table 8 Regression results from R4

At first, one should note that the payout ratio is eliminated from the regression model since its coefficient was insignificant in the R2. The (adjusted) R-squared is more or less the same as in the R2 and all of the coefficients are significantly different from zero with the confidence level of 0.95. Moreover, the signs of the coefficients remain the same as in the R2 except the coefficient of relative spread is now negative. Especially, the coefficient of RES-SIZE/RELSP is positive and implies that the equity beta does not take into account all the size aspects of risk (Loderer & Roth 2005). According to Loderer and Roth (2005), a positive sign there means that larger companies are less risky and the valuation is higher, all else being the same compared to smaller companies. In addition, table 8 shows that the coefficient of relative bid-ask spread, RELSP, has a negative sign which is one of the most important results in this study. It means that a larger bid-ask spread reduces the equity value.

However, the residuals from the regression R4 are not normally distributed as shown in the figure 5. Therefore, the results from t- and F-tests are not reliable. The remedy for that is to use a robust regression model by Huber (1964) (Loderer & Roth 2005).

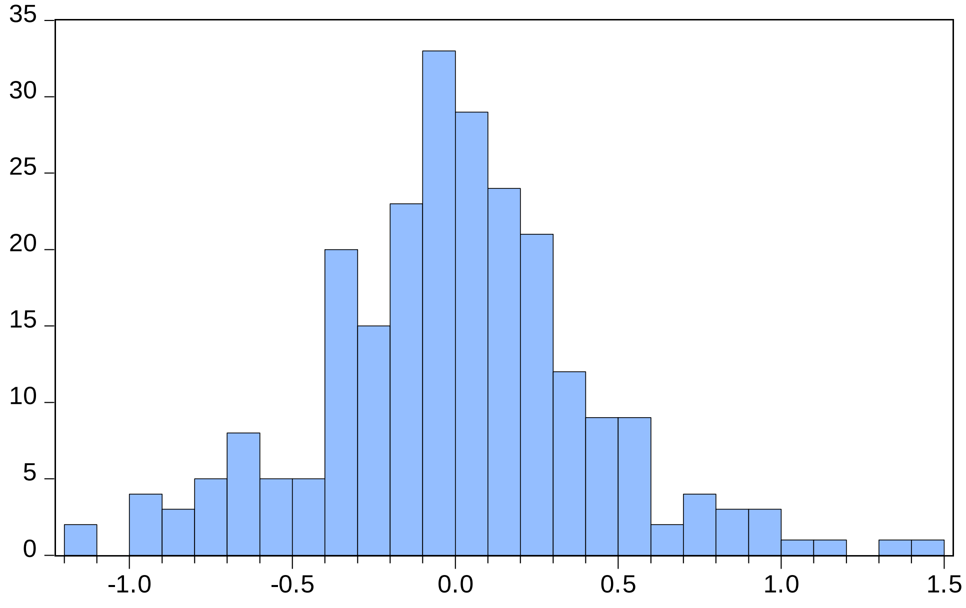

Figure 5 Residuals from the regression R4

As the figure 5 presents, the residuals are right-skewed. Skewness and Kurtosis of the distribution presented in the figure 5 are 0.17 and 3.86, respectively. Moreover, the figure 6 depicts the normal Q-Q plot of the R4.

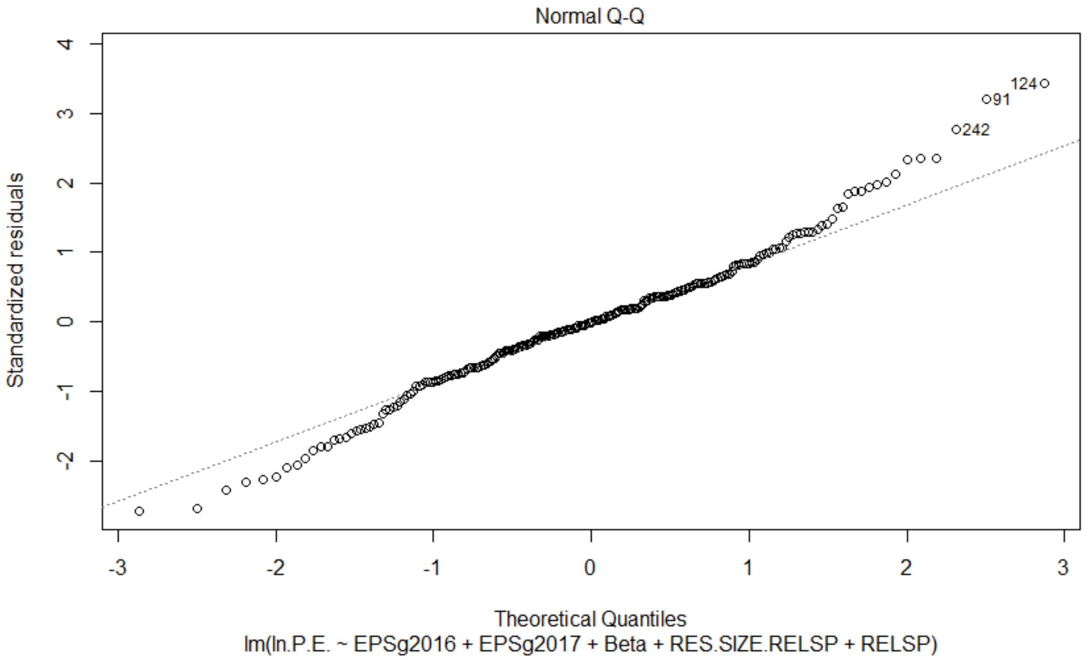

Figure 6 Normal Q-Q plot of the regression R4

The diagnostic analysis reveals the fact of skewed residuals. Table 9 presents the results related to the M-estimated R4, i.e. the robust regression by Huber.

Variable	**Regression coefficient**	**z-Statistics**	**p-values**
Intercept	2.7906	37.8425	0.0000
EPSg2016	0.5333	11.8029	0.0000
EPSg2017	0.9428	9.4537	0.0000
Beta	-0.1841	-2.5453	0.0109
RES-SIZE/RELSP	0.0946	5.2843	0.0000
RELSP	-4.8512	-1.9798	0.0477
Number of observations	243		
R-squared	0.3812		
Adjusted R-squared	0.3689		

Table 9 Regression results from M-estimated R4

The regression coefficients have same signs as before and those are significantly different from zero with the confidence level of 0.95.

One economic concern in this study is related to multicollinearity when the regression model includes multiple factors. The collinearity is a phenomenon in which two variables are highly correlated in the regression and the multicollinearity occurs if there are more than two correlated variables. However, almost always there is some correlation between variables and it is not a problem. If the correlation between variables is strong, the multicollinearity might be a problem. In general, one should check the multicollinearity of variables if R-squared is high but coefficients of variables are not statistically significant. (Stock & Watson 2007, 206-210.) Table 10 presents the multicollinearity test for R4.

	LN(P/E)	EPSg2016	EPSg2017	Beta	RES-SIZE/RELSP	RELSP
LN(P/E)	1.000	0.533	0.388	-0.184	0.045	-0.002
EPSg2016	0.533	1.000	0.059	-0.168	-0.135	0.062
EPSg2017	0.388	0.059	1.000	-0.085	-0.176	0.084
Beta	-0.184	-0.168	-0.085	1.000	0.177	-0.242
RES-SIZE/RELSP	0.045	-0.135	-0.176	0.177	1.000	0.000
RELSP	-0.002	0.062	0.084	-0.242	0.000	1.000

Table 10 Multicollinearity test for R4 (White procedure)

Table 10 interprets that there is no multicollinearity between variables since the values are small, i.e. smaller than 0.6. Moreover, to test multicollinearity, variance inflation factors (VIF) are studied also. If the VIF values are close to one, it means that there is no multicollinearity. (Weisberg 2005, 216-217.) Table 11 presents the VIF values for R4.

Variable	Centered VIF
EPSg2016	1.0759
EPSg2017	1.1078
Beta	1.2251
RES-SIZE/RELSP	1.2382
RELSP	1.1918

Table 11 VIF values for R4 (White procedure)

As one can observe, all the values are really close to one, i.e. there is no multicollinearity based on the VIF values and the multicollinearity matrix.

6.1.1 Holdout samples

In this chapter, the whole sample is divided into two holdout samples and it will be tested whether the results are same as before. First, the whole sample is divided as follows: sample 1 includes 33 companies from OMX HEL, 71 OMX STO and 18 from OMX COP; sample 2 includes 33 companies from OMX HEL, 71 from OMX STO and 17 from OMX CPH. The regression R4 is used with White heteroscedasticity-consistent standard errors and covariance. Table 12 presents the results

Variable	Regression coefficient	t-Statistics	p-values
Intercept	2.6706	23.7529	0.0000
EPSg2016	0.5425	7.3949	0.0000
EPSg2017	0.8158	3.5639	0.0005
Beta	-0.0513	-0.4996	0.6183
RES-SIZE/RELSP	0.0298	0.9630	0.3375
RELSP	-7.0351	-1.3418	0.1823
Number of observations	122		
F-statistics (p-value)	19.7217 (0.000)		
R-squared	0.4595		
Adjusted R-squared	0.4362		

Table 12 Holdout sample 1 results from R4

As one can observe, Beta, RES-SIZE/RELSP and RELSP coefficients are not statistically significant. This might be the result of a small sample size. Still all the signs are same as before. However, the table 13 presents the results regressed from the holdout sample 2.

Variable	Regression coefficient	t-Statistics	p-values
Intercept	2.9560	25.5856	0.0000
EPSg2016	0.5357	5.4142	0.0000
EPSg2017	0.9827	5.4548	0.0000
Beta	-0.3488	-2.9210	0.0042
RES-SIZE/RELSP	0.1351	5.1293	0.0000
RELSP	-5.4365	-2.1543	0.0333
Number of observations	121		
F-statistics (p-value)	24.3512 (0.000)		
R-squared	0.5143		
Adjusted R-squared	0.4931		

Table 13 Holdout sample 2 results from R4

All the coefficients are significantly different from zero with the confidence level of 0.95. Moreover, the signs of coefficients are same as before.

6.1.2 Regression model for private companies

One regression model is used in addition to models used by Loderer and Roth (2005). The results from the fifth regression model R5 are presented in the table 14.

