



Turun yliopisto
University of Turku



ESSAYS ON NON-SEARCH
UNEMPLOYMENT AND
MONETARY POLICY

Oskari Vähämaa



Turun yliopisto
University of Turku

ESSAYS ON NON-SEARCH UNEMPLOYMENT AND MONETARY POLICY

Oskari Vähämaa

University of Turku

Turku School of Economics

Department of Economics

Subject - Economics

Doctoral Programme of Turku School of Economics

Supervised by

Professor Jouko Vilmunen
Turku School of Economics
Finland

Professor Matti Viren
Turku School of Economics
Finland

Reviewed by

Professor Ludo Visschers
University of Edinburgh
United Kingdom

Dr. Shigeru Fujita
Federal Reserve Bank of Philadelphia
United States

Custos

Professor Jouko Vilmunen
Turku School of Economics
Finland

Opponent

Professor Ludo Visschers
University of Edinburgh
United Kingdom

The originality of this thesis has been checked in accordance with the University of Turku quality assurance system using the Turnitin OriginalityCheck service.

ISBN 978-951-29-7280-7 (PRINT)

ISBN 978-951-29-7281-4 (PDF)

ISSN 2343-3159 (Painettu/Print)

ISSN 2343-3167 (Verkojulkaisu/Online)

Painosalama Oy - Turku, Finland 2018

ABSTRACT

This doctoral dissertation explores the role that institutions and policies can have in shaping aggregate economic outcomes. The thesis is comprised of an introductory chapter and three independent essays. All essays set up a clear structure that specifies how economic agents react to a changing environment. That is, each essay builds on the general equilibrium modeling of Macroeconomics.

The first essay examines the equilibrium effects of occupational human capital protection during mass layoffs in a setup where human capital can depreciate during unemployment spells and commitment problems prevent markets from allocating layoffs optimally. As the consequences of the policy are tightly related to occupational mobility, the paper focuses on modeling reallocation incentives of heterogeneous workers. In a calibrated model, a policy that concentrates involuntary unemployment incidences to inexperienced workers, decreases workers' incentives to reallocate, compared to an equilibrium where everyone faces an identical unemployment risk, leading also to a decrease in aggregate unemployment. Moreover, this policy change increases the market output and on average does not harm the inexperienced workers.

The second essay explores the effects of unionization in an island model of Lucas and Prescott (1974) with different union structures. When a model with competitive labor markets is set to match the empirical fact that a large number of unemployment spells ends with recalls, an introduction of a large labor union, that represents all workers and sets a common economy-wide minimum wage, increases unemployment substantially. Moreover, the whole increase is about non-search unemployment as search unemployment actually reduces marginally. If the same degree of unionization is generated by a continuum of small unions, the aggregate unemployment reaction is somewhat smaller. However, the increase in non-search unemployment is still considerable. The workings of a large union are also explored when the union is assumed to bargain over the minimum wage with an employers' organization. This environment leads to a considerably lower increase in aggregate unemployment. Yet again, the search intensity of unemployed workers drops significantly.

In the third essay we show that the cancellation of income and substitution effect implied by King-Plosser-Rebelo (1988) preferences breaks tight coefficient restriction between the slope of the Phillips curve and the elasticity of consumption with respect to real interest rate in a sticky price macro model. This facilitates the estimation of intertemporal elasticity of substitution using full information Bayesian Maximum Likelihood techniques within a structural model.

The US data from the period 1984–2007 supports low intertemporal elasticity of substitution and strongly rejects a logarithmic and an additively separable utility specification commonly applied in the New Keynesian literature.

Keywords: Layoff order, occupational mobility, unemployment, labor union, monetary policy, non-separable utility.

TIIVISTELMÄ

Tämä väitöskirja tarkastelee instituutioiden ja politiikkatoimenpiteiden vaikutuksia koko talouden tulemien kannalta, keskittyen erityisesti työmarkkinoihin. Työ koostuu johdannosta ja kolmesta itsenäisestä esseestä. Tarkastelu pohjautuu yleisen tasapainon malleihin.

Ensimmäinen essee tutkii ammatillisen osaamisen suojelun vaikutuksia irtisanomistilanteissa, kun inhimillinen pääoma voi rapautua työttömyysjaksojen aikana ja epätäydellisistä sopimuksista johtuen markkinat eivät kohdista irtisanomisia tehokkaasti. Koska politiikkamuutoksen seuraukset ovat läheisesti sidoksissa ammatilliseen liikkuvuuteen, essee keskittyy heterogeenisten työntekijöiden uranvaihtojen mallintamiseen. Kalibroituun malliin pohjautuvat tulokset indikoivat, että irtisanomissääntö, joka kohdistaa työttömyysjaksot ensisijassa kokemattomiin työntekijöihin, laskee ammattien välistä liikkuvuutta verrattuna tilanteeseen, jossa kaikki työntekijät kohtaavat saman työttömyysriskin. Toisaalta talouden työttömyys on pienempää ja tuotanto suurempaa, kun irtisanomiset kohdistuvat kokemattomiin työntekijöihin.

Väitöskirjan toinen luku tarkastelee erilaisten ammattiliittorakenteiden merkitystä Lucasin ja Prescottin (1974) mallin avulla. Kun kalibroinnissa huomioidaan, että suuri osa työttömyysjaksoista päättyy paluuseen samalle työnantajalle, suuren ja kaikkia työntekijöitä edustavan ammattiliiton vaikutus minipalkkaan ja työttömyyteen on merkittävä. Lisäksi koko työttömyyden lisäys on työnhaun poissulkevaa työttömyyttä. Jos vastaava järjestäytymisaste saavutetaan monien ammattiliittojen toimesta, työttömyyden ja minimipalkan reaktiot ovat pienempiä. Kuitenkin myös tässä tapauksessa työnhaun poissulkevan työttömyyden kasvu on huomattavaa. Tutkimuksessa tarkastellaan myös tilannetta, jossa suuri ammattiliitto neuvottelee minimipalkasta työnantajajärjestön kanssa. Tässä tapauksessa työttömyyden reaktiot ovat selvästi maltillisempia.

Kolmannessa esseessä tarkastellaan uuskeynesiläistä mallia, jossa työn tarjonnan tulo- ja substituutiovaikutus kumoavat toisensa, kun taloudenpitäjien preferenssejä kuvataan tasaisen kasvun mukaisilla ns. King–Plosser–Rebelo-preferensseillä (1988). Tällöin talouden inflaatiodynamiikkaa kuvaavan Phillips-käyrän kulmakertoimen – inflaation kustannusherkkyyden – ja kulutuksen korkojoustop välinen tiivis yhteys rikkoontuu. Tämä ominaisuus helpottaa kulutuksen kasvuvauhdin korkoherkkyyttä mittaavan parametrin eli intertemporaalisen substituutiojoustop estimointia rakenteellisissa malleissa, kun estimoinneissa käytetään täyden informaation bayesilaista suurimman uskottavu-

uden estimointimenetelmää. Yhdysvalloista kerätyssä, ajanjakson 1984–2007 kattavassa aineistossa intertemporaalisen substituutiojouston arvo estimoituu pieneksi.

Asiasanat: irtisanomisjärjestys, ammatillinen liikkuvuus, työttömyys, ammat-
tiliitot, rahapolitiikka, ei-separoituva hyöty.

ACKNOWLEDGEMENTS

I would like to thank my advisors Jouko Vilmunen and Matti Viren for their support through my PhD studies. It was Professor Viren's entertaining and insightful lectures that got me interested in pursuing a PhD track and asking policy oriented questions in the first place. Later on, his sharp intuition and general knowledge have proved to be invaluable. I feel highly indebted to Professor Vilmunen for taking the time to guide me even before his appointment to the University of Turku. Having had the chance to work with him on one of the chapters of my dissertation has also been priceless. His impact on my modelling philosophy and techniques and more generally on my approach to research has been immense. I am also in debt to Juha Kilponen for his willingness to work as a co-author with me while I was just starting to learn my trade. This gave me a unique opportunity to build my human capital through learning-by-doing. I would also like to express my gratitude to my pre-evaluators Ludo Visschers and Shigeru Fujita for taking the time to read my work and giving such insightful comments.

During the process I have had a chance to interact with numerous people at the Turku School of Economics, the Bank of Finland and the University of Helsinki. I would especially like to thank Marlène Isoré, Heikki Kauppi, Mitri Kitti, Ville Korpela, Tomi Kortela and Antti Ripatti for numerous discussions about my work and Economics in general. These discussions have undoubtedly made my work better. Conversations with Luis Alvarez, Matti Heikkonen, Olli Lappalainen, Erik Mäkelä, Eero Mäkynen have also been fun and educating.

I thank the OP Group Research Foundation and the Yrjö Jahnesson Foundation for financial support.

Finally and most importantly, I would like to acknowledge the support of my wife Laura Englund and my parents Annikki Vähämaa and Timo Vähämaa.

TABLE OF CONTENTS

ABSTRACT

TIIVISTELMÄ

ACKNOWLEDGEMENTS

I	SYNTHESIS	11
1	THE GENERAL EQUILIBRIUM APPROACH TO ASSESSING THE AGGREGATE EFFECTS OF POLICIES AND INSTITUTIONS	13
2	ENDOGENOUS MISMATCH	15
2.1	Time Diary Evidence on Time Use of Unemployed Workers	16
2.2	Exogenous Mismatch	19
2.3	A Simple Model of Endogenous Mismatch	24
2.4	Studies on Endogenous Mismatch	28
3	THE IMPORTANCE OF LONG-RUN RESTRICTIONS FOR ESTIMATED BUSINESS CYCLE MODELS	33
4	SUMMARY OF THE ESSAYS	36
	REFERENCES	38
II	ESSAYS	41
	ESSAY 1	
	LAYOFF ORDERS, HUMAN CAPITAL AND OCCUPATIONAL MOBILITY VIA UNEMPLOYMENT	43
	ESSAY 2	
	UNIONIZING NON-SEARCH UNEMPLOYMENT	75
	ESSAY 3	
	ESTIMATING INTERTEMPORAL ELASTICITY OF SUBSTITUTION IN A STICKY PRICE MODEL	99

LIST OF ORIGINAL RESEARCH PAPERS

- (1) - Layoff Orders, Human Capital and Occupational Mobility via Unemployment
- (2) - Unionizing Non-Search Unemployment
- (3) - Estimating Intertemporal Elasticity of Substitution in a Sticky Price Model

Part I
SYNTHESIS

1 THE GENERAL EQUILIBRIUM APPROACH TO ASSESSING THE AGGREGATE EFFECTS OF POLICIES AND INSTITUTIONS

This dissertation explores the role that institutions and policies can have in shaping aggregate economic outcomes. The questions covered in this thesis, such as how monetary policy affects output and employment, or how labor market policies contribute to unemployment and efficient allocation of resources, are important not just for macroeconomists but also for policy makers.

The last decades in economic research have been characterized by a strong emphasis on detailed micro-level data. Along with this trend, atheoretical methods have been gaining popularity. Unfortunately, data alone cannot shed light on the aggregate issues that area at the focus of this dissertation. As the economic environment changes, due to, for example, a policy change, the behavior of agents is likely to change as well. This argument, known as the Lucas critique, is generally accepted in modern Macroeconomics¹. Thus, a more structural approach is considered a necessity for macroeconomic analysis. This will, among other things, ensure internal consistency.

All essays in this thesis build on the general equilibrium modeling of Macroeconomics. That is, the essays make explicit assumptions about preferences, endowments, information sets and restrictions faced by agents. Aggregate outcomes are then generated jointly by optimizing agents, and an equilibrium concept that ties together the behavior of individual agents.

Unfortunately the complexity of models rises rapidly as more "bells and whistles" are added, especially in dynamic settings. Thus, one has to make decisions about what the essential features for the question on hand are and leave some other aspects out from the model. Additionally, an equilibrium concept has to be chosen based on the problem at hand. For example, in models where the problem being analyzed is the co-movement between output, employment and interest rates, it is typical to assume that agents are free to trade with each other, while in models where unemployment is the variable of interest, it is common that not all potential trading partners are able to meet. These modeling decisions are, of course, always going to be imperfect and somewhat subjective. Ideally, one should make decisions that can be tightly linked to empirical evidence and avoid adding too many free parameters. In this introduction, I am going to discuss the empirical evidence that has guided the modeling work in the thesis, as well as review the relevant existing theoretical studies that position

¹ See Lucas (1976) for the famous argument in favor of a more structural modeling.

my work.

The common theme in this thesis is that I consider the effects of policies and institutions in frameworks where the economic environments are characterized by frictions. The frictions imply that, at least in some cases, the policies and institutions set up by the society can lead to welfare improvements. Case in point, a central bank with a coherent interest rate policy. However, some other arrangements can reinforce the original sources of problems. Both of these arguments are highlighted in this dissertation.

The dissertation contains three studies out of which the first two cover endogenous mismatch unemployment while the last paper builds on the New Keynesian paradigm. The first essay explores the equilibrium effects of protecting more experienced workers in layoffs over inexperienced ones. The analysis shows that the seniority rule type of practice can be useful when there are no markets for allocating layoffs optimally. The second essay continues with equilibrium unemployment modeling and shows that unionization offers a powerful mechanism to reinforce mismatch unemployment. The last essay considers a central bank's ability to affect real variables in a small scale New Keynesian model when the consumer preferences are chosen, so that the model is in line with the long run growth facts.

Given the two topics, the introduction is split accordingly. The next section covers mismatch unemployment, while the New Keynesian literature is discussed in Section 3. As the equilibrium models that allow for mismatch unemployment are not as well established as the sticky price models. The focus of this introduction chapter is on mismatch as a source of unemployment. Finally, a summary of the essays is presented in Section 4.

2 ENDOGENOUS MISMATCH

Recent empirical evidence on the time use of unemployed workers has revealed that the unemployed spend a relatively minor fraction of their increased time on search. Moreover, there is a substantial heterogeneity among unemployed workers. It seems that only a small fraction of unemployed workers, 20 percent in the US, spends any time at all on search. However, those who do, spend several hours searching.

Standard models in the macro-labor research focus on search frictions. The canonical search models (e.g., McCall, 1970, and Mortensen and Pissarides, 1994) assume that unemployed workers are actively searching for a new job, typically spending an equivalent amount of time on search that they would on work².

The discrepancy between theory and empirical evidence of time use has led to a development of new models of unemployment where the mismatch between unemployed workers and vacant jobs in dimensions such as skill, location or occupation is at the heart of unemployment. That is, if unemployed workers are attached to labor markets where jobs are scarce, they, instead of searching, end up waiting for jobs that fit their profile. Borrowing an example from Shimer (2007) in mismatch unemployment³, a former steel worker stays close to the closed plant in hopes of it to reopen, while in search unemployment the worker is actively trying to find a job as a nurse in another city.

Of course, search unemployment and mismatch unemployment are likely to be complementary. Recent research has combined the two types of unemployment by allowing workers to choose whether to stay attached to their current local markets or search for jobs in other markets. Endogenous mismatch is at the core of the first two studies contained in this dissertation.

The next subsection reviews empirical studies of time use, while the second subsection utilize Shimer's (2007) model in order to highlight the essence of non-search unemployment. The model is highly stylized but yet it is able to explain some of the key features of unemployment. The third subsection introduces the idea of endogenous mismatch with the help of a modified version of

² Some variants allow unemployed workers to choose their search intensity (see e.g. Mortensen 1977). However, even with these models it is hard to explain how a large fraction of unemployed workers would find it optimal not to search at all.

³ Currently many different terms are used for non-search unemployment. In this dissertation, I use "mismatch unemployment", "non-search unemployment", "rest unemployment" and "waiting unemployment". While the concepts are theoretically somewhat different, e.g in waiting unemployment workers cannot work, while in rest unemployment they choose not to work, they all lead to observably equivalent behavior.

the island model of Lucas and Prescott. The model captures the essence of the endogenous mismatch, unemployed workers' decisions to stay or to go. The subsection also gives an algorithm for solving the model quantitatively. The final subsection discusses some of the existing studies, utilizing endogenous mismatch.

2.1 Time Diary Evidence on Time Use of Unemployed Workers

The emergence of detailed time use surveys has enabled researchers to have a closer look at how unemployed workers allocate the increased non-work time. For example, American Time Use Survey (ATUS) provides representative survey data about how individuals over 15 years spend their time. The individuals in the sample are chosen from households that completed the eighth interview for Current Population Survey. These individuals are interviewed once about their time use on the day before the interviewing day.

Krueger and Mueller (2010) utilize the ATUS data from 2003 to 2007 in order to provide evidence on the search activity of the unemployed workers. Somewhat surprisingly, unemployed workers spend on average only 32 minutes per day searching for a job. This is, however, considerably more than the average search time for individuals classified as out of the labor force (0.8 minutes) or the employed (0.6 minutes). This suggests that unemployment is indeed characterized by increased search activity even though the average search time is not comparable to the average time spent at work which is 325 minutes according to Krueger and Mueller (2012).

Interestingly, Krueger and Mueller also show that there is substantial heterogeneity in the search activity of unemployed workers. First of all, for a given day, a large fraction of unemployed workers does not participate in job search at all. Only 19.3 percent of unemployed workers report that they have spent some time on search activities. Secondly, those who search, devote a substantial part of their day on job search. The average duration of job search conditional on participation is 167 minutes. Moreover, 25 percent of searchers spent more than 240 minutes on job seeking activities.

It also seems that the reason for unemployment matters. For example, those who have lost their job without an indication of recall search for about 45 minutes on average, while those who expect to be recalled only search for 13 minutes. Moreover, the authors' micro regressions reveal that, everything else equal, those who expect a recall hardly search at all.

Krueger and Mueller (2012) provide cross-country evidence on the time use of unemployed workers. They draw on the time diary data for 14 countries in North America and Europe. In general, it seems that the unemployed spend considerably more time on activities classified as leisure than the employed.

Especially, hours spent on sleeping and watching television increase substantially when comparing the two groups. For example, in the US, an unemployed worker sleeps almost an hour more than an employed worker. In Europe, the difference in sleeping hours varies from 0.6 hours to an hour. For most of the countries, the time spent on watching TV increases by about an hour.

Another observation that holds for all countries is that the time spent on home production and caring for others is much higher for the unemployed than for the employed. Across the regions, the difference is between 0.6 hours and 1.7 hours.

Interestingly, Krueger and Mueller (2012) find that there are substantial differences across countries on time spent on job search activities. While the average search time in the US and Canada is around 30 minutes, it is only a little above ten minutes in Europe. In the Nordic Countries, an unemployed person spends only 4 minutes per day on job search.

A closer look at the search activity of the unemployed workers reveals that the cross-country variation in the fraction of the unemployed who search is also large. In Finland, for example, the participation in job search is only 1/4th of the participation rate in the US (5% and 20.2%, respectively). The correlation between the participation rate and the average search times is 0.88. There are also substantial differences between the countries in the intensive margin of search⁴.

Krueger and Mueller (2012) also explore the role of institutional factors in explaining the cross country variation in search times. Given that they have data for only 14 countries, the results have to be interpreted with caution. They show that the relationship between unemployment benefits and search time is mildly negative and insignificant. The bivariate relationship between unemployed search activity and income dispersion, measured with the 90-10 wage ratio, is strongly positive and significant. They also find that the correlation is even stronger if the wage inequality is measured using the 50-10 wage ratio. In regressions where other controls, such as a measure of benefit escalation and average years of schooling, are also included, the wage inequality has a robust positive effect. Furthermore, the authors use micro data for eight countries to explore the robustness of the country regressions and conclude that the effects of the 90-10 wage ratio are consistent with the country-level results.

Krueger and Mueller (2011) utilize a longitudinal survey data to measure the evolution of job search and emotional well-being over the unemployment spells. They convey a repeated weekly survey of time use of the unemployed workers in New Jersey. The advantage of repeated surveys for the same individuals is that it enables researchers to explore whether the unemployed become discouraged on their job search as the duration of their unemployment spell increases. That is, the use of individual specific fixed effects makes it possible to separate between the composition effect and the behavioral responses as an explanation behind

⁴ The intensive margin of search is defined as the time spent on search for those who participate in job search.

declining hazard rates from unemployment.

Krueger and Mueller (2011) show that when using only the entry week survey, a setup similar to cross-sectional survey data, the relationship between the search time and the duration of the unemployment spell is positive. However, when individual-specific fixed effects are introduced, the relationship is strongly negative and statistically significant. Based on this, the authors suggest that the declining search activity during an unemployment spell seems to be associated with behavioral responses. Moreover, the exhaustion of unemployment benefits does not increase the search activity. Krueger and Mueller (2011) also decompose search activity into extensive and intensive margins and show that the search activities measured both ways decrease as the duration of the unemployment spell increases.

The study also gives a detailed decomposition of the search activities. According to the decomposition, looking at help wanted ads (27%), Sending out résumés or applications (24%) and placing and answering ads (14%) take most of the aggregate search time. Utilizing the network of friends and relatives is the fourth most popular form of job search. This form of search has been shown to be highly efficient but it makes up just about 9% of the aggregate search time.

Additionally, Krueger and Mueller (2011) ask about the subjective well-being of the survey participants during their unemployment spell. Consistently with earlier studies, the unemployed express less life satisfaction than employed workers. Moreover, as the unemployment spell lengthens, the share of time spent in bad mood increases and the share of time spent in good mood decreases.

The survey respondents were also asked about their well-being during various activities. According to the results, job search seems to be by far the most troublesome activity. It has the lowest reported scores on happiness, with and without person-specific fixed effects, by some distance. Additionally, the job search periods have the highest average ratings in stress and sadness. Moreover, the authors find a strong negative relationship between general life satisfaction and time spent on job search. When a binary variable indicating life satisfaction was regressed with a variable of search time, the effect was negative and significant.

Aguiar, Hurts and Karabarbounis (2013) document the cyclical properties of different time use categories using ATUS. They focus on how decreased market work hours were allocated across other forms of time use during the great recession. It is well documented that there are long run trends in non-market work and leisure (see e.g. Aguiar and Hurst, 2007). This makes it harder to explore the business cycle properties, as the standard filtering techniques are not feasible, given the short time dimension of the ATUS data. In order to overcome this, Aguiar et al (2013) utilize state level variation in the severity of the recession.

According to their results, job search absorbs only about 2 to 6 percent of the increased time when the market work hours decrease. Most of the increased time

is allocated to leisure (50 percent) and home production (30 percent). Aguiar et al (2013) also decompose increased leisure and home production into subcomponents. For leisure, the majority of the increased time is allocated to sleeping and watching TV, around 42 percent and 23 percent, respectively. The margin associated with socializing with one's spouse and friends does not increase substantially. Finally, a residual category called "other leisure" covers 28 percent of the increased leisure. This category includes activities such as playing video games and listening to music.

Decomposition of the increased home production reveals that core home production, a category that includes activities such as preparing meals and indoor household cleaning, takes up almost 40 percent of the increased time spent on home production. Shopping time and home ownership activities also go up significantly. Moreover, time spent on child care, a category that the authors treat separately, also increases.

Aguiar et al (2013) also calculate elasticities of substitution between different time use categories at the business cycle frequency. As home production absorbs a large fraction of foregone market work but counts for only about 11 percent of the total time endowment, the substitutability between the two uses of time is high. This elasticity is, in fact, much higher than the elasticity between market work and leisure (0.5 versus 0.15). In their working paper version of the paper, using a standard RBC model with home production (see Benhabib, Rogerson and Wright, 1991), the authors show that this elasticity implies that the elasticity of substitution between the market and the home good is as high as 2.5. In reduced form models where home production is not considered, this would imply that leisure and consumption are strongly complementary.

2.2 Exogenous Mismatch

As the previous subsection illustrated, the majority of unemployed workers are not actively searching for new jobs. A potential line of research that takes this into account assumes that unemployment is caused by a poor match between vacant jobs and job seekers. That is, if there is a mismatch between labor demanded and supplied along dimensions such as skill requirements, occupations or geographic locations, unemployed workers may just have to wait for jobs that suit their profile.

In this subsection, I highlight the idea of mismatch relying on the model of Shimer (2007). In the model, labor markets are segmented to a large number of submarkets. Production technology in each market is described by the Leontief production function, i.e., jobs and workers can only produce in pairs. Workers and jobs in each submarket are able to meet freely and wages on these markets are determined competitively. However, vacancies and unemployed workers

cannot meet across the submarkets. This implies that there are vacant jobs in a submarket if there are more jobs than workers. Markets where the opposite holds true are characterized by unemployment. Firms decide how many vacant jobs they create but they are not able to decide for which submarket their vacancies are allocated. Moreover, the workers' movement across the segments of the economy is exogenous.

Shimer (2007) uses the model to show that this simple structure of mismatch, where the only endogenous decision relates to firms' job creation, is able to generate some key empirical regularities observed in the business cycle frequencies. To be more precise, the author shows that the model generates a Beveridge curve that matches the one calculated from the US data. Moreover, the model can also reproduce an empirically relevant aggregate matching function (correlation between market tightness and job finding probability).

The origin of this type of model of unemployment can be traced back to the urn-ball structure where workers are randomly allocated to jobs. Any job that receives no workers leads to an unfilled vacancy. Conversely, when a job receives more than one worker, unemployment is generated (see, e.g., Hall, 1977). If this structure would be interpreted via segmented labor markets, there would be as many markets as there are jobs.

Going into detail of Shimer's (2007) model, consider an economy where there are L workers and many firms. Time is continuous. Both workers and firms are infinitely lived, risk neutral and discount future at rate r . The labor markets are divided into S segments. Each worker is attached to some labor market independently of the attachments of other workers. A firm may have any discrete number of jobs starting from 0. $M(t)$ denotes the aggregate number of jobs in the economy at time t . This variable depends on the endogenous decisions of firms to create jobs and the exogenous destruction of jobs. The jobs are also allocated randomly and independently across labor markets. Thus, the distribution of workers and jobs across labor markets are described by independent multinomial distributions.

Denote $L = L/S$ and $M(t) = M(t)/S$ and let $S \rightarrow \infty$. Now, the fraction of labor markets with $i \in \{0, 1, \dots\}$ workers can be described with the Poisson distribution

$$\mu(i; L) = \frac{e^{-L} L^i}{i!} \quad (1)$$

and the fraction of labor markets with $j \in \{0, 1, \dots\}$ jobs is given

$$\mu(j; M(t)) = \frac{e^{-M(t)} M(t)^j}{j!}. \quad (2)$$

As the amount of workers and jobs are independent random variables, the joint distribution $\mu(i, j; M(t))$, takes the following form

$$\mu(i, j; M(t)) = \frac{e^{-(L+M(t))} L^i M(t)^j}{i! j!}$$

A worker-job pair produces $y(t)$ units of the market good. Workers who are not matched with jobs can engage in home production generating b_r units of the home good. The market and home good are assumed to be perfect substitutes.

Within each labor market, jobs and workers meet freely, that is, perfect competition determines the wage rate. If there are more workers than there are vacant jobs ($j < i$), there is unemployment in the local market ($i - j$ workers remain without a job) and the wage paid to employed workers is equal to b_r . If the opposite holds true, the wage rate is $y(t)$ and all workers are employed. Moreover, some jobs remain vacant ($j - i$ jobs). Thus, unemployed workers and vacant jobs never co-exist in the same market. However, at the aggregate level, the economy is characterized by both vacancies and unemployment. One can use $\mu(i, j; M(t))$ in order to describe the number of the unemployed and the vacant jobs

$$U_{M(t)} = \sum_{i=1}^{\infty} \sum_{j=0}^i (i - j) \mu(i, j; M(t)),$$

$$V_{M(t)} = \sum_{j=1}^{\infty} \sum_{i=0}^j (j - i) \mu(i, j; M(t)).$$

In the Shimer's (2007) model it is the restricted mobility that creates a situation where unemployed workers and vacant jobs co-exists in the aggregate level. It is assumed, that the workers' mobility is described by an exogenous mobility shock that follows a Poisson process. A shock that forces a worker to move out from her current local market arrives at rate λ . When a worker moves across markets, the next labor market is allocated randomly. Thus, the arrival rate of workers to a market is λL .

Now one can describe the evolution of the probability distribution of locations. A measure of islands with i workers, $\mu(i, L)$, increases when a worker moves out from a location of $i + 1$ workers or a new worker arrives to a market with $i - 1$ workers. Additionally, the amount of markets with i workers decreases when a worker moves out from one of these locations or a new worker arrives to any of these locations. Taken together,

$$\dot{\mu}(i; L) = \lambda(i + 1)\mu(i + 1; L) + \lambda L[\mu(i - 1; L) - \mu(i; L)] - \lambda i \mu(i; L).$$

Assuming that the current probability density function obeys the Poisson distribution stated in equation (1) and plugging it into the previous equation, gives $\dot{\pi} = 0$. That is, the flow equation is consistent with the stock equation (1).

Job destruction is also exogenous. A job destruction shock arrives at rate δ . When this shock hits a job, the job disappears from the economy. Firms can create jobs by paying a fixed job creation cost k . Note that unlike in Mortensen-Pissarides matching models, this is a fixed cost that only needs to be paid once. A new job is randomly allocated to a local market. The job remains attached

to this market as long as it exists. When jobs are created at rate $m(t)$, the flow of aggregate jobs, $\dot{M}(t)$, is given by $\dot{M}(t) = m(t) - \delta M(t)$. The evolution of the probability distribution of jobs, $\dot{\mu}(j, M(t))$, is similar to the evolution of workers and takes the following form

$$\dot{\mu}(j; M(t)) = \delta(j+1)\mu(j+1; M(t)) + m(t)\mu(j-1; M(t)) - (\delta j + m(t))\mu(j; M(t))$$

Using equation (2) and the definition of $\dot{M}(t)$, this can be rewritten in the following form

$$\dot{\mu}(j; M(t)) = e^{-M(t)} \frac{M(t)^j}{j!} \left(\frac{j}{M(t)} - 1 \right) \dot{M}(t)$$

This is equal to the derivative of equation (2) with respect to t . That is, the flow equation is consistent with stock equation (2).

The productivity of matches, $y(t)$, is assumed to follow a mean reverting jump process where the arrival rate of innovation is ρ . Moreover, $y(t)$ takes values on a discrete grid, where the smallest value is so high that the economy is always producing something.