Variable	Regression coefficient	t-Statistics	p-values
Intercept	1.3440	3.6308	0.0003
EPSg2016	0.5377	9.1791	0.0000
EPSg2017	0.8867	0.1585	0.0000
Beta	-0.2051	-2.7395	0.0066
LNSIZE	0.0610	3.8602	0.0001
Number of observations	243		
F-statistics (p-value)	50.6833 (0.000)		
R-squared	0.4600		
Adjusted R-squared	0.4509		

Table 14 Regression results from R5

The (adjusted) R-squared is more or less same as before and all the signs remain as before. The coefficients are significantly different from zero with the confidence level of 0.95. The multicollinearity matrix is shown in the table 15.

	LN(P/E)	EPSg2016	EPSg2017	Beta	LNSIZE
LN(P/E)	1.000	0.533	0.388	-0.184	0.037
EPSg2016	0.533	1.000	0.059	-0.168	-0.144
EPSg2017	0.388	0.059	1.000	-0.085	-0.190
Beta	-0.184	-0.168	-0.085	1.000	0.290
LNSIZE	0.037	-0.144	-0.190	0.290	1.000

Table 15 Multicollinearity test for R5 (White procedure)

Table 15 interprets that there is no multicollinearity between variables since the values are pretty small, i.e. smaller than 0.6. Table 16 presents the VIF values for R5.

Variable	**Centered VIF**
EPSg2016	1.068986
EPSg2017	1.100808
Beta	1.218409
LNSIZE	1.396057

Table 16 VIF value for R5 (White procedure)

As one can observe, all the values are really close to one, i.e. there is no multicollinearity based on the VIF values and the multicollinearity matrix.

6.1.3 Trading volume as a liquidity proxy

In addition to the bid-ask spread, the trading volume is used also as a liquidity proxy in this study, as Loderer & Roth (2005) did. Table 17 presents the results from the regression model R4.2. The RELSP is replaced with LNVOLUME and RES-SIZE/RELSP is now RES-SIZE/LNVOL, where residuals are as before. The R4.2 is regressed by using M-estimation since the residuals are not normally distributed as shown in the figure 7.

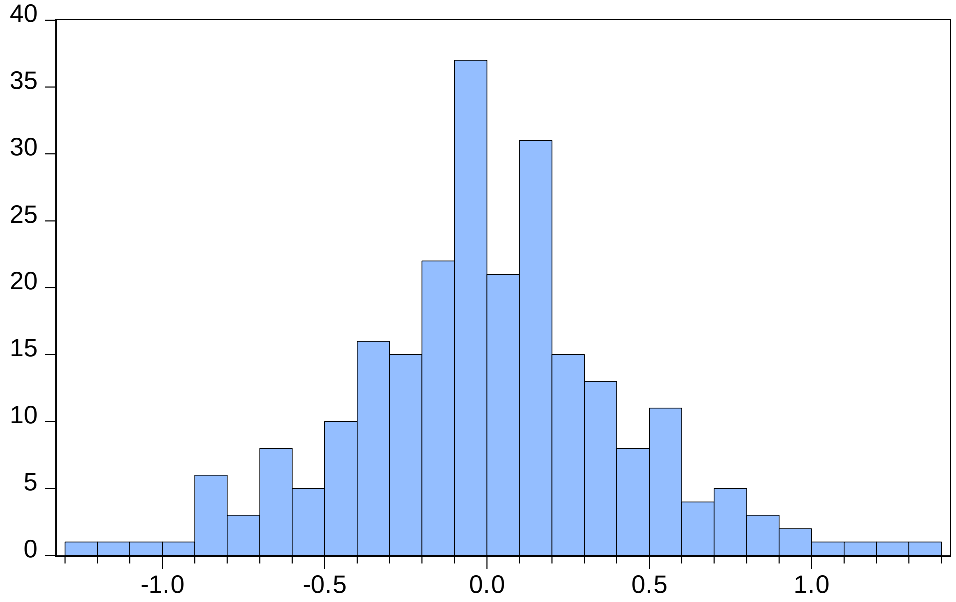

Figure 7 Residuals from the regression R4.2 without M-estimation

Skewness and Kurtosis of the distribution presented in the figure 7 are 0.14 and 3.64, respectively. Table 17 presents the results from the M-estimated regression R4.2.

Variable	**Regression coefficient**	**z-Statistics**	**p-values**
Intercept	2.0076	12.6251	0.0000
EPSg2016	0.5277	11.4384	0.0000
EPSg2017	0.9328	9.1499	0.0000
Beta	-0.2120	-2.8150	0.0049
RES-SIZE/LNVOL	0.0587	2.2388	0.0252
LNVOLUME	0.0499	4.8134	0.0000
Number of observations	243		
R-squared	0.3765		
Adjusted R-squared	0.3634		

Table 17 Regression results from M-estimated R4.2

As it can be seen from the table 17 above, the beta still has a negative sign (i.e. beta measures risk) and LNVOLUME has a positive sign, i.e. higher volume increases the company value. All the coefficients are statistically significant with the confidence level of 0.95.

Table 18 presents the multicollinearity matrix from the regression R4.2 where LNVOLUME is the liquidity proxy.

	LN(P/E)	EPSg2016	EPSg2017	Beta	RES-SIZE/LNVOL	LNVOLUME
LN(P/E)	1.000	0.533	0.388	-0.184	0.005	0.041
EPSg2016	0.533	1.000	0.059	-0.168	-0.067	-0.128
EPSg2017	0.388	0.059	1.000	-0.085	-0.052	-0.191
Beta	-0.184	-0.168	-0.085	1.000	-0.019	0.352
RES-SIZE/LNVOL	0.005	-0.067	-0.052	-0.019	1.000	0.000
LNVOLUME	0.041	-0.128	-0.191	0.352	0.000	1.000

Table 18 Multicollinearity test for R4.2

Table 18 interprets that there is no multicollinearity between variables since the values are pretty small, i.e. smaller than 0.6.

6.2 Practical implications

In this chapter, the above results are interpreted into practice. The liquidity discount for the 10 most liquid shares and less liquid shares in the sample are presented in the appendix 2. Note that the size variable is presented in euros. The liquidity discount for the firm i is calculated following

$$Illiquidity\ discount_i = 1 - \exp(-4.8512 \times RELSP_i), \quad (44)$$

where -4.8512 is the regression coefficient of M-estimated R4 for the relative bid-ask spread and RELSP is the relative bid-ask spread for the firm i (Loderer & Roth 2005). The range of the liquidity discount of the whole sample is presented in the table 19.

Variable	Sample size	Min	Lower quartile	Median	Average	Upper quartile	Max	Standard deviation
RELSP	243	0.06%	0.13%	0.48%	0.74%	1.02%	9.20%	1.05%
Liquidity discount relative to a situation of perfect liquidity								
		0.3%	**0.6%**	**2.3%**	**3.5%**	**4.8%**	**36.0%**	

Table 19 Liquidity discount relative to a situation of perfect liquidity

As the table 19 above presents, the liquidity discount varies from 0.3% to 36.0% in the sample, i.e. in the Nordic stock markets. Basically, it would be said that 0.3% reflects the transaction costs such as brokerage commissions. Moreover, 36.0% includes also the indirect effect of illiquidity. In conclusion, the most liquid firms sell at a 0.3% discount and the least liquid firms sells at a 36.0% discount. Loderer and Roth (2005) found the liquidity discount range of 1.6%-33.4%, respectively. They studied the Swiss stock markets.

Moreover, the equation 43 is used to derive a pricing discount range for perfectly liquid equity value. The benchmark equity value is the lowest value of the 10 most liquid market capitalizations from the appendix 2. This discount range can be used in private company valuations where the perfectly liquid equity value is calculated as proposed in the valuation section of this study. Based on coefficients of LNSIZE and RES-SIZE/RELSP from the R4 (White procedure), the discount range is presented in the table 20.

	Discount range	
Perfectly liquid equity value	**Min**	**Max**
50,000	58.3%	65.3%
100,000	56.0%	63.0%
250,000	52.9%	59.8%
500,000	50.4%	57.2%
750,000	48.8%	55.5%
1,000,000	47.7%	54.4%
2,500,000	44.0%	50.4%
5,000,000	41.0%	47.2%
7,500,000	39.2%	45.2%
10,000,000	37.8%	43.7%
25,000,000	33.4%	38.8%
50,000,000	29.8%	34.8%
100,000,000	26.1%	30.6%
250,000,000	20.8%	24.6%
500,000,000	16.5%	19.6%

Table 20 Illiquidity discount for private companies

As one can see, the illiquidity discount is significant for private companies as previous literature has also stated. The discount range varies between 16.5%-65.3% for companies with the perfectly liquid equity value of 500,000,000€ and 50,000€, respectively. This means that if the equity value of the company in question is e.g. 1 million euro calculated without a small company risk premium or an illiquidity premium, therefore the pricing discount should be approximately 50%. In other words, an investor demands 50% discount from the purely liquid equity value in order to have compensation of lack of liquidity of an asset or a company in question. In this case, the investor would pay only 500,000€ from the company's equity and it corresponds a fair market equity value.

6.3 Interpretation and discussion

The main research question in this study is: how much an investor demands discount from perfectly liquid equity value in the case of limited liquidity? The main research question contains two sub-questions as following

- How to define a fair market equity value and what characteristics should be taken into account in its valuation?
- How to measure limited liquidity in valuation?

The fair market equity value can be calculated by using an income approach valuation or a market approach valuation. The income approach includes the discounted cash flow model as a main method, the residual earnings model and the abnormal earnings growth model. These models are based on projected cash flows, earnings and dividends over an explicit forecasting period. However, the cost of capital has significant impact on the valuation results. Therefore, the analysis of an appropriate weighted average cost of capital is needed in the valuation. The market approach covers trading and transaction multiple analysis where the peer group has significant impact on results.

In short, the answer to the second sub-research question includes two main approaches to limited liquidity. First, the illiquidity premium in the cost of capital is highly used but also the illiquidity discount can be applied. In this study, the focus is on the illiquidity discount which reflects the difference between the purely liquid equity value and the illiquid equity value.

The main research question is handled following. This study has shown that investors demand liquidity discount which ranges from 0.3% to 36.0% in the Nordic stock markets. The mean and the median discounts are 2.3% and 3.5%, respectively. With this information different market participant and managers of companies can take into account liquidity of the company and can affect its costs in order to increase the share value, i.e. to maximize the shareholder value. For example, the results from this study can be used for valuation purposes. Corporate finance companies or departments that perform valuations a lot can use the results of this study to add customer's value and reflect on their professional valuation methods. Companies with limited liquidity can get valuable information from this study to reduce the cost of capital by affecting liquidity of securities owned by the company. Thus, the companies can also maximize the shareholder value which is its main objective usually. Moreover, with management forecasts companies can increase liquidity and therefore increase the value of the company by reducing the bid-ask spreads.

Theoretically this study gives new information about liquidity proxies, especially in the Nordic markets. This study has confirmed that the relative bid-ask spread can work as a liquidity proxy but also the trading volume works as a liquidity proxy. The research gap in the Nordic markets is fulfilled with this study but the sample size is still limited since the number of listed companies is relatively low. Moreover, this study deepens the knowledge of the Nordic stock markets liquidity in a whole new way and supports the results of previous studies.

However, in this study the median and the mean illiquidity discounts are much smaller compared to the previous studies where the median discounts went up to 30% in many cases. I believe that the main reason for smaller median and mean discounts is more efficient stock markets globally. During recent decades, the trading has been facilitated and the cost of trading has decreased significantly. Therefore, with the higher number of investors in the markets, liquidity increases and the discounts decrease. Moreover, the average of relative bid-ask spreads can vary between different stock exchanges and therefore, the stock exchanges with higher spreads have higher discounts (see the difference between the Swiss stock exchange and the NASDAQ in the study of Loderer & Roth (2005)). This also shows that there is not one specific liquidity discount for all the companies. It is highly important to take into account the firm-specific characteristics and apply an appropriate discount for each company. Previous literature has focused on median and mean pricing discounts even though it would be more fruitful to look at the whole discount range.

The deviation between results of previous studies is significant. For example, Loderer and Roth (2005) found that the median pricing discount for limited liquidity in the Swiss stock exchange was 6.4% in 1997 and 21.3% in 1999. Hence, the liquidity of the markets can vary across time substantially. Since my study does not cover the time aspect of limited liquidity, the results of my study can reflect the time of high liquid period of the markets. Still the difference between the lowest median discount (6.4% in 1997) from Loderer and Roth (2005) and my study (2.3%) is multifold. Thus, an assumption of more efficient markets is valid.

The practical results from the regression model 5 show that the illiquidity discounts for private companies are still enormous, i.e. the investors demand high illiquidity discounts for private companies. It can be believed that the number of private equity investors are still much more limited in comparison with investors who operate with the publicly traded companies. Briefly, there are not enough buyers or sellers in the private equity markets and therefore, the illiquidity discounts increase. The results of my study support evidence of Paglio and Harjoto's (2010) study where the discount for lack of marketability was averagely 65%-70% and exceeded 80% in some sectors of the economy. Therefore, one should question sizes of discounts resulted from restricted stock, IPO and acquisition studies in valuation engagements for private companies.

This study has shown that liquidity has significant impact on company valuation not only in the case of private companies but also for listed companies in the Nordic stock markets. However, liquidity increases if a share is listed on a stock exchange and therefore, the illiquidity discount is much smaller as it is observed from the results of this study. For example, Kemira, a Finnish-based listed company with the market capitalisation of 1,610 million euro and the relative bid-ask spread of 0.15%, has an illiquidity discount of 0.7% based on the equation 44 and 8.9% if the company would not be listed on the stock exchange based on the equation 43.

6.4 Evaluation and further research topics

It is highly important that the results are reliable. To keep the methods open, the data set is shown in the appendices. However, the reliance of this study is a broad concept. First, to solve the research questions, the theoretical framework has been chosen to support the analyses of the empirical methods but also the empirical data is selected carefully from the available databases. Second, the reliance of the study is based on the considered and reasoned interpretation of the literature, methods and data. The literature related to this study is written and studied by well-known academic researchers and the journals are high classified academic research journals. Together the previous studies form a strong base for this study and therefore the plausibility of the results from this study can be believed to be correct and useful. Third, the results are tried to be presented in a logical order so that the interpretations stay also logical.

In the following the plausibility of the study is discussed at a terminology level, concerning the reliability, stability and validity of the study including internal, external and face validity. First, the reliability of the study seems to be quite strong. The signs of regression coefficients remain the same and they are logically and statistically significant through the whole study. The repeatability is hard to test since the number of observations is limited but as seen before, one of the sample confirmed the results for the whole study but the other sample does not. The randomness increases when the number of observations is low and therefore, the results from the holdout samples are valid with discretion. Moreover, the results from this study concern the Nordic stocks markets, especially Finland and Sweden where the number of observations are high. However, the capital market structures in the Nordics are quite similar and therefore the results can be reflected also to illiquidity effects in Denmark, Norway and Iceland.

The stability of the measure of illiquidity is questionable in this study since Loderer and Roth (2005) have proved that the liquidity discount varies across time. Therefore, the results are valid only in the near future and should be updated when the market conditions change.

The validity of the results is high. Previous literature and logical interpretations confirm that the liquidity discount range is measured as in this study and the results are similar. The internal validity of the study is strong also since the results in the studied population are plausible and reflect the population as a whole. However, since the population is still quite limited, the external validity of the results is doubtful. Even if the results might be doubtful, the results reflect the best possible and available data and furthermore, studied population is two times larger than the population studied by Loderer & Roth (2005). The methods of this study are easy to test with other data from different markets and this supports the external validity of this study. Thus, the results can be seen also good to use in practice since better estimations are not available. The face validity of the results is strong since previous literature supports finding of this study. Moreover, based on own experience from capital markets and valuations, the illiquidity discount resulted from this study is consistent and equal with the discounts used before.

Even if this study fulfilled one research gap, at the same time it opened other ones. It would be interesting to know how the illiquidity discount has varied across time in the Nordics and would it be possible to construct a method which takes into account the future expectations related to the illiquidity discount better. Moreover, from qualitative point of view it would be interesting to have some perspective on how much analysts and managers actually use this liquidity discount on own valuations and management decision-making. It would be also valuable to research how the Finnish tax authority treats illiquidity premiums and illiquidity discounts in valuations that realize tax payments afterwards.

7 CONCLUSION

This study researches the illiquidity effects on company valuation, i.e. the purpose of this study is to measure the pricing discount for limited liquidity by using cross-sectional regression based on the observed market information from the Nordic stock markets. Earlier studies have found that the equity value is negatively correlated with the relative bid-ask spread and positively correlated with liquidity characteristics like the number of shareholders, trading volume and the number of market makers trading the share. Moreover, there is evidence that illiquidity can explain differences in expected returns across shares and it has been proposed that over time expected illiquidity has a positive effect on expected stock returns. In corporate finance, liquidity and especially lack of liquidity, has substantial consequences for decision-making and valuation conclusions. In general, limited liquidity can reduce potential equity value more than 50%. Hence, limited liquidity can be seen as a critical element on corporate finance and financial markets overall.

The illiquidity effects are studied by using different cross-sectional regression models, mostly based on Loderer and Roth's (2005) methods. The models include regression arguments for each particular firm. These arguments are expected rate of earnings growth, firm's payout ratio, risk of the share, market value of the firm's equity and share's liquidity. The regression model includes the firm size due to the evidence that risk-adjusted returns are affected by the size of the firm. In this study, the bid-ask spread is the main liquidity proxy based on its previous empirical evidences but also the trading volume is used as a liquidity proxy. The data is analyzed from OMX Helsinki, OMX Stockholm and OMX Copenhagen stock exchanges between 1 January 2013 and 30 June 2016. The sample includes 243 companies, 66 companies from Finland, 142 from Sweden and 35 from Denmark.

The theoretical framework of the study constructs of components of a valuation methodology. First, the cost of capital is studied. Second, income approach valuation methodologies are studied and with these two components a roadmap to valuation is built. After that, the illiquidity effects are researched and taken into account in valuation.

The main finding of this study is that the illiquidity discount in the Nordic stock markets varies between 0.3% and 36.0%. The most liquid shares sell at a 0.3% discount and the least liquid shares sell at a 36.0% discount compared with a purely liquid asset. The discount of 0.3% can reflect transaction costs such as brokerage commissions but the discount of 36.0% includes also price-impact liquidity costs. However, based on the lower quartile and upper quartile of the sample's relative bid-ask spread, the illiquidity discount ranges between 0.6% and 4.8%. The mean illiquidity discount is 3.5% in the sample. In this study the range of the results is line with the previous studies even though the mean and median discounts are smaller. This can be seen as a result of developed financial markets. Moreover, this study shows that the illiquidity discount can be significant for private companies. The discount range varies between 16.5% and 65.3% for private companies with the perfectly liquid equity value of 500,000,000€ and 50,000€, respectively.

In conclusion, the results of this study show that the market are more efficient nowadays based on a lower median illiquidity discount compared with the median discounts of previous studies. However, still liquidity plays an important role in company valuation and decision-making and the results of this study suggest that illiquidity should be taken into account, especially in company valuations in the Nordic environment. This study has also proved that the illiquidity discount would be smaller if a company was listed on a stock exchange, i.e. the risk of the company would be smaller due to increased liquidity and therefore the equity value would be higher. Managers could use this information to maximize the shareholder value.