In the model, the only relevant decision is the firms' job creation. In order to describe the evolution of jobs, $M(t)$, one needs to consider the expected value of a new job that has not yet been assigned to a local market, $J(M, y)$. Firms are able to create as many jobs as they want. This implies that at equilibrium $J(M, y) \leq k$. When this condition is not true for a pair (M, y) , the number of jobs created jumps to the value that equates $J(M, y)$ and k . Thus, M is a jump variable. When $J(M, y) < k$, firms do not create new jobs and $\dot{M} = -\delta M(t)$.

The evolution of the job's value, when no new jobs are created, is described by the following HJB-equation

$$rJ(M, y) = (y - b_r) \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} \mu(i, j; M(t)) - \delta J(M, y) + J'(M, y) \dot{M} + \rho(E_y J(M, y') - J(M, y))$$

when $M > M^*$. The first term on the right hand side describes the expected flow profit from a filled vacancy ($\sum_{i=1}^{\infty} \sum_{j=0}^{i-1} \mu(i, j; M(t))$ is the probability that a new job is allocated to a market where there are unemployed workers). The second term accounts for the possibility that a job can be destroyed exogenously. The third term measures the deterministic evolution of the value of the asset. Note that when no new jobs are created $\dot{M} = -\delta M(t)$. Finally, the last terms measures the change in value associated with potential changes in aggregate productivity.

When the number of jobs in the economy is smaller than the critical value, M^* , the HJB takes the following form

$$rJ(M, y) = rk$$

In order to solve $M^*(y)$, one also needs the value matching and smooth pasting conditions

$$J(M^*, y) = k, J'(M^*, y) = 0.$$

Shimer (2007) proves that there is a unique equilibrium and that $M^*(y)$ are increasing in y . He also provides a simple recursive algorithm for solving $M^*(y)$. Note that after one knows $M^*(y)$ for all productivity levels, simulating the evolution of $M(t)$ is easy. One just needs to simulate the productivity process and for a given $y(t)$, let $M(t)$ evolve as $M = \delta M(t)$ if $M^*(y(t)) < M(t)$ otherwise set $M(t) = M^*(y(t))$. After one has a path of aggregate jobs, solving other variables, such as aggregate vacancy and unemployment rates, is straightforward. In the model, business cycle variation in unemployment and unfilled vacancies is generated due to the interaction of job creation, restricted mobility and aggregation. An increase in M decreases unemployment in markets where there is excess labor supply. However, as new jobs are allocated randomly, part of the new jobs are attached to markets where there is excess demand for labor. This implies that also unfilled vacancies in the economy increase. That is consistent with the US data, the model generates strong negative co-movements between unemployment and vacancies. Note also that the model's ability to generate a negatively sloped Beveridge curve is not dependent on the type of shock, as any shock that will affect the endogenous variable M , will create a negative correlation between unemployment and vacancies. This contrasts with the matching models where even a productivity shock does not necessarily produce a negative correlation. Moreover, as argued by Shimer (2005), other realistic shocks, such as separation shocks, induce a negative relationship in a standard matching model.

Shimer (2007) uses comparative statics analysis to explore the mismatch model's ability to amplify productivity shocks. When L and M are calibrated based on average unemployment and vacancy rates and only permanent movements in productivity are considered, the elasticity of labor market tightness, V/U , with respect to productivity is $4.25 \frac{y}{y-b_r}$. This is over four times larger than the same elasticity in a realistically calibrated search and matching model (see Shimer, 2005). Because in the US data V/U -ratio is highly cyclical while labor productivity is only weakly procyclical, the strong amplification in the model helps to match the model with the data. Part of this amplification is assumed since the vacancy cost needs to be paid only once per vacancy. However, roughly half of the amplification is due to the limited mobility and the wage determination in local markets. This happens because, unlike in matching models, wages do not change "one-to-one" with productivity⁵. In the model considered here, a productivity increase causes wages to increase one-to-one in locations where there are excess vacancies. In markets where there is unemployment, wages stay unchanged. Furthermore, some markets move from having unemployment to having unfilled vacancies. Due to the market structure, the firms' job creation incentives are stronger in the mismatch model.

⁵ Shimer (2005) claims that in search and matching models the reaction of wages to a productivity shock is almost one-to-one. This reduces firms incentives to job creation and thus reduces the amplification of the models.

Shimer (2007) also shows that the model generates a reduced form matching function⁶ that is almost isoelastic. This is consistent with the empirical observation that the “matching function” is a Cobb-Douglas. Note that unlike matching models this reduced form function is not assumed, it is generated jointly by explicit micro structure and aggregation.

The author also uses a quantitative version of the model to show that previous steady state comparative static results generalize to a dynamic setting. Additionally, he shows that the stylized model of mismatch is able to explain around 25 percent of the volatility in job finding rate and around 1/3 of the volatility in market tightness.

Hawkins (2015) considers different wage determination schemes to the Bertrand style competition in Shimer (2007). He shows that the wage determination is highly important for the mismatch models’s ability to amplify productivity shocks.

Another class of models that gives rise to mismatch, similar to the one discussed here, is Stock-Flow matching models (see e.g. Coles and Muthoo, 1998, or Ebrahimi and Shimer, 2010). In these models, workers and jobs are heterogeneous and only a fraction of potential worker-job pairs can produce. When a worker loses her job she observes the requirements of vacant jobs and if there is a suitable match, she becomes employed immediately. Otherwise, she becomes unemployed and will wait for a vacancy that fits her profile. Firms decide how many vacancies they create but cannot decide on the type of the job. These features lead to the coexistence of unemployed workers and open vacancies. Note that also in this case the mismatch unemployment is exogenous. Ebrahimi and Shimer (2010) show that a Stock-Flow matching model leads to a larger amplification than the mismatch model considered here. However, they also emphasize that with similar costs of finding a worker, the amplification properties would be equal.

2.3 A Simple Model of Endogenous Mismatch

In the model discussed in the previous subsection the allocation of vacancies and workers was exogenous. In this subsection, we endogenize searchers’ decisions to wait for jobs similar to the ones they had before, or search for new kind of jobs. A few different approaches have been explored in the literature. For example, Lagos (2000) analyzes a spatial equilibrium of taxicabs and passengers where passengers move exogenously and taxicabs choose their locations in each period in a response to local conditions. Free movement means that mismatch is not assumed, but it arises endogenously as a response to price frictions.

Part of the assumptions that give rise to mismatch in Lagos (2000) are some-

⁶ the relationship between aggregate market tightness and job finding probability

what specific to the taxicab-passenger search problem⁷. Given this, I follow Alvarez and Shimer (2011) and generate endogenous mismatch with the help of fixed moving costs that workers have to pay in order to switch local labor markets. The fixed costs imply that some workers will choose to wait for better conditions in their current location rather than search for a new location.

The considered model builds on the island search models pioneered by Lucas and Prescott (1974). As before, the economy is characterized by a large amount of local labor markets. The conditions in local markets are varying. This means that some workers would like to reallocate. However, workers are unable to move instantly between markets and have to spend time and resources in order to find a new local market. One can understand the search time, for example, as time needed to acquire the skills required in a new occupation. In this class of models, the workers' decision to stay or to search for a new market is at the heart of the model. Unlike in the original model where agents can work or search, I follow Alvarez and Shimer (2011) and also allow unemployed workers a possibility to wait for the local conditions to improve. Due to the moving costs, some unemployed workers prefer mismatch unemployment over search unemployment.

As in Shimer (2007) local markets are characterized by competitive labor markets. However, unlike in the model of exogenous mismatch, varying local conditions, endogenous mobility and more flexible production technology imply that the model considered here generates a rich endogenous wage distribution.

ENVIRONMENT. Time is discrete and the economy is populated by a measure one of workers distributed over a continuum of separated labor markets, islands. Also the mass island is set to unity. Let us denote the amount of workers on an island i by $x(i)$. During a period out of these workers $n(i)$ work, $r(i)$ are mismatch unemployed and $x(i) - n(i) - r(i)$ search for a new location.

There is also a large number of firms in each location. The aggregate production function in an island is given by

$$y(i) = z(i)f(n(i)),$$

where $z(i)$ is islands specific productivity shock. The production function is assumed to satisfy the standard assumptions about positive but decreasing marginal returns. Moreover, it is assumed that Inada conditions hold. Following Ljungqvist and Sargent (2012) we can simplify the environment and assume that the productivity, z , is taking discrete values $z_1 < z_2 < \dots < z_n$ and that the transition probabilities between z and z' are given by $Q(z, z')$. Moreover, assume that the productivity shocks are persistent in a sense that the cumulative density function, $Pr(z' < z_k | z)$, is a decreasing function of z .

⁷ For example, the prices of the rides are typically heavily regulated. This makes the idea that mismatch is caused due to imperfect price responses more plausible in taxi markets than, for example, in labor market context.

Local labor markets are competitive, i.e, the wage is equal to the marginal product of labor.

$$w(z(i), x(i)) = zf'(n(i))$$

Moreover, the amounts mismatch unemployed and employed workers are restricted by the mobility frictions. That is

$$n(i) + r(i) \leq x(i).$$

Workers can decide to reallocate but it will take time. To be more precise, if a worker decides to reallocate, she will not be able to work during the period and in the beginning of the next period she will be relocated to a new island randomly. Given the undirected search, the next period labor force $x'(i)$ is given by

$$x'(i) = n(i) + r(i) + U_S,$$

where U_S is the amount of searchers in the economy.

TIMING. At the beginning of the period productivities in each location are revealed. After this, agents decide whether to work, to engage in non-search unemployment or to reallocate. Next, production in each location takes place. Finally, agents who decided to search are randomly allocated to a new island.

WORKERS' PROBLEM. Workers are assumed to be risk neutral and thus they maximize the expected present value of their lifetime earnings. Workers make decisions about whether they work, stay on their island without working or search for a new island. They discount future earning at rate β . Unemployed workers who decide to be non-search unemployed earn flow payoff b_r . This flow payoff is not available to searchers. Based on time diary evidence one can understand this as a time that search takes that non-searchers can allocate to leisure and home production. Note that this term plays an important part in making search unemployment more costly than non-search unemployment. It is assumed that agents can move instantly between mismatch unemployment and work.

We can summarize the worker's problem in an island (x, z) with following the Bellman equation

$$V(x, z) = \max\{\theta, \max\{w(n(x, z)), b_r\} + \beta E[V(z', x') | z, x]\},$$

where θ is the value of search. Moreover, a worker takes $n(x, z)$ and $r(x, z)$ as given

EQUILIBRIUM. Given that local markets are competitive, at the equilibrium it must be that

$$b_r \leq w(n(x, z), z).$$

When the previous condition holds as an equality, some workers decide to be non-search unemployed. Thus, in the model considered here mismatch unemployment is voluntary. However, as demonstrated later in this dissertation, non-search unemployment can easily be made non-voluntary. Some authors, e.g.,

Alvarez and Shimer (2011) call voluntary mismatch unemployment rest unemployment⁸. Birchenall (2011) uses a term waiting unemployment for the type of mismatch unemployment where workers would like to work in their current labor market but those jobs are not currently available.

At the equilibrium local policies $n(x, z)$ and $r(x, z)$ have to be consistent with the individual behavior. We can identify four possible, qualitatively different, equilibrium candidates. First if $n(x, z) = x$, then $V(x, z) > \theta$. Second if $n(x, z) + r(x, z) = x$ and $r(x, z) > 0$ then $V(x, z) > \theta$ and $w(n(x, z), z) = b_r$. Thirdly if $n(x, z) < x$ and $r(x, z) = 0$, then $V(x, z) = \theta$ and $w(n(x, z), z) > b_r$. Finally if $n(x, z) + r(x, z) < x$ and $r(x, z) > 0$, then $V(x, z) = \theta$ and $w(n(x, z), z) = b_r$.

For now we have only described workers' actions on an island while we have taken aggregate variables θ and U_S as given. In order to define these variables we have to state how the distribution of islands evolves. In this dissertation, I focus on stationary equilibria where the distribution of islands stays unchanged in each period. That is, workers' actions and productivity process generate a distribution of islands

$$\mu(X', Z') = \sum_{i=1}^n Q(z_i, Z') \int_{\{(x,z): (n(x,z)+r(x,z)+U_S) \in X'\}} \mu(dx, z_i)$$

such as for all possible sets of (X', Z') the measure of islands $\mu(X', Z')$ does not change. Given this measure we can describe the value of search

$$\theta = \beta \sum_{i=1}^n \int V(x, z_i) \mu(dx, z_i).$$

Aggregate employment is given by

$$N = \sum_{i=1}^n \int n(x, z_i) \mu(dx, z_i)$$

and the amount of mismatch unemployment is

$$U_R = \sum_{i=1}^n \int r(x, z_i) \mu(dx, z_i).$$

N and U_R pin down the amount of search unemployed

$$U_S = 1 - N - U_R.$$

COMPUTATION. The model cannot be solved analytically. Thus, in order to analyze it, a numerical solution is required. The computational algorithm is as follows

1. Set up a grid for X and choose the potential values for productivity, z , and transition probabilities $Q(z, z')$.

⁸ The term is borrowed from Jovanovic (1987)

2. Guess the value of search, θ , and the amount of search unemployed, U_S .
3. Solve the value function, $V(x, z)$ for each (x, z) .
4. Given the value function and the exogenous productivity process, generate a stationary distribution by simulating a large number of labor markets.
5. Compute the value of search and the amount of search unemployed from the simulated stationary distribution. If the simulated values are equal to the values used in step 2, stop. Otherwise, return to step 2.

In step 3 value function iteration is a convenient approach. Start with an initial guess for the value function and for each point on the grid work through the potential equilibrium candidates. If an equilibrium candidate is consistent with the current approximation of value function, save the implied policies $n(x, z)$ and $r(x, z)$ and update the value function accordingly. That is, given the i th iteration's value, $V_i(x, z)$, at (x, z) , assume that all workers stay and work, and check if $w(x, z) > b_r$ and $w(x, z) + \beta EV(x + U_S, z') > \theta$. If this holds the set $V_{i+1}(x, z) = w(x, z) + \beta EV(x + U_S, z')$ and $n(x, z) = x$ and $r(x, z) = 0$. Otherwise, move on to the next equilibrium candidate. For the candidates for which some worker reallocate one has to use an equation solver, such as bisection method, to find amount of agents staying that equates the value(s) of staying with θ . After working through all points on the grid compare $V_i(x, z)$ and $V_{i+1}(x, z)$. If they are "close enough", stop. Otherwise move on to the next iteration round.

The model decomposes unemployment into search and mismatch unemployment. For the implications of the model, it is important to think of the dimension along which search frictions are modeled. The original paper by Lucas and Prescott (1974) considered spatially separate markets. Later studies have interpreted islands, for example, as occupations⁹ or industries¹⁰. Empirical studies on mismatch, such as Sahin et al (2014), seem to suggest that the geographical interpretation of mismatch is not as important as occupational or industrial.

2.4 Studies on Endogenous Mismatch

This subsection highlights existing studies that explore the importance of endogenous mismatch. The exact set-up that is used to generate mismatch vary somewhat between the papers. However, all the discussed studies build on a similar structure to the one considered in the previous subsection in a sense that they start from island-search models, where unemployed workers can choose whether to stay or to go.

⁹ see. Kambourov and Manovskii (2009)

¹⁰ See Alvarez and Shimer (2011)

Alvarez and Shimer (2011) develop a continuous time version of the island model where, as in the previous subsection, workers can stay inactive in their current local market. They assume that the productivity in local markets follows a geometric Brownian motion. This enables them to reduce the number of state variables from two to one, the real wage given that all agents present would work, and solve their model analytically¹¹. The workers' search restricts the evolution of wages. In the case of undirect search, there exists a threshold wage level which triggers the reallocation. This implies that there is a reflecting barrier, \underline{w} , for wages in each local market. Moreover, for those markets, where the wage is higher than \underline{w} , incoming workers put a downward drift to the geometric Brownian motion that governs the wage dynamic in local markets. The larger the amount of searchers, U_S , the stronger is the downward drift. A possibility of rest unemployment also implies that there is another threshold that determines whether there are non-search unemployed workers present in a market. This is similar to the equilibrium condition $b_r \leq w(n(x, z), z)$ in the last subsection. The authors also present a version of model where search is directed. In this version, there is also an upper barrier to the wage rate.

Alvarez and Shimer (2011) show that the parameters of the model and the amount of search and rest unemployment generated by the model can be linked with an estimated AR(1)-process of log-wages in local markets. To be more precise, they show that high search unemployment implies relatively low persistence for AR(1)-process as the wage hits the reflecting barrier often.

Next they interpreted islands as industries and estimate the persistence in wages using the panel data of industry wages in the US. Alvarez and Shimer (2011) show that with realistic parameter values, the model is not able to generate the observed persistence of wages unless most of unemployment is non-search unemployment. That is, if search unemployment is higher than 1.3 percent, it is difficult to generate the observed high persistence of industry wages given their set-up.

Carrillo-Tudella and Visschers (2014) build on Alvarez and Shimer (2011). They introduce vacancies into a set-up similar to the one explored in the previous subsection. That is, they assume that local labor markets are characterized with search friction modeled with a Mortensen-Pissarides style reduced form matching function. The fact that they explicitly model job creation takes their model closer to the idea of mismatch laid out in Shimer (2007). However, they also assume that, unlike in Shimer (2007), the firms are free to choose at which submarket they create vacancies. Their model decomposes unemployment to three different forms of unemployment: search between local markets, search within local markets and non-search unemployment. The model also allows

¹¹ Their approach is similar to mean field games that have recently gained attention in mathematics (see e.g. Lasry and Lions, 2007), where a HJB-equation and a Kolmogorov forward equation determine an equilibrium.

workers' location specific productivity to evolve in learning-by-doing-style.

Carrillo-Tudela and Visschers (2014) reduce the complexity of the model by assuming that the production function is linear. This leads to a block recursive equilibrium structure (see Menzio and Shi, 2010) where the heterogeneous agents behave independently. The assumption is important as their focus is on out-of-steady-state dynamics as they are interested in the model's ability to explain the business cycle variation of unemployment.

On the empirical side Carrillo-Tudela and Visschers (2014) focus on occupational reallocations. That is, they assume that the relevant form of mismatch is between occupations. They show, using the Survey of Income and Program Participation, that on average over 50 percent of unemployed workers change occupations during an unemployment spell¹². Thus for understanding aggregate unemployment, modeling unemployed workers' occupational mobility is important.

The authors calibrate their model by targeting the long run empirical facts on occupational mobility, and returns on occupational tenure. They also match their model along other steady state features of unemployment such as the duration distribution, and the reduced form aggregate matching function. Their steady state results indicate that even though a considerable fraction of unemployed workers change occupations during an unemployment spell, non-search unemployment is still the most predominant form of unemployment accounting 68 percent of aggregate unemployment, while reallocation makes up only 10 percent of total unemployment. That is, also those workers who switch occupations typically spend considerable time being non-search unemployed. Note that this steady state result, which was not targeted, is qualitatively in line with Alvarez and Shimer (2011) and the time diary studies discussed earlier.

The model performs well along several dimensions on the business cycle frequency. It is able to replicate procyclical occupational reallocations. Furthermore consistent with the data, the model implies that occupational mobility is more volatile and persistent than output per worker. It also generates a strongly cyclical aggregate job finding rate. Moreover, the correlation between aggregate market tightness and job finding rate is high. Correlation between unemployment and vacancies is -0.61. Thus, as with Shimer (2007), the mismatch model is able to generate reduced form aggregate matching function and a Beveridge curve that fits the US data reasonably well. The model's ability to generate a negative correlation between vacancies and unemployment is noteworthy as it allows endogenous separations.

Since Carrillo-Tudela and Visschers (2014) model reallocations endogenously, they can explore the relative importance of mismatch and search unemployment for the unemployment variation. According to their model non-search unemployment is the most sensitive component of unemployment for the aggre-

¹² The exact amount depends on the occupational criterion used.

gate shocks. This supports the idea proposed in Shimer (2007) that mismatch can be highly important for the business cycle variation of unemployment.

Wiczer (2015) utilize a model similar to Carrillo-Tudela and Visschers (2014)¹³ in order to analyze connections between long-term unemployment and occupational mobility. He shows that his calibrated model is capable of explaining 80 percent of the strong increase in long-term unemployment during the great recession and 80 percent of the cross sectional dispersion in unemployment durations.

Birichenall (2011) builds a model of competitive mismatch. In the model, as in Carrillo-Tudela and Visscher (2014) and Wiczer (2015), vacancies are modeled explicitly. Moreover, in his model it is not just workers who can reallocate but also capital is mobile. In the model demand shocks generate mismatch and thus, unlike in the other papers discussed in this subsection, mismatch unemployment is not voluntary.

Birichenall (2011) uses his model to explain medium-run behavior of labor markets. Unlike in the business cycle frequencies the correlation between vacancies and unemployment is positive in the trends of HP-filtered series for the US data. Interestingly, for the aggregate volatility this low frequency movement is as important as the business cycle variation. Birichenall's (2011) model is qualitatively consistent with these features of the US labor market data.

In the papers discussed the focus have been on exploring whether and to what extend non-search unemployment can help to explain observed labor market features from the steady state aspects to business cycle properties. Given the encouraging results it is noteworthy that policy analysis and considerations of institutional set-ups are absent from the literature. It is highly probable that, for example, the effects of institutional change could be different depending on whether non-search unemployment is allowed or not. Moreover, mismatch unemployment gives rise to a whole set of new policy relevant questions, such as to what extend policy makers should aim for increasing the mobility of unemployed workers¹⁴.

The on-going work of Alvarez and Shimer (2014) is a notable exception. In their project, they utilize a model similar to Alvarez and Shimer (2011) in order to understand to what extend small unions can generate rest unemployment. The first two essays of this dissertation continue towards this direction. The first paper analyzes the effects of human capital protection in a situation where demand shocks generate mismatch unemployment and markets are unable to allocate layoffs optimally. The second essay explores the role of unionization, with different union structures, in a model where non-search unemployment is

¹³ The main difference being that in Carrillo-Tudela and Visschers (2014) workers productivity in an occupations is purely idiosyncratic while Wiczer (2015) focuses on occupational shocks that affect the whole group of workers

¹⁴ As the first essay highlights the answers are not always straight forward when, for example, the policy's effects on human capital have been taken into account

calibrated based on the high recall rate observed in the US. The main difference between this study and Alvarez and Shimer (2014) is that my focus is on quantitative work in a framework where union's policy is an equilibrium outcome, while their work is more theory oriented and, for the most part, they take the behavior of unions as exogenous. Additional difference is that I am mainly interested in the effects of European style large union while they concentrate on the US style small unions.

3 THE IMPORTANCE OF LONG-RUN RESTRICTIONS FOR ESTIMATED BUSINESS CYCLE MODELS

The real business cycle literature pioneered by Kydland Prescott (1982) and Long and Plosser (1983) has traditionally taken the long-run growth restrictions extremely seriously. This is probably partly explained by the fact that the computational complexity required that the early stage versions of RBC models had to be calibrated instead of estimated. For calibration exercises, the long-run restrictions are usually, right after a credible micro data based evidence, the second most convincing source of calibration targets. However, as computers and estimation strategies have evolved, the emphasis has slowly moved from long-run targets to the direct fit of the models in business cycle frequencies. This is especially true for New Keynesian models (see, e.g., Smets and Wouters, 2003 or Christiano, Eichenbaum and Evans, 2005).

The aim of my third essay (co-authored with Juha Kilponen and Jouko Vilmunen) is to show that long run restrictions can still be highly important, not just because they give additional information, but also because ignoring them can severely impact the potential parameter space that is plausible in the estimation process. We highlight this with the help of balanced growth path restrictions in the context of a sticky price model. To be more precise, we focus on the estimation of intertemporal elasticity of substitution in a small scale New Keynesian model when preferences are chosen in a way that is in line with long run trends in consumption, wages and hours worked. We use the stylized model because it enables us to emphasize the links between the preferences and the model-implied dynamics in a tractable way.

In real business cycle models a sufficiently strong intertemporal elasticity of substitution (IES), which links consumption and hours worked to real intertemporal prices, plays a highly significant role in ensuring that the models generate realistic covariation in key economic variables (see King and Rebelo, 1999). The role of IES is even more important in standard representative agent New Keynesian models. In these models, the central bank's ability to affect real economy is directly related to consumers' sensitivity to intertemporal prices (see discussion in Kaplan, Moll and Violante, forthcoming). For this reason, realistic values for IES are highly important for the policy analysis. Typically, the literature has focused on additively separable preferences and either assumed a high intertemporal elasticity or ignored the balanced growth path restrictions.

Assuming additively separable preferences while keeping the business cycle

model in line with the growth facts, implies that IES has to be fixed to one. This is, however, at the odds with empirical estimates based on consumers' Euler equations that typically find that intertemporal elasticity is much smaller than one (see, e.g., Hall, 1988; Basu and Kimball, 2002 or Yogo, 2002).

The literature has often assumed a more flexible parametrization while still keeping the assumption of additive separability (see, e.g., Smets and Wouters, 2003) However, the additive form implies the zero elasticity between hours and consumption which is at the odds with micro data based literature that suggests that consumption reacts to known changes in labor supply, such as retirement (see, e.g., Bernheim et al, 2001). The cross elasticity could be non-zero if households like to consume more when they work more (e.g. households dine out more when they work more). Time diary studies, such as Aguiar et al (2013) discussed earlier, give an additional explanation for the complementarity between consumption and leisure in traditional business cycle models. That is, if the elasticity of home production and market production is high, in the models where home production is ignored we should see strong cross elasticity between leisure and market good consumption (see Auaiar et al, 2013, for further discussion).

In the third essay, we apply the balanced growth path consistent preferences, similar to King, Plosser and Rebelo (1988), and show that the tight relationship between IES and the slope of the Phillips curve, present when preferences are additively separable, is broken down. That is, it is possible that IES is low without imposing a steep Phillips Curve. We also estimate our small scale model and show that data favors a small IES. Thus, our results are in line with the aggregate time series analysis discussed above.

It is important to note that the modest responses to the intertemporal price changes do not imply that the monetary policy is inefficient. However, the low value of IES could imply that the monetary transmission mechanism needs to be reconsidered. Interestingly, a recent study by Kaplan et al (forthcoming) proposes an alternative mechanism through which the monetary shocks could affect consumption. The authors explore the monetary transmission mechanism in a Heterogeneous Agent New Keynesian (HANK) model. In their model, the direct effects of interest rate change on consumption are small. However, the equilibrium responses that operate through changing labor demand are far more important. This happens because in their calibration there is a large number of hand-to-mouth and wealthy hand-to-mouth consumers whose consumption reacts strongly to increased labor income. There is also another potentially strong indirect channel that works through fiscal policy. When the central bank changes the interest rate, it directly affects the cost of the government debt. Additionally, as hours worked change, the government's tax revenues also change. As there is a large number of non-Ricardian households, the change in fiscal policy will affect consumption, amplifying the indirect effects of monetary policy.

If the equilibrium reactions in the labor market are important for the transmission of nominal interest rate shocks to the reactions in real variables, the structural modeling of these markets in monetary policy context could be important not just for labor market outcomes, such as unemployment, but also for other equilibrium objects. For example, questions such as to what extent monetary policy works through extensive or intensive margins, could be highly relevant to the aggregate effects of monetary shocks.

4 SUMMARY OF THE ESSAYS

In this section, I present a brief summary of the three essays that make up the dissertation.

Essay 1: Layoff Orders, Human Capital and Occupational Mobility via Unemployment

In this essay, I examine the equilibrium effects of occupational human capital protection during mass layoffs. In the model, workers are distributed along a continuum of local labor markets, occupations. Occupation-specific productivities and demand levels vary, creating reallocations and layoffs. Workers accumulate occupation-specific human capital in a learning-by-doing-fashion. This human capital is assumed to be fragile, in the sense that it can depreciate during unemployment spells. Moreover, a market failure prevents the optimal allocation of layoffs in the laissez-faire economy. As the potential advantages and disadvantages are tightly related to workers' reallocation decisions, the essay focuses on modeling the reallocation behavior of heterogeneous workers under different layoff rules.

In a calibrated model, a policy that concentrates involuntary unemployment incidences to inexperienced workers, decreases workers' incentives to reallocate, compared to an equilibrium where everyone faces an identical unemployment risk, leading also to a decrease in aggregate unemployment. However, due to increased human capital, this policy change increases the market output and, on average, does not harm inexperienced workers.

Essay 2: Unionizing Non-search Unemployment

Essay 2 explores the effects of unionization in an island model of Lucas and Prescott (1974) with different union structures. When a model with competitive labor markets is set to match the empirical fact that a large number of unemployment spells ends with recalls, the introduction of a large labor union that represents all workers and sets a common economy-wide minimum wage increases unemployment substantially. Moreover, the whole increase is about non-search unemployment as search unemployment actually reduces marginally.

If the same degree of unionization is generated by a continuum of small unions, the aggregate unemployment reaction is somewhat smaller. However,

the increase in non-search unemployment is still considerable.

The workings of a large union are also explored when the union is assumed to bargain over the minimum wage with an employers' organization. This environment leads to a considerably lower increase in aggregate unemployment. Yet again, the search intensity of unemployed workers drops significantly.

Essay 3: Estimating Intertemporal Elasticity of Substitution in a Sticky Price Model

Cancellation of income and substitution effect implied by King-Plosser- Rebelo (1988) preferences breaks tight coefficient restriction between the slope of the Phillips curve and the elasticity of consumption with respect to real interest rate in a sticky price macro model. This facilitates the estimation of intertemporal elasticity of substitution using full information Bayesian Maximum Likelihood techniques within a structural model. The US data from the period 1984-2007 supports low intertemporal elasticity of substitution and strongly rejects a logarithmic and an additively separable utility specification commonly applied in the New Keynesian literature.