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APPENDICES

Appendix 1

Ticker	P/E ratio	ln(P/E)	EPSg2016	EPSg2017	Payout ratio	Beta	Size €m	LNSIZE	RELSP	RELSP^2	LNVOLUME	RES-SIZE/RELSP	RES-SIZE/LNVOL
RUTAV.HE	8.0	2.1	-14.6 %	-20.0 %	48.8 %	0.31	43.0	17.6	0.96 %	0.01 %	10.4	-2.7282	-0.0215
AHL1V.HE	37.4	3.6	236.1 %	-12.8 %	157.3 %	0.33	355.3	19.7	0.87 %	0.01 %	11.8	-0.7094	1.2664
VAIAS.HE	18.8	2.9	-4.4 %	31.6 %	63.1 %	0.54	368.6	19.7	0.89 %	0.01 %	12.2	-0.6571	1.0948
PKC1V.HE	75.2	4.3	264.3 %	48.0 %	311.0 %	1.04	378.3	19.8	0.30 %	0.00 %	13.6	-1.2749	0.2730
TLT1V.HE	13.9	2.6	8.1 %	13.1 %	37.7 %	0.43	161.0	18.9	0.91 %	0.01 %	11.1	-1.4634	0.8909
AKTRV.HE	12.6	2.5	5.8 %	0.6 %	69.2 %	0.14	214.2	19.2	5.53 %	0.31 %	9.9	3.9349	1.8770
NDA1V.HE	8.3	2.1	-9.8 %	6.4 %	70.4 %	0.87	35449.1	24.3	0.14 %	0.00 %	16.3	3.0831	3.2538
SRV1V.HE	16.1	2.8	-11.5 %	104.5 %	40.2 %	0.66	198.6	19.1	1.03 %	0.01 %	11.3	-1.1195	0.9799
KCR1V.HE	43.1	3.8	92.0 %	48.9 %	199.6 %	1.00	1347.8	21.0	0.11 %	0.00 %	15.5	-0.2201	0.4419
UNR1V.HE	27.5	3.3	26.8 %	37.9 %	85.3 %	1.47	971.5	20.7	0.30 %	0.00 %	13.9	-0.3366	1.0787
YTY1V.HE	17.3	2.9	-10.8 %	50.9 %	59.1 %	0.71	691.7	20.4	0.15 %	0.00 %	15.3	-0.8372	-0.0818
METSB.HE	11.6	2.4	-9.8 %	32.6 %	43.3 %	1.48	1759.5	21.3	0.37 %	0.00 %	14.3	0.3311	1.4180
LAT1V.HE	16.9	2.8	6.4 %	6.2 %	86.0 %	0.48	625.0	20.3	0.39 %	0.00 %	13.2	-0.6764	1.0150
RAP1V.HE	25.1	3.2	79.4 %	17.0 %	85.9 %	0.48	170.3	19.0	1.06 %	0.01 %	10.8	-1.2391	1.1498
INVEST.HE	5.2	1.7	-39.8 %	1.5 %	15.5 %	0.02	19.8	16.8	3.13 %	0.10 %	10.3	-1.1007	-0.7165
OKDBV.HE	16.3	2.8	4.5 %	7.4 %	51.9 %	0.38	527.6	20.1	0.73 %	0.01 %	12.9	-0.4699	1.0492
RMR1V.HE	19.0	2.9	18.5 %	15.0 %	110.6 %	0.80	665.8	20.3	0.29 %	0.00 %	13.9	-0.7277	0.6744
TPS1V.HE	9.3	2.2	31.0 %	0.0 %	44.5 %	0.82	390.1	19.8	0.55 %	0.00 %	13.0	-0.9713	0.6608
ATRAV.HE	18.2	2.9	67.4 %	16.2 %	81.6 %	0.98	164.7	18.9	0.63 %	0.00 %	11.7	-1.7477	0.5681
AKTAV.HE	10.5	2.4	5.8 %	0.6 %	69.2 %	0.55	424.1	19.9	0.49 %	0.00 %	12.5	-0.9600	1.0400
AMEAS.HE	23.8	3.2	27.3 %	16.0 %	53.3 %	0.73	3006.2	21.8	0.11 %	0.00 %	15.5	0.5829	1.2692
ALN1V.HE	29.1	3.4	85.2 %	40.3 %	92.6 %	0.30	280.8	19.5	1.02 %	0.01 %	11.5	-0.7825	1.2309
KEMIRA.HE	22.9	3.1	44.0 %	23.0 %	113.8 %	0.88	1610.2	21.2	0.15 %	0.00 %	15.0	0.0019	0.9380
STERV.HE	7.0	1.9	-28.6 %	6.2 %	32.3 %	1.42	4613.1	22.3	0.10 %	0.00 %	17.0	1.0019	0.8095
AFE1V.HE	9.9	2.3	24.6 %	11.8 %	58.6 %	0.80	64.7	18.0	0.94 %	0.01 %	11.2	-2.3419	-0.0734
REG1V.HE	40.0	3.7	19.9 %	35.4 %	89.7 %	1.08	201.0	19.1	0.69 %	0.00 %	12.1	-1.4815	0.5492
SCL1V.HE	22.5	3.1	-9.6 %	184.6 %	55.6 %	1.22	211.1	19.2	1.47 %	0.02 %	11.0	-0.5742	1.2498
CGCBV.HE	16.5	2.8	18.5 %	8.4 %	36.2 %	1.60	1717.2	21.3	0.13 %	0.00 %	15.6	0.0453	0.6347
SDA1V.HE	4.9	1.6	-52.0 %	-12.0 %	38.6 %	0.71	1266.4	21.0	0.33 %	0.00 %	14.3	-0.0403	1.1128
CRA1V.HE	16.5	2.8	30.6 %	10.9 %	57.9 %	0.98	814.7	20.5	0.24 %	0.00 %	14.3	-0.5843	0.6324
AFAGR.HE	13.2	2.6	-3.4 %	0.0 %	64.4 %	0.07	110.8	18.5	2.66 %	0.07 %	10.5	0.1037	0.8912
ETT1V.HE	15.3	2.7	12.9 %	18.6 %	48.4 %	0.55	109.4	18.5	1.28 %	0.02 %	10.8	-1.4336	0.6945
UPM1V.HE	9.6	2.3	-16.2 %	-3.2 %	43.7 %	1.61	8513.6	22.9	0.10 %	0.00 %	17.1	1.6073	1.3605
OLVAS.HE	23.5	3.2	34.8 %	16.9 %	65.1 %	0.28	409.9	19.8	0.58 %	0.00 %	12.2	-0.8902	1.1799
NESTE.HE	14.7	2.7	24.3 %	-10.6 %	45.8 %	1.18	7398.0	22.7	0.10 %	0.00 %	16.7	1.4677	1.4604
PON1V.HE	15.5	2.7	6.0 %	7.2 %	37.3 %	0.85	565.3	20.2	0.66 %	0.00 %	12.1	-0.4769	1.5662
ELISA.HE	22.6	3.1	6.0 %	5.7 %	91.8 %	0.59	5551.7	22.4	0.10 %	0.00 %	16.2	1.1813	1.4533
RAIVV.HE	17.4	2.9	15.6 %	2.9 %	71.8 %	0.31	548.4	20.1	0.51 %	0.00 %	13.0	-0.6720	1.0010
CTY1S.HE	14.6	2.7	37.5 %	-8.9 %	53.6 %	0.97	1944.3	21.4	0.41 %	0.00 %	14.0	0.4782	1.6909
MEO1V.HE	7.1	2.0	-64.0 %	13.6 %	35.6 %	1.23	3075.3	21.8	0.08 %	0.00 %	16.7	0.5685	0.5847
TELIA1.HE	21.2	3.1	48.8 %	11.2 %	164.9 %	0.67	18376.9	23.6	0.17 %	0.00 %	15.5	2.4541	3.0509
ORNBV.HE	23.5	3.2	0.0 %	2.0 %	87.9 %	0.66	3148.2	21.9	0.10 %	0.00 %	15.8	0.6137	1.1097
SIILI.HE	18.1	2.9	43.9 %	18.6 %	56.9 %	0.49	52.7	17.8	1.60 %	0.03 %	11.0	-1.8134	-0.1241
FIS1V.HE	15.7	2.8	-50.4 %	60.2 %	67.4 %	0.30	1406.0	21.1	0.40 %	0.00 %	12.9	0.1454	2.0385
CTL1V.HE	43.4	3.8	77.6 %	27.3 %	72.7 %	0.60	167.9	18.9	1.68 %	0.03 %	11.8	-0.5714	0.5567
ASPO.HE	9.9	2.3	-21.8 %	21.1 %	63.3 %	0.84	213.7	19.2	0.52 %	0.00 %	11.6	-1.6045	0.8839
TIE1V.HE	19.9	3.0	17.5 %	7.1 %	109.7 %	0.63	1760.8	21.3	0.16 %	0.00 %	14.7	0.1020	1.1597
WRT1V.HE	17.1	2.8	5.2 %	8.6 %	56.1 %	0.56	7549.1	22.7	0.07 %	0.00 %	16.7	1.4637	1.4794
NRE1V.HE	17.8	2.9	-0.7 %	9.3 %	83.3 %	1.24	4192.6	22.2	0.07 %	0.00 %	16.8	0.8756	0.7799
HUH1V.HE	26.0	3.3	33.5 %	6.4 %	46.2 %	1.09	3612.3	22.0	0.13 %	0.00 %	15.4	0.7932	1.4833
EQV1V.HE	19.7	3.0	24.5 %	8.5 %	105.2 %	0.76	215.2	19.2	1.36 %	0.02 %	11.3	-0.6749	1.0672
SUY1V.HE	13.4	2.6	20.0 %	11.1 %	33.3 %	0.82	224.9	19.2	1.72 %	0.03 %	12.3	-0.2291	0.5357
PNA1V.HE	41.3	3.7	288.9 %	25.0 %	243.1 %	0.60	46.0	17.6	1.76 %	0.03 %	10.0	-1.7722	0.3334
KESBV.HE	37.2	3.6	71.6 %	19.5 %	243.9 %	0.65	2460.1	21.6	0.09 %	0.00 %	15.7	0.3627	0.9316
KNEBV.HE	20.6	3.0	-4.6 %	3.6 %	69.9 %	0.86	18042.0	23.6	0.07 %	0.00 %	17.3	2.3300	1.9643
MARAS.HE	10.3	2.3	27.9 %	23.1 %	41.0 %	0.11	19.0	16.8	1.44 %	0.02 %	9.8	-3.0139	-0.4478
FSC1V.HE	34.5	3.5	3.3 %	41.7 %	154.9 %	0.61	409.5	19.8	0.82 %	0.01 %	13.0	-0.6306	0.7477
IFA1V.HE	17.3	2.8	26.3 %	33.3 %	0.0 %	1.02	29.8	17.2	1.66 %	0.03 %	10.6	-2.3207	-0.4653
DIG1V.HE	17.0	2.8	-19.6 %	63.3 %	39.4 %	0.70	61.6	17.9	1.40 %	0.02 %	11.3	-1.8799	-0.1753

Ticker	P/E ratio	ln(P/E)	EPSg2016	EPSg2017	Payout ratio	Beta	Size €m	LNSIZE	RELSP	$RELSP^2$	LNVOLUME	RES-SIZE/RELSP	RES-SIZE/LNVOL
LEM1S.HE	41.9	3.7	209.2 %	47.9 %	38.7 %	0.51	310.9	19.6	1.14 %	0.01 %	11.4	-0.5517	1.3760
EXL1V.HE	20.5	3.0	-20.5 %	100.0 %	92.0 %	0.43	70.0	18.1	1.44 %	0.02 %	11.4	-1.7046	-0.1100
CPMBV.HE	17.1	2.8	46.6 %	5.9 %	120.7 %	0.48	76.9	18.2	1.24 %	0.02 %	11.2	-1.8370	0.0966
TIK1V.HE	17.3	2.9	5.5 %	10.1 %	85.1 %	1.01	679.7	20.3	0.33 %	0.00 %	13.6	-0.6617	0.8487
MMO1V.HE	71.8	4.3	175.4 %	65.6 %	352.6 %	0.29	62.9	18.0	1.20 %	0.01 %	10.7	-2.0801	0.2132
ILK2S.HE	13.8	2.6	99.2 %	46.4 %	71.2 %	0.20	43.3	17.6	1.13 %	0.01 %	10.2	-2.5258	0.1077
FIA1S.HE	13.9	2.6	177.8 %	3.3 %	0.0 %	0.77	655.2	20.3	0.69 %	0.00 %	12.7	-0.2961	1.3649
FINGb.ST	32.0	3.5	192.4 %	3.9 %	0.0 %	0.47	3219.1	21.9	0.39 %	0.00 %	19.8	0.9548	-1.2241
SCAb.ST	26.4	3.3	24.4 %	10.8 %	57.7 %	0.84	17304.3	23.6	0.07 %	0.00 %	19.7	2.2921	0.4936
MYCR.ST	13.6	2.6	-33.3 %	46.6 %	88.5 %	1.64	713.4	20.4	0.65 %	0.00 %	16.3	-0.2568	-0.6550
ARP.ST	12.5	2.5	42.5 %	32.1 %	0.0 %	0.89	69.2	18.1	2.64 %	0.07 %	11.7	-0.3933	-0.3156
HOLMb.ST	39.6	3.7	130.0 %	17.9 %	157.8 %	0.82	1734.0	21.3	0.09 %	0.00 %	17.3	0.0152	-0.3935
BEIAb.ST	15.8	2.8	-6.2 %	10.4 %	80.9 %	1.25	568.7	20.2	0.66 %	0.00 %	14.7	-0.4707	0.0367
FAG.ST	23.6	3.2	19.6 %	11.8 %	45.9 %	0.71	698.3	20.4	1.42 %	0.02 %	13.8	0.5708	0.8079
ELOSSb.ST	34.7	3.5	103.2 %	54.9 %	37.6 %	0.72	53.3	17.8	1.49 %	0.02 %	12.9	-1.9233	-1.2728
SWECb.ST	33.3	3.5	40.1 %	29.6 %	81.6 %	0.78	1536.9	21.2	0.68 %	0.00 %	15.4	0.5364	0.6278
BORG.ST	17.8	2.9	18.0 %	44.0 %	0.0 %	0.99	82.0	18.2	0.89 %	0.01 %	14.3	-2.1581	-1.6270
SAS.ST	9.7	2.3	139.7 %	-15.0 %	0.0 %	0.72	798.7	20.5	0.58 %	0.00 %	16.7	-0.2289	-0.8084
IFSb.ST	42.1	3.7	75.2 %	23.2 %	38.0 %	0.98	941.3	20.7	0.59 %	0.00 %	16.4	-0.0505	-0.4449
BIOGb.ST	28.1	3.3	-2.5 %	10.3 %	62.9 %	0.73	394.4	19.8	0.60 %	0.00 %	15.3	-0.9114	-0.6785
SWOLb.ST	32.7	3.5	125.9 %	44.2 %	68.1 %	0.34	117.4	18.6	1.31 %	0.02 %	13.5	-1.3299	-0.8262
VICPA.ST	3.4	1.2	-83.2 %	12.5 %	3.6 %	0.83	143.8	18.8	1.53 %	0.02 %	13.9	-0.8869	-0.8693
NEWAb.ST	16.9	2.8	54.1 %	32.0 %	46.2 %	1.69	173.6	19.0	0.62 %	0.00 %	15.0	-1.7019	-1.3246
BEIJb.ST	22.1	3.1	11.8 %	13.4 %	60.8 %	0.76	816.1	20.5	1.18 %	0.01 %	14.6	0.4628	0.4753
GHP.ST	24.1	3.2	18.4 %	40.5 %	45.5 %	0.33	58.7	17.9	1.87 %	0.04 %	13.1	-1.4046	-1.2655
NDA.ST	8.3	2.1	-8.1 %	4.9 %	70.4 %	1.12	35440.9	24.3	0.08 %	0.00 %	20.4	3.0168	0.7924
SEBa.ST	9.5	2.2	-33.8 %	51.3 %	69.8 %	0.86	18734.9	23.7	0.08 %	0.00 %	20.0	2.3802	0.4207
SHBb.ST	12.6	2.5	-2.4 %	2.7 %	73.0 %	1.08	420.9	19.9	0.26 %	0.00 %	15.6	-1.2138	-0.7800
ANODb.ST	15.7	2.8	-14.6 %	17.0 %	70.7 %	0.74	171.4	19.0	1.52 %	0.02 %	13.2	-0.7207	-0.2748
FPAR.ST	5.6	1.7	-34.6 %	-34.9 %	16.6 %	1.03	769.2	20.5	1.34 %	0.02 %	13.9	0.5823	0.8022
AFb.ST	18.0	2.9	10.3 %	6.4 %	49.1 %	0.74	1131.7	20.8	0.40 %	0.00 %	16.3	-0.0689	-0.2095
ARCM.ST	171.8	5.1	105.1 %	71.2 %	0.0 %	0.58	401.1	19.8	0.51 %	0.00 %	17.0	-0.9892	-1.6321
TRELb.ST	18.8	2.9	198.7 %	-53.6 %	51.8 %	1.32	3920.8	22.1	0.10 %	0.00 %	18.6	0.8388	-0.3202
VITb.ST	23.5	3.2	-0.8 %	9.9 %	33.8 %	0.72	176.6	19.0	0.94 %	0.01 %	13.7	-1.3393	-0.5556
SYSR.ST	16.7	2.8	-29.7 %	25.6 %	33.6 %	0.77	577.9	20.2	0.76 %	0.01 %	14.7	-0.3535	0.0778
ITABb.ST	20.1	3.0	0.6 %	10.4 %	49.4 %	1.54	692.1	20.4	0.86 %	0.01 %	14.9	-0.0597	0.1500
AZN.ST	26.2	3.3	-14.4 %	0.2 %	125.4 %	0.58	66920.4	24.9	0.08 %	0.00 %	19.5	3.6492	2.0071
SHBa.ST	12.1	2.5	-3.9 %	0.2 %	73.0 %	0.90	21506.6	23.8	0.07 %	0.00 %	19.8	2.5034	0.6791
KLOVa.ST	5.1	1.6	-9.9 %	-39.5 %	19.1 %	1.11	77.2	18.2	0.63 %	0.00 %	15.6	-2.5095	-2.4769
UNIBsdb.ST	25.4	3.2	38.2 %	10.3 %	98.2 %	0.27	2304.6	21.6	0.27 %	0.00 %	17.5	0.4989	-0.2165
AVANZ.ST	22.4	3.1	-3.5 %	15.8 %	74.5 %	1.24	1054.6	20.8	0.39 %	0.00 %	16.3	-0.1572	-0.2950
HEXAb.ST	23.4	3.2	15.5 %	9.4 %	31.0 %	1.08	11250.2	23.1	0.08 %	0.00 %	18.8	1.8719	0.5903
SWECa.ST	34.6	3.5	40.1 %	29.6 %	81.6 %	0.44	149.9	18.8	4.78 %	0.23 %	11.1	2.7405	0.8483
CONIC.ST	14.6	2.7	-9.4 %	11.1 %	50.5 %	1.51	424.9	19.9	0.61 %	0.00 %	15.7	-0.8227	-0.8144
LAGRb.ST	26.2	3.3	20.2 %	8.5 %	50.2 %	1.00	526.8	20.1	0.84 %	0.01 %	14.4	-0.3497	0.1270
ORIFL.ST	36.6	3.6	83.3 %	20.4 %	64.2 %	1.08	903.7	20.6	0.26 %	0.00 %	17.3	-0.4501	-1.0101
SECTb.ST	36.4	3.6	-1.5 %	12.4 %	0.0 %	1.20	414.7	19.8	1.07 %	0.01 %	14.0	-0.3389	0.1215
UFLXb.ST	11.6	2.5	88.6 %	-3.7 %	96.5 %	0.18	31.6	17.3	2.12 %	0.05 %	12.8	-1.7482	-1.7408
SAGAa.ST	8.0	2.1	-41.3 %	1.4 %	11.7 %	1.19	102.3	18.4	2.35 %	0.06 %	12.6	-0.3174	-0.4076
AVEGb.ST	11.2	2.4	46.5 %	-13.8 %	67.7 %	0.16	20.8	16.9	1.27 %	0.02 %	12.9	-3.1073	-2.1830
RATOb.ST	30.9	3.4	192.3 %	3.4 %	251.0 %	0.75	1197.5	20.9	0.13 %	0.00 %	17.5	-0.3136	-0.8715
HIQ.ST	18.1	2.9	12.8 %	6.8 %	0.0 %	0.32	276.0	19.4	0.54 %	0.00 %	15.0	-1.3310	-0.8758
DUNI.ST	14.3	2.7	-1.3 %	10.7 %	67.9 %	0.42	617.3	20.2	0.53 %	0.00 %	15.3	-0.5379	-0.2509
LUC.ST	13.0	2.6	81.8 %	-43.4 %	14.1 %	1.19	763.5	20.5	0.66 %	0.00 %	15.9	-0.1813	-0.3816
ADDTb.ST	22.1	3.1	9.8 %	4.2 %	68.8 %	1.05	772.7	20.5	0.68 %	0.00 %	15.3	-0.1505	-0.0205
NNb.ST	12.4	2.5	-16.5 %	23.0 %	63.6 %	1.04	588.6	20.2	0.63 %	0.00 %	15.5	-0.4770	-0.3991
SKISb.ST	19.6	3.0	24.8 %	-1.9 %	63.1 %	0.04	473.2	20.0	0.59 %	0.00 %	14.9	-0.7343	-0.2368
ALIVsdb.ST	20.2	3.0	29.2 %	12.4 %	43.3 %	1.02	7091.1	22.7	0.11 %	0.00 %	19.0	1.4373	0.0042
ELUXb.ST	41.2	3.7	173.4 %	13.1 %	119.9 %	1.08	6736.8	22.6	0.07 %	0.00 %	19.5	1.3439	-0.3426
KNOW.ST	12.9	2.6	34.2 %	7.9 %	70.9 %	0.46	120.9	18.6	0.99 %	0.01 %	14.3	-1.6592	-1.2578
BBTOb.ST	15.4	2.7	18.4 %	0.4 %	36.8 %	0.79	434.1	19.9	0.62 %	0.00 %	15.4	-0.7900	-0.6377
NCCb.ST	15.6	2.7	62.4 %	-21.9 %	24.6 %	0.75	1865.1	21.3	0.09 %	0.00 %	18.2	0.0812	-0.8383
SKAb.ST	14.9	2.7	6.3 %	-0.6 %	65.0 %	1.09	7449.3	22.7	0.09 %	0.00 %	19.1	1.4663	0.0112
LIAB.ST	15.8	2.8	13.3 %	13.4 %	31.3 %	0.83	543.8	20.1	0.27 %	0.00 %	16.3	-0.9547	-0.9327
MBPH.ST	20.3	3.0	-73.9 %	-42.6 %	0.0 %	1.11	74.5	18.1	1.06 %	0.01 %	14.5	-2.0664	-1.8905
MSONb.ST	12.4	2.5	35.8 %	13.6 %	40.7 %	0.47	100.9	18.4	1.84 %	0.03 %	13.3	-0.8991	-0.8655
SKFb.ST	15.4	2.7	16.6 %	11.4 %	64.5 %	0.93	6378.3	22.6	0.08 %	0.00 %	19.8	1.2984	-0.5641
NOLAb.ST	13.6	2.6	-10.5 %	8.5 %	62.6 %	0.51	557.8	20.1	0.43 %	0.00 %	15.8	-0.7431	-0.6078
HPOLb.ST	20.8	3.0	-1.4 %	5.8 %	42.0 %	1.13	2964.6	21.8	0.17 %	0.00 %	17.6	0.6303	0.0156