REFERENCES

- Aguiar, M. – Hurst, E. (2007) Measuring trends in leisure: The allocation of time over five decades. *The Quarterly Journal of Economics*, Vol. 122 (3), 969–1006.
- Aguiar, M. – Hurst, E. – Karabarbounis, L. (2013) Time use during the great recession. *American Economic Review*, Vol. 103 (5), 1664–1696.
- Alvarez, F. – Shimer, R. (2011) Search and rest unemployment. *Econometrica*, Vol. 79 (1), 75–122.
- Alvarez, F. – Shimer, R. (2014) Unions and unemployment, Unpublished manuscript, University of Chicago.
- Basu, S. – Kimball, M. (2002) Long run labour supply and the elasticity of the intertemporal substitution for consumption, Working Paper, University of Michigan.
- Benhabib, J. – Rogerson, R. – Wright, R. (1991) Homework in macroeconomics: Household production and aggregate fluctuations. *Journal of Political Economy*, Vol. 99 (6), 1166–1187.
- Bernheim, D. – Skinner, J. – Weinberg, S. (2001) What accounts for the variation in retirement wealth among u.s. households? *American Economic Review*, Vol. 91 (4), 832–857.
- Birchenall, J. (2011) A competitive theory of mismatch, Unpublished manuscript, University of California at Santa Barbara.
- Carrillo-Tudela, C. – Visschers, L. (2014) Unemployment and endogenous reallocation over the business cycle, ESE Discussion Paper 241, Edinburgh School of Economics, University of Edinburgh.
- Christiano, L. – Eichenbaum, M. – Evans, C. (2005) Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, Vol. 113 (1), 1–45.
- Coles, M. – Muthoo, A. (1998) Strategic bargaining and competitive bidding in a dynamic market equilibrium. *Review of Economic Studies*, Vol. 65 (2), 235–260.
- Ebrahimi, E. – Shimer, R. (2010) Stock flow matching. *Journal of Economic Theory*, Vol. 145 (4), 1325–1353.
- Hall, R. (1977) An aspect of the economic role of unemployment. In *Microeconomic Foundations of Macroeconomics*, ed. G. C. Harcourt, 354–372, London: Macmillan.
- Hall, R. E. (1988) Intertemporal substitution in consumption. *Journal of Political Economy*, Vol. 96, 339–357.

- Hawkins, W. B. (2015) A note on wage determination under mismatch. *Macroeconomic Dynamics*, Vol. 19 (8), 1866–1879.
- Jovanovic, B. (1987) Work, rest, and search: Unemployment, turnover, and the cycle. *Journal of Labor Economics*, Vol. 5 (2), 131–148.
- Kambourov, G. – Manovskii, I. (2009) Occupational mobility and wage inequality. *Review of Economic Studies*, Vol. 76 (2), 731–759.
- Kaplan, G. – Moll, B. – Violante, G. L. (Forthcoming) Monetary policy according to HANK. *American Economic Review*.
- King, R. – Plosser, C. – Rebelo, S. (1988) Production, growth and business cycles: The basic neoclassical model. *Journal of Monetary Economics*, Vol. 21 (2-3), 195–232.
- King, R. – Rebelo, S. (1999) Resuscitating real business cycles. In *Handbook of Macroeconomics*, eds. J. Taylor – M. Woodford, Vol. 1, chap. 14, 927–1007, North Holland.
- Krueger, A. – Mueller, A. (2010) Job search and unemployment insurance: New evidence from time use data. *Journal of Public Economics*, Vol. 94 (3-4), 298–307.
- Krueger, A. – Mueller, A. (2011) Job search, emotional well-being, and job finding in a period of mass unemployment: Evidence from high-frequency longitudinal data. *Brookings Paper on Economic Activity*, 1–70.
- Krueger, A. – Mueller, A. (2012) The lot of the unemployed: A time use perspective. *Journal of European Economic Association*, Vol. 10 (4), 765–794.
- Kydland, F. – Prescott, E. (1982) Time to build and aggregate fluctuations. *Econometrica*, Vol. 50 (6), 1345–1370.
- Lagos, R. (2000) An alternative approach to search frictions. *Journal of Political Economy*, Vol. 108 (5), 851–873.
- Lasry., J. M. – Lions, P.-L. (2007) Mean field games. *Japanese Journal of Mathematics*, Vol. 2 (1), 229–260.
- Ljungqvist, L. – Sargent, T. (2012) *Recursive Macroeconomic Theory*. The MIT Press, 3rd edn.
- Long, J. – Plosser, C. (1983) Real business cycles. *Journal of Political Economy*, Vol. 91 (1), 39–69.
- Lucas, R. E. (1976) Econometric policy evaluation: A critique. In *The Phillips Curve and Labor Markets*, eds. K. Brunner – A. H. Meltzer, Vol. 1 of *Carnegie-Rochester Conference series on Public Policy*, North-Holland.
- Lucas, R. E. – Prescott, E. C. (1974) Equilibrium search and unemployment. *Journal of Economic Theory*, Vol. 7 (2), 188–209.
- McCall, J. J. (1970) Economics of information and job search. *The Quarterly Journal of Economics*, Vol. 84 (1), 113–126.

- Menzio, G. – Shi, S. (2010) Block recursive equilibria for stochastic models for search on the job. *Journal of Economic Theory*, Vol. 145 (4), 1453–1494.
- Mortensen, D. (1977) Unemployment insurance and job search decisions. *ILR Review*, Vol. 30 (4), 505–517.
- Mortensen, D. – Pissarides, C. (1994) Job creation and job destruction in the theory of unemployment. *The Review of Economic Studies*, Vol. 61, 397–415.
- Sahin, A. – Song, J. – Topa, G. – Violante, G. L. (2014) Mismatch unemployment. *American Economic Review*, Vol. 104 (11), 3529–3564.
- Shimer, R. (2005) The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review*, Vol. 95 (1), 25–49.
- Shimer, R. (2007) Mismatch. *American Economic Review*, Vol. 97 (4), 1074–1101.
- Smets, F. – Wouters, R. (2003) An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of European Economic Association*, Vol. 1 (5), 1123–1175.
- Wiczer, D. (2015) Long-term unemployment: Attached or mismatched?, Working Paper no. 2015-042A, Federal Reserve Bank of St. Louis.
- Yogo, M. (2002) Estimating the elasticity of intertemporal substitution when instruments are weak. *The Review of Economics and Statistics*, Vol. 86 (3), 797–810.

Part II
ESSAYS

ESSAY 1

Oskari Vähämaa

Layoff Orders, Human Capital and Occupational Mobility via Unemployment

Preprint

Layoff Orders, Human Capital and Occupational Mobility via Unemployment*

Oskari Vähämaa[†]

Abstract

This paper examines the equilibrium effects of occupational human capital protection during mass layoffs in a setup where human capital can depreciate during unemployment spells and commitment problems prevent markets from allocating layoffs optimally. As the consequences of the policy are tightly related to occupational mobility, the paper focuses on modeling reallocation incentives of heterogeneous workers. In a calibrated model, a policy that concentrates involuntary unemployment incidences to inexperienced workers, decreases workers' incentives to reallocate, compared to an equilibrium where everyone faces an identical unemployment risk, leading also to a decrease in aggregate unemployment. Moreover, this policy change increases the market output and on average does not harm the inexperienced workers.

1 INTRODUCTION

The costs of unemployment go well beyond the lost wages during the unemployment spells; the earning losses after re-employment are severe and persistent¹. Interestingly, it also seems that the costs of layoffs vary with worker characteristics, as the earning losses due to the job displacement increase with job tenure²

* This paper was previously circulated under the title "Layoff Orders and Occupational Mobility via Unemployment".

[†] *Department of Economics, University of Turku, FI-20014, Finland. Email: oskari.vahamaa@gmail.com. I would like to thank Marlène Isoré, Antti Kauhanen, Mitri Kitti, Stefan Pitschner, Thijs van Rens, Richard Rogerson, Kjetil Storesletten, Jouko Vilmunen and Matti Viren for helpful comments and suggestions. I would also like to thank conference and seminar participants at the Oslo-BI-NHH Workshop in Macroeconomics, EALE conference and the Macroeconomics and Development seminar at University of Helsinki.*

¹ See, for example, Jacobson, Lalonde and Sullivan (1993) and Couch and Placzek (2010).

² see Hamermesh (1989) for a summary of these studies.

and age³. Yet, there do not seem to exist markets that would allocate layoffs to the ones who suffer the least. On the other hand, informal and formal seniority practices are common.

If the earning losses reflect the loss of production potential, the cost of inefficient allocation of layoffs could be high, not just for the individuals involved but for the whole society. In this paper, I look at the equilibrium effects of introducing a layoff order that protects experienced workers over inexperienced ones, in a stationary environment where experience is specific to local labor markets. This implies that changing the layoff rule will also affect the amount and decomposition of unemployment. Especially, reallocation unemployment in the economy will change, partly due to the decomposition effect, as the value of waiting is higher for the experienced unemployed workers, implying they are less mobile compared to the inexperienced ones. Furthermore, the policy will also affect the reallocation incentives of heterogeneous workers. The change in reallocation unemployment could be important in its own right as a large macro-oriented body of research emphasizes efficient reallocation to the aggregate economic outcomes⁴. Moreover, reallocations obviously restrict the policy's ability to protect experienced workers.

The paper focuses on occupational human capital and its connections with occupational mobility via unemployment. The importance of occupational human capital is motivated by the work of Kambourov and Manovskii (2009b). They find large returns on occupational tenure, while firm and industry tenures play a relatively minor role in explaining wages⁵. Another reason for focusing on occupations is that unemployment spells with occupational mobility make up for a large fraction of aggregate unemployment⁶, and so policies that affect reallocation unemployment could play a significant role for the aggregate unemployment as well as being important for efficient reallocation of labor force.

In order to formalize the idea, I build an equilibrium island model in the spirit of Lucas and Prescott (1974), where employed and unemployed workers make endogenous reallocation decisions. The model contains a continuum of occupations and a continuum of infinitely-lived workers. Workers accumulate occupation-specific human capital while working. To consider different layoff rules, I assume that the goods market is also segmented in the sense that shoppers enter into local markets with a license to consume a fixed amount. More-

³ see e.g. Couch, Jolly and Placzek (2009) or Carrington (1993).

⁴ For example, firm dynamic models, such as Hopenhayn and Rogerson (1993), emphasize the efficient transition of labor force between firms. In growth theory, for instance, Restuccia, Yang and Zhu (2008), highlight the factor reallocation across sectors as a possible explanation for income and growth differences across countries.

⁵ Estimation results by Sullivan (2010) also support the view that occupational human capital is important. He also emphasizes the importance of industry specific human capital if within-firm occupational mobility is allowed.

⁶ According to Carrillo-Tudela and Visschers (2014) 50% of completed unemployment spells in the US involves a occupational reallocation between major occupational groups.

over, entering into local markets is uncertain. This generates demand shocks that cannot be sorted out by adjusting prices. From the workers' point of view this means that the local labor markets are like labor markets for stevedores in the past. Workers come to the harbor in the morning and, depending on how many ships have arrived or will arrive during the day, some of them will be hired. This structure creates a situation where some workers have to be laid-off (or not be hired for that day) when the demand is low. Unemployed workers can then either decide to wait for a job or to reallocate. Employed workers make the same decisions, but are, in general, less mobile since reallocation would require them to forgo the current period wage. To make the costs of unemployment heterogeneous in a simple human capital framework, I follow Ljungqvist and Sargent (1998), and assume that human capital can erode during unemployment spells.

To keep the model tractable, I assume that experience levels are perfectly substitutable in production. This imposes a clear structure on workers' value functions (i.e., experienced workers are always better off than inexperienced workers with a similar job market status), which simplifies the computational process. Note also that this assumption, together with the perfect competition in local labor markets, implies that firms are indifferent over different layoff orders. In locations, where the demand is low, buyers and firms are assumed to meet randomly. Experienced workers would be willing to pay for inexperienced workers to reduce their unemployment risk⁷. However, a large number of anonymous workers together with random meetings and free mobility of workers within local markets make it impossible for workers to make such arrangements. This implies that without rules set by the society, each worker faces an identical unemployment risk.

When the model with an equal unemployment risk is calibrated to match the empirical evidence on the occupational human capital, the average wage losses and the reallocation patterns of unemployed workers in the US, it turns out that, even though the inexperienced workers are more willing to change occupations during unemployment spells, the introduction of the human capital rule actually reduces occupational mobility from 4% to 2.9-3.5%⁸. Moreover, the aggregate level of human capital, i.e., the amount of experienced workers in the economy, increases as experienced workers are now better protected against human capital depreciation. This "insurance" also reduces the experienced workers' incentives to reallocate since returns on experience increase. Same holds for

⁷ Since workers can freely move from one firm to another, it is not enough that experienced workers would just have smaller wages without inexperienced workers being fully compensated.

⁸ The introduction of the new layoff order opens up the possibility of multiple equilibria. This happens since now the actions of inexperienced workers are partially strategic complements. Unemployment probability in an occupation is smaller if all inexperienced workers decide to stay in their current occupations. Results reported are related to the percentage changes associated with the equilibria where the occupational mobility is at its lowest and highest levels possible.

the inexperienced workers, given that they have a job. The effects of increasing human capital are large enough to offset the reduction of average labor productivity caused by the reduced mobility. Due to these effects, the market output produced in the economy increases by 1.5-1.6%. Moreover, the inexperienced workers are also, on average, better off.

The model considered in this paper builds on the island framework of Kambourov and Manovskii (2009a). In their paper, islands are interpreted as occupations and workers accumulate occupational human capital in a learning-by-doing-fashion. Different levels of occupation-specific human capital imply that workers' reallocation costs are different. However their focus is quite different as they use the model to study the connections of occupational mobility and wage inequality. I also add demand shocks to this island structure in order to consider layoff practices. Because of this, my model contains elements of mismatch unemployment *à la* Shimer (2007), even though it does not contain vacancies typically present in mismatch papers. Here it is the inability of shoppers and sellers to meet that leads to a situation where some unemployed workers will rather stay close to their previous job than search for a new one.

By combining mismatch and reallocation unemployment the model developed in this paper is also closely related to a branch of literature that concentrates on the endogenous reallocation decisions of unemployed workers across local labor markets. In these papers the aim is to understand why unemployed workers may prefer staying attached to their current labor markets even though some other locations could offer better labor market conditions. For example, Alvarez and Shimer (2011) decompose unemployment to rest and reallocation unemployment. In their model, rest unemployment is generated since reallocation to better industries is costly time- and resource-wise, implying that some workers find it optimal to stay inactive in their current industry and wait for industry conditions to improve. Carrillo-Tudela and Visschers (2014) take into account the effects of different experience levels at the top of employment status when considering occupational reallocation.⁹ However, unlike Carrillo-Tudela and Visschers (2014), who model intra-island unemployment with the setup of Mortensen and Pissarides (1994) at the top of rest unemployment, I, similar to Birchenall (2011), use demand shocks as a driving force behind involuntary intra-island unemployment. These papers have been successful in replicating a variety of labor market facts.¹⁰ My contribution to this literature is to look at how policies implemented by the society, *i.e.*, different layoff orders, can affect the composition of unemployment and other aggregate equilibrium variables,

⁹ With this respect the work by Alvarez and Shimer (2012) is also closely related. Their paper concentrates on the nature of unemployment for experienced workers.

¹⁰ For business cycles properties, see Carrillo-Tudela and Visschers (2014), and for links between unemployment durations and occupations, see Wiczer (2015). For medium run trends, see Birchenall (2011).

such as the output produced and the average labor market productivity¹¹. With this respect, my work is perhaps closest to Alvarez and Shimer (2014) who explore how island specific unions can generate non-search unemployment. In their model, they assume that unions’ apply a strict last-in-first-out-rule when they decide who are allowed to work. However, in their model workers are identical in their productivity. This implies that in their analysis the seniority rule only protects the “union bosses” from the negative effects of rationing the local labor supply and thus cannot have socially beneficial effects.

The remainder of the paper is organized as follows. Section 2 presents the economy with equal unemployment probability and the economy in which experienced workers are protected. Section 3 analyzes a simplified framework analytically while section 4 states the calibration of the model. Results are reported in section 5 while section 6 looks at the sensitivity of the results. Section 7 concludes the paper.

2 MODEL

In this section I introduce a model that is used to analyze the effects of layoff orders. Section 2.1 describes a model with an equal unemployment risk while section 2.2. presents a model where experienced workers face an unemployment risk only if there are not enough inexperienced workers to absorb the demand shock. For brevity, I will call this layoff rule a “seniority rule”, even though it is the amount of human capital that workers have that affects their unemployment probability and not the length of their tenure.

2.1 Economy with an equal unemployment probability

ENVIRONMENT. There is a mass one of infinitely-lived workers distributed along a continuum of locations, islands. Workers differ in location-specific productivity, experience and employment status. I interpret islands as different occupations. The log of productivity level in each location, $\ln(z(l))$, is assumed to follow an AR(1)-process. Demand shocks generate involuntary unemployment. With probability p , demand in location l is below the full employment level. When this happens, fraction D of production capacity remains unused. Finally, workers can be either experienced or inexperienced. When an inexperienced

¹¹ The use of an island model in order to explore effects of a labor market policy means my set up is in spirit similar to Alvarez and Veracierto (1999) who, among other things, look at the general equilibrium effects of minimum wages and firing taxes.

worker is working, there is a positive probability, α , that she will become an experienced worker. Experience is location-specific in the sense that if a worker decides to reallocate, she will become an inexperienced worker in her new island. An experienced worker can also become an inexperienced worker during unemployment spells. In each period of unemployment that the experienced worker waits for a job in her current occupation, there is a probability γ that she will become an inexperienced worker. Reallocation to a new location is possible but it will take time. If a worker decides to reallocate, she will not be able to work during that period and at the beginning of the next period she will be allocated to a new location randomly.

TIMING. I assume that the productivities are revealed at the beginning of the period. Next, nature chooses the demand levels for each location. At this point workers discover whether they have a job or not. After this, employed workers decide whether they would like to work for a given wage or whether they prefer to reallocate. Unemployed workers can decide to wait in their current location or to reallocate. After the workers have made their decisions, production takes place. At the end of the period, a fraction of the inexperienced workers that had worked will become experienced and a part of the unemployed experienced workers that decided to wait, as well as all workers that decide to reallocate, become inexperienced.

DEMAND SHOCKS. Workers belong to large families that fully insure them against differences in experience, productivity levels and employment status. Goods produced in different locations are assumed to be perfect substitutes. However, I assume that goods markets are segmented in the sense that household members have to travel through the archipelago to collect their consumption shares. Each shopper has time to visit in one island only and gets a chance to meet with only one randomly picked seller/worker. The weather in front of each island can be good or bad. The probability of bad weather is p . This probability is independent of past realizations and weather conditions in other locations. When the weather is bad, only a fraction $1 - D$, of all buyers sent to that island find their way to the island and are able to meet a seller/worker while a fraction D of workers don't meet a shopper. Moreover, overbooking is not possible. Given this a household sends shoppers to islands based on number of workers present in each locations and gives them license to buy the whole production of an worker. Later on the products bought are shared equally between members of the family.

The demand shock structure is set up to parsimoniously capture the fact that specialized production often requires tailoring the products based on the customer's individual needs; and if this customer temporarily or permanently does not turn up, it takes time to find new customers and adjust products to match the needs of the new buyers. For example, Finland had a vibrant textile industry at the end of the 1980's that primarily sold its products to the local market and to

the Soviet Union. When the Soviet Union collapsed, finding new markets and restructuring the production and design was a slow process, during which the unemployment rose.

The set up is close to the structure presented in Birchenall (2011). However, in his paper demand shocks are a combination of aggregate shocks and idiosyncratic shocks, while here demand shocks are purely idiosyncratic. Furthermore, in his specification $D = 1$, while I assume D to take an intermediate value between zero and one in order to consider the effects of different lay-off schemes.

PRODUCTION. Production in location l is given by linear production technology

$$y(l) = z(l)(k_{ie} * g_{ie,job} + k_e * g_{e,job}),$$

where k_e and k_{ie} are the levels of human capital for experienced and inexperienced workers, respectively. $g_{ie,job}$ gives the measure of inexperienced workers who were offered a job and decided to work in occupation l while $g_{e,job}$ gives the measure of experienced workers that are working. Due to the local labor markets $g_{i,job}$, $i = \{ie, e\}$, cannot be larger than the measure of type i workers present in that location at the beginning of the period, m_i^b . The evolution of productivity shock is given by

$$\ln(z'(l)) = (1 - \rho)a + \rho \ln(z(l)) + \epsilon'_l,$$

$0 < \rho < 1$ and $\epsilon'_l \sim N(0, \sigma^2)$. Markets in each location are assumed to be locally competitive, implying that wages are given by

$$w_{ie} = z(l)k_{ie}$$

$$w_e = z(l)k_e$$

The assumption of perfect substitutability between experience levels ensures that wages for experienced workers are always higher than wages for inexperienced workers.¹² Moreover, given the competitive labor markets and perfect substitutability between inexperienced and experienced workers, firms are indifferent with respect to who gets a possibility to work. As production technology is linear, one could think firms having just one worker. During low demand I assume that each worker faces an identical probability, $1 - D$, of meeting a potential buyer and with probability D they do not meet a buyer. Note that this probability is equal to the amount of consumers getting lost when the weather is bad. So, during the low demand periods, $g_{i,job}$ is further restricted, i.e. $g_{i,job} \leq (1 - D)m_i^b$ if the demand is low. As an example of the market structure, think taxicabs at a taxicab stand where taxicabs are organized to a queue randomly and the amount of passengers is random.

¹² This implies that the value functions for experienced workers always take values higher or equal than the value functions for inexperienced workers with a similar job market status (employed or unemployed). This helps the solution of the equilibrium with “seniority rule” since it greatly reduces the potential equilibrium candidates that I have to consider during the computation of the model.

Given the random meetings and the large number of anonymous workers, contracts that would reduce the unemployment risk for experienced workers are not credible. Workers would have to collectively agree how the generated surplus would be shared between all experienced and inexperienced workers present in the location. After that all experienced workers should pay to each inexperienced worker. However, given that all others are paying, an experienced worker would always prefer not to pay at least to one inexperienced worker as this would not affect the unemployment probability and anonymity would prevent the punishment.

WORKERS' PROBLEM. Given the insurance arrangement and the assumption that market goods and home goods are perfect substitutes, workers simply maximize their lifetime incomes. I assume that workers earn income b_r when they are waiting and workers who decide to switch occupations (i.e. searchers) do not earn anything during the reallocation process. Workers take the wages and the value of search, θ , as given. A worker who is offered a job, makes her decision between working, W , waiting, R , and reallocation, and a worker who cannot work, makes a decision between reallocation and waiting. The value function for inexperienced workers with a job offer at the beginning of the production stage is

$$\begin{aligned} V_{ie,job}(z) &= \max\{W_{ie}(z), R_{ie}(z), \theta\} \\ W_{ie}(z) &= w_{ie}(z) + \beta[\alpha EV_e^b(z') + (1 - \alpha)EV_{ie}^b(z')] \\ R_{ie}(z) &= b_r + \beta EV_{ie}^b(z'), \end{aligned}$$

where $V_{ie}^b(z')$ gives the value of being an inexperienced worker in the current occupation at the beginning of the next period before the demand shocks are revealed, α is the probability of skill evolution and $V_e^b(z)$ gives the value of being an experienced worker.

The value function for an inexperienced worker who does not have a job is given by

$$V_{ie,nojob}(z) = \max\{R_{ie}(z), \theta\},$$

where $R_{ie}(z)$ is the same as before.

Value functions for an experienced worker are similar

$$\begin{aligned} V_{e,job}(z) &= \max\{W_e(z), R_e(z), \theta\} \\ V_{e,nojob}(z) &= \max\{R_e(z), \theta\} \\ W_e(z) &= w_e(z) + \beta E_t V_e^b(z') \\ R_e(z) &= b_r + \beta[(1 - \gamma)E_t V_e^b(z') + \gamma E_t V_{ie}^b(z')], \end{aligned}$$

where γ is the probability of skill loss during the unemployment period.

Note that, given our assumption about the demand shocks, we can rewrite the beginning of the period value functions as

$$E_t V_i^b(z') = (1 - pD)EV_{i,job}(z') + pDEV_{i,nojob}(z'),$$

where $i = \{ie, e\}$. I assume that $b_r < w_{ie}(z)$ for all z . This implies that workers who are offered a job always make their decisions between working and reallocation.

EQUILIBRIUM. Note that, since I assumed linear production function, I can solve the value functions without knowing the distribution of agents over the islands. In other words, I can solve the value functions first and then generate the distribution of workers over islands based on endogenous reallocation decisions implied by value functions, stochastic productivity levels and search technology. Menzio and Shi (2010) call this type of equilibrium a block recursive equilibrium. In models that consider endogenous reallocation across occupations, Carrillo-Tudela and Visschers (2014) also use this type of equilibrium structure in order to consider aggregate and idiosyncratic shocks jointly¹³.

The measure of different types of workers after productivities and demand uncertainty have been revealed in location 1, is given by $m = (m_{ie,job}, m_{ie,nojob}, m_{e,job}, m_{e,nojob})$. I also need to define a beginning of the period measure of inexperienced and experienced workers: $m^b = (m_{ie}^b, m_e^b)$. The mapping from m^b to m is trivial, once the demand level is known, that is,

$$m_{i,job} = m_i^b - \delta(Dm_i^b)$$

$$m_{i,nojob} = \delta Dm_i^b,$$

where $i = \{ie, e\}$ and δ is an indicator function taking the value of 0 if demand is high and of 1 if demand is low. Let us denote this mapping as $m = F(m^b, \delta)$. Next, a mapping from m to $m^{b'}$ (measure of workers at the beginning of the next period in location 1) is needed. The undirected search implies that in a stationary equilibrium, a constant measure, S , of unemployed occupation switchers, arrives on an island at the beginning of each period. If $g(m, z) = (g_{ie,job}, g_{ie,nojob}, g_{e,job}, g_{e,nojob})$ gives the amount of workers who decided to stay attached in the occupation, we see that

$$m_{ie}^{b'} = (1 - \alpha)g_{ie,job} + g_{ie,nojob} + \gamma g_{e,nojob} + S$$

$$m_e^{b'} = g_{e,job} + (1 - \gamma)g_{e,nojob} + \alpha g_{ie,job}$$

Let us denote this as $m^{b'} = G(m, z)$.

Finally, $P(\delta)$ gives the probabilities for demand states. The equilibrium is as in Kambourov and Manovskii (2009a) once the block recursive structure and the demand shocks have been taken into account. It can be defined as

1. Given θ and wages, $V_{ie,job}$, $V_{ie,nojob}$, $V_{e,job}$ and $V_{e,nojob}$ satisfy the Bellman equations above.
2. Wages are determined competitively.

¹³ Veracierto (2000) also uses a linear production function in an island setup in order to make a worker's value function independent of the quantity of agents present in the local labor market.

3. Reallocation rules $g_{ie,job}$, $g_{ie,nojob}$, $g_{e,job}$ and $g_{e,nojob}$ are implied by the value functions.

- (a) If $V_{ie,nojob}(z) > \theta$, then $g_{ie,nojob}(m, z) = m_{ie,nojob}$, $g_{ie,job}(m, z) = m_{ie,job}$, $g_{e,nojob}(m, z) = m_{e,nojob}$ and $g_{e,job}(m, z) = m_{e,job}$ (No-one reallocates).
- (b) If $V_{ie,nojob}(z) = \theta$, $V_{ie,job}(z) > \theta$ and $V_{e,nojob}(z) > \theta$, then $g_{ie,nojob}(m, z) = 0$, $g_{ie,job}(m, z) = m_{ie,job}$, $g_{e,nojob}(m, z) = m_{e,nojob}$ and $g_{e,job}(m, z) = m_{e,job}$ (all unemployed inexperienced workers reallocate and everyone else stays).
- (c) If $V_{ie,job}(z) = \theta$ and $V_{e,nojob}(z) > \theta$, then $g_{ie,nojob}(m, z) = 0$, $g_{ie,job}(m, z) = 0$, $g_{e,nojob}(m, z) = m_{e,nojob}$ and $g_{e,job}(m, z) = m_{e,job}$ (all inexperienced workers reallocate).
- (d) If $V_{e,nojob}(z) = \theta$ and $V_{ie,job}(z) > \theta$, then $g_{ie,nojob}(m, z) = 0$, $g_{ie,job}(m, z) = m_{ie,job}$, $g_{e,nojob}(m, z) = 0$ and $g_{e,job}(m, z) = m_{e,job}$ (all unemployed workers reallocate).
- (e) If $V_{ie,job}(z) = V_{e,nojob}(z) = \theta$ and $V_{e,job}(z) > \theta$, then $g_{ie,nojob}(m, z) = g_{ie,job}(m, z) = g_{e,nojob}(m, z) = 0$ and $g_{e,job}(m, z) = m_{e,job}$ (all inexperienced and unemployed experienced workers reallocate).
- (f) If $V_{e,job}(z) = \theta$, then $g_{ie,nojob}(m, z) = g_{ie,job}(m, z) = g_{e,nojob}(m, z) = g_{e,job}(m, z) = 0$ (all workers reallocate).

4. $G(\cdot)$, that embeds the policy rules together, with $F(\cdot)$, $Q(dz, Z')$ and $P(\delta')$, generates a stationary distribution of islands

$$\mu(M', Z') = \sum_{\delta' \in \Delta'} P(\delta') \int_{(m, z): F(G(m, z), \delta') \in M'} Q(z, Z') \mu(dm, dz),$$

where M' is a set of experience-unemployment distribution, Z' and Δ' are sets of shocks and $Q(dz, Z')$ is the transition function for the productivity shocks.

- 5. Aggregate employment is given by $E = \int (g_{ie,job} + g_{e,job}) \mu(dm, dz)$.
- 6. Aggregate waiting unemployment is given by $WU = \int (g_{ie,nojob} + g_{e,nojob}) \mu(dm, dz)$.
- 7. The number of agents that search for a new occupation, S , is given by the feasibility constraint $1 = E + WU + S$.
- 8. The value of search is $\theta = \beta \int V_{ie}^b(z) QS(z)$, where $QS(z)$ is the stationary distribution of productivity shocks.