Ticker	P/E ratio	ln(P/E)	EPSg2016	EPSg2017	Payout ratio	Beta	Size €m	LNSIZE	RELSP	RELSP^2	LNVOLUME	RES-SIZE/RELSP	RES-SIZE/LNVOL
INDT.ST	21.8	3.1	6.7 %	10.5 %	40.3 %	1.10	2034.8	21.4	0.33 %	0.00 %	16.9	0.4325	0.0014
SAABb.ST	20.0	3.0	1.7 %	24.3 %	39.1 %	0.52	3065.1	21.8	0.14 %	0.00 %	17.4	0.6341	0.1377
GETIb.ST	28.9	3.4	20.7 %	40.8 %	48.0 %	0.94	4376.1	22.2	0.08 %	0.00 %	18.9	0.9237	-0.3697
HUFVa.ST	7.7	2.0	-50.8 %	-33.1 %	18.4 %	0.83	2714.0	21.7	0.13 %	0.00 %	17.1	0.4992	0.1968
INVEb.ST	12.0	2.5	0.6 %	-6.2 %	43.8 %	1.31	13993.4	23.4	0.06 %	0.00 %	19.5	2.0661	0.4178
AAK.ST	26.4	3.3	7.1 %	13.3 %	35.0 %	1.29	2709.6	21.7	0.21 %	0.00 %	16.9	0.5898	0.2992
INVEa.ST	11.9	2.5	0.6 %	-6.2 %	43.8 %	1.28	9428.9	23.0	0.15 %	0.00 %	16.9	1.7646	1.5849
NETb.ST	52.1	4.0	31.0 %	19.7 %	0.0 %	0.60	1772.7	21.3	0.38 %	0.00 %	16.7	0.3567	-0.0138
VOLVb.ST	10.9	2.4	-12.1 %	10.5 %	40.5 %	1.30	15139.0	23.4	0.07 %	0.00 %	20.4	2.1588	-0.0418
TELIA.ST	21.3	3.1	59.9 %	4.7 %	164.9 %	0.69	18369.4	23.6	0.07 %	0.00 %	20.0	2.3456	0.3950
BEFsdb.ST	5.7	1.7	-15.2 %	64.5 %	0.0 %	1.03	86.1	18.3	1.02 %	0.01 %	14.4	-1.9622	-1.6719
HLDX.ST	18.3	2.9	13.9 %	19.0 %	46.8 %	1.38	332.6	19.6	0.41 %	0.00 %	16.7	-1.2917	-1.6518
IJ.ST	16.1	2.8	7.0 %	5.5 %	51.8 %	0.51	2162.1	21.5	0.16 %	0.00 %	17.7	0.3084	-0.4014
CORE.ST	2.9	1.1	-57.5 %	-1.6 %	15.0 %	0.92	226.0	19.2	2.02 %	0.04 %	13.9	0.1019	-0.3891
MTGb.ST	29.2	3.4	-4.0 %	126.3 %	154.5 %	1.15	1546.0	21.2	0.07 %	0.00 %	18.2	-0.1224	-1.0112
ERICb.ST	15.2	2.7	-10.9 %	28.9 %	89.6 %	0.99	23957.8	23.9	0.08 %	0.00 %	20.4	2.6196	0.4242
SWEDa.ST	12.1	2.5	13.4 %	-3.3 %	75.7 %	0.85	21247.7	23.8	0.07 %	0.00 %	20.1	2.4936	0.4752
BILL.ST	14.0	2.6	-3.8 %	9.1 %	48.6 %	1.30	2927.1	21.8	0.14 %	0.00 %	17.8	0.5911	-0.1418
ECEX.ST	25.5	3.2	-71.8 %	30.2 %	35.2 %	0.62	169.3	18.9	0.75 %	0.01 %	14.8	-1.5878	-1.2061
NIBEb.ST	24.3	3.2	4.3 %	11.9 %	29.9 %	1.02	2841.2	21.8	0.18 %	0.00 %	17.1	0.6004	0.2625
CVTEC.ST	30.4	3.4	21.7 %	42.1 %	35.3 %	0.16	193.4	19.1	1.52 %	0.02 %	14.0	-0.5978	-0.6259
HMb.ST	19.2	3.0	-6.1 %	13.7 %	77.2 %	1.00	43019.7	24.5	0.06 %	0.00 %	20.5	3.1827	0.9695
BILIa.ST	14.1	2.6	-11.5 %	1.3 %	52.2 %	1.28	986.9	20.7	0.40 %	0.00 %	16.4	-0.2061	-0.3825
JM.ST	14.4	2.7	26.0 %	10.7 %	57.7 %	1.21	1711.6	21.3	0.14 %	0.00 %	18.0	0.0533	-0.8273
WALLb.ST	8.1	2.1	-58.9 %	-12.3 %	18.2 %	0.96	2183.1	21.5	0.19 %	0.00 %	16.6	0.3550	0.2932
PEABb.ST	23.0	3.1	93.3 %	3.7 %	96.1 %	1.22	1874.7	21.4	0.14 %	0.00 %	17.1	0.1388	-0.1733
ALFA.ST	14.1	2.6	-16.3 %	-6.1 %	46.4 %	1.03	5989.6	22.5	0.09 %	0.00 %	19.1	1.2490	-0.2129
TIEN.ST	20.0	3.0	15.8 %	7.7 %	109.7 %	0.62	1760.5	21.3	1.17 %	0.01 %	14.3	1.2211	1.4260
STEa.ST	7.8	2.1	-28.4 %	5.5 %	32.3 %	1.54	593.8	20.2	1.67 %	0.03 %	13.3	0.6818	0.8908
OPUS.ST	22.5	3.1	-27.2 %	58.0 %	43.1 %	0.61	151.3	18.8	0.78 %	0.01 %	15.9	-1.6644	-1.9668
ICAA.ST	15.1	2.7	-9.4 %	1.6 %	54.9 %	0.51	5973.4	22.5	0.12 %	0.00 %	18.4	1.2758	0.2379
FABG.ST	7.1	2.0	-25.3 %	-48.4 %	17.9 %	0.88	2385.2	21.6	0.14 %	0.00 %	17.3	0.3870	-0.0235
AXFO.ST	24.3	3.2	9.2 %	4.2 %	138.8 %	0.60	3367.9	21.9	0.14 %	0.00 %	17.2	0.7286	0.3324
ATCOa.ST	22.1	3.1	8.6 %	7.4 %	65.5 %	1.33	18202.6	23.6	0.07 %	0.00 %	20.1	2.3371	0.3367
CAST.ST	7.7	2.0	-38.0 %	-15.0 %	27.9 %	0.63	3253.9	21.9	0.12 %	0.00 %	17.7	0.6680	0.0124
SAND.ST	29.4	3.4	62.8 %	12.5 %	89.7 %	1.22	10716.8	23.1	0.08 %	0.00 %	20.0	1.8172	-0.1636
SECUb.ST	18.9	2.9	5.5 %	8.6 %	52.4 %	1.08	4734.7	22.3	0.10 %	0.00 %	18.6	1.0233	-0.1385
KAHL.ST	25.7	3.2	90.5 %	12.6 %	51.4 %	0.60	264.1	19.4	0.48 %	0.00 %	15.9	-1.4452	-1.3980
TEL2b.ST	25.4	3.2	-29.3 %	77.2 %	189.4 %	0.81	3430.6	22.0	0.09 %	0.00 %	19.1	0.6918	-0.7477
LOOMb.ST	14.1	2.6	5.5 %	4.7 %	49.3 %	0.94	1793.9	21.3	0.26 %	0.00 %	17.2	0.2277	-0.2607
HMSN.ST	40.4	3.7	36.7 %	39.9 %	46.9 %	0.52	287.4	19.5	1.50 %	0.02 %	13.7	-0.2322	-0.0631
DORO.ST	21.5	3.1	50.6 %	0.0 %	0.0 %	0.59	163.5	18.9	0.48 %	0.00 %	15.3	-1.9189	-1.5388
ACANb.ST	12.1	2.5	16.1 %	4.0 %	94.4 %	0.61	188.9	19.1	0.94 %	0.01 %	14.5	-1.2652	-0.9582
DIOS.ST	8.3	2.1	-4.4 %	1.7 %	40.2 %	1.18	473.4	20.0	0.49 %	0.00 %	15.9	-0.8481	-0.8381
NOBI.ST	14.5	2.7	17.3 %	9.3 %	51.0 %	1.04	1635.6	21.2	0.41 %	0.00 %	16.8	0.3098	-0.1326
ATRLJb.ST	6.4	1.9	-67.1 %	26.3 %	17.0 %	0.65	1832.0	21.3	0.41 %	0.00 %	15.8	0.4160	0.5562
SWMA.ST	19.7	3.0	21.5 %	-8.4 %	55.2 %	0.62	5524.0	22.4	0.07 %	0.00 %	19.1	1.1429	-0.2503
MEKO.ST	15.1	2.7	4.5 %	15.4 %	59.4 %	0.98	711.8	20.4	0.39 %	0.00 %	16.4	-0.5463	-0.7212
ASSAb.ST	24.3	3.2	-0.4 %	12.5 %	38.3 %	0.95	18747.1	23.7	0.07 %	0.00 %	19.5	2.3652	0.6820
ENEA.ST	14.1	2.6	1.2 %	7.0 %	0.0 %	1.81	149.3	18.8	1.15 %	0.01 %	13.7	-1.2714	-0.6807
WIHL.ST	5.7	1.7	-57.9 %	7.4 %	17.7 %	0.97	1380.3	21.0	0.31 %	0.00 %	16.7	0.0228	-0.2172
MQH.ST	9.5	2.2	-16.5 %	29.7 %	51.3 %	0.91	153.3	18.8	0.81 %	0.01 %	14.9	-1.6187	-1.4073
CLOEb.ST	21.0	3.0	3.4 %	23.8 %	37.1 %	0.54	800.6	20.5	0.65 %	0.00 %	16.6	-0.1432	-0.7125
GUNN.ST	18.9	2.9	48.3 %	18.7 %	46.0 %	1.17	346.8	19.7	0.56 %	0.00 %	15.0	-1.0759	-0.6073
BOL.ST	16.5	2.8	6.7 %	34.5 %	33.7 %	1.57	3949.8	22.1	0.09 %	0.00 %	19.6	0.8315	-0.8988
RAYb.ST	56.8	4.0	56.1 %	33.1 %	12.2 %	1.26	293.6	19.5	0.90 %	0.01 %	14.8	-0.8686	-0.6935
MEDCAP.ST	50.3	3.9	183.5 %	78.1 %	0.0 %	0.27	44.5	17.6	1.43 %	0.02 %	13.1	-2.1684	-1.5639
BETSb.ST	11.4	2.4	-2.9 %	8.1 %	0.0 %	0.74	1510.3	21.1	0.24 %	0.00 %	17.8	0.0378	-0.8076
PROB.ST	45.0	3.8	55.1 %	14.5 %	18.6 %	0.61	149.9	18.8	1.15 %	0.01 %	13.7	-1.2715	-0.6906
BIOT.ST	25.4	3.2	8.1 %	4.0 %	110.4 %	0.75	185.6	19.0	1.23 %	0.02 %	14.3	-0.9608	-0.8075
NMAN.ST	15.0	2.7	-4.1 %	10.0 %	38.4 %	0.83	265.1	19.4	1.97 %	0.04 %	13.4	0.2145	0.0491
BALDb.ST	7.2	2.0	-52.8 %	-13.4 %	0.0 %	1.10	3461.3	22.0	0.30 %	0.00 %	17.2	0.9364	0.3878
CONSb.ST	15.7	2.8	77.0 %	20.5 %	35.6 %	0.52	104.4	18.5	1.38 %	0.02 %	13.7	-1.3728	-1.0709
KLED.ST	19.3	3.0	113.2 %	-6.2 %	72.1 %	1.09	1107.3	20.8	0.43 %	0.00 %	16.7	-0.0612	-0.4436
RROS.ST	4.1	1.4	-65.6 %	7.7 %	34.2 %	1.38	106.7	18.5	1.19 %	0.01 %	14.7	-1.5586	-1.6174
MSONa.ST	19.1	2.9	6.6 %	8.6 %	40.7 %	0.99	1.6	14.3	9.02 %	0.81 %	10.2	2.8827	-3.1605
DGCO.ST	21.9	3.1	5.6 %	21.3 %	94.8 %	0.27	118.8	18.6	1.29 %	0.02 %	12.2	-1.3417	-0.0296
ABB.ST	21.3	3.1	4.5 %	22.6 %	0.0 %	0.78	38047.7	24.4	0.08 %	0.00 %	19.5	3.0853	1.4114
BULTEN.ST	14.2	2.7	26.1 %	9.4 %	58.0 %	1.48	174.0	19.0	0.80 %	0.01 %	15.4	-1.5050	-1.5369

Ticker	P/E ratio	ln(P/E)	EPSg2016	EPSg2017	Payout ratio	Beta	Size €m	LNSIZE	RELSP	$RELSP^2$	LNVOLUME	RES-SIZE/RELSP	RES-SIZE/LNVOL
REJLb.ST	21.6	3.1	90.8 %	26.4 %	50.5 %	0.55	121.2	18.6	1.11 %	0.01 %	13.3	-1.5287	-0.6784
EKTAb.ST	46.9	3.8	0.6 %	43.8 %	34.5 %	0.93	2478.5	21.6	0.10 %	0.00 %	18.9	0.3832	-0.9297
CLASb.ST	19.8	3.0	-9.6 %	19.6 %	72.5 %	0.71	970.8	20.7	0.37 %	0.00 %	16.2	-0.2660	-0.3116
LAMMb.ST	13.8	2.6	61.9 %	25.2 %	53.2 %	0.91	34.3	17.4	2.21 %	0.05 %	12.4	-1.5671	-1.3977
REZT.ST	18.8	2.9	17.2 %	38.8 %	35.4 %	1.34	617.7	20.2	0.64 %	0.00 %	15.7	-0.4149	-0.4756
FPIP.ST	47.5	3.9	70.5 %	44.1 %	57.8 %	0.31	47.8	17.7	1.15 %	0.01 %	13.6	-2.4053	-1.7724
BMAX.ST	14.7	2.7	-1.3 %	15.4 %	50.0 %	1.00	448.6	19.9	0.50 %	0.00 %	16.3	-0.8858	-1.1532
VITR.ST	53.9	4.0	15.4 %	30.7 %	28.5 %	0.69	801.8	20.5	0.66 %	0.00 %	15.7	-0.1296	-0.1692
IFSa.ST	43.1	3.8	75.2 %	23.2 %	38.0 %	0.29	40.3	17.5	9.20 %	0.85 %	14.1	6.3191	-2.2443
HUSQb.ST	18.6	2.9	11.2 %	19.0 %	50.3 %	0.91	2944.3	21.8	0.11 %	0.00 %	18.2	0.5562	-0.3851
CCORb.ST	4.2	1.4	-1.0 %	-34.8 %	13.7 %	0.88	79.7	18.2	1.21 %	0.01 %	13.8	-1.8371	-1.3586
SAS.CO	9.8	2.3	131.7 %	-15.7 %	0.0 %	0.89	799.1	20.5	0.81 %	0.01 %	15.8	0.0345	-0.2605
JYSK.CO	9.7	2.3	-5.3 %	30.5 %	20.1 %	0.38	3593.7	22.0	0.10 %	0.00 %	18.0	0.7508	-0.0660
PAALb.CO	8.1	2.1	-14.6 %	22.1 %	16.7 %	0.65	514.2	20.1	0.56 %	0.00 %	15.4	-0.6913	-0.4632
HHDC.CO	12.3	2.5	29.7 %	46.3 %	0.0 %	1.64	110.1	18.5	1.21 %	0.01 %	14.1	-1.5093	-1.2518
SYDB.CO	10.6	2.4	21.5 %	10.8 %	70.2 %	0.74	1818.6	21.3	0.14 %	0.00 %	17.4	0.1152	-0.3906
NDA.CO	8.3	2.1	-8.0 %	4.2 %	70.4 %	0.95	35418.4	24.3	0.11 %	0.00 %	18.1	3.0535	2.1730
SPNO.CO	7.5	2.0	-24.9 %	21.3 %	41.9 %	0.88	945.9	20.7	0.79 %	0.01 %	15.8	0.1734	-0.0867
DANSKE.CO	13.9	2.6	42.9 %	5.9 %	63.2 %	0.74	24183.8	23.9	0.10 %	0.00 %	19.6	2.6605	0.8920
MAERSKb.CO	34.4	3.5	110.9 %	42.9 %	118.9 %	1.07	11676.4	23.2	0.12 %	0.00 %	19.6	1.9539	0.2042
MTb.CO	14.3	2.7	53.3 %	5.1 %	44.2 %	0.51	141.1	18.8	1.71 %	0.03 %	13.2	-0.7035	-0.4521
NOVOb.CO	26.6	3.3	14.3 %	11.1 %	47.3 %	1.43	97841.0	25.3	0.06 %	0.00 %	20.5	4.0117	1.7556
ALMB.CO	14.8	2.7	18.2 %	2.5 %	99.0 %	1.27	1076.7	20.8	0.56 %	0.00 %	15.7	0.0508	0.1190
DFDS.CO	17.8	2.9	35.2 %	6.8 %	18.2 %	0.60	2108.8	21.5	0.36 %	0.00 %	16.4	0.4993	0.3289
SCHO.CO	13.6	2.6	1.0 %	10.7 %	36.5 %	0.59	1323.5	21.0	0.45 %	0.00 %	15.7	0.1363	0.3077
HOEJb.CO	12.9	2.6	17.1 %	35.1 %	0.0 %	0.63	52.9	17.8	1.75 %	0.03 %	12.7	-1.6428	-1.1516
VWS.CO	19.8	3.0	13.7 %	1.4 %	29.7 %	1.19	13348.4	23.3	0.09 %	0.00 %	20.0	2.0456	0.0887
DSV.CO	23.5	3.2	-30.8 %	61.1 %	14.2 %	0.20	7033.9	22.7	0.10 %	0.00 %	18.4	1.4160	0.3524
RILBA.CO	13.7	2.6	15.1 %	10.6 %	29.9 %	0.44	848.7	20.6	0.49 %	0.00 %	14.9	-0.2669	0.3444
ROCKb.CO	38.7	3.7	61.4 %	24.4 %	36.7 %	0.59	1536.7	21.2	0.20 %	0.00 %	16.9	0.0095	-0.2565
GN.CO	23.5	3.2	48.6 %	13.0 %	19.3 %	0.76	2694.4	21.7	0.12 %	0.00 %	18.3	0.4831	-0.5033
NZYMb.CO	35.2	3.6	12.3 %	7.7 %	38.4 %	1.20	10353.2	23.1	0.08 %	0.00 %	18.8	1.7899	0.5507
WDH.CO	24.5	3.2	17.9 %	11.6 %	0.0 %	0.42	4574.2	22.2	0.11 %	0.00 %	17.9	1.0058	0.2664
TOP.CO	13.5	2.6	-1.0 %	11.5 %	0.0 %	0.41	2370.2	21.6	0.11 %	0.00 %	17.6	0.3440	-0.2414
TRYG.CO	17.7	2.9	12.9 %	8.8 %	89.1 %	0.66	4847.8	22.3	0.12 %	0.00 %	17.9	1.0688	0.2862
SOLARb.CO	18.2	2.9	25.9 %	21.8 %	58.6 %	0.17	323.5	19.6	0.58 %	0.00 %	15.1	-1.1266	-0.7262
SIM.CO	34.3	3.5	-3.0 %	9.0 %	54.7 %	0.70	1771.4	21.3	0.38 %	0.00 %	16.5	0.3530	0.1015
FLS.CO	19.5	3.0	0.2 %	25.6 %	32.7 %	0.75	1696.9	21.3	0.09 %	0.00 %	18.5	-0.0116	-1.0941
PNDORA.CO	29.7	3.4	64.3 %	20.2 %	42.4 %	0.89	13884.6	23.4	0.10 %	0.00 %	19.6	2.0975	0.3622
OSSR.CO	31.4	3.4	13.8 %	13.3 %	15.2 %	0.40	1438.6	21.1	1.20 %	0.01 %	14.6	1.0536	1.0135
CHRH.CO	48.1	3.9	15.6 %	17.4 %	123.4 %	0.48	7369.9	22.7	0.09 %	0.00 %	18.3	1.4529	0.5135
IC.CO	18.5	2.9	28.1 %	13.8 %	43.0 %	0.36	467.7	20.0	0.72 %	0.01 %	14.6	-0.6028	-0.0992
ALKb.CO	33.6	3.5	-23.4 %	41.3 %	14.3 %	0.59	1280.8	21.0	0.38 %	0.00 %	16.0	0.0311	0.0645
RBREW.CO	23.1	3.1	9.4 %	7.5 %	55.7 %	0.38	2094.7	21.5	0.25 %	0.00 %	16.8	0.3802	0.1180
AMBUb.CO	90.5	4.5	58.0 %	27.4 %	31.0 %	0.14	1285.7	21.0	0.64 %	0.00 %	15.7	0.3188	0.2491
HARBb.CO	25.1	3.2	46.5 %	13.5 %	38.8 %	0.59	82.1	18.2	1.27 %	0.02 %	13.4	-1.7369	-1.1330