Note that at point 3 of the definition we utilize the fact that perfect substitutability and $b_r < w_{ie}(z)$ for all z imply that $V_{i,nojob} \leq V_{i,job}$ and $V_{ie,j} \leq V_{e,j}$ where $i =$

$\{ie, e\}$ and $j = \{no\ job, job\}$. The equality only applies when reallocation is optimal. So if, for example $V_{ie,no\ job} > \theta \Rightarrow V_{ie,job}(z) > \theta$, $V_{e,no\ job}(z) > \theta$ and $V_{e,job}(z) > \theta$.

2.2 Economy with a layoff rule that protects experienced workers

Here I describe an economy that is otherwise identical to the one presented in the previous section, but the layoff order in all locations is changed so that experienced workers are more protected against involuntary unemployment spells. The lay-off process starts from inexperienced workers and moves on to experienced workers only if there are not enough inexperienced workers to absorb the demand shock. Within each group (experienced or inexperienced workers present in a certain location), everyone is facing an identical unemployment risk. Continuing with the taxicab example one could think that all taxicabs are organized in a queue where experienced drivers are at the beginning of the queue and inexperienced drivers at the end of the queue. Incoming passengers are directed to start from the beginning of the queue.

Given this structure, there are three types of islands. If the demand is high, both inexperienced and experienced (if present) workers are offered a job. When the demand is low and there are enough inexperienced workers to absorb the shock, some inexperienced workers will receive an offer while some will not. All experienced workers (if present) receive a job offer. Finally, if there are not enough inexperienced workers to absorb the demand shock, none of the inexperienced workers are able to work while some experienced workers will get a job offer and some will not.

The layoff process changes the value functions, which now also depend on the distribution of workers, m , since the next periods' unemployment probability is a function of $m^{b'}$. The value functions for different workers are

$$\begin{aligned}
 V_{ie,job}(z, m) &= \max\{W_{ie}(z, m), R_{ie}(z, m), \theta\}, \\
 V_{ie,no\ job}(z, m) &= \max\{R_{ie}(z, m), \theta\}, \\
 W_{ie}(z, m) &= w_{ie}(z) + \beta[\alpha E_t V_e^b(z', m^{b'}) + (1 - \alpha) E_t V_{ie}^b(z', m^{b'})], \\
 R_{ie}(z, m) &= b_r + \beta E_t V_{ie}^b(z', m^{b'}), \\
 V_{e,job}(z, m) &= \max\{W_e(z, m), R_e(z, m), \theta\}, \\
 V_{e,no\ job}(z, m) &= \{R_e(z, m), \theta\}, \\
 W_e(z, m) &= w_e(z) + \beta E_t V_e^b(z', m^{b'}), \\
 R_e(z, m) &= b_r + \beta[(1 - \gamma) E_t V_e^b(z', m^{b'}) + \gamma E_t V_{ie}^b(z', m^{b'})].
 \end{aligned}$$

Where

$$E_i V_i^b(z', m') = (1 - pD_i)E_i V_{i,job}(z', m') + pD_i E_i V_{i,nojob}(z', m'), i = \{ie, e\}$$

where

$$D_{ie} = \min \left\{ \frac{D(k_{ie}m_{ie}^{b'} + k_e m_e^{b'})}{k_{ie}m_{ie}^{b'}}, 1 \right\}$$

$$D_e = \begin{cases} \frac{D(k_{ie}m_{ie}^{b'} + k_e m_e^{b'}) - D_{ie}k_{ie}m_{ie}^{b'}}{k_e m_e^{b'}} & \text{if } m_e^{b'} \neq 0 \\ 0 & \text{if } m_e^{b'} = 0 \end{cases}$$

EQUILIBRIUM. Since the next period's unemployment probabilities now depend on the distribution of workers in that location, the equilibrium is no longer block recursive. That is, agents need to take into account the reallocation rules $g(m, z)$ when making their decisions and individuals' behavior must be consistent with the assumed reallocation rules. Furthermore, actions of inexperienced workers are partly strategic complements, since an inexperienced worker who decided to stay in her current location is usually better off if other inexperienced workers also decided to stay.¹⁴ This allows for a possibility of multiple equilibria. When solving the model, I consider two cases. For the first case, among possible equilibrium candidates during the computational process, I always select the one that leads to the lowest possible mobility. This is an equilibrium candidate that maximizes the present value of production in the current location. In this case, a worker believes that the rest of the workers present on the island that belong to the same group (e.g. unemployed inexperienced) stay put and reallocation in this group only starts if given this belief the worker wants to reallocate.¹⁵ As for the second case, I look at the other extreme equilibrium, where during the computation the candidate that leads to the highest possible mobility is always chosen.

With seniority rules of firing, we need to modify our definition of equilibrium a little relating to parts 1, 3 and 8. Also D_{ie} and D_e have to be used in $F(\cdot)$.

1. given θ , S , wages and policies $g_{i,j}(z, m)$, where $i = \{ie, e\}$ and $j = \{job, nojob\}$, $V_{ie,job}$, $V_{ie,nojob}$, $V_{e,job}$ and $V_{e,nojob}$ satisfy the Bellman equations above

3. The reallocation rules in each occupation are consistent with value functions

(a) demand is high

¹⁴ In locations where there are only few experienced workers, this does not always hold true, since a higher amount of inexperienced workers with a job also implies a higher amount of experienced workers present in the next period.

¹⁵ This is an equilibrium that would result if workers would be able to coordinate their actions.

- i. If $g_{ie,job}(z, m) = m_{ie,job}$ and $g_{e,job}(z, m) = m_{e,job}$, then $V_{ie,job}(z, m) > \theta$.
 - ii. If $g_{ie,job}(z, m) < m_{ie,job}$ and $g_{e,job}(z, m) = m_{e,job}$, then $V_{ie,job}(z, m) = \theta$ and $V_{e,job}(z, m) > \theta$.
 - iii. If $g_{ie,job}(z, m) = 0$ and $g_{e,job}(z, m) < m_{e,job}$, then $V_{e,job}(z, m) = \theta$.
- (b) demand is low and $D_{ie} < 1$
- i. If $g_{ie,nojob}(z, m) = m_{ie,nojob}$, $g_{ie,job}(z, m) = m_{ie,job}$ and $g_{e,job}(z, m) = m_{e,job}$, then $V_{ie,nojob} > \theta$.
 - ii. If $g_{ie,nojob}(z, m) < m_{ie,nojob}$, $g_{ie,job}(z, m) = m_{ie,job}$ and $g_{e,job}(z, m) = m_{e,job}$, then $V_{ie,nojob} = \theta$ and $V_{ie,job} > \theta$.
 - iii. If $g_{ie,nojob}(z, m) = 0$, $g_{ie,job}(z, m) < m_{ie,job}$ and $g_{e,job}(z, m) = m_{e,job}$, then $V_{ie,job} = \theta$ and $V_{e,job} > \theta$.
 - iv. If $g_{ie,nojob}(z, m) = 0$, $g_{ie,job}(z, m) = 0$ and $g_{e,job}(z, m) < m_{e,job}$, then $V_{e,job} = \theta$.
- (c) demand is low and $D_{ie} = 1$
- i. If $g_{ie,nojob}(z, m) = m_{ie,nojob}$, $g_{e,nojob}(z, m) = m_{e,nojob}$, $g_{e,job}(z, m) = m_{e,job}$, then $V_{ie,nojob} > \theta$.
 - ii. If $g_{ie,nojob}(z, m) < m_{ie,nojob}$, $g_{e,nojob}(z, m) = m_{e,nojob}$, $g_{e,job}(z, m) = m_{e,job}$, then $V_{ie,nojob} = \theta$ and $V_{e,nojob} > \theta$.
 - iii. If $g_{ie,nojob}(z, m) = 0$, $g_{e,nojob}(z, m) < m_{e,nojob}$, $g_{e,job}(z, m) = m_{e,job}$, then $V_{e,nojob} = \theta$ and $V_{e,job} > \theta$.
 - iv. If $g_{ie,nojob}(z, m) = 0$, $g_{e,nojob}(z, m) = 0$ and $g_{e,job}(z, m) < m_{e,job}$, then $V_{e,job} = \theta$.

8. The value of search θ is given by $\theta = \beta \int V_{ie}^b(z, m) \mu^b(dz, dm)$.

Otherwise the definition is identical to the equilibrium in the previous section.

3 DISCUSSION

Before moving to the calibration and results of the full scale model, it is useful to highlight some of the model properties in a simplified one-island-framework where the outside option, θ , is assumed to be fixed and independent of the different layoff orders. The productivity on the island is constant, subject to only one once-in-a-lifetime change. I assume that the productivity has been so high that before the permanent drop the whole population was working at this location. This implies that there are no incoming workers. Movement between different

experience levels during employment and unemployment spells is not possible, i.e., $\alpha = \gamma = 0$.

Given this setup, I look at the size of the productivity drop needed, in order to start the reallocation process for different worker groups when the demand is low. The workers' optimal reallocation policies can be described with reservation wages, i.e., cut-off values that will start the reallocation process.¹⁶ In an equilibrium, reservation wages turn into reservation productivities. Each worker type has its own reservation productivity. Based on the properties of the value functions and assumptions made in the last section, we can conclude that the reservation productivity has to be higher for the unemployed workers compared with the workers who have a job. Inexperienced workers are also more mobile than experienced workers in the sense that experienced workers are willing to accept lower wages per units of human capital than inexperienced workers.

Let us start by looking at the reservation productivities in the economy with an equal unemployment risk. Detailed derivations are available in the appendix. First, assume that the productivity changes permanently into a new level where unemployed inexperienced workers are indifferent between reallocation and staying attached to their current location. Given that $V_{ie,nojob}(z_{ie}^+) = \theta$, the value of working takes following form

$$W_{ie}(z_{ie}^+) = \frac{z_{ie}^+ k_{ie} + \beta p D \theta}{1 - \beta(1 - pD)}.$$

Substituting this into equation $R_{ie}(z_{ie}^+) = \theta$ and solving for z_{ie}^+ gives:

$$z_{ie}^+ = \left[\frac{1 - \beta}{\beta(1 - pD)} \theta - \frac{1 - \beta(1 - pD)}{\beta(1 - pD)} b_r \right] / k_{ie}.$$

Employed inexperienced workers decide to move out from the island when $V_{ie,job}(z_{ie}^-) = W_{ie}(z_{ie}^-) = \theta$. Since unemployed workers cannot be better off than employed workers, $V_{ie,nojob}(z_{ie}^-) = \theta$. Using these two equations the following threshold productivity is obtained:

$$z_{ie}^- = \frac{1 - \beta}{k_{ie}} \theta.$$

Note that the reallocation decisions of employed workers do not depend on the unemployment risk. These workers will stay on the island as long as the flow value of working (the wage) is higher than the flow value of reallocation.

Since the only difference between inexperienced workers and experienced workers is the amount of location specific human capital they have, the cut-off

¹⁶ In the economy with an equal unemployment risk, these are independent of the beginning of the period masses of experienced and inexperienced workers and all workers in a certain group make identical decisions. In the economy with the "seniority rule", it is not always the case that similar workers end up making identical decisions. However, in the simplified example considered in this section, the economy is a bang-bang one, even with the "seniority rule".

values are also equal once the differences in human capital has been taken into account

$$z_e^+ = \left[\frac{1-\beta}{\beta(1-pD)}\theta - \frac{1-\beta(1-pD)}{\beta(1-pD)}b_r \right] / k_e,$$

$$z_e^- = \frac{1-\beta}{k_e}\theta.$$

Next, assume that the society introduces the layoff order considered in section 2.2. Moreover, assume that there are enough inexperienced workers to absorb the demand shock. This implies that there are three types of workers present on the island: unemployed inexperienced, employed inexperienced and employed experienced workers. Since now the unemployment risk tomorrow depends on the distribution of workers present tomorrow, $m^{b'}$, the value functions could also depend on the distribution of workers. As before derivations are available in the appendix. Let us start by looking at the case where no one moves given the beliefs that no-one moves. The lowest possible productivity level for which all worker are willing to stay, $z_{ie}^+(m)$, is such that $R_{ie}(z_{ie}^+(m), m) = \theta$. For this productivity level the value of working can shown to be

$$W_{ie}(z_{ie}^+(m), m) = \frac{z_{ie}^+(m)k_{ie} + \beta p D_{ie} \theta}{1 - \beta(1 - p D_{ie})}.$$

Using this in the equation that equates the value of waiting and the value of reallocation yields to the following threshold value:

$$z_{ie}^+(m) = \left[\frac{1-\beta}{\beta(1-pD_{ie})}\theta - \frac{1-\beta(1-pD_{ie})}{\beta(1-pD_{ie})}b_r \right] / k_{ie}.$$

From the previous equation we see that $z_{ie}^+(m)$ is increasing in D_{ie} given that one assumes that the flow value of outside option, $(1-\beta)\theta$, is larger than b_r . This implies that $z_{ie}^+(m)$ is higher if the amount of inexperienced workers today, m_{ie} , relative to the amount of experienced workers today, m_e , is lower.

All inexperienced workers move when productivity takes a value that is smaller than $z_{ie}^-(m)$, where $W(z_{ie}^-(m), m) = \theta$. This threshold value takes familiar form

$$z_{ie}^- = \frac{(1-\beta)\theta}{k_{ie}}.$$

Also in this case, the cut-off productivity for employed inexperienced workers is independent of the probability of unemployment and so it is also independent of the distribution of workers. This holds true as long as we do not allow for the possibility of skill-evolution. In the full-scale model, the reallocation rules for inexperienced workers with a job are not independent of the distribution of workers present on the island. Finally, when $W(z_e^-(m), m) = \theta$, all the inexperienced workers will reallocate, and so the next period, the unemployment risk for an experienced worker is D . Given this, the cut-off rule for experienced workers today is exactly the same as with the case of equal unemployment risk.

Table 1: An illustrative calibration of the simplified model

m_e^b	m_{ie}^b	β	k_{ie}	k_e	p	D	b_r	θ
0.3	0.7	0.99	0.7	1	0.5	0.1	0.2	100

To obtain some idea about the solutions, let us try a calibration presented in Table 1. Without the seniority rule we get the following cut-off values

$$z_{ie}^+ = 1.5009$$

$$z_{ie}^- = 1.4286$$

$$z_e^+ = 1.0506$$

$$z_e^- = 1$$

When $z > z_{ie}^+$, no one is willing to reallocate. When $z_{ie}^- < z < z_{ie}^+$, unemployed inexperienced workers move. When $z_e^+ < z < z_{ie}^-$, all inexperienced workers move. When $z_e^- < z < z_e^+$, all inexperienced and unemployed experienced workers move. If z drops below one, all workers move.

With the “seniority rule”, the threshold values for workers are

$$z_{ie}^+ = 1.76$$

$$z_{ie}^- = 1.4286$$

$$z_e^- = 1$$

This simple example illustrates how the “seniority rule” considered in section 2.2 could affect reallocation unemployment. First of all, there is a simple direct channel: more workers with lower cut-off values are being laid off when the waiting unemployment concentrates on the inexperienced workers. The introduction of the “seniority rule” also reduces the cut-off value for inexperienced unemployed workers, implying that smaller productivity changes are likely to trigger the reallocation process. The smaller the amount of inexperienced workers respect to the experienced workers, the larger this channel. In section 5 we will see that even though these channels are present also in the general equilibrium, where movement between skill levels is possible, the effect of human capital accumulation drives the results and reallocation unemployment actually decreases. Finally, note that in order to solve z_{ie}^+ , I made the assumption that no one is moving and checked what the lowest productivity level consistent with this is. Had I assumed that every inexperienced worker without a job is moving and checked what the highest productivity level consistent with this is, the cut-off value would have been higher than z_{ie}^+ , since D_{ie} would have been higher.

4 CALIBRATION

The decision of time period used in the model fixes the duration of unemployment spells without reallocation. I set this to three months. This is motivated by the monthly outflow rates of unemployment, without occupational switches, ranging from 0.305 to 0.327, depending on the occupational definition used, reported by Carrillo-Tudela and Visschers (2012). Given the decision of the time period, β is chosen to be consistent with 4% annual interest rate.

The level of human capital for experienced workers, k_e , is normalized to 1. I set $k_{ie} = 0.84$ to be broadly consistent with the OLS estimation results for 1 digit occupational returns reported in Kambourov and Manovskii (2009b). Based on their results, I also assume that it takes on average 10 years of working for an inexperienced worker to become an experienced one, implying that $\alpha = 0.025$. I focus on 1-digit occupations in order to ensure that reallocation really implies destruction of occupational human capital.

It is well documented that unemployment spells are costly not only in terms of foregone wages but also in terms of the re-employment wages which tend to be considerably lower (see, e.g. Jacobson, Lalonde and Sullivan , 1993 and Couch and Placzek, 2010). Furthermore, for example, Neal (1995) conditions the wage changes, among other controls, on the industry switch, and shows that in general workers that switch industries suffer greater wage losses. The empirical evidence on wage losses conditional on the occupational tenure and occupational stay, however, is scarce. Carrillo-Tudela and Visschers (2012) report that the median wage changes for prime aged and older workers are about -10%, when there are no occupational changes, while young workers' re-employment wages do not change conditional on occupational stay. According to Kambourov and Manovskii (2009b), the workers who stay in the same occupation, suffer a 6 % drop in their earnings. I set the probability of skill erosion during unemployment spells without reallocation, γ , to 0.5. This implies that the average wage loss for an experienced worker is 8% if he decides to stay in the same occupation. At the equilibrium where everyone faces an identical unemployment risk, this implies an average loss of 4.5% for those unemployed workers who do not change occupations.

Since I do not allow for the possibility of rest unemployment I have to make sure that workers prefer work over home production at all levels of technology. Following Shimer (2007), I set b_r to 0.4. Identifying D and p is difficult since at the aggregate level we only observe their combination, which, together with the productivity parameters, pins down the waiting unemployment in the economy. In the baseline calibration I fix D to 0.05. Later I explore the sensitivity of my results along this dimension.

The rest of the parameters are chosen so that the model, where all workers

face an equal unemployment risk, matches the aggregate unemployment patterns with and without reallocation. I take the model of equal unemployment risk to be representative of the US economy. The motivation for this is twofold. Firstly, the seniority rules used in practice vary a lot between firms and occupations (industries) ranging from clear layoff orders to practices where seniority is one of the many things considered. Moreover, these practices are typically quite different from the layoff rules I am looking at. For example, if firms use some sort of explicit firing order, it relates to the firm tenure not to the occupational experience. Secondly, as unionization has decreased, the use of written rules that protect senior workers has decreased.¹⁷

I choose the persistence and volatility of the productivity process and the probability of demand shock to match the aggregate unemployment, the fraction of unemployed workers who will find a new job in a new occupation and the fraction of workers who reallocate after they also switched occupation during their previous unemployment spell. The decision of the targets used is motivated by Kambourov and Manovskii (2009a) who target aggregate mobility and repeated mobility. However, they do not separate between reallocation with and without unemployment, while here I am interested in the responses of the unemployment. From Xiong (2008), we can conclude that a large fraction of occupational switches does not involve unemployment spells. When deciding the empirical targets for reallocation patterns of unemployed workers, I rely on the evidence presented on Carrillo-Tudela and Visschers (2014), who look at the reallocation patterns at the 1 digit level using SIPP data from 1986 to 2011. For all working ages, 50% of the completed unemployment spells involved an occupational switch between major occupational groups. My target for the repeated mobility, 0.56, is also from Carrillo-Tudela and Visschers (2014). The target for aggregate unemployment is 6%. These variables together determine the aggregate reallocation through unemployment, repeated reallocation and unemployment without reallocation. I give an equal weight to each target. When solving the model, I use the discretization method of Tauchen (1986) to approximate the productivity process with a 15 stage Markov process.

Table 2 reports the parametrization used and Table 3 reproduces the targets and shows the model counterparts for the preferred parametrization. From Table 3, it can be seen that our stylized model is able to match the data quite well in terms of aggregate unemployment and the fraction of the unemployed who change occupations. However, the model's ability to match the data when considering repeated mobility is somewhat weaker. It seems that, in reality, the reallocation is a little more persistent than that which the model is able to generate.

¹⁷ For the use of written layoff rules in unionized and non-unionized firms see Abraham and Medoff, 1984.

Table 2: Parameter values

k_e	k_{ie}	α	γ	p	D	ρ	σ^2	a	β	b_r
1	0.84	0.025	0.5	0.428	0.05	0.977	0.03	0	0.99	0.4

Table 3: Targets and model counterparts

	unemployment	mobility	repeated mobility
targets	0.06	0.50	0.56
model	0.0594	0.5043	0.6177

5 RESULTS

Table 4 assembles the main results. The first column reports the equilibrium statistics for the equal unemployment risk case and the second and third columns give the results for the policy experiment. Low mobility refers to the equilibrium where, during the computation, in the case of multiple equilibrium candidates the one that leads to the lowest reallocation is selected. While high mobility refers to the equilibrium that among potential equilibria leads to the highest possible reallocation.

First of all, looking at the decomposition of unemployment for the benchmark economy, we can see that a little more than 4% of the labor force is in the process of finding a new occupation and 1.90% of the work force is waiting for a job in their current occupation. This means that the majority of unemployment, 68%, is reallocation unemployment. Figure 1 presents the densities of the different types of workers over the productivity levels at the production stage. From the figure it can be seen that experienced workers are staying on islands where inexperienced workers are not willing to stay. This implies that a larger fraction of the inexperienced workers are on the high productivity islands. With the parametrization used, the cut-off productivities for inexperienced workers are same whether they are employed or not. There are, however, islands where the experienced labor force is gradually moving out as demand shocks trigger the reallocation of unemployed workers, while the employed experienced workers stay put.

Comparing the first and second columns in Table 4 and Figures 1 and 2, it can be seen how the stationary equilibrium variables change in the case of low mobility seniority rule equilibrium. It turns out that there is a large drop in unemployment, which is now 4.78 %. This drop comes from the reduced realloca-

Table 4: Results

	Equal unemployment risk	seniority	
		low mobility	high mobility
S	4.035	2.907	3.504
WU, inexperienced	0.839	1.475	1.446
WU, experienced	1.069	0.397	0.427
total unemployment	5.943	4.779	5.377
m_e^b aggregate	57.008	70.091	69.832
m_{ie}^b aggregate	42.992	29.909	30.168
$g_{e,job}$ aggregate	55.522	69.243	68.978
$g_{ie,job}$ aggregate	38.536	25.978	25.645
aggregate output	101.904	103.507	103.4
Y/N	1.083	1.087	1.093
θ	98.704	99.140	99.014

Notes: The size of the population is fixed to 100. Y/N is the average labor market productivity.

tion unemployment, which is reduced from 4.04% to 2.91%, representing 61% of the total unemployment. Contrary to what one could have assumed based on the partial equilibrium analysis, the introduction of the seniority rule practice reduces mobility in the economy. Next, it should be noted that the amount of experienced workers in the economy increases from 57% to 70%. There are, essentially, two interrelated mechanisms that could lead to the reduced mobility and higher aggregate human capital. First, it could be that the workers' incentives to reallocate reduce, i.e. the distributions of different types of workers move towards the lower productivities. Second, when inexperienced workers are laid off first, the amount of exogenous human capital destruction is much smaller than in the case of equal unemployment risk. Given that experienced workers are less mobile, this reduces reallocation unemployment.

It can be seen that both mechanisms are at work. Consistent with the latter explanation, the amount of waiting unemployment for experienced workers is more than 50% lower even though the amount of experienced workers has increased. Comparing the distributions of unemployed experienced workers in Figures 1 and 2 one can see that the new layoff order is mainly protecting human capital in high productivity islands. This occurs since these are locations where inexperienced workers, that now face higher unemployment risk, are willing to stay even though there would be lot of experienced workers present. The next section further highlights this "insurance argument" by exploring how the results react to changes in the human capital depreciation parameter γ .

Looking closer to the reallocation behavior of inexperienced workers reveals that, consistent with the previous partial equilibrium analysis, unemployed inexperienced workers are now pushed out from the islands where they were willing to stay before the new policy. Note, though, that there are still unemployed inexperienced workers willing to stay on islands where productivity is taking the

value 1 (the lowest productivity level where unemployed inexperienced workers in an equal unemployment case where still willing to wait for jobs). This, however, requires that there are not too many experienced workers on the island. In fact, some unemployed inexperienced workers are forced away from the islands that have a productivity level of almost 30% higher than the original cut-off value. However, this requires that the unemployment probability is extremely high for future periods. Given the random search and mobility of inexperienced workers, the amount of these types of islands is small and so their contribution to the aggregate equilibrium statistics is small.

Increased mobility of unemployed inexperienced workers is offset by decreased mobility of employed inexperienced workers. These workers are now willing to stay on the islands where they were not willing to stay in the case of an equal unemployment risk. This happens because returns of investment in human capital are now higher since when inexperienced workers become experienced they are likely to spend less time waiting and consequently to stay experienced for a longer period. However, once again inexperienced workers with a job are only willing to stay in these locations given that the distribution of work force is such that the future unemployment risk is not too high. The relative amount of inexperienced workers that decides to reallocate reduces marginally (from 0.084 to 0.081) indicating that an increasing mobility for the unemployed inexperienced workers and a decreasing mobility for the employed inexperienced workers more or less cancel each other.

Finally, turning attention to the reallocation behavior of experienced workers reveals that some unemployed workers are now staying in locations where they were not willing to stay when everybody was treated equally. The reallocation patterns of experienced workers with a job do not change.

There is a substantial increase in waiting unemployment for the inexperienced workers, about 75%. However, the inexperienced workers are not, on average, worse off. The value of θ , which gives the average value of being an inexperienced worker in a randomly picked island at the beginning of a period, actually increases somewhat.

Even though more workers are now staying on “low productivity islands”, the effects of increasing human capital dominate the reduced mobility. Production increases by about 1.6% and, unlike in island models without human capital accumulation, reduced mobility does not imply decreasing average labor productivity. The average labor market productivity increases from 1.083 to 1.087.

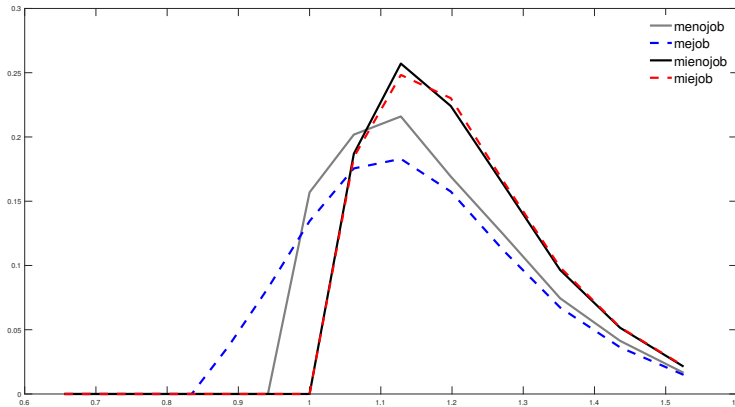


Figure 1: Worker densities over the productivity levels, equal unemployment probability

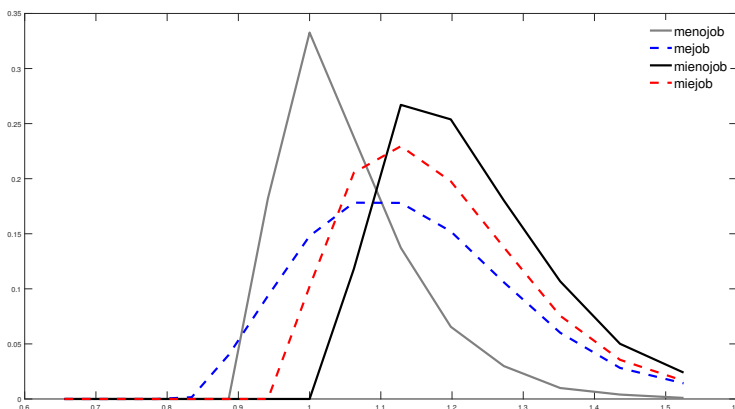


Figure 2: Worker densities over productivity level, seniority rule low mobility

Notes: These densities are reported given that the demand is low. The policy rules for high demand are almost identical to the policy rules of low demand given that you are offered a job.

The results reported so far are only related to the stationary equilibrium where workers behaved as if they could co-ordinate their actions within each group. Next, we look at how our results change when we examine the other extreme: a stationary equilibrium where a candidate leading to the highest possible mobility is chosen during the computation process. Comparing columns two and three in Table 4, one can see that the main implications from the policy experiment stay unchanged. The layoff order scheme that protects experienced workers reduces reallocation unemployment and increases the output produced and the aggregate amount of human capital in the economy. However, now 3.5 % of the workers are in the process of finding a new occupation. This is almost 0.6%

more than in the case of the low mobility equilibrium. This notably increased mobility does not affect output. In the case of high mobility equilibrium, the output produced increases by about 1.5% relative to the economy with an equal unemployment risk. In the high mobility equilibrium increased unemployment is compensated by the increased productivity. Moreover, the human capital in the economy is more or less as in the low mobility equilibrium. These results imply that even though multiple equilibria can have an important effect for the aggregate amount of reallocation unemployment, increased mobility is mainly taking place in locations where it does not have a large effect on workers' value functions.

6 SENSITIVITY

The results of the baseline calibration highlighted the potential gains of the human capital protection in the absence of markets for layoffs. Unemployment dropped while reduced human capital depreciation, especially in the high productivity occupations, increased the average labor productivity and output. In this section, I will explore the sensitivity of the results to the calibration and the main driving forces behind the results reported in the last section. I will only concentrate on the low mobility equilibrium.

The gains of the policy are tightly related the fragility of the human capital. In order to highlight this, I now explore how the results change when the parameter that governs the human capital destruction during the waiting unemployment, γ , takes different values while other parameters are kept unchanged. Table 5 collects these results. For convenience, the results for the baseline calibration, $\gamma = 0.5$, are also reported in the table. First of all, it can be seen that the speed of human capital depreciation plays an important part in explaining the strong reduction in reallocation unemployment caused by the policy introduction. Even though the workers' incentives to reallocate reduce even when the human capital destruction channel is closed down, since the experienced workers still spend less time being unemployed in the seniority rule economy, the reallocation unemployment is only reduced by 15.8%. Next, it should be noted that if γ takes small values, the policy starts hurting the inexperienced workers, which is reflected in the reduction of θ . Finally, in order to have a positive effect on the market output at least some human capital depreciation has to take place during the unemployment spells. However, even with the modest values of fragility, such as $\gamma = 0.2$, the gains in output are sizable.

Table 5: Relative change of the equilibrium variables between two policies when γ takes different values

γ	0	0.2	0.3	0.5	0.7	0.8
reallocation	-0.158	-0.180	-0.223	-0.280	-0.263	-0.281
unemployment	-0.098	-0.146	-0.175	-0.195	-0.218	-0.228
output	-0.001	0.007	0.010	0.016	0.023	0.026
θ	-0.010	-0.004	0.000	0.004	0.010	0.012

Notes: The table reports how the (stationary) equilibrium variables change in relative terms when the economy moves from an equal unemployment risk to a layoff practice where inexperienced workers are laid-off first, concentrating on the low mobility equilibrium. The only parameter whose value is allowed to change is the capital depreciation parameter, γ .

In the baseline calibration, the parameter that governs the severity of a demand shock when it strikes, D , was set at 0.05, and the model was calibrated to match aggregate unemployment patterns by allowing the probability of demand shocks to take different values. Next, I explore how sensitive the results are for different values of D by allowing D to take different values while adjusting p so that pD stays unchanged. This should keep the results for the equal unemployment risk unchanged. However, in the “seniority rule” economy, the size of D affects the unemployment risk of the experienced workers and so it could alter the equilibrium variables. The results in Table 6 indicate that, for plausible values of D , the effects of the policy change are not sensitive to the exact value of D selected. Although, as one would assume, output gains of the policy are larger when D takes small values, since then smaller numbers of inexperienced workers are enough to protect experienced workers.

Table 6: Relative change of the equilibrium variables between two policies when the severity of demand shock, D , is allowed to change while $p * D$ is kept unchanged

D	0.025	0.05	0.1	0.15	0.2
reallocation	-0.274	-0.280	-0.284	-0.254	-0.266
unemployment	-0.203	-0.195	-0.200	-0.185	-0.197
output	0.019	0.016	0.017	0.016	0.013
θ	0.004	0.004	0.006	0.006	0.006

Table 7 explores how the results of the policy experiment relate to the parameters governing the productivity process. When convergence to the mean is fast, i.e. ρ takes low values, or variance of innovations is moderate, the introduction of the “seniority” rule reduces reallocation the most.¹⁸ As the persistence of the productivity process increases, reallocation unemployment increases faster in

¹⁸ Obviously with extremely low values of persistence or variance the two economies would yield equal reallocation unemployment as the returns of reallocation would not be high enough to support occupational mobility.

the seniority rule economy than in the economy where everyone faces an equal unemployment risk. When turbulence of the economy is caused by high values of σ^2 , the policy change could even increase the reallocation unemployment. This, however, only happens when reallocation unemployment is almost as high as the long-run average US unemployment rate.

In general, the smaller the reallocation unemployment, the larger the increase in output caused by the policy. However, if the reallocation incentives are increased by increasing persistence, the relationship is u-shaped. In this type of an environment, the potential gains of reducing human capital depreciation in the best occupations, together with a higher arrival rate of new workers to these locations, are large enough to compensate for less effective human capital protection in the middle of the productivity distribution. This highlights the fact that in order to understand the effects of the policy experiment modeling reallocation incentives is important.

Table 7: Relative change of the equilibrium variables between two policies when ρ or σ^2 takes different values

ρ	0.957	0.967	0.977	0.987
reallocation	-0.385	-0.363	-0.280	-0.064
unemployment	-0.207	-0.230	-0.195	-0.042
output	0.022	0.014	0.016	0.025
θ	-0.006	0.000	0.004	0.002
σ^2	0.02	0.03	0.04	0.05
reallocation	-0.146	-0.280	-0.054	0.076
unemployment	-0.113	-0.195	-0.012	0.062
output	0.022	0.016	0.015	0.012
θ	0.002	0.004	0.003	0.005

7 CONCLUSIONS

This paper explored the equilibrium effects of occupational human capital protection during mass layoffs in a setup where commitment problems prevented markets from allocating layoffs optimally. As the inexperienced workers are not compensated for the increased unemployment risk, the “seniority rule” is, at its best, only partially able to compensate for the non-existing layoff markets. This happens because unemployed inexperienced workers can always move out from a labor market where future unemployment risk is too high. Given that the

change in the layoff order affects the reallocation incentives and thus the distribution of labor force and human capital over occupations, the paper focused on modeling the workers' reallocation decisions.

It turned out that even though inexperienced workers are more willing to change occupations during unemployment spells, the introduction of a layoff order that concentrates the involuntary unemployment incidences to these workers does not increase the reallocation unemployment in the economy. The baseline calibration indicates that reallocation unemployment drops from 4% to 2.9-3.5%, depending on how the beliefs of workers are set. This together with decreased exogenous human capital depreciation implies that the amount of experienced workers in the economy increases. The human capital depreciation during layoffs is especially reduced in high productivity occupations where inexperienced workers are willing to accept the increased unemployment risk. The increased human capital is enough to offset the negative effects of decreased mobility to the average labor market productivity. Because of the decreased unemployment and non-decreasing labor productivity, the output increases by around 1.5% as a result of the introduction of the "seniority rule". Moreover, inexperienced workers are not, on average, worse off.

The human capital depreciation during the waiting unemployment plays a major role in generating these results. If the human capital depreciation channel is closed, the reduction in reallocation is halved and the market output decreases marginally. In addition, persistence and variance of innovations in the occupational productivity process are important for the strong reduction of reallocation unemployment.

On a more general level, this paper illustrates that when considering labor market policies, it is important to model how these policies affect the human capital accumulation. Even though reallocation of employment is important for aggregate outcomes, setting policy targets only based on fast reallocation might be counterproductive. As it might be that unemployed workers prefer waiting unemployment for reasons that are also socially desirable.

References

- Abraham, K. G. – Medoff, J. L. (1984) Length of service and layoffs in union and nonunion work groups. *Industrial and Labor Relations Review*, Vol. 38 (1), 87–97.
- Alvarez, F. – Shimer, R. (2011) Search and rest unemployment. *Econometrica*, Vol. 79 (1), 75–122.
- Alvarez, F. – Shimer, R. (2012) Unemployment and human capital, Unpublished manuscript, University of Chicago.
- Alvarez, F. – Shimer, R. (2014) Unions and unemployment, Unpublished manuscript, University of Chicago.
- Alvarez, F. – Veracierto, M. (1999) Labor market policies in an equilibrium search model. *NBER Macroeconomics Annual*, Vol. 14, 265–304.
- Birchenall, J. (2011) A competitive theory of mismatch, Unpublished manuscript, University of California at Santa Barbara.
- Carrillo-Tudela, C. – Visschers, L. (2012) Unemployment and endogenous reallocation over the business cycle, Unpublished manuscript.
- Carrillo-Tudela, C. – Visschers, L. (2014) Unemployment and endogenous reallocation over the business cycle, ESE Discussion Papers 241, Edinburgh School of Economics, University of Edinburgh.
- Carrington, W. (1993) Wage losses for displaced workers: Is it really the firm that matters? *Journal of Human Resources*, Vol. 28, 435–462.
- Couch, K. A. – Jolly, N. A. – Placzek, D. W. (2009) Earnings losses of older displaced workers: A detailed analysis with administrative data. *Research on aging*, Vol. 31 (1), 17–40.
- Couch, K. A. – Placzek, D. W. (2010) Earnings losses of displaced workers revisited. *American Economic Review*, Vol. 100 (1), 572–589.
- Hamermesh, D. (1989) What do we know about worker displacement in the U.S.? *Industrial Relations*, Vol. 28, 51–59.
- Hopenhayn, H. – Rogerson, R. (1993) Job turnover and policy evaluation: A general equilibrium analysis. *Journal of Political Economy*, Vol. 101 (5), 915–938.
- Jacobson, L. S. – LaLonde, R. J. – Sullivan, D. G. (1993) Earnings losses of displaced workers. *American Economic Review*, Vol. 83 (4), 685–709.
- Kambourov, G. – Manovskii, I. (2009a) Occupational mobility and wage in-

- equality. *Review of Economic Studies*, Vol. 76 (2), 731–759.
- Kambourov, G. – Manovskii, I. (2009b) Occupational specificity of human capital. *International Economic Review*, Vol. 50 (1), 63–115.
- Ljungqvist, L. – Sargent, T. J. (1998) The European unemployment dilemma. *Journal of Political Economy*, Vol. 106 (3), 514–550.
- Lucas, R. E. – Prescott, E. C. (1974) Equilibrium search and unemployment. *Journal of Economic Theory*, Vol. 7 (188-209).
- Menzio, G. – Shi, S. (2010) Block recursive equilibria for stochastic models of search on the job. *Journal of Economic Theory*, Vol. 145 (4), 1453–1494.
- Mortensen, D. T. – Pissarides, C. A. (1994) Job creation and destruction in the theory of unemployment. *Review of Economic Studies*, Vol. 61 (3), 397–415.
- Neal, D. (1995) Industry-specific human capital: Evidence from displaced workers. *Journal of Labor Economics*, Vol. 13 (4), 653–677.
- Restuccia, D. – Yang, D. T. – Zhu, X. (2008) Agriculture and aggregate productivity: A quantitative cross-country analysis. *Journal of Monetary Economics*, Vol. 55 (2), 234–250.
- Shimer, R. (2007) Mismatch. *American Economic Review*, Vol. 97 (4), 1074–1101.
- Sullivan, P. (2010) Empirical evidence on occupational and industry specific human capital. *Labour Economics*, Vol. 17 (3), 567–580.
- Tauchen, G. (1985) Finite state markov-chain approximations to univariate and vector autoregressions. *Economic Letters*, Vol. 20 (2), 177–181.
- Veracierto, M. (2002) What are the short-run effects of increasing labor market flexibility, Working Paper no. 2000-29, Federal Reserve Bank of Chicago.
- Wiczer, D. (2015) Long-term unemployment: Attached and mismatched?, Working Paper no. 2015-042A, Federal Reserve Bank of St. Louis.
- Xiong, H. (2008) The U.S. occupational mobility from 1988 to 2003: Evidence from SIPP, Unpublished manuscript, University of Toronto.

Appendix A DERIVATION OF THE CUT-OFF PRODUCTIVITIES

This appendix derives the cut-off productivities shown in section 3. It relies on the fact that both the value of working and that of waiting are continuously increasing in productivity.

A.1 An equal unemployment risk

First, assume that the productivity changes permanently into a new level where unemployed inexperienced workers are indifferent between reallocation and staying attached to their current location. I assumed that working is always preferred over resting, i.e., $W_{ie}(z_{ie}^+) > R_{ie}(z_{ie}^+) = \theta$:

$$\begin{aligned} V_{ie,job}(z_{ie}^+) &= W_{ie}(z_{ie}^+) = z_{ie}^+ k_{ie} + \beta E V_{ie}^{b_i}(z_{ie}^+) \\ W_{ie}(z_{ie}^+) &= z_{ie}^+ k_{ie} + \beta(1-pD)W_{ie}(z_{ie}^+) + \beta p D \theta \\ W_{ie}(z_{ie}^+) &= \frac{z_{ie}^+ k_{ie} + \beta p D \theta}{1 - \beta(1-pD)}. \end{aligned}$$

z_{ie}^+ can then be solved by using the fact that for this productivity level an inexperienced worker is indifferent between waiting and reallocation

$$\begin{aligned} \theta &= R_{ie}(z^+) = b_r + \beta(1-pD)W_{ie}(z^+) + \beta p D \theta \\ \theta &= b_r + \beta(1-pD) \frac{z^+ k_{ie} + \beta p D \theta}{1 - \beta(1-pD)} + \beta p D \theta \\ \frac{1-\beta}{1-\beta(1-pD)} \theta &= b_r + \frac{\beta(1-pD)}{1-\beta(1-pD)} z^+ k_{ie} \\ z_{ie}^+ &= \left[\frac{1-\beta}{\beta(1-pD)} \theta - \frac{1-\beta(1-pD)}{\beta(1-pD)} b_r \right] / k_{ie}. \end{aligned}$$

Next, let us find the productivity level when the employed inexperienced workers also decide to move out from the island. This happens when $V_{ie,job}(z^-) = W_{ie}(z^-) = \theta = V_{ie,nojob}(z^-)$:

$$\begin{aligned} W_{ie}(z^-) &= z^- k_{ie} + \beta(1-pD)\theta + \beta p D \theta \\ \theta &= z^- k_{ie} + \beta \theta \\ z_{ie}^- &= \frac{1-\beta}{k_{ie}} \theta. \end{aligned}$$

Reservation productivities for experienced workers can be derived based on a similar approach.

A.2 A layoff rule that protects experienced workers

Assume that there are enough inexperienced workers to absorb the initial demand shock. This implies that there are three types of workers present on the island: unemployed inexperienced, employed inexperienced and employed experienced workers. Let us start by looking at the case where no one moves. Moreover, we assume that workers believe that no one moves and the reallocation only starts when, given this belief, a worker with the lowest value of remaining (an inexperienced worker without a job) finds it optimal to reallocate. The lowest possible productivity level, $z_{ie}^+(m)$, when all workers are willing to stay, is such that $R(z_{ie}^+(m), m) = \theta$. For this productivity level

$$\begin{aligned} V_{ie,job}(z_{ie}^+(m), m) &= W_{ie}(z_{ie}^+(m), m) \\ &= zk_{ie} + \beta(1 - pD_{ie})W_{ie}(z_{ie}^+(m), m') + \beta pD_{ie}\theta. \end{aligned}$$

Note that m' is not necessarily the same as m since demand levels can be different. However, m^b and unemployment probabilities will stay the same for all periods since no one leaves and there is no movement between the different skill levels. This implies that the value of working will stay unchanged.

$$W_{ie}(z_{ie}^+(m), m) = \frac{z_{ie}^+(m)k_{ie} + \beta pD_{ie}\theta}{1 - \beta(1 - pD_{ie})}.$$

Given this, we can solve the threshold productivity by equating the value of waiting and the value reallocation:

$$\theta = R_{ie}(z_{ie}^+(m), m) = b_r + \beta(1 - pD_{ie})W_{ie}(z_{ie}^+(m), m) + \beta pD_{ie}\theta$$

$$z_{ie}^+(m) = \left[\frac{1 - \beta}{\beta(1 - pD_{ie})}\theta - \frac{1 - \beta(1 - pD_{ie})}{\beta(1 - pD_{ie})}b_r \right] / k_{ie}.$$

All inexperienced workers move when productivity takes a value that is smaller than $z_{ie}^-(m)$, where $W(z_{ie}^-(m), m) = \theta$.

$$\begin{aligned} \theta &= z_{ie}^-(m)k_{ie} + \beta(1 - p)V_{ie,job}(z_{ie}^-(m), m') + \beta p(1 - D_{ie})V_{ie,job}(z_{ie}^-(m), m') \\ &\quad + pD_{ie}V_{ie,nojob}(z_{ie}^-(m), m') \end{aligned}$$

$$\theta = z_{ie}^-k_{ie} + \beta\theta$$

$$z_{ie}^- = \frac{(1 - \beta)\theta}{k_{ie}}.$$

ESSAY 2

Oskari Vähämaa

Unionizing Non-Search Unemployment

Preprint

Unionizing Non-Search Unemployment

Oskari Vähämaa*

Abstract

This paper explores the effects of unionization in an island model of Lucas and Prescott (1974) with different union structures. When a model with competitive labor markets is set to match the empirical fact that a large number of unemployment spells ends with recalls, an introduction of a large labor union, that represents all workers and sets a common economy-wide minimum wage, increases unemployment substantially. Moreover, the whole increase is about non-search unemployment as search unemployment actually reduces marginally. If the same degree of unionization is generated by a continuum of small unions, the aggregate unemployment reaction is somewhat smaller. However, the increase in non-search unemployment is still considerable.

1 INTRODUCTION

The existence of powerful labor unions is a prominent feature of labor markets across a wide variety of countries and industries. Typically, these unions have an influence over a large group of firms with varying conditions, an extreme example being the nationwide cooperation over the whole economy sometimes applied, for example, in the Nordic countries. Quite often the unions seem to be unable or unwilling to take into account these varying local conditions potentially leading to increased unemployment. On the other hand, as some studies suggest, the increased coordination and better internalization of the adverse effects can lead to a preferable outcome compared with a set-up where many unions operate independently¹.

* *Department of Economics, University of Turku, FI-20014, Finland. Email: oskari.vahamaa@gmail.com. I would like to thank Petri Böckerman, Tomi Kortela, Jouko Vilminen and Matti Viren for comments.*

¹ For example, the study by Calmfors and Driffill (1988) suggest that cost of centralized wage setting systems are hump shaped, the extremes working the best.

Somewhat surprisingly, studies that explore the issue in an equilibrium set-up, while modeling unemployment explicitly, are scarce. In this paper, I examine the workings of a large union by assuming that it sets a common minimum wage over heterogeneous firms. Moreover, moving frictions between firms with different productivities generate search unemployment and prevent the instant reallocation of labor force over production units. In order to gain a better understanding of the potential advantages and disadvantages of a large union, I compare this with a case where there is a continuum of small unions that focus solely on their members and take the rest of the economy as given, but take the local conditions into account.

A major feature in the analysis is the fact that I allow unemployed workers to engage in non-search unemployment. Recent time diary studies have revealed that unemployed workers spend surprisingly little time looking for a job². A potential explanation is that unemployed workers have a high probability of landing a job even without active search. In line with this, Fujita and Moscarini (2017) show that recalls are extremely common, widely exceeding the amount of temporarily laid off workers. However, the commonly used models of unemployment are based on an assumption of active search by the unemployed, a notable exception being Alvarez and Shimer (2011) who use an analytically tractable island search model and show that in order to explain the large wage differences between sectors, non-search unemployment has to be prominent. Additionally, Fujita and Moscarini (2017) and Fernandez-Blanco (2013) analyze non-search unemployment using the search and matching models à la Mortensen and Pissarides, while Shimer (2007) shows that many business cycle properties of unemployment can be generated with a model where non-search unemployment is the only form of unemployment.

Unions offer a powerful mechanism of generating non-search unemployment in a frictional labor market. When a union sets a minimum wage, it rations the job supply in firms where the productivity is low. However, as the upper tail of the wage distribution is unaffected, the returns of search do not change substantially. Given that non-search unemployment is less costly than search unemployment, an idea supported by the time diary evidence of Krueger and Mueller (2011), this should imply that a large part of the increased unemployment is non-search unemployment. The theoretical work of Alvarez and Shimer (2014) about the connections of non-search unemployment and unions suggests that this type of mechanism is indeed strong.

The cross-country comparison of Krueger and Mueller (2012) offers support for the idea that different labor market institutions and policies are important for unemployed search activity as they report large differences in the time devoted to search by unemployed workers. Interestingly for the mechanism outlined above, search intensities are relatively low in Europe where the labor unions are tradi-

² See, e.g., Krueger and Mueller (2010)

tionally strong. For example, in the US and Canada, unemployed workers spend more than eight times more time on search than in the Nordic countries. Moreover, Krueger and Mueller (2012) also find that income variability is a strong predictor of search intensity. This, together with the empirical evidence of labor unions suppressing wage distributions³, further suggests that the connection between unionization and non-search unemployment could be empirically relevant.

In order to explore the role of unionization, I formulate an equilibrium island model à Lucas and Prescott (1974) where firms and workers meet in locally competitive labor markets with varying conditions. In the economy with a large labor union, this union is assumed to set an archipelago-wide minimum wage.⁴ In locations where this minimum wage is binding, jobs are rationed to a point where firms are willing to pay the minimum wage. Workers are attached to a certain local labor market. Moving between labor markets is possible but it takes time. Following Alvarez and Veracierto (1999), the search between islands is undirected. Moreover, instead of just working or searching, agents can also, as in Alvarez and Shimer (2011), stay inactive in their current labor market and enjoy increased leisure. Alvarez and Shimer (2011) call this type of unemployment rest unemployment. The objectives of the union and the workers differ in the sense that workers concentrate on their income maximization while the union looks at the consumption and the leisure of all workers. This means that the union has to take the workers incentives into account.

I also compare the case of a large union with a set up where there is a continuum of island specific unions. These unions are able to modify their policies according to local conditions and take the rest of the economy as given. Moreover, they only care about the workers that stay on the island. This, together with the assumption that rents are not shared equally between members, means that unions are not willing to restrict employment when the real wage is low, as some of their members might reallocate and become members of another union. Unions also find it optimal to allow all agents present to work when the wage is high enough.

When the model without a large union is made consistent with the empirical observation of Fujita and Moscarini (2017) that a large number of unemployment spells ends with recalls, the introduction of a large labor union representing all workers increases unemployment by 22.4%. The whole increase is non-search unemployment. This implies that unionization substantially drops unemployed workers search intensity. If the union is assumed to bargain over wages with a large union of firms, arguable a more empirically relevant sce-

³ See Card, Lemieux and Riddell (2003)

⁴ As Krusell and Rudanko (2016) point out one can understand a large union set-up representing to the whole economy or think about it as an industry equilibrium where moving between industries is difficult.

nario, the aggregate unemployment responses are not as severe, but non-search unemployment still increases substantially. For example, 50-50 Nash bargaining solution, yields to a unemployment rate that is “only” 4.4 percent higher than the unemployment rate in the *laissez-faire* economy.

The structure of unionization matters; the aggregate unemployment increases by about 17% as response to a continuum of small unions. Thus the unionization with a small unions is causing a smaller response in aggregate unemployment even though these unions do not internalize the aggregate effects of their actions. The key to this is unions desire to avoid losing members when the local labor market conditions are weak. Due to this reason unions find it optimal to restrict labor supply only in a small range at the middle of the wage distribution. However, the whole increase in aggregate unemployment is still caused by non-search unemployment.

There are a few papers that study the effects of labor unions which model unemployment in an equilibrium framework. Alvarez and Veracierto (1999) explore, among other things, the effects of small labor unions in an island model. They look at the effects of unions when the gains of unionization are shared equally between all members, a coalition model, and when the monopoly rents are paid to a union boss. They find that while the coalition model leads to a strong increase in unemployment, the union boss model decreases unemployment.

In terms of concentrating on the connection between non-search unemployment and labor unions, my analysis is close to Alvarez and Shimer (2014). They explore the workings of unions in an island economy when unions apply strict seniority rules in hiring and firing decisions. They utilize a continuous time model where island specific productivities follow a geometric Brownian motion. This allows them to solve the model analytically. My work can be seen as complementary to theirs, as my focus is on the quantitative side of unionization and non-search unemployment, while also taking into account the large recall rate that implies that non-search unemployment has to be considerably less costly. Another difference is that while they explore the effects of small, the US style, unions, my focus is on the union structure more common in Europe where a large union is having an influence over a large group of varying firms.

There are also some studies that analyze the effects of a union in Mortensen-Pissarides models. Pissarides (1986) studies the effect of a union in a steady state of a canonical search and matching model, exploring conditions under which a large union internalizes the congestion externality, while Krusell and Rudanko (2016), explore the dynamic effects of a large union. Both of these papers, however, abstract away from firm heterogeneity, the main focus of this paper.

The rest of the paper is organized as follows. Section 2 represents the model with and without labor unions. Section 3 states the calibration of the model.

While results are given in Section 4. Finally, Section 5 concludes.

2 MODEL

This section introduces the model used to explore the effects of unionization. The basic framework is essentially a standard island model in discrete time à la Lucas and Prescott (1974) and Alvarez and Veracierto (1999). Following Alvarez and Shimer (2011 and 2014), I allow non-search unemployment that is less costly than search unemployment in terms of flow values and allows workers to stay in touch with their current local labor market.

2.1 General environment

There is a mass one of infinitely lived workers distributed along a continuum of separate locations, islands, index by $i \in [0, 1]$. On location i , there are $x_t(i)$ workers present at the beginning of period t . During the period, out of these agents $n_t(i)$ work, $r_t(i)$ are non-search unemployed and $x_t - n_t - r_t$ search for a new location. Agents who work or engage in non-search unemployment, start the next period in their current location, while searchers are randomly dropped on a new island at the beginning of the next period.

Each location produces identical goods and the production technology on each location is characterized by a production function of the form

$$y_t(i) = z_t(i)n_t(i)^\alpha,$$

where $0 < \alpha < 1$ and the productivity shock, $z_t(i)$, is independent across locations. The logarithm of the productivity follows an AR(1) process

$$\log(z_t(i)) = (1 - \rho)a + \rho \log(z_{t-1}(i)) + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2), \quad (1)$$

where $0 < \rho < 1$.

Each worker belongs to a large family that gives them a full insurance against idiosyncratic shocks. The family's utility function is given by additively separable balanced growth path consistent preferences as follows

$$\sum_{t=0}^{\infty} \beta^t (\ln(C_H) + b_r U_r), \quad (2)$$

where C_H is the amount of market goods consumed and U_r is the amount of non-search unemployed workers, while b_r is the flow value of increased leisure

that non-search unemployed can get and $\beta \in [0, 1]$. I have dropped the time indices from the aggregate variables as I will focus on stationary equilibria. Following Alvarez and Shimer (2011), I assume that searching is more costly than non-search unemployment in the sense that the searchers are not able to enjoy increased leisure.

I assume that there is a mass one of capitalist who owns the firms and to whom the profits are paid. This implies that the household can only use wage income to finance its consumption, i.e.,

$$\int_0^1 w_t(i)n_t(i)di = C_H, \quad (3)$$

where $w_t(i)$ is the wage rate, to be specified in later subsections, in location i .

The amount of agents working and engaging in non-search unemployment is restricted by the mobility frictions and thus cannot be larger than the amount of workers present at location i at the beginning of the period, i.e.

$$n_t(i) + r_t(i) \leq x_t(i).$$

Given the insurance, each family member makes decisions about whether to work, stay on the island without working or search in a way that the discounted present value of flow payoffs is maximized.

The indirect search implies that the amount of workers present at the beginning of the next period will be

$$\mathbf{m}_{t+1}(i) = (m_{1,t+1}, m_{2,t+1}) = (n_t(i), r_t(i) + U_{s,t}), \quad (4)$$

where U_s is the amount of searchers in the economy. Given this, the beginning of period labor force in location i is

$$x_t(i) = \mathbf{m}_t(i) * \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

The aggregate amount of non-search unemployment is given by

$$U_r = \int r_t(i)di \quad (5)$$

and the aggregate employment rate is

$$N = \int_0^1 n_t(i)di \quad (6)$$

As I do not allow for the possibility of out of labor force, the amount of searchers is given by the aggregate feasibility condition

$$N + U_s + U_r = 1. \quad (7)$$

2.2 Competitive local labor markets

In this subsection, I set up the benchmark wage mechanism under which a large amount of workers and firms meet in competitive local labor markets in each location. Wages are then determined competitively. That is

$$w_t(x, z) = \alpha z_t n_t(x, z)^{\alpha-1}. \quad (8)$$

Since workers can costlessly move between non-search unemployment and work, the wage rate measured in units of marginal utility cannot be smaller than the flow value of non-search unemployment

$$b_r \leq w_t(x, z) \frac{1}{C}. \quad (9)$$

As in Alvarez and Shimer (2011), non-search unemployment is generated when the wage rate measured in units of utility would be lower than the flow value of leisure given that all workers that are willing to stay would work. In this case, the amount of agents working adjusts to a level that ensures that flow values work and non-search unemployment are equal and workers engaging in non-search unemployment and work are indifferent between the two activities.

The voluntary nature of non-search unemployment also means that it is not necessary to keep track of workers employment statuses. The relevant location specific state variables are the beginning of the period labor force $x_t(i)$ and productivity $z_t(i)$. An agent in location (x, z) has to decide whether to work, engage in non-search unemployment or search. The problem of an agent can be described with the following Bellman equation

$$V(x, z) = \max\{\omega(n(x, z)) + \beta \int V(x'(x, z), z') Q(z, dz') \\ b_r + \beta \int V(x'(x, z), z') Q(z, dz'), \theta\}$$

where $x'(x, z) = n(x, z) + r(x, z) + U_s$, $\omega(n(x, z))$ is the wage rate measured in the units of marginal utility, i.e., $\omega(x, z) = \frac{1}{C} w(x, z)$, $Q(z, dz')$ is the transition function for the productivity shocks and θ is the value of search. Moreover, individuals take policies $n(x, z)$ and $r(x, z)$ as given.

At the equilibrium, policy functions have to be consistent with the individual decisions⁵

1. If $n(x, z) = x$ and $r(x, z) = 0$ then $V(x, z) > \theta$ and $\omega(n(x, z), z) > b_r$.
2. If $n(x, z) + r(x, z) = x$ and $n(x, z) < x$ then $V(x, z) > \theta$ and $\omega(n(x, z), z) = b_r$.
3. If $n(x, z) < x$ and $r(x, z) = 0$ then $v(x, z) = \theta$ and $\omega(n(x, z), z) > b_r$.

⁵ As in this subsection the expected future values of non-search unemployment and work are equal, the decision between work and non-search is directly related to the flow values of both activities. Given this I use the flow values to describe the values of work and non-search unemployment.

4. If $n(x, z) + r(x, z) < x$ and $r(x, z) > 0$ then $v(x, z) = \theta$ and $\omega(n(x, z), z) = b_r$.

The workers' policy rules together with the search technology and the law of motions for productivity, stated by condition (1), generate a stationary distribution of islands

$$\mu(X', Z') = \int_{\{(x, z): (n(x, z) + r(x, z) + U_s) \in X'\}} Q(z, Z') \mu(dx, dz), \quad (10)$$

for all sets of beginning of the next period labor forces, X' , and productivities, Z' . Given the stationary measure, the value of search satisfies

$$\theta = \beta \int V(x, z) \mu(dx, dz). \quad (11)$$

The competitive stationary equilibrium, conditional on the general structure stated in the previous subsection⁶, is such that workers' value functions satisfy the Bellman equations above and the island specific employment, $n(x, z)$, and non-search unemployment rules, $r(x, z)$, are consistent with conditions 1-4. Wages are defined by equations (8) and (9), the agents behavior together with the productivity process generate an invariant distribution that is consistent with equation (10) and the value of search is given by equation (11).