Appendix 2

Ticker	P/E ratio	ln(P/E)	EPSg2016	EPSg2017	Payout ratio	Beta	Size	LNSIZE	RELSP	RELSP^2	LNVOLUME	RES-SIZE/RELSP	RES-SIZE/LNVOL	Liquidity discount
KNEBV.HE	20.6	3.0	-5 %	4 %	70 %	0.86	18042	23.6	0.07 %	0.00 %	17.31	2.3300	1.9643	0.3 %
SHBa.ST	12.1	2.5	-4 %	0 %	73 %	0.90	21507	23.8	0.07 %	0.00 %	19.79	2.5034	0.6791	0.3 %
ELUXb.ST	41.2	3.7	173 %	13 %	120 %	1.08	6737	22.6	0.07 %	0.00 %	19.55	1.3439	-0.3426	0.3 %
INVEb.ST	12.0	2.5	1 %	-6 %	44 %	1.31	13993	23.4	0.06 %	0.00 %	19.50	2.0661	0.4178	0.3 %
TELIA.ST	21.3	3.1	60 %	5 %	165 %	0.69	18369	23.6	0.07 %	0.00 %	20.00	2.3456	0.3950	0.3 %
HMb.ST	19.2	3.0	-6 %	14 %	77 %	1.00	43020	24.5	0.06 %	0.00 %	20.47	3.1827	0.9695	0.3 %
ATCOa.ST	22.1	3.1	9 %	7 %	66 %	1.33	18203	23.6	0.07 %	0.00 %	20.08	2.3371	0.3367	0.3 %
SWMA.ST	19.7	3.0	21 %	-8 %	55 %	0.62	5524	22.4	0.07 %	0.00 %	19.06	1.1429	-0.2503	0.3 %
ASSAb.ST	24.3	3.2	0 %	12 %	38 %	0.95	18747	23.7	0.07 %	0.00 %	19.55	2.3652	0.6820	0.3 %
NOVOb.CO	26.6	3.3	14 %	11 %	47 %	1.43	97841	25.3	0.06 %	0.00 %	20.53	4.0117	1.7556	0.3 %
Min	12.0	2.5	-6 %	-8 %	38 %	0.62	5524	22.4	0.06 %	0.00 %	17.31	1.1429	-0.3426	0.3 %
Lower quartile	19.3	3.0	-3 %	1 %	49 %	0.87	15006	23.4	0.06 %	0.00 %	19.51	2.1320	0.3513	0.3 %
Median	**21.0**	**3.0**	**5 %**	**6 %**	**68 %**	**0.98**	**18286**	**23.6**	**0.07 %**	**0.00 %**	**19.67**	**2.3413**	**0.5485**	**0.3 %**
Average	**21.9**	**3.0**	**26 %**	**5 %**	**76 %**	**1.02**	**26198**	**23.7**	**0.07 %**	**0.00 %**	**19.58**	**2.3629**	**0.6607**	**0.3 %**
Upper quartile	23.7	3.2	20 %	12 %	76 %	1.25	20817	23.8	0.07 %	0.00 %	20.06	2.4688	0.8976	0.3 %
Max	41.2	3.7	173 %	14 %	165 %	1.43	97841	25.3	0.07 %	0.00 %	20.53	4.0117	1.9643	0.3 %

Ticker	P/E ratio	ln(P/E)	EPSg2016	EPSg2017	Payout ratio	Beta	Size	LNSIZE	RELSP	RELSP2	LNVOLUME	RES-SIZE/RELSP	RES-SIZE/LNVOL	Liquidity discount
AKTRV.HE	12.6	2.5	6 %	1 %	69 %	0.14	214.2	19.2	5.53 %	0.31 %	9.95	3.9349	1.8770	23.5 %
INVEST.HE	5.2	1.7	-40 %	2 %	16 %	0.02	19.8	16.8	3.13 %	0.10 %	10.31	-1.1007	-0.7165	14.1 %
AFAGR.HE	13.2	2.6	-3 %	0 %	64 %	0.07	110.8	18.5	2.66 %	0.07 %	10.50	0.1037	0.8912	12.1 %
ARP.ST	12.5	2.5	42 %	32 %	0 %	0.89	69.2	18.1	2.64 %	0.07 %	11.75	-0.3933	-0.3156	12.0 %
SWECa.ST	34.6	3.5	40 %	30 %	82 %	0.44	149.9	18.8	4.78 %	0.23 %	11.09	2.7405	0.8483	20.7 %
UFLXb.ST	11.6	2.5	89 %	-4 %	96 %	0.18	31.6	17.3	2.12 %	0.05 %	12.84	-1.7482	-1.7408	9.8 %
SAGAa.ST	8.0	2.1	-41 %	1 %	12 %	1.19	102.3	18.4	2.35 %	0.06 %	12.57	-0.3174	-0.4076	10.8 %
MSONa.ST	19.1	2.9	7 %	9 %	41 %	0.99	1.6	14.3	9.02 %	0.81 %	10.17	2.8827	-3.1605	35.5 %
LAMMb.ST	13.8	2.6	62 %	25 %	53 %	0.91	34.3	17.4	2.21 %	0.05 %	12.39	-1.5671	-1.3977	10.2 %
IFSa.ST	43.1	3.8	75 %	23 %	38 %	0.29	40.3	17.5	9.20 %	0.85 %	14.10	6.3191	-2.2443	36.0 %
Min	5.2	1.7	-41 %	-4 %	0 %	0.02	1.6	14.3	2.12 %	0.05 %	9.95	-1.7482	-3.1605	9.8 %
Lower quartile	11.8	2.5	-1 %	1 %	21 %	0.15	32.3	17.3	2.43 %	0.06 %	10.36	-0.9238	-1.6550	11.1 %
Median	**12.9**	**2.6**	**23 %**	**5 %**	**47 %**	**0.36**	**54.8**	**17.8**	**2.90 %**	**0.08 %**	**11.42**	**-0.1068**	**-0.5621**	**13.1 %**
Average	**17.4**	**2.7**	**24 %**	**12 %**	**47 %**	**0.51**	**77.4**	**17.6**	**4.37 %**	**0.26 %**	**11.57**	**1.0854**	**-0.6366**	**18.5 %**
Upper quartile	17.7	2.9	57 %	25 %	68 %	0.91	108.7	18.5	5.35 %	0.29 %	12.52	2.8471	0.5573	22.8 %
Max	43.1	3.8	89 %	32 %	96 %	1.19	214.2	19.2	9.20 %	0.85 %	14.10	6.3191	1.8770	36.0 %