2.3 A large union

This subsection introduces an equilibrium with a large labor union that represents all workers. The union has monopoly power over firms and it is fully aware of the general structure of the economy but it cannot observe time-varying local labor market conditions. Given this, the union sets an economy-wide minimum wage and forbids its members to work unless the wage offered meets the minimum requirement. When the minimum wage is not binding, competition on local markets determines the wage rate. This mimics the real life examples of the workings of unions that typically allow firms to pay more than the wage agreed. Ideally, the union would like to set a state-contingent policy for each locations based on the value of the best outside option, reallocation or non-search unemployment. However, due to the moving frictions this policy would still be a minimum wage policy.

I focus on the stationary minimum wage policy of the union. It is assumed that the union cares for the welfare of the representative household and thus it fully internalizes the unemployment effects caused by the minimum wage policy. However, the union cannot directly control reallocation or hiring decisions and thus has to take the objectives of workers and firms as given. Here, I assume

⁶ Especially, productivities follow condition (1) and aggregate conditions stated in equations (5)-(7) are satisfied.

that the rents of the union are shared only at the household level and each worker is, as before, maximizing her present and expected future earnings measured in units of marginal utility. The agents make decisions on working, searching and waiting for jobs in their current location.

Assuming that the union sets a minimum wage that is binding at least in some locations, i.e., $\omega_{min} > b_r$, the nature of non-search unemployment changes from voluntary to non-voluntary as now the non-search unemployed agents would rather work in their current location. Thus, the employment status becomes a state variable.

Given the local mass of workers at the beginning of the period m , productivity z , policies $n(m, z)$ and $r(m, z)$, the value of search θ , aggregate consumption C_H and the amount of searchers U_r , the problems of agents can be described with the help of the following Bellman equations

$$V_{job}(m, z; \omega_{min}) = \max\{\omega(n(m, z; \omega_{min}) + \beta \int Q(z, dz') [p_{job}(m', z') V_{job}(m', z'; \omega_{min}) + (1 - p_{job}(m', z')) V_{nojob}(m', z'; \omega_{min})], \theta\}$$

$$V_{nojob}(m, z; \omega_{min}) = \max\{b_r + \beta \int Q(z, dz') [p_{nojob}(m', z') V_{job}(m', z; \omega_{min}) + (1 - p_{nojob}(m', z')) V_{nojob}(m', z'; \omega_{min})], \theta\}$$

where the evolution of m is defined based on equation (3). While, $p_{job}(m', z')$ and $p_{nojob}(m', z')$ are probabilities of being offered a job at the beginning of the next period, conditional on the current job market status, the amount of workers m' and productivity z' .

With the minimum wage $w_{min} = C_H * \omega_{min}$ and productivity z , the maximum amount of workers that can be offered a job is given by

$$e(z; \omega_{min}) = \left(\frac{w_{min}}{\alpha z}\right)^{\frac{1}{\alpha-1}}$$

If $x \leq e(z, \omega_{min})$, all agents are offered a job, i.e., $p_{job}(m, z) = p_{nojob}(m, z) = 1$ and the wage rate is determined by equation (7). If $e(z; \omega_{min}) < x$ only $e(z; \omega_{min})$ workers get a job offer. I assume that the agents that worked in the last period are given a preferred position. That is, given the amount of agents who worked in the last period m_1 and agents who were waiting m_2 ,

$$p_{job} = \min\left\{\frac{e}{m_1}, 1\right\}.$$

and

$$p_{nojob} = \max\left\{\frac{e - m_1}{m_2}, 0\right\},$$

when $m_2 > 0$.

At the equilibrium the workers' behavior has to be consistent with the Bellman equations above. That is

1. If $x \leq e(z; \omega_{min})$

- (a) If $n(m, z; \omega_{min}) = x$ and $r(m, z; \omega_{min}) = 0$ then $V_{job}(m, z; \omega_{min}) > \theta$.
 (b) If $n(m, z; \omega_{min}) < x$ and $r(m, z; \omega_{min}) = 0$ then $V_{job}(m, z; \omega_{min}) = \theta$

2. If $x \geq e(z; \omega_{min})$

- (a) if $n(m, z; \omega_{min}) = e$ and $r(m, z; \omega_{min}) = x - e$ then $V_{nojob}(m, z; \omega_{min}) > \theta$
 (b) if $n(m, z; \omega_{min}) = e$ and $0 < r(m, z; \omega_{min}) < x - e$ then $V_{nojob}(m, z; \omega_{min}) = \theta$
 (c) If $n(m, z; \omega_{min}) < e$ and $r(m, z; \omega_{min}) = 0$ then $V_{job}(m, z; \omega_{min}) = \theta$

The workers' policies together with the productivity process generates a stationary distribution of islands

$$\mu(M', Z') = \int_{\{(m, z): (n(m, z), r(m, z) + U_s) \in M'\}} Q(z, Z') \mu(dm, dz), \quad (12)$$

for all sets of beginning of the period labor forces, M' , and productivities, Z' . The value of search is given by

$$\theta = \beta \int [p_{nojob}(m, z) V_{job}(m, z; \omega_{min}) + (1 - p_{nojob}(m, z)) V_{nojob}(m, z; \omega_{min})] \mu(dx, dz). \quad (13)$$

The labor union takes into account the workers' and the firms' behavior and the general structure of the economy and chooses the minimum wage, ω_{min} , such that the stationary utility of the representative household is maximized. That is, the union's problem is given by

$$\begin{aligned} & \max_{\omega_{min}} \ln(C_H) + b_r U_r \\ & st \ C_H = \int w(m, z) n(m, z) \mu(dm, dz) \\ & \quad U_r = \int r(m, z) \mu(dm, dz) \end{aligned}$$

where, for a given minimum wage, the variables $w(m, z)$, $n(m, z)$ and $r(m, z)$ are determined by the workers' and firms' behavior, as stated above.

The equilibrium is such that the workers' value functions are consistent with the Bellman equations stated above, the state dependent policies are in line with conditions 1 a-b and 2 a-c, the wages are defined by equation (8) or the minimum wage, the stationary distribution of islands is given by equation (12), the value of search is given by equation (13). Finally, the minimum wage is set by the labor union.

2.4 Bargaining

In countries where large labor unions are common, e.g. in continental Europe, employers have also typically set up employer's organizations to bargain with the labor unions. In order to explore this type of set-up, I assume that rents from the fixed factor are paid to a continuum of capitalists who have diversified their ownings over the whole economy. Moreover, these capitalists have formed a coalition that negotiates with the labor union.

The aim of this coalition is to maximize the utility of a representative capitalist when the periodic utility function takes the following form

$$U(C_{C,t}) = \ln(C_{C,t})$$

The capitalist is only able to use profits from the fixed factor to finance her consumption, i.e.,

$$C_{C,t} = \int \pi(m, z) \mu(dm, dz) = \int (zn(m, z)^\alpha - w(m, z)n(m, z)) \mu(dm, dz). \quad (14)$$

The labor union and the coalition of firm owners negotiate over the minimum wage in the economy. The union's objective function and restrictions are as in the previous subsection. Moreover, for a given minimum wage the workers behave as before. I also assume that the coalition of firms is not able to prevent firms from competing against each other in local markets. That is, in markets where the minimum wage constraint is not binding, wages are set according to equation (8).

The minimum wage rate is determined via Nash bargaining over the stationary equilibria. That is,

$$\underset{\omega_{min}}{\operatorname{argmax}} (\ln(C_H) + b_r U_r - b_r)^\gamma * (\ln(C_C))^{1-\gamma}, \quad (15)$$

where γ is the bargaining power of the labor union. The outside option is assumed to be the one where no one is producing market goods and all workers are rest unemployed.

The equilibrium is similar to the one in last subsection, the only difference being that now the bargaining solution determines the minimum wage.

2.5 Many small unions

To contrast the results related to a large union with a more traditional analysis of many small unions (see, e.g., Alvarez and Veraciero, 1999 and Alvarez and Shimer, 2014), I set up an economy with a continuum of unions that takes the

rest of the world as given. As in previous subsections, I assume universal coverage ,i.e., all workers belong to some union. To be more precise, I assume that there is a union on every island.

The union on island (x,z) maximizes the expected present discounted value of earnings of its current members, measured in units of marginal utility. I assume that when an agent enters the island, she becomes a member of the union. The membership ends when the agent reallocates. As before, each agent gets her own income and makes her own reallocation decisions. The future employment statuses are allocated randomly. The union chooses the amount of workers allowed to work, taking into account how its decisions affect non-search unemployment. As the union is small, it takes the aggregate consumption, the value of search and the amount of incoming new workers as given.

The value function of a union satisfies the following Bellman equation

$$\begin{aligned}
 V_U(x,z) &= \max_{n,r} \frac{1}{C} wn + b_r r + \beta \frac{x}{x'} EV_U(x',z') \\
 \text{s.t. } x' &= n + r + U_s \\
 0 &\leq x - n - r \\
 0 &\leq r \\
 w &= z \alpha n^{\alpha-1},
 \end{aligned}$$

Moreover, as the union cannot force agents to stay on the island, it has to be the case that the amounts of workers and non-search unemployed agents are consistent with the value functions of agents. Due to the insurance given by the family and the fact that the next period jobs are allocated randomly, we can write these constraints with the help of the union's value function. This happens because the next period continuation value for a worker is equal to the continuation value of the union divided by the amount of workers present in the next period.

$$0 \leq r(b_r + \beta \frac{1}{x'} EV_U(x',z') - \theta)$$

and

$$\theta \leq w + \beta \frac{1}{x'} EV_U(x',z')$$

Given the policy of the union and the workers' behavior, there are four possibilities

1. All agents stay and work
2. All agents stay and some are non-search unemployed
3. Some agents go, while some are non-search unemployed and others work

4. Some agents go while those who stay, work

If case 1 is observed on island (x,z) , the union finds it optimal not to restrict labor supply. This can happen when the competitive wage rate is high enough. As the union does not care about the workers who reallocate, in case 4, the union also finds it optimal not to restrict labor supply. Finally, from the first order conditions it can be concluded that in cases where the union restricts labor force, i.e. cases 2 and 3, the union sets the amount of workers according to

$$n = \left(\frac{b_r}{\alpha^2 z} C\right)^{\frac{1}{\alpha-1}} \quad (16)$$

This implies that the union chooses the following wage rate

$$w = \frac{b_r}{\alpha} C. \quad (17)$$

That is, when the union finds it optimal to restrict labor supply, it will generate non-search unemployment and set the wage rate in a way that equates it with the flow value of the outside option multiplied with a constant mark up.

To sum it up, the union's optimal policy at the stage (x,z) is either to allow wages to be determined competitively or set the wage on the island according to equation (17). The union chooses between these two actions in a way that solves the Bellman equation above.

Note that the union's problem in this section is simplified by the random allocation of the next period job opportunities. If, as in the previous subsection, employed workers would have been offered a preferred position, the next period continuation values would be present in the optimal wage policy. Moreover, the mapping between the union's value function and the workers' value functions would be more complicated. However, as the union prefers to keep all workers present on the island, the randomization of future jobs is in the interest of the union as it makes the values of non-search unemployment and work more similar. In addition to the randomization, other major difference is the fact that small unions do not internalize the effects of its actions on the aggregate consumption, the value of search and the amount of searchers.

The policy of the union resembles the union wage rule in Alvarez and Shimer (2014), who, at the top of an exogenous minimum wage, also consider the wage policy of a small union when it is maximizing over the flow values of utility. In their analysis, they explore a set up where the employment opportunities are decided based on a strict seniority rule. This implies that workers become search unemployed only through a period of rest unemployment. This means that the union's policy is a minimum wage policy where jobs are rationed when the competitive wage would be lower than the target rate of the union. In my case, the risk of losing members means that a union may also find it optimal not to ration labor supply when local labor market conditions are weak. That is,

unions are restricting labor force only in a small range at the middle of the wage distribution.

Finally, the stationary measure of islands is similar to the stationary measure of islands in the competitive economy (see equation (10)). Given this measure, the value of search can be expressed with the help of the union's value function

$$\theta = \beta \int \frac{1}{x} V_U(x, z) \mu(dx, dz).$$

3 CALIBRATION

I utilize the method of simulated moments in order to calibrate the model. I first fix the values of certain parameters based on external evidence and then set the rest of the parameters in a way that the model without unions matches the stylized facts of the unemployment patterns in the US. To be more precise, I minimize the relative distance between simulated moments and moments calculated from the data using the identity matrix as a weighting matrix.

The model period is set to a month and the annual real interest rate is assumed to be 4%. Taken together these decisions imply that β is fixed to 0.9966. The curvature parameter of the production function, α , is set to 0.66, based on the labor share of output. I use Tauchen's method with 15 stages to approximate the AR(1) process of logged productivity. The unconditional mean of the productivity is normalized to 1.

The remaining parameters, i.e., the flow value of non-search unemployment and the parameters of the productivity process, the persistence, ρ , and the volatility of innovations, σ , are calibrated internally. These parameters determine unemployment and its decomposition. In order to take a conservative approach to non-search unemployment, I assume that only workers who return to their previous employer are non-search unemployed. This, as suggested by Fernandez-Blanco (2013), is one interpretation of Alvarez and Shimer's (2011) "rest unemployment".

When deciding on the empirical target for non-search unemployment, I rely on the empirical evidence of Fujita and Moscarini (2017). From the Survey of Income and Program Participation (SIPP), they conclude that a large share of jobless workers returns to their previous employer. Depending on the SIPP panel and the type of non-employment, the recall rates vary between 0.32 and 0.553. As I do not allow the out of labor force option, I focus on the recall rate related to workers who do not leave the labor force. My target for completed unemployment spells that end with recalls is 0.48175 and it is calculated as a non-weighted average from the SIPP panels of 1996 to 2008 (see Fujita and

Table 1: Parameter values

α	β	ρ	σ^2	b_r	a
0.66	0.9966	0.9571	0.051	0.0095	0

Moscarini, 2017). Given that this interpretation would imply that the non-search unemployed always return to their previous employer, this calibration strategy gives the lower bound for the non-search unemployment.

My next target is the average duration of completed unemployment spells for workers who stay in the labor force. This is set to 2.96 months. It is again from Fujita and Moscarini (2017) and calculated as an unweighted mean of the SIPP panels. Note that the *laissez-faire* economy considered in Section 2.2 is silent about the duration of non-search unemployment due to the fact that agents are indifferent between work and non-search unemployment in those islands where part of them decide to “rest”. When calculating the duration of unemployment spells, I assume, consistently with the analysis of the large union, that employed workers are always offered jobs first in the next period. My last empirical target is the aggregate unemployment rate, 6 per cent, calculated from the period covered by the SIPP panels from 1996 to 2008.

When calculating the simulated moments, I generate pseudo SIPP panels for a large number of individuals at the stationary equilibrium of the competitive economy. Table 1 presents the parameter values associated with the preferred calibration, while Table 2 recaps the empirical targets and summarizes the model counterparts.

Before moving on to the effects of different types of union structures, I first highlight some results related to the competitive economy that were not targeted. For any given period, 16% of the unemployed workers are search unemployed. Given that over 50% of completed unemployment spells involves periods of active search, this reflects the fact that a considerable fraction of those workers who switch locations also spend time being non-search unemployed. To compare this to the empirical evidence, Krueger and Mueller (2012) report, based on time diary evidence, that for a given day, 20% of the unemployed in the US searches for a job. However, the time they spent on search is less than half the time spent on work for employed workers (159.7 minutes vs 325 minutes). In the model, it is assumed that the time spent on search is the same that the time spent at work. Calculating the average search time per day in the model by multiplying the fraction of search unemployed by the average minutes spent on work for employed agents, reported in Krueger and Mueller (2012), gives 52 minutes. This is somewhat higher than the average search times in the data (32 minutes) but relatively close to the time that unemployed workers spend on search and work-like activities, 42 minutes, on average.

Table 2: Empirical targets and the model counterparts

	unemployment	recall rate	mean duration
Data	6	0.4818	2.96
Model	6.0152	0.4752	2.9826

4 RESULTS

This section quantitatively assesses the effects of unionization with different union structures introduced in section 2. Instead of using simulated panels, the aggregate unemployment rate and its composition are calculated from the stationary distributions. In order to facilitate comparisons, the results related to wages, consumption and output are normalized to the competitive economy levels.

4.1 A large union

Table 3 reports the effects of the large union. The union sets a minimum wage that is almost 20 percent higher than the lowest wage paid in the laissez-faire economy. In fact, the minimum wage is 7.4 percent higher than the average wage paid in the competitive economy.

Due to the policy of the union, the average income of a worker goes up by 11%. This is consistent with the body of empirical evidence that suggests that union membership increases the wage rate. For example, Lewis (1986) reviews the empirical literature of the union wage differentials in the US and concludes that (the upper bound for) the union wage gap is around 14%.

Achieving the substantial increase in minimum wage rates requires unions to heavily restrict labor supply. This leads to a strong increase in unemployment which rises to 28.3 percent. All of this increase is non-search unemployment, which goes up to 27.6 percent. The search unemployment reduces marginally to 0.7 per cent. Now around 97.5% of the unemployed workers are not searching for a job in any given period. Repeating the back of the envelope calculation of average search times for unemployed workers implies that an unemployed worker spends on average 8.15 minutes per day on search. That is, the search intensity of an average unemployed worker drops substantially compared to 52 minutes in the competitive economy. Moreover, the search time is comparable with the average search times in Europe, reported by Krueger and Mueller (2012)⁷.

Even though the increase in unemployment is strong, it is in terms of mag-

⁷ According to Krueger and Mueller (2012), the average search time per day is 12 minutes in Western Europe and 11 minutes in Eastern Europe.

nitudes broadly in line with other studies that explore unions in frictional labor markets. For example, in the coalition model of Alvarez and Veracierto (1999), the introduction of an 80% unionization increases unemployment from 5.3% to 16.3%. Since in their model less costly non-search unemployment is not possible, a milder increase in unemployment is to be expected, especially given the high flow value of non-search unemployment that is needed to achieve the high recall rate. In Krusell and Rudanko (2016), a large union causes unemployment to rise from 5% to 16%. Once again, the possibility of recall is likely to give unionization more power in my analysis, even though directly comparing the results is more difficult, as Krusell and Rudanko (2016) build their analysis on the search and matching framework.

The strong increase in the lowest wage paid in the economy, together with increased non-search unemployment, implies that the majority of firms are now paying the minimum wage; only 20% of the islands are paying more than the minimum wage. However, decreased search unemployment increases the highest wages paid in the economy. For example, the maximum wage in the economy goes up by 3.9%. Taken together, the suppression of the lower tail dominates the widening of the upper tail and so the standard deviation of wage distribution decreases by almost 30%. This is qualitatively consistent with the empirical evidence that unions reduce wage inequality (see Card et al, 2003).

To summarize, the model implies that the unionization decreases the search intensity of unemployed workers and suppresses the wage distribution. This is in line with Krueger and Mueller (2012), who by using a cross country data, show that the wage dispersion is a strong predictor of unemployed workers' search activity. Interestingly, they also find that the lowest search activities among the countries they examine are found in Scandinavian countries, where the centralized wage mechanisms are particularly popular.

Increased unemployment and reduced search activity also coexist with increased unemployment duration. The average duration of unemployment spells goes up from 2.98 months to 6.8 months.

The increasing unemployment causes a 15.5% reduction in the market output produced. In line with this, the workers' and the capitalists' consumption also go down, roughly equivalently. Finally, looking at the welfare of the workers, measured in units of consumption at the competitive economy, we can see that the welfare gain for workers from a large union is equivalent to a 4.7 percent increase in the representative household's consumption.

4.2 Bargaining

This section analyzes the model where the union is assumed to bargain over the minimum wage with the coalition of firms. The results for the bargaining

Table 3: Model with a large union

	Competitive economy	Large union
Search unemployment	0.94	0.71
Non-search unemployment	4.99	27.6
Total unemployment	5.93	28.3
Search/total unemployment %	15.9	2.51
Avg. duration	2.98	6.80
Avg wage	100	110.8
Std. of wages	100	70.3
Min. wage	100	119.5
Max. wage	100	103.9
Market output	100	84.5
Household's consumption	100	84.5
Capitalists's consumption	100	84.6
Household's welfare	100	104.7

solutions, when the bargaining power of the union takes the values of 0.5 and 0.75, are given in Table 4.

As one would expect, the bargaining results are more moderate than the results in the last subsection where the union decided the minimum wage unilaterally. Compared to the competitive economy, the minimum wage increases by around 9% when the union's bargaining power is equal to 0.75, while the 50/50 bargaining rule increases the minimum wage by only about 2%. The restrictions on labor supply increase the average wage in the economy to 4.9% or 1.8%, depending on the bargaining power.

It can be seen that the unemployment responses are considerably weaker when at least some bargaining power is given to the coalition of firms. When γ is set to 0.75, aggregate unemployment increases to 16%, while the bargaining model with $\gamma = 0.5$ is associated with aggregate unemployment around 10%. The whole increase is non-search unemployment, as in both cases search unemployment drops to 0.5%. Due to these effects, the relative amount of search unemployed agents increases as the bargaining power of the union decreases. The average search times for unemployed workers are 8.7 minutes and 13.3 minutes for $\gamma = 0.75$ and $\gamma = 0.5$, respectively. Consistently with smaller unemployment responses, also the increase in the average duration of unemployment spells is milder in comparison with the economy analyzed in the previous subsection.

Decreasing search unemployment increases the highest wages paid in the economy. However, for $\gamma = 0.75$, the increasing minimum wage dominates and wage inequality measured with the standard deviation of the wage distribution decreases by 8.3 percent. When the union's bargaining power is 0.5, wage inequality increases 4.7 percent, as the effects on the left tail are not strong enough to compensate for the increased inequality caused by the reduced search.

Table 4: Bargaining model

γ	Competitive economy		Bargaining	
			75	50
Search unemployment	0.94		0.44	0.42
Non-search unemployment	4.99		16.0	9.86
Total unemployment	5.93		16.5	10.3
Search/total unemployment %	15.9		2.67	4.08
Avg. duration	2.98		5.29	4.38
Avg wage	100		104.9	101.8
Std. of wages	100		91.7	104.7
Min. wage	100		108.9	102.3
Max. wage	100		103.9	104.7
Market output	100		93.3	97.3
Household's consumption	100		93.3	97.3
Capitalists' consumption	100		93.3	97.3
Household's welfare	100		103.6	101.9

The market output reduces by 6.7 and 2.7 percent depending on the value of bargaining power. The workers' welfare increases is 3.6% compared with the competitive economy when the bargaining power is set to 0.75. When $\gamma = 0.5$, the welfare increases by less than 2%.

4.3 Many small unions

Table 5 assembles the results of the model where labor supply on each island is controlled by a local labor union. In order to keep as many workers as possible attached to their current locations, small unions do not restrict employment in locations where conditions are weak. Moreover, due to the moving frictions, restricting labor supply is not optimal when local conditions are good enough. At the equilibrium, 31% of all locations have an active labor union. Compared with the case of a large union, in which firms in almost 80% of the locations restricted labor supply, there is a major drop in the amount of local markets where unionization causes direct effects. This, in turn, implies that non-search unemployment does not increase as much. Though, the non-search unemployment still increases to 21.9% reflecting the fact that the active unions still represent a large group of labor force. Search unemployment is lower than in the competitive economy. This shows that small unions are able to reduce mobility.

Aggregate unemployment goes up to 22.7 percent. That is, even though small unions do not take into account how their actions contribute to the aggregate variables, such as the amount of the search unemployed, the unionization with a continuum of small unions leads to five percent smaller increase in aggregate unemployment than in the case of a large union. The unions' desire to retain their

Table 5: Many small unions

	Competitive economy	Continuum of unions
Search unemployment	0.94	0.82
Non-search unemployment	4.99	21.9
Total unemployment	5.93	22.7
Search/total unemployment %	15.9	3.62
Avg wage	100	106
Std. of wages	100	164
Min. wage	100	88.2
Max. wage	100	109
Market output	100	87.0
Household's consumption	100	87.0
Capitalists's consumption	100	87.0
Household's welfare	100	102.1

members by not restricting labor supply in bad times creates a strong enough effect to counter for the negative consequences of non-existing coordination. Note that, the assumption of randomization of jobs is likely to increase the unions' use of monopoly power compared with what would be the case if previously employed workers were preferred. That is, the difference in unemployment between one large union versus many small unions could be even larger.

Decreasing search unemployment, together with increasing non-search unemployment, means that only 3.62% of the unemployed are searching for a job per period. Model consistent search time of an average worker is 11.8 minutes per day when the daily average working hours from Krueger and Mueller (2012) are used to measure search times in the model.

Since the unions let competitive markets determine the wage rate when local conditions are weak, the minimum wage is determined in a similar way as in the competitive model. Given that the aggregate consumption is lower than in the benchmark equilibrium, the minimum wage is actually 11.8% lower. Moreover, due to decreased search the highest wage paid in the economy increases by 9%. Taken together, the dispersion of wages increases, standard deviation being 64% higher. This raises the question of whether the assumption that unions are willing to allow competitive wage rates during bad times, is realistic. For example, the union boss model à Alvarez and Veracierto (1999) might lead to decreased wage dispersion and higher unemployment.

The market output, the workers' and the capitalists' consumption all drop by 13%. These reactions are somewhat smaller than in the equilibrium with a large union. Workers' welfare goes up by 2.1 percent.

5 CONCLUSIONS

This paper explored the equilibrium effects of the unionization with different union structures. The main focus was on a union that affects a large group of heterogeneous firms by setting a common minimum wage. The results were also compared against a union structure of many small unions. A key element of the analysis was the fact that unemployed workers could engage in non-search unemployment that allowed the unemployed to enjoy increased leisure time. The model was calibrated based on the empirical evidence of recall unemployment in the US. Even though this can be seen as a natural lower bound of non-search unemployment, the resulting search intensity was broadly comparable to the search activity measures of time diary evidence.

The quantitative results of a large union suggest that the steady state unemployment increases strongly, by around 22%, as a response to the introduction of a minimum wage set by the union. The whole increase is non-search unemployment, as search unemployment actually decreases a bit. Taken together, the search intensity of unemployed agents decreases strongly. This is qualitatively consistent with the empirical observation of Krueger and Mueller (2012) that in Europe the unemployed workers search substantially less than in North America. The union also decreases wage inequality in the economy. When it is assumed that the union negotiates over wages with an employer's organization, the increase in aggregate unemployment is much more subdued. For example, the Nash bargaining solution, when the bargaining power of the union is set to 0.5, generates unemployment increase by 4.4 percent. However, the non-search unemployment still increases and the search unemployment decreases.

The unionization with a continuum of small unions leads to an increase in aggregate unemployment that is about 17%. That is, even though small unions do not properly internalize their actions as they take the aggregate variables as given, the increase in unemployment is less severe than with the large union. The important factors behind this are the assumptions that the union membership is determined based on the workers' current location and that the unions care only about workers who stay put. These assumptions imply that unions are not willing to restrict employment in times when the wage rate is low. However, this set-up also leads to an increase in wage inequality which is at the odds with the empirical evidence and so casts some doubts on whether small unions truly react to weak local conditions.

Overall, the results highlight the fact that unions offer a powerful channel for generating non-search unemployment. To be more precise, the natural level of non-search unemployment is complementary with the union generated non-search unemployment, as the possibility of an additional, less costly form of unemployment, allows the union(s) to push minimum wages higher.

References

- Alvarez, F. – Shimer, R. (2011) Search and rest unemployment. *Econometrica*, Vol. 79 (1), 75–122.
- Alvarez, F. – Shimer, R. (2014) Unions and unemployment, unpublished manuscript, University of Chicago.
- Alvarez, F. – Veracierto, M. (1999) Labor market policies in an equilibrium search model. *NBER Macroeconomics Annual*, Vol. 14, 265–304.
- Calmfors, L. – Driffill, J. (1988) Bargaining structure, corporatism and macroeconomic performance. *Economic Policy*, Vol. 3, 13–61.
- Card, D. – Lemieux, T. – Riddell, C. (2003) Unions and the wage structure. In *International Handbook of Trade Unions*, eds. J. T. Addison – C. Schnabel, 246–292, Edward Elgar.
- Fernandez-Blanco, J. (2013) Labor market equilibrium with rehiring. *International Economic Review*, Vol. 54, 885–914.
- Fujita, S. – Moscarini, G. (2017) Recall and unemployment. *American Economic Review*, Vol. 107, 3875–3916.
- Krueger, A. – Mueller, A. (2010) Job search and unemployment insurance: New evidence from time use data. *Journal of Public Economics*, Vol. 94, 298–307.
- Krueger, A. – Mueller, A. (2011) Job search, emotional well-being, and job finding in a period of mass unemployment: Evidence from high-frequency longitudinal data. *Brookings Paper on Economic Activity*.
- Krueger, A. – Mueller, A. (2012) The lot of the unemployed: A time use perspective. *Journal of European Economic Association*, Vol. 10, 765–794.
- Krusell, P. – Rudanko, L. (2016) Unions in a frictional labor market. *Journal of Monetary Economics*, Vol. 80, 35–50.
- Lewis, G. (1986) Union relative wage effects. In *Handbook of Labor Economics*, eds. O. Ashenfelter – R. Layard, chap. 20, 1139–1181, Elsevier.
- Lucas, R. E. – Prescott, E. C. (1974) Equilibrium search and unemployment. *Journal of Economic Theory*, Vol. 7 (188-209).
- Shimer, R. (2007) Mismatch. *American Economic Review*, Vol. 97 (4), 1074–1101.

ESSAY 3

Juha Kilponen, Jouko Vilmunen & Oskari Vähämaa

Estimating Intertemporal Elasticity of Substitution in a Sticky Price Model

Preprint

Estimating Intertemporal Elasticity of Substitution in a Sticky Price Model*

Juha Kilponen[†] Jouko Vilmunen[‡] Oskari Vähämaa[§]

Abstract

Cancellation of income and substitution effect implied by King-Plosser-Rebelo (1988) preferences breaks tight coefficient restriction between the slope of the Phillips curve and the elasticity of consumption with respect to real interest rate in a sticky price macro model. This facilitates the estimation of intertemporal elasticity of substitution using full information Bayesian Maximum Likelihood techniques within a structural model. The US data from the period 1984–2007 supports low intertemporal elasticity of substitution and strongly rejects a logarithmic and an additively separable utility specification commonly applied in the New Keynesian literature.

1 INTRODUCTION

There are two important conditions that the preferences must fulfill in order for the balanced growth path to exist in the neoclassical growth model. First, the intertemporal elasticity of substitution must be invariant to the scale of consumption and the income and substitution effects associated to sustained growth in the labour productivity must not change the labour supply (cf. King-Plosser-Rebelo, 1988). The latter condition states that, in the long-run, the income and substitution effects must cancel the each other¹.

In order to fulfill these restrictions one possibility is to assume additively

* We are indebted to Michael Andrieu, Efram Castelnuovo, Ivan Jaccard and Thomas Laubach for many useful comments and suggestions.

[†] Corresponding author. *Bank of Finland, P.O. Box 160, FI-00101 Helsinki. email: juha.kilponen@bof.fi.*

[‡] *University of Turku*

[§] *University of Turku*

¹ Consensus from a large number of empirical work on labour supply elasticities also suggests that the income elasticity cannot be much larger than the substitution elasticity. Hence, the preferred estimate of the uncompensated wage elasticity is *weakly* positive.

separable preferences, where, in addition, consumption enters logarithmically so that the cross-elasticity between consumption and hours worked is zero and the intertemporal elasticity of substitution is one. In this case, the household utility function typically takes a form $\ln(C_t) - v(N_t)$ where C_t is consumption and $v(N_t)$ is some strictly increasing function of the quantity of labour, N_t , representing disutility from work. However, both of these assumptions can be challenged on empirical grounds.

First, under this specification of preferences the elasticity of consumption growth with respect to the real interest rate should be one. However, estimates based on the consumption Euler equation yields consistently much lower values (see e.g. Hall, 1988, Campbell and Mankiw, 1989; Basu and Kimball, 2002; Fuhrer and Rudebusch, 2004, Yogo, 2002). Second, the zero cross-elasticity between consumption and hours is generally rejected by the empirical research based on micro-data based literature. In particular, the level of consumption tends to fall after a retirement or after a person becomes unemployed² The latter evidence is consistent with the complementarity between consumption and work: households like to consume more when they work more. Aguiar et al. (2013) study based on American Time Use Surveys gives also support to macroeconomic models in which consumption and labour are strong complements.

Furthermore, the additively separable logarithmic in consumption preferences³ also imply that the Frisch elasticity of labour supply and the consumption-constant elasticity of labour supply coincide. The Frisch elasticity of labour supply⁴ primarily tells how a transitory change in the real wage impacts on the labour supply while the consumption-constant elasticity gives the impact of a permanent change in the real wage on the labour supply. When these two elasticities coincide, the same model can not say why a permanent increase in the marginal tax rate leads to a substantial decline in hours, but a transitory movement in the real wage does not cause as big a change in hours worked at the business cycle frequency. Much of the large literature on Frisch elasticity in the general equilibrium macroeconomic models hovers around this tension.

In this paper, we apply the class of preferences similar to King-Plosser-Rebelo (1988, henceforth KPR), that relax the assumption of additive separability between consumption and labour, and estimate the equilibrium intertemporal elasticity of substitution within the structural New Keynesian model⁵.

² See e.g. Banks et al., (1998), Hamermesh (1984), Bernheim, (2001), Browning and Crossley (2001), Ameriks et al. (2007), Hurst (2008).

³ In what follows we shall refer to these preferences with somewhat less precise terminology of 'additively separable log(arithmetic) preference (utility)'.

⁴ As argued in Kimball and Shapiro (2008) the Frisch elasticity governs, in a frictionless world, the intertemporal substitution in labour supply and is tightly linked to the effects of real interest rate on labour supply.

⁵ Independently from us, also Bilbiie (2009) studies the New Keynesian model with non-separable preferences. His main interest is to explore the implications of non-separable preferences for fiscal

KPR preferences allow for a non-zero cross-elasticity between consumption and labour and this elasticity is tightly linked to curvature of utility⁶. We show analytically that with KPR preferences the relationship between inflation and output gap i.e. the slope of the Phillips curve depends only weakly on the curvature of utility in contrast to more usual additively separable preference specification. Under additively separable preferences the link between slope of the Phillips curve and curvature of utility is far tighter such that high values of utility curvature would yield to unrealistically high slope of the Phillips curve. In our view, there is no particular reason why curvature of utility and slope of the Phillips curve should be tightly linked.

Given the structural model, we can make use of Full Information Bayesian Maximum Likelihood methods, instead of relying on GMM estimation as is widely done in the consumption Euler equation literature. Full-information methods impose more restrictions on the estimated model and thus potentially make more efficient use of the information in the data (see e.g. Magnusson and Mavroeidis, 2010). As shown e.g. in Yogo (2002), Kiley (2010) and Kilponen (2012), weak instrument problem makes it difficult to identify IES using GMM techniques from macro data.⁷ To the extent that the structural model and the restriction on preferences are correct, full information methods provide more reliable inference than limited information methods such as GMM. This enhances the model and preference validation.

Using Bayesian Maximum Likelihood Methods and the U.S. data from the period 1984Q1-2007Q4, we find that the real interest rate elasticity of output is 0.22. This value is in stark contrast to unitary real interest elasticity of output implied by the logarithmic and additively separable utility, but better in line with Euler consumption equation based estimates. The 90% credibility intervals are rather tight, ranging between 0.09 and 0.36, suggesting that the IES is different from zero, but significantly lower than unity. The respective curvature of the utility with respect to consumption is far higher than in Smets and Wouters (2007), who estimate a fully fledged DSGE model with KPR preferences. We show that differing results are driven by the choice of priors, the data preferring a high curvature. An important implication of our result is that high curvature and complementarity considerably weakens the real interest rate channel of monetary policy, while strengthening the transmission of productivity shocks to the economy. At the same time, given the preferences specification, a low IES

policy.

⁶ When the assumption of additive separability is relaxed, there is no longer a unique way of measuring the consumers' willingness to substitute consumption over time. By equilibrium intertemporal elasticity of substitution we effectively mean the elasticity of consumption growth to real interest rate after general equilibrium features of the macro model are accounted for. This elasticity can be expressed as a function of curvature of utility, cross-elasticity between consumption and labour, and concavity of the production function.

⁷ Campbell and Ludvigson (2001) use the state level data for the U.S. to evaluate the degree of IES. They find that the IES could take any value between 0 and 1.5.

implies a strong complementarity between consumption and labour. The earlier literature which have studied the consumption-leisure non-separability provides some evidence on complementarity at the aggregate data, but this evidence also suffers from the weak instrument problem since it has used the GMM or IV estimation methods, see e.g. Eichenbaum, Hansen and Singleton (1988), Campbell and Mankiw (1990).

The remaining paper is organized as follows. Section 2 describes the model. Section 3 discusses the properties of the model with KPR preferences. Section 4 provide the estimation results including robustness analysis and section 5 concludes.

2 THE MODEL

This section develops a stylized sticky price monetary policy model featuring King-Plosser-Rebelo preferences. We follow closely a text book type derivation of the sticky price monetary policy model (see e.g. Goodfriend and King, 1997; Walsh, 2010; Woodford, 2003; Gali, 2008).

2.1 Households

The economy is populated by identical infinitely-lived households who solve the following problem

$$\begin{aligned} & \max_{C_t(i), N_t} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \\ & \text{s.t.} \\ & \int_0^1 P_t(i) C_t(i) di + Q_t B_t = B_{t-1} + W_t N_t + T_t \\ & C_t \equiv \left(\int C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \\ & \lim_{T \rightarrow \infty} \mathbb{E}_t(B_T) \geq 0. \end{aligned}$$

where $C_t(i)$ is the quantity of good i consumed by the representative household in period t ; $P_t(i)$ is the price of good i ; N_t is quantity of labour; W_t is nominal wage, B_t represents purchases of one period bonds of which price is Q_t ; T_t is lump sum component of income and finally ϵ is the elasticity of substitution between the differentiated goods.⁸ Following King-Plosser-Rebelo (1988),

⁸ ϵ also denotes the absolute value of the own price elasticity of the demand for a good.

Kimball (1995), Basu and Kimball (2002), we assume that the additively time-separable felicity function $U(C_t, N_t)$ takes a form

$$U(C_t, N_t) = \frac{C_t^{1-\gamma}}{1-\gamma} e^{(\gamma-1)v(N_t)}, \quad (1)$$

where $\gamma \neq 1$ controls the concavity of the utility function.⁹ We shall in the following focus on the case where $\gamma > 1$. In this formulation $s \equiv 1/\gamma$ denotes the labour-held-constant intertemporal elasticity of consumption. It is important to notice that γ is (up to scaling) equal to the usual risk aversion measure only in the special case of exogenously fixed labour, as shown by Swanson (2012, corollary 1, p. 1671). That is, the usual measure of risk aversion ignores the household's ability to offset income shocks by the adjustment of labour. As discussed further by Swansson (2012), high values of γ (or low values of s) are not ruled out by empirical micro estimates of risk aversion when labour margin is taken into account. $v(N_t)$ is some strictly increasing function of quantity of labour, representing the disutility from work.¹⁰ Note that in the limiting case where $s \equiv \gamma^{-1} = 1$, the function $U(C_t, N_t) - \frac{1}{1-\gamma}$ converges, by l'Hopital's rule, to $\ln(C_t) - v(N_t)$.

In the first step, the household makes a decision on consumption and labour supply. The optimal choice of consumption and labour supply yields the following consumption Euler equation and the labour supply equation

$$Q_t = \beta \mathbb{E}_t \left\{ \frac{U_C(C_{t+1}, N_{t+1})}{U_C(C_t, N_t)} \frac{P_t}{P_{t+1}} \right\} \quad (2)$$

$$\frac{W_t}{P_t} = - \frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} \quad (3)$$

where $U_C(C_{t+j}, N_{t+j}) = C_{t+j}^{-\gamma} e^{(\gamma-1)v(N_{t+j})}$; $U_N(C_t, N_t) = \frac{C_t^{1-\gamma}}{1-\gamma} e^{(\gamma-1)v(N_t)} (\gamma - 1)v'(N_t)$. \mathbb{E}_t is the usual conditional expectation operator. As usual, the optimal labour supply condition states that the intratemporal marginal rate of substitution between labour and consumption is equal to the real wage. The representative household must also decide on the allocation of her consumption expenditure among the differentiated goods. This gives rise to the familiar demand equations:

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t. \quad (4)$$

⁹ Jaimovich and Rebelo (2009) further extend this class of preferences by considering also a time-non-separability. The original KPR preferences arise as a special case of their preferences. Note that Greenwood, Hercowitz and Huffman (1998) (GHH) preferences are not consistent with the balanced growth path due to lack of income effect. Hence, a permanent change in productivity would lead into a permanent change of the labour supply.

¹⁰ Smets and Wouters (2007) use the similar utility function with the additional assumption of a particular functional form for $v(N_t)$, namely $v(N_t) = (\sigma - 1)/(1 + \xi_l) N_t^{(1+\xi_l)}$, where ξ_l is the labour supply elasticity.

where $P_t \equiv \left(\int P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$ is the aggregate price index.

Focusing on the first-order terms in the Taylor expansion and assuming homoscedasticity of the stochastic processes for $c_t \equiv \ln(C_t)$, $p_t = \ln(P_t)$ and $n_t = \ln N_t$, the optimal consumption and labour supply dynamics can be re-parameterized as

$$c_t = \mathbb{E}_t c_{t+1} - s(i_t - \mathbb{E}_t \pi_{t+1} - \rho) - (1-s)\tau \mathbb{E}_t \Delta n_{t+1} \quad (5)$$

$$w_t - p_t = c_t + \varphi n_t + \iota, \quad (6)$$

where $\pi_{t+1} \equiv \ln P_{t+1} - \ln P_t$, $i_t \equiv -q_t$, $s \equiv 1/\gamma$, $\rho \equiv -\ln \beta$, $\varphi \equiv \frac{v''(N)/N}{v'(N)}$ and $\iota \equiv \ln \tau - (1 + \varphi)n$ and $\tau = WN/PC = v'(N)N$.

Equation (6) shows that KPR preferences imply that there is a unitary elasticity between the real wage and consumption. The unitary elasticity is important, since by the definition of the balanced growth path, the real wage and the consumption must grow at the same rate in the long-run. At the same time, the elasticity between the real wage and labour depends on the term $\varphi \equiv \frac{v''(N)/N}{v'(N)}$. This can be interpreted as an inverse of the consumption-constant elasticity of labour supply, not the Frisch elasticity of labour supply.¹¹ However, it is possible to relate this term to Frisch labour supply elasticity ξ i.e. to the labour supply elasticity which keeps the marginal utility of consumption constant (see appendix A for detailed derivation). Specifically, we show in the appendix that

$$\xi = \frac{1}{\varphi + \tau(1-s)}. \quad (7)$$

From (7) it is easy to see that the Frisch elasticity of labour supply ξ is in general lower than the consumption-constant elasticity of labour supply φ^{-1} . The difference between these two elasticities depends directly on the cross-elasticity of consumption and labour (and thus on the long-run-labour share, τ , and the IES, s). It can be shown that $(1-s)\tau = -\frac{UCN}{UCC} \frac{N}{C} = -\frac{dC}{dN} \frac{N}{C}$ is the cross-elasticity of consumption and labour. Hence, $(1-s)\tau$ parameterizes elasticity of consumption w.r.t. labour supply. Equation (7) also reveals that the intertemporal aspects of consumption and the labour supply elasticity are tightly linked within this class of preferences.¹²

¹¹ Kimball (1995) argues that inverse of the consumption-constant labour supply elasticity φ can be calibrated on the basis of marginal expenditure share of leisure being equal to the ratio of marginal expenditure share of consumption to leisure times the wage income consumption share. His preferred value for φ^{-1} is one.

¹² As discussed by Kimball and Shapiro (2008), the consumption-constant labour supply elasticity is most useful for understanding how a permanent change in the real wage impacts on labour supply. The Frisch elasticity gives the impact of a temporary change in the real wage on labour supply. This means that the Frisch elasticity is a more useful concept at the business cycle frequency. In accordance with this interpretation, it is also natural to find that the Frisch elasticity of labour supply is lower than the consumption-constant elasticity. Finally, note that when $s = 1$, these two elasticities coincide.

Non-separability between consumption and labour also implies that there is no unique way to measure consumers' willingness to substitute consumption over time. Under KPR preferences s is the labour-held-constant intertemporal elasticity of substitution. Allowing the household also to use the labour margin in response to changes in the real interest rate while holding the expected real wage constant gives following (near steady state approximation for) IES

$$\psi^* = s\varphi\xi \quad (8)$$

As in the case of labour elasticities, ψ^* is lower than s when consumption and leisure are complements. In the spirit of Frisch elasticity, we can derive yet another measure of IES by keeping the marginal disutility of labor constant $\psi^{**} = \xi(\varphi s + (1 - s)\tau)$. In section 2.3 we introduce the equilibrium IES, ψ , that in our opinion, is the most relevant elasticity in macroeconomic context. This elasticity effectively measures the elasticity of consumption growth to the real interest rate after the general equilibrium implications of the model are taken into consideration.

Finally, notice that letting $s \rightarrow 1$, the optimal consumption and labour supply equations given in equations (5)-(6) collapse to

$$c_t = \mathbb{E}_t c_{t+1} - (i_t - \mathbb{E}_t \pi_{t+1} - \rho), \quad (9)$$

$$w_t - p_t = c_t + \varphi n_t + \iota, \quad (10)$$

and where φ can now be interpreted directly as inverse of the Frisch elasticity of labour supply. These equations are also consistent with the balanced growth path but with two important differences. First, employment is no longer part of the dynamic IS equation. Second, the elasticity of consumption with respect to real interest rate is restricted to unity. As is also well known, in this case the intertemporal elasticity of consumption is equal to unity.

2.2 Firms, optimal price setting and inflation equation

Specification of the supply side of the model follows the standard setup. We assume that there is a continuum of firms indexed by $i \in [0, 1]$. Each firm produces a differentiated good using homogenous technology. Firms' production possibilities are given by the production function:

$$Y_t(i) = A_t N_t(i)^{1-\alpha}. \quad (11)$$

A_t represents the common stochastic level of technology. All firms face identical isoelastic demand schedule (4) and they take aggregate price and quantities as given. In this model, the absence of (nominal) rigidities would imply that movements in technology, A_t , would not induce any movements in hours worked:

output would move hand-in-hand with the technology. Hours worked would not be affected, because substitution and income effect cancels each other, the key property of the preferences which we have discussed above. Consequently, price rigidity is the sole reason why variations in technology induce movements in hours.

In order to introduce price rigidity into the model, we make the typical assumption that each firm may re-set its price only with probability $1 - \theta$. Thus a measure of $1 - \theta$ producers reset their prices in each period. The average duration of price is given by $1/(1 - \theta)$. In this framework, (log linearized) optimal price setting rule of the firms can be characterized as

$$p_t^* - p_{t-1} = (1 - \theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \{\widehat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})\} \quad (12)$$

where $\mu \equiv \ln \mathcal{M} = \ln \frac{\epsilon}{\epsilon-1}$, and $\widehat{mc}_{t+k|t}$ denotes the log deviation of real marginal cost from its steady state value in period $t+k$ for a firm whose price was last set in period t . Combining the optimal price setting rule of the firms with the goods market and the labour market clearing conditions as well as with the dynamic IS curve in equation (5) delivers a inflation equation:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \lambda \widehat{mc}_t, \quad (13)$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \Theta$ and $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$. Importantly, the slope of the marginal cost term λ is independent of the parameters of the utility and, hence, up to a first order approximation, the relationship between inflation and marginal costs is independent on the choice of the utility functional and intertemporal elasticity of substitution¹³.

This independence of inflation equation from the utility function in the first order approximation breaks down once the real marginal cost term is related to the measure of economic activity. Eliminating real wage from the definition of log real marginal costs $mc_t \equiv w_t - p_t - mpn_t$, (where mpn refers to marginal productivity of labour) and imposing market clearing conditions, we obtain

$$mc_t = \frac{(1 + \varphi)}{1 - \alpha} (y_t - a_t) - (\ln(1 - \alpha) - \iota). \quad (14)$$

Note then that under flexible prices, the real marginal cost is constant and is given by $mc = -\mu$. Defining the natural level of output as the equilibrium level of output under flexible prices, y_t^n , it follows from (14) that

$$y_t^n = a_t + \vartheta_y^n, \quad (15)$$

¹³ Eichenbaum and Fisher (2007) derive a model in which the elasticity of demand facing firms is variable, capital is firm-specific and costly to adjust. This leads to specification of the inflation equation in which the firm specific capital reduces the response of inflation to marginal cost i.e. leads to a smaller slope of the marginal cost term in equation (13).

where $\vartheta_y^n \equiv \frac{(1-\alpha)(\ln(1-\alpha)-\iota-\mu)}{(1+\varphi)}$ and $\mu \equiv \ln \mathcal{M} = \ln \frac{\varepsilon}{\varepsilon-1}$. Furthermore, we have that

$$\widehat{mc}_t = \frac{(1+\varphi)}{1-\alpha}(y_t - y_t^n). \quad (16)$$

Finally, by combining (16) with (13), and defining $\tilde{y}_t \equiv (y_t - y_t^n)$, we obtain the New Keynesian Phillips curve

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \lambda \frac{(1+\varphi)}{1-\alpha} \tilde{y}_t. \quad (17)$$

Equation (15) says that the movements in the flexible price equilibrium output are solely due to movements in the productivity shifter. As discussed above, this is what we should expect at the flexible price equilibrium due to the KPR preferences. Furthermore, the marginal cost term (16) now depends on labour supply elasticity φ . However, note that the relationship between the marginal cost term and the output wedge is still independent of s , and hence on curvature of utility, if φ is taken as a parameter.¹⁴ With additively separable log preferences where $U(C_t, N_t) = \ln C_t - \nu(N_t)$, this also holds true. If we write φ following equation (7) then it is the case that the slope of the Phillips curve does depend on s . This dependence is however rather weak (see section 3)

An important distinction, however, is that the additively separable preferences that are consistent with the balanced growth path, constrains the IES to equal unity. In a more general case of additively separable constant relative risk aversion utility function the relationship between the real marginal costs and output wedge can be written as $\widehat{mc}_t = \left(\gamma + \frac{\xi+\alpha}{1-\alpha}\right) \tilde{y}_t$. This implies that IES and the slope of the Phillips curve are much more tightly linked. A lower intertemporal elasticity of substitution (a higher γ), given other parameter values, implies that inflation is more responsive to fluctuations in output wedge (see e.g. Gali, 2008).

2.3 IS equation

The final step in the derivations is to express the IS curve in terms of the output wedge \tilde{y}_t and to define the natural rate of interest. Using the approximate log linear production function $y_t = (1-\alpha)n_t + a_t$ and substituting Δn_{t+1} away from the IS curve yields:

$$y_t = \mathbb{E}_t y_{t+1} + \frac{(1-s)\tau}{1-\alpha-(1-s)\tau} \mathbb{E}_t \Delta a_{t+1} - \frac{(1-\alpha)s}{1-\alpha-(1-s)\tau} (i_t - \mathbb{E}_t \pi_{t+1} - \rho) \quad (18)$$

Re-writing above in terms of the output wedge $\tilde{y}_t \equiv (y_t - y_t^n)$ by subtracting y_t^n from both sides, and using the fact that $y_t^n = a_t + \vartheta_y^n$ we arrive to:

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \psi (i_t - \mathbb{E}_t \pi_{t+1} - r_t^n), \quad (19)$$

¹⁴ The independence of slope of the Phillips curve from IES under KPR preferences is not itself a new result, but is not emphasised in the literature (see e.g. Smets and Wouters, 2007).

where r_t^n denotes the natural (real) rate of interest:

$$r_t^n = \rho + \frac{1}{s} \mathbb{E}_t \Delta a_{t+1}. \quad (20)$$

and $\psi \equiv \frac{(1-\alpha)s}{1-\alpha-(1-s)\tau}$. The natural rate of interest given in equation (20) is the equilibrium real rate of return in the flexible price economy.

Equation (19) takes exactly the same form as in the model with additively separable log preferences, but with the following important difference: The elasticity of output wedge with respect to the real interest rate ψ is different from unity. In this set up ψ can be interpreted as (equilibrium) intertemporal elasticity of substitution. At given $\tau < (1-\alpha)$, the relationship between ψ and the labour-held-constant-intertemporal elasticity of substitution s is concave. In particular, as the curvature of the utility function increases (s declines), ψ decreases less than proportionately. This implies that a high curvature of utility can be supported by empirically reasonable values of ψ . However, this comes at the cost of introducing a stronger complementarity between consumption and work.

2.4 Alternative formulations of the IS curve

Another way of formulating the IS equation is to express it in terms of expected growth in labour. This alternative formulation can be achieved by using $y_t = (1-\alpha)n_t + a_t$ to substitute for output in equation (18). This yields

$$\mathbb{E}_t \Delta n_{t+1} = \psi' (i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \quad (21)$$

where $r_t^n \equiv \rho + \frac{1}{s} \mathbb{E}_t \Delta a_{t+1}$, as defined earlier and $\psi' = \frac{s}{[(1-\alpha)-(1-s)\tau]}$. ψ' now gives the elasticity of labour w.r.t. the real interest rate. It is equal to ψ in the special case where $\alpha = 0$. Otherwise, due to concavity of the production function, $\psi' > \psi$. Equation (21) shows that labour can be used as an observable in the estimation instead of the output wedge $y_t - y_t^n$, which requires a proxy for the unobservable natural rate of output y_t^n . A clear benefit of using labour is that there is a much less controversy on how to measure labour than how to measure y_t^n , or how to treat the growth component of output in the estimation. Yet another way of writing the IS curve is not to substitute for labour, but simply subtract y_t^n from both sides of (18) and use the fact that $y_t^n = a_t + \vartheta_y^n$. This yields

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - s(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) - (1-s)\tau \mathbb{E}_t \Delta n_{t+1} \quad (22)$$

where $r_t^n \equiv \rho + \frac{1}{s} \mathbb{E}_t \Delta a_{t+1}$.

Table 1: Parameterization

ξ	α	θ	β	ϵ	τ
1	0.33	0.67	0.99	6	$0.5^{(a)}$

Note: Except for τ , these calibrated parameters are taken from Gali (2008, Ch. 3, p. 52). a) This value is chosen to reflect roughly the narrow measure of (after tax) labour share in the US.¹⁶

3 DISCUSSION

We highlight the impact of different values of s on the elasticity of output with respect to the real interest rate ψ , and on the slope of the Phillips curve κ in the model with additively separable preferences (the standard model) and in the model which allows non-zero cross elasticity between consumption and labour (KPR preferences). The first model is referred to as the standard NK model, where $\psi = s$ and $\kappa \equiv \lambda[s^{-1} + \frac{(\xi+\alpha)}{1-\alpha}]$ and where ξ is Frisch elasticity of labour supply. The key equations of the standard model are re-produced in the appendix B. The key equations and parameter definitions of the model with KPR preferences are given below:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t \quad (23)$$

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \psi (i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \quad (24)$$

where $r_t^n \equiv \rho + \frac{1}{s} \mathbb{E}_t \Delta a_{t+1}$ and $\psi \equiv \frac{(1-\alpha)s}{1-\alpha-(1-s)\tau}$, $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \Theta$, $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$, $\kappa = \lambda \frac{(1+\varphi)}{1-\alpha}$, $\tilde{y}_t \equiv (y_t - y_t^n)$, $y_t^n \equiv a_t + \vartheta_y^n$, $\vartheta_y^n \equiv \frac{(1-\alpha)(\ln(1-\alpha)-\iota-\mu)}{(1+\varphi)}$, $\iota \equiv \tau - (1+\varphi)n$.

Baseline calibration is shown in Table 1 and the results from comparisons are reported in Table 2. The main result is that the standard NK model with additively separable preferences yields (in an empirical sense) implausibly steep Phillips curve at low values of s (and of course, is also inconsistent with the balanced growth path requirement). On the contrary, the slope of the Phillips curve is practically invariant to different values of s ¹⁵ in the model with KPR preferences.

Furthermore, in the model with KPR preferences, the relationship between ψ and s is concave. As the labour-held-constant intertemporal elasticity of substitution falls from unity to 1/10, the interest rate elasticity of output ψ only falls from unity to roughly 0.30. Therefore, the model permits a low value s , yet to achieve empirically plausible degree of real interest rate elasticity of output. Very low values of s , in turn generates a strong complementarity between consumption and leisure.

Because the slope of the Phillips curve is not very sensitive to different values

¹⁵ The variability of κ is due to the fact that we have fixed Frisch elasticity of labour supply equal to unity, and instead let φ vary in accordance with the equation (7).

Table 2: Key Tensions

	Additively Separable Preferences				KPR Preferences			
s	1	1/2	1/5	1/10	1	1/2	1/5	1/10
ψ	1	0.50	0.20	0.1	1	0.80	0.50	0.30
κ	0.13	0.19	0.38	0.69	0.13	0.11	0.10	0.09
φ^{-1}	1	1	1	1	1	1.33	1.67	1.82

Note: This table shows the key tensions in the standard model with additively separable preferences and in the model with KPR preferences when s varies from unity to 1/10, and otherwise the parameter values are chosen according to Table 1.

of s , this allows us to identify it primarily from the relationship between ex-ante real interest rate and output, just like it is done in the consumption Euler equation estimations by GMM methods. Conversely, when using the standard preference specification, low values of s should be associated with empirically unrealistically high slope of the Phillips curve.

However, the advantage is that we have a structural model for inflation and interest rates, permitting us to use full information maximum likelihood based methods, instead of GMM methods. GMM methods applied to estimate s from aggregate data typically suffer from weak instrument problems and lack of identification. Furthermore, if one would be willing to assign prior directly on the consumption-constant elasticity of labour supply, the slope of the Phillips curve and IES would be structurally independent in the case of KPR preferences (see equation (17)).

4 ESTIMATION

Relaxing the assumption of non-zero cross elasticity between consumption and labour permits a wide range of values for the curvature of utility with respect to consumption without distorting the relationship between output wedge and inflation in the sticky price monetary policy model, and yet keep the model consistent with the long-run labour supply facts. Typical values found in the empirical macro literature for the elasticity of consumption growth to the real interest rate are closer to zero than one. At the same time, one of the key weaknesses of estimating this elasticity directly from the consumption Euler equation is that some form of instrumental variable estimation needs to be employed (e.g. Hall (1988), Cambell and Mankiw, (1989), Fuhrer and Rudebusch, 2004 and Yogo, 2002). As shown for instance in Yogo (2002), Kiley (2010) and Kilponen

(2012), the weak instrument problem makes it difficult to identify this elasticity. In order to rest on more reliable inference, the weak instrument problem is addressed by using weak instrument robust confidence intervals, which are typically much wider than the classical ones (see e.g. Yogo, 2002, Stock and Yogo, 2005). But then, large confidence intervals do not allow to statistically discriminate between alternative consumption preference specifications. The Bayesian Maximum Likelihood method applied to the structural model does not suffer from a similar problem. As can be seen later on, credible sets around the point estimates of the equilibrium IES and s are rather tight even when uninformative priors are used.

In this section, we estimate s together with the other key parameters of the model using the structural equilibrium relations given in (23)-(24), and the respective definitions given underneath these equations. For comparison, we also estimate the model with additively separable logarithmic utility. We label these models as $M1$ and $M2$ in what follows. In order to make our estimation exercise more comparable to many other studies, we write the policy rule by allowing interest rate smoothing and assuming that the shocks to interest rate rule are i.i.d. over time:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)[\rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t] + v_t, \quad v_t \sim N(0, \sigma_v^2). \quad (25)$$

We also allow AR(1) shocks to the inflation equation (mark-up shocks) and to productivity shifter a_t as is standard in the literature:

$$\epsilon_t^\pi = \rho_\pi \epsilon_{t-1}^\pi + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2). \quad (26)$$

$$a_t = \rho_a a_{t-1} + \epsilon_t^a, \quad \epsilon_t^a \sim N(0, \sigma_{\epsilon^a}) \quad (27)$$

4.1 The Data

As observable variables, we use hours worked, interest rate and inflation. Interest rate is quarterly federal funds rate and inflation is measured as quarterly log difference of the consumer price index. In contrast to many others, our observable vector does not contain the output gap. A clear benefit of using labour as observable is that there is much less controversy on how to measure labour than how to measure y_t^n , or how to treat the deterministic growth component of output in the estimation.¹⁷ Hours worked are calculated following Hall (2009). Specifically, we average over monthly series of hours ($H=LNU02033120$) and unemployment ($U=LNS14000000$) from the Bureau of Labour statistics and compute total hours as $N=H*(1-U/100)$, where $(1-U/100)$ is the employment rate. Our measure of quarterly hours then represents (seasonally adjusted) hours worked

¹⁷ See e.g. Canova (1998)

at non-agricultural industries in the US.¹⁸ The corresponding observable variables are shown in Figure 1. Parameters α , τ and ϵ are fixed according to Table 1. Estimation sample is 1984Q1-2007Q4.

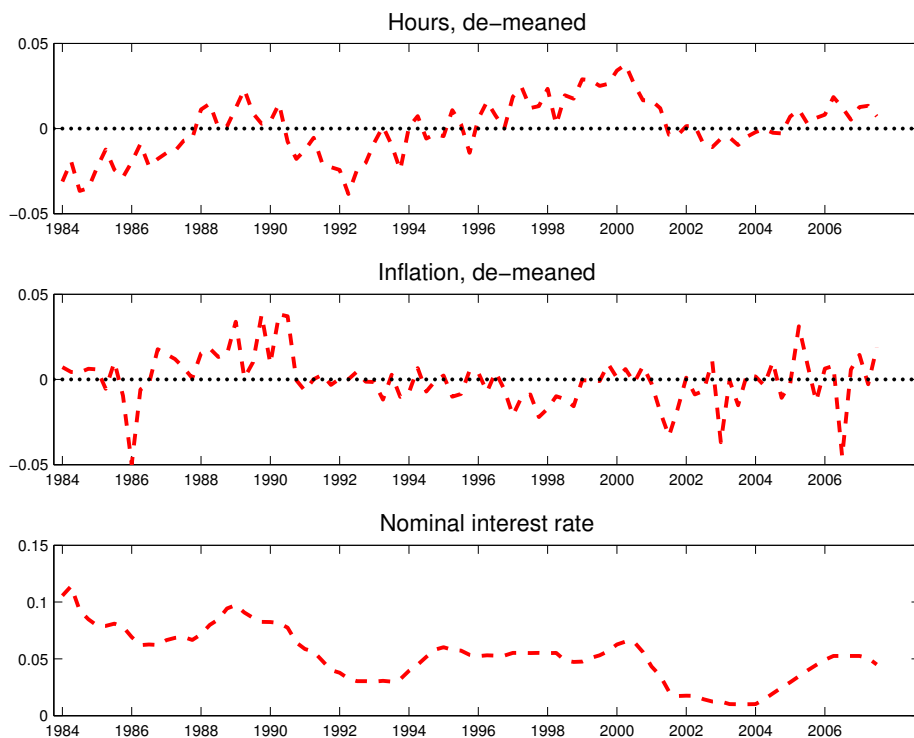


Figure 1: The Observable Variables Used in the Estimation

Note: This figure shows the quarterly data from the U.S. Hours is seasonally adjusted and de-meanded hours worked at non-agricultural industries. Inflation is annualized quarterly difference of log consumer price index. Nominal interest rate is annualized Federal Funds Rate. See section 4.1 for more details of the data.

4.2 Choice of priors

We rely primarily on the evidence summarized in Hall (2009) when choosing the priors for the key labour market parameters and s . Hall's "priors" for the Frisch elasticity and the interest rate elasticity are as follows: $\xi = 0.7$ and $\psi = 0.5$. These priors translate to following prior values for the IES and the (inverse) consumption-constant labour supply elasticity, $s = 0.20$ and $\varphi = 1.03$ (at given

¹⁸ Smets and Wouters (2007) use a different measure of labour supply i.e. they compute $n = \ln((H/L)*(E/100))$, where H=average weekly hours in non-farm business (PRS85006023), E = Employment of 16 years of age and older (CE16OV) and L = population of 16 years and older (LNS10000000). We have estimated our model also with this measure of labour. The results are qualitatively similar i.e. we obtain small intertemporal elasticity of substitution.

$\tau = 0.5$, $\alpha = 0.33$), respectively. The implied prior for the cross-elasticity between consumption and hours is $\frac{dC}{dN} \frac{N}{C} = (1-s)\tau = 0.4$. Direct empirical evidence on this cross-elasticity is clearly more scarce than on the Frisch elasticity. Hall (2009) provides a brief summary of the empirical literature which attempts to identify this elasticity by looking at what happens to the level of consumption when a person stops working. This means that the cross elasticity is identified from the correlation of consumption and the exogenous movements in the labour supply (due to e.g. unemployment, disability or retirement). Based on this literature, Hall's (2009) preferred value of this cross-elasticity is 0.3. Kimball and Shapiro (2008) use a specific survey evidence on the response of hours to a large wealth shock to estimate different labour supply elasticities. Unfortunately, they are not able to uncover the cross-elasticity discussed herein. However, their baseline value needed to infer the other labour supply elasticities is also 0.3. Chetty (2006) argues that upper bound of this elasticity is 0.15, considerably lower than the values preferred by Hall (2009) and Kimball and Shapiro (2008).

In comparison to Smets and Wouters (2007), who also use KPR preferences to estimate a more fully specified DSGE model, our prior mean of the curvature of the utility function is quite a bit higher. Otherwise, our priors are rather standard (for comparison, see for instance Del Negro and Schorfheide, 2008 and Smets and Wouters, 2007). Prior densities and estimation results are summarized in Table 3.

4.3 The results

To begin with, the main result for $M1$ is that the data supports a low value for the s . Posterior mean of s is as low as 0.07 with a relatively tight 90% credible set, ranging from 0.02 to 0.12. This implies together with the other estimated and calibrated parameters of the model that the posterior mean estimate for the elasticity of output with respect to real interest rate is $\hat{\psi} = 0.21$ (0.09, 0.36). This accords well with the consumption Euler equation based literature. Such a low value of s implies a very strong complementarity between consumption and labour and hence a rather large difference between Frisch and consumption-constant elasticity of labour supply. The posterior mean estimate for the Frisch labour supply elasticity is $\hat{\xi} = 0.96$ (0.50, 1.73) while the posterior mean estimate for the consumption-constant elasticity of labour supply $\hat{\varphi}^{-1} = 1.32$ (0.68, 2.38).¹⁹

Finally, the posterior mean estimate for the slope of the Phillips curve $\hat{\kappa}$ is 0.014 (0.003, 0.033). The estimated parameters of the policy rule are $\rho_i = 0.90$, $\hat{\phi}_x = 0.32$, $\hat{\phi}_\pi = 2.10$. These are relatively close to e.g. Erceg, Guerrieri and Gust

¹⁹ The numbers in the brackets provide 90% probability sets.

Table 3: Priors and Summary of Posteriors

Parameter	Density	Prior		Posterior					
		Mean	P	Mean	M1 90% CI	Mean	M2 90% CI		
FIRMS AND HOUSEHOLDS									
θ	\mathcal{B}	0.60	0.15	0.88	0.82	0.95	0.84	0.77	0.94
$s^{(*)}$	\mathcal{G}	0.20	0.10	0.07	0.02	0.12	–	–	–
ξ^{-1}	\mathcal{G}	1.40	0.50	1.22	0.50	1.90	1.27	0.52	1.97
β	\mathcal{B}	0.99	0.001	0.987	0.985	0.989	0.987	0.986	0.989
INTEREST RATE RULE									
ρ_i	\mathcal{B}	0.50	0.15	0.89	0.86	0.92	0.84	0.80	0.88
ϕ_x	\mathcal{G}	0.20	0.10	0.32	0.15	0.59	0.32	0.10	0.53
ϕ_π	\mathcal{G}	1.50	0.40	2.10	1.32	2.83	2.53	1.66	3.33
EXOGENOUS SHOCK PROCESSES									
ρ_a	\mathcal{B}	0.40	0.15	0.83	0.75	0.92	0.89	0.84	0.94
ρ_π	\mathcal{B}	0.40	0.15	0.48	0.32	0.63	0.42	0.27	0.58
σ_a	\mathcal{G}^{-1}	0.80	0.50	0.32	0.21	0.43	2.16	1.49	2.38
σ_π	\mathcal{G}^{-1}	0.30	0.50	0.20	0.14	0.25	0.23	0.18	0.28
σ_v	\mathcal{G}^{-1}	0.20	0.20	0.13	0.12	0.14	0.14	0.12	0.16
<i>LMD</i>				1173.07				1157.97	

Note: \mathcal{B} , \mathcal{G} and \mathcal{G}^{-1} correspond to Beta, Gamma and inverse Gamma distributions. Mean corresponds to mean and P is the standard deviation of the respective prior distribution. Fixed parameters are $\alpha = 0.33$, $\tau = 0.5$, $\epsilon = 6$ as in Table 1. Prior and posterior moments for the standard errors (σ) are in percentage form. *) In *M2*, s is fixed to 1. Hence, prior and posterior moments are irrelevant. *LMD* is log marginal density. Estimation sample is 1984Q1-2007Q4 and estimations were done using Dynare version 4.3.1. Posterior distribution was obtained by Metropolis–Hastings algorithm.

(2006). Taylor (1993) coefficients are within 90% credible set. In comparison to Smets and Wouters (2007), our estimated value for s is far much lower, implying strong curvature of utility. In section 4.5, we show that this is primarily due to a choice of priors.

As for the shocks, the monetary policy shock has an (annualized) standard deviation of 48 basis points, while the cost-push shock has an (annualized) standard deviation of 88 basis points and a low persistence. The technology shock is strongly serially correlated and the standard error of innovations is equal to 37 basis points in quarterly terms. This is somewhat smaller than given by most of the estimates based on Solow residuals.

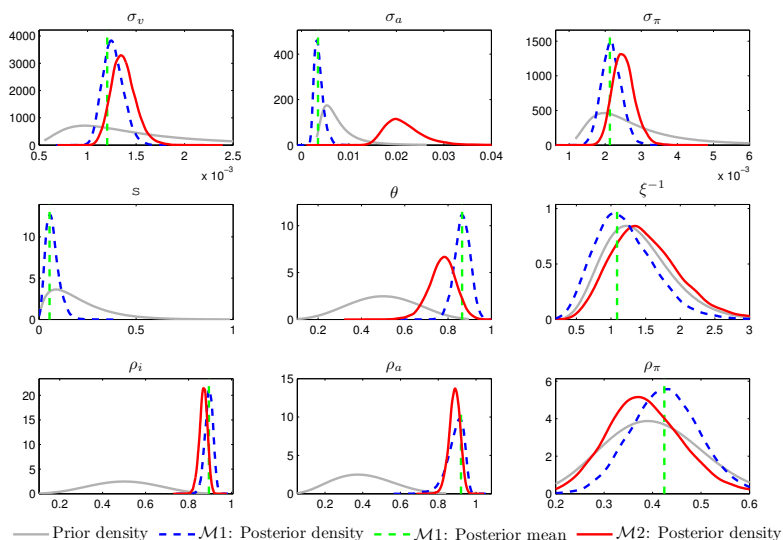


Figure 2: The Prior and Posterior Densities.

Note: This figure compares the prior and posterior densities after estimation of the model with KPR preferences ($M1$) and with logarithmic utility ($M2$).

Now compare these results to the model where the intertemporal elasticity of substitution is restricted to unity ($M2$). Figure 2 compares the posterior densities against the common prior densities in the two models. The main difference between the two models is that the standard error of innovations to technology shocks in $M2$ (at posterior mean) is almost 6(!) times larger than the respective standard error in $M1$. Furthermore, the data prefers the model $M1$. The ratio of marginal likelihood values (LMDs) between the two models, in favour to $M1$, is equal to 1.013. (see Table 3). As for the other form parameters, the slope of the Phillips curve in $M2$ is quite much larger. The posterior mean estimate for the slope is 0.05 (0.014, 0.11).

4.4 Equilibrium responses to technology and monetary policy shocks

Figures 3-4 show the equilibrium responses (at posterior mean) to one standard deviation technology and monetary policy shock in the two models. As discussed above, the standard deviations of technological innovations are 0.32% and 2.16% in *M1* and *M2* respectively. Note also that due to cancelling out of the income and substitution effect the response of the natural output (output under flexible prices) tracks the exogenous response of technology to its innovation (not shown in the Figure) in the both models exactly.

The first important difference between the two models is the response of output to technology shocks. Technology shock does open a negative output wedge in the both models and leads to a fall in employment (hours worked). However, the sign of the response of output in *M1* is the same as that of the output wedge. That is, the productivity shock generates a negative response to output in the short-run. Output will eventually be pushed to a positive territory (after 10 quarters or so) as the negative employment response fades away and technology shock persists. Negative short-run output reaction in *M1* is explained by the strong degree of complementarity between output and employment. Our results suggest that the cross-elasticity between consumption and hours is 0.47(0.43,0.49), which is a rather high number. This is manifested by much stronger relative response of employment to technology shock in *M1* when compared to *M2*. In *M1* employment falls on impact roughly 1% given a one standard deviation shock (0.37%) to technology. In *M2*, the size of the technology shock is 2.16%, but employment falls on impact only 1%.

Note furthermore that the natural rate of interest falls far more in *M1* than in *M2*. Since the real rate, due to slow reaction of the nominal rate, follows the fall in the natural rate with a considerable delay, technology shock opens up a *positive* interest rate gap between the real rate and the natural rate of interest. This contributes negatively to output due to usual interest rate channel. However, this contribution is undermined by low interest rate sensitivity of output (due to the low intertemporal substitution) in *M1*.

In *M2* both employment and the natural rate reacts relatively little such that the output response is clearly positive. Technology shocks opens up a small positive interest rate gap also in *M2*. On the contrary to *M1*, this has relatively strong negative contribution to output since the intertemporal substitution is high.

The models *M1* and *M2* also show a clear difference with respect to strength of the response to a monetary policy shock. Note that, in contrast to technology shocks, monetary policy shocks in the two models have almost exactly equal standard deviations. Thus the differences in the responses in Figure 4 can be read directly as the differences in the strength of the equilibrium responses. It is clear that in *M2* responses of all the variables are much stronger. This is explained

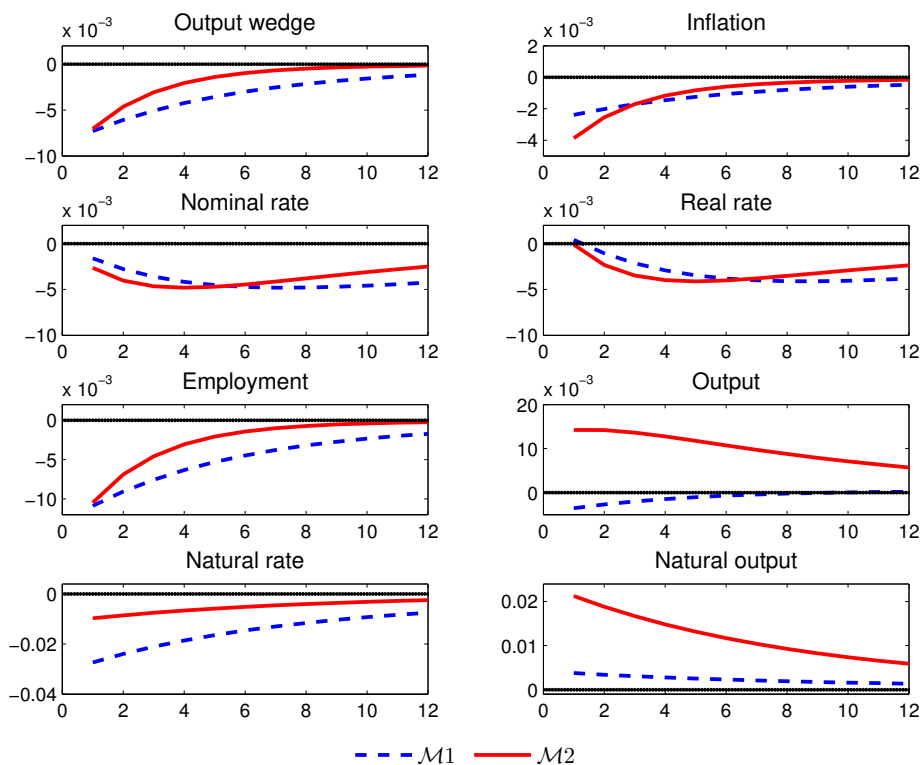


Figure 3: Equilibrium Responses to One Standard Deviation Technology Shock at Posterior Mean.

Note: This Figure shows the impulse responses of selected variables to technology shock in the model estimated under KPR preferences ($M1$) and under logarithmic utility ($M2$). Interest rates and inflation rates are annualised.

simply by the restrictive assumption that the real-interest rate elasticity of output is unity in $M2$. In $M1$ the real interest rate elasticity of output is roughly 0.21. In summary, $M1$ emphasizes the labour market responses, while $M2$ puts emphasis on the nominal side, and the reaction of the monetary policy.

4.5 Sensitivity to priors and habit persistence

Our results imply a considerably higher curvature of utility function, due to low values of s , than those obtained by Smets and Wouters (2007). Although the results are not directly comparable due to various different modelling assumption, we demonstrate that the key reason for the differing results is the choice of priors. Smets and Wouters (2007) impose a relatively informative prior to the curvature parameter γ such that high values of labour-held-constant risk aversion are practically ruled out in their estimation. In order to demonstrate this,

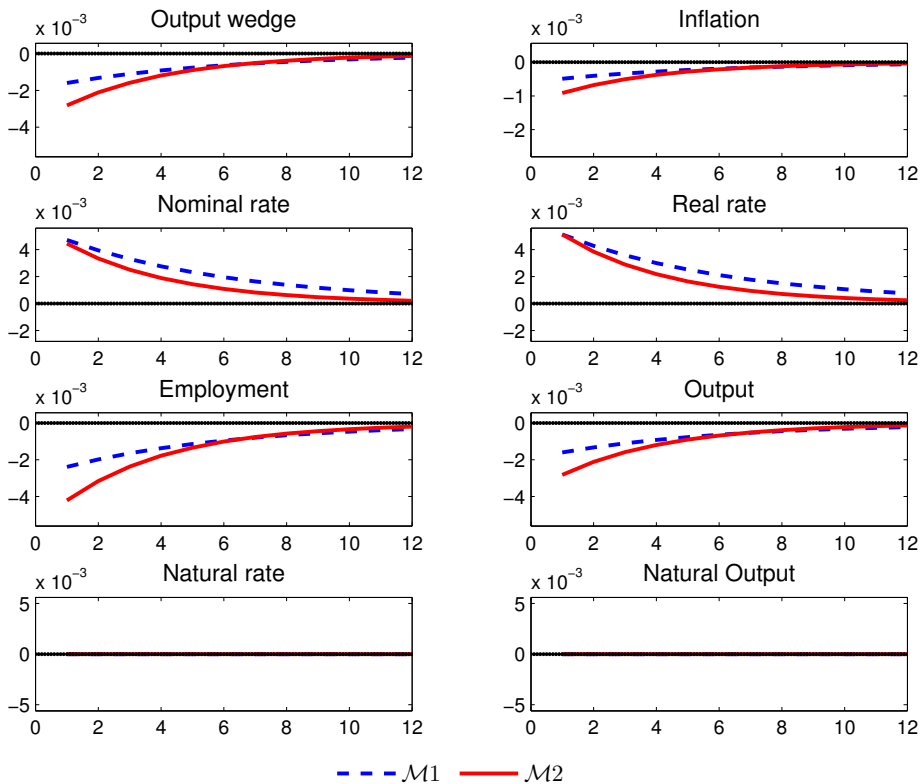


Figure 4: Equilibrium Responses to One Standard Deviation Monetary Policy Shock at Posterior Mean.

Note: This Figure shows the impulse responses of selected variables to monetary policy shock in the model estimated under KPR preferences ($M1$) and under logarithmic utility ($M2$). Interest rates and inflation rates are annualised rates.

we have extended the model by introducing external habit persistence into consumption and re-estimated the complete model using priors for γ and habit persistence similar to Smets and Wouters.²⁰ We have introduced habit formation in the model, since it is present in Smets and Wouters (2007). Habit formation alters the parametrisation of the dynamic IS and AS curves and it further complicates the relationship between equilibrium IES and curvature of utility. IES and γ reflect distinct characteristics of preferences when the utility function is not time-separable, as is the case with habits.

With habit persistence in consumption, the parameter which governs the sensitivity of consumption to the real interest rate can be expressed as

$$\psi \equiv \frac{(1-b)(1-\alpha)s}{(1+b)(1-\alpha) - (1-s)\tau} \quad (28)$$

and where b measures the intensity of external habit persistence. A given real

²⁰ See Appendix C for description of the model with external habits.

interest rate sensitivity of output is consistent with different combinations of s and habit intensity parameter b . Furthermore, it is clear from equation (28) that ψ is decreasing in b and increasing in s . This suggest then that setting a high (and informative) prior for the degree of habit persistence makes it more likely that the estimation also produces relatively high values for s . As for the New Keynesian Phillips curve, the introduction of habits introduces a current period output gap difference to the right hand side of the equation. Furthermore, the relationship between the Frisch elasticity of labour supply and the consumption constant elasticity of labour supply now depends on the intensity of habits. When the intensity of habits increases, a difference between the Frisch elasticity of labour supply and the consumption constant elasticity of labour supply increases. Similarly, more intense habits increases the dependence of inflation on the output gap difference (see appendix C).

Smets and Wouters (2007) set the prior mean for habit persistence parameter to 0.7 with a standard error of 0.1, while their prior for γ is 1.5 with standard error of 0.37. Using (28) these priors imply a prior mean for ψ approximately equal to 0.138, given $\tau = 0.5$ and $\alpha = 0.33$ in our setup.

Table 4 shows the estimation results from the extended model with habit persistence²¹. Column I reproduces the benchmark results from table 3, while columns II-III shows the results using the priors comparable to Smets and Wouters (2007). In column II, habit persistence is fixed to zero, while in column III the prior mean for the habit persistence parameter has been set equal to 0.6. This is close to the value used by Smets and Wouters (2007)²² while the prior for s have been translated from original prior for γ in Smets and Wouters (2007). Column IV shows the estimation results by using our own prior on s (from our benchmark model) and using a lower prior for habit persistence, where we rule out the high values of habit persistence.

The results show that estimated value for s is sensitive to the choice of priors. This also translates into different values of posterior mean of ψ , ranging from 0.06 to 0.28 in different specifications. Allowing for habit persistence leads in general to a lower value of ψ , but this lower value can be obtained with strikingly different values of s . As expected, with the priors from Smets and Wouters, the estimated values of s tend to be higher and with habit persistence the posterior estimates of both s and b are close to their prior contributions (see column III). However, based on log marginal likelihood, the data weakly supports the combination of low s and moderate degree of habit persistence (see column IV). Even if the implied real interest rate elasticity of output is very similar in both

²¹ Appendix C shows the key equations of the model with habit persistence.

²² We have chosen this lower value due to the fact that the model does not permit much higher initial values of b . Otherwise, the model becomes unstable. We have also added into interest rate rule equation the term $\phi_{\Delta y} \Delta \tilde{y}_t$, which appears in the original contribution by Smets and Wouters (2007). We estimate the parameter $\phi_{\Delta y}$ alongside with the other parameters. This helps to reconcile the stable equilibria in the model, even with relatively high value of IES and habit persistence.

Table 4: Sensitivity of Intertemporal Elasticity of Substitution to Priors

	I	II	III	IV
Prior s	$\mathcal{B}(0.2, 0.15)$	$\mathcal{N}(0.67, 0.15)$	$\mathcal{N}(0.67, 0.15)$	$\mathcal{B}(0.2, 0.15)$
Prior b	fixed to zero	fixed to zero	$\mathcal{N}[0.6, 0.1]$	$\mathcal{N}[0.45, 0.05]$
Parameter	Posterior Distribution			
s	0.07 (0.02, 0.12)	0.10 (0.02, 0.23)	0.56 (0.23, 0.89)	0.09 (0.02, 0.15)
b			0.79 (0.58, 0.95)	0.47 (0.41, 0.52)
ψ	0.21 (0.09, 0.36)	0.28 (0.03, 0.64)	0.08 (0.01, 0.21)	0.06 (0.02, 0.11)
<i>LMD</i>	1173.07	1166.15	1181.68	1183.75

Note: \mathcal{B} and \mathcal{N} correspond to Beta and Normal distributions. Fixed parameters are $\alpha = 0.33$, $\tau = 0.5$, $\epsilon = 6$ as in Table 1. *LMD* is log marginal density. Estimation sample is 1984Q1-2007Q4 and estimations were done using Dynare version 4.3.1. Posterior distribution was obtained by Metropolis–Hastings algorithm.

cases, this difference is important. Namely, combination of low s and moderate habit persistence considerably weakens the transmission of monetary policy in the sticky price model. In this case, the maximum impact of the monetary policy shock is about two times larger with Smets and Wouters priors when compared to our case. Note further that the credible sets in columns I and IV are remarkably tight relative to those in columns II and III. Consequently, using our own priors the posterior estimates of the key parameters tend to be in much tighter range than under the priors from Smets and Wouters. This applies also to implied credible set for equilibrium IES, ψ .

5 CONCLUSIONS

One of the most common assumptions in sticky price monetary policy models is the additively separable utility in consumption and labour. In order to make this particular class of utility functions consistent with balanced growth path, consumption enters the utility function in a logarithmic form, the implications

of which are not well supported by empirical evidence. Allowing for non-zero cross-elasticity between consumption and labour, for example along the lines of KPR type preferences employed in this paper, leads to empirically more plausible results and allows to estimate IES with full information maximum likelihood based methods. Key to this result is that KPR preferences break the tight coefficient restriction between the slope of the Phillips curve and curvature of utility in the model economy studied here. The Bayesian estimation results suggest that the real interest rate elasticity of output is in the range 0.1 – 0.4 in the US during period 1984-2007. This suggests a very high curvature of utility. In the model with habit persistence, the data weakly supports the combination of moderate degree of habit persistence and high curvature of utility as opposed to relatively low curvature of utility and high habit persistence as in Smets and Wouters (2007). This difference is important, since a combination of high curvature and moderate degree of habit persistence considerably weakens the transmission of monetary policy in the sticky price model. At the same time, our estimates suggest a strong complementarity between consumption and labour. Aguiar et al. (2013) study based on American Time Use Surveys gives support to macroeconomic models in which consumption and labour are strong complements, but this does not accord with all micro evidence on labour supply. Hence, further work on testing alternative models of aggregate consumption and labour supply behaviour is needed with the ultimate target of finding specifications that would feature consumption-labour complementarity, a moderate responsiveness of consumption to real interest rates and yet be consistent with the long-run labour supply facts and micro evidence on household's inter and intratemporal behaviour.

References

- Aguiar, M. – Hurst, E. – Karabarbounis, L. (2013) Time use during the great recession. *American Economic Review*, Vol. 103 (5), 1664–1696.
- Ameriks, J. – Caplin, A. – Leahy, J. (2007) Retirement consumption: Insights from a survey. *The Review of Economics and Statistics*, Vol. 89 (2), 265–274.
- Banks, J. – Blundell, R. – Tanner, S. (1998) Is there a retirement-savings puzzle? *The American Economic Review*, Vol. 88 (4), 769–788.
- Basu, S. – Kimball, M. (2002) Long run labour supply and elasticity of the intertemporal substitution for consumption, Working Paper, University of Michigan.
- Bernheim, D. – Skinner, J. – Weinberg, S. (2001) What accounts for the variation in retirement wealth among u.s. households? *The American Economic Review*, Vol. 91 (4), 832–857.
- Bilbie, F. (2009) Non-separable preferences and frisch labor supply: One solution to a fiscal policy puzzle, HEC Paris Business School, Paris School of Economics and ParisTech. mimeo.
- Browning, M. – Crossley, T. F. (2001) Unemployment insurance benefit levels and consumption changes. *Journal of Public Economics*, Vol. 80 (1), 1–23.
- Campbell, J. Y. – Mankiw, N. G. (1989) Consumption, income, and interest rates: Reinterpreting the time series evidence. *NBER Macroeconomics Annual*, Vol. 4, 185–216.
- Campbell, Y., John – Ludvigson, S. (2001) Elasticities of substitution on real business cycle models with home production. *Journal of Money, Credit and Banking*, Vol. 33, 847–875.
- Canova, F. (1998) Detrending and business cycle facts. *Journal of Monetary Economics*, Vol. 41 (3), 475–512.
- Chetty, R. (2006) A new method of estimating risk aversion. *American Economic Review*, Vol. 96 (5), 1821–1834.
- Del Negro, M. – Schorfheide, F. (2008) Forming priors for dsge models (and how it affects the assessment of nominal rigidities). *Journal of Monetary Economics*, Vol. 55 (7), 1191–1208.
- Eichenbaum, M. – Fisher, J. D. M. (2007) Estimating the frequency of price re-optimization in calvo-style models. *Journal of Monetary Eco-*

- nomics*, Vol. 54 (7), 2032–2047.
- Eichenbaum, S., Martin – Hansen, P., Lars – Singleton, J., Kenneth (1988) A time series analysis of representative agent models of consumption and leisure choice under uncertainty. *The Quarterly Journal of Economics*, Vol. 103 (1), 51–78.
- Erceg, C. J. – Guerrieri, L. – Gust, C. J. (2006) Sigma: A new open economy model for policy analysis. *International Journal of Central Banking*, Vol. 2, 1–50.
- Fuhrer, J. C. – Rudebusch, G. D. (2004) Estimating the euler equation for output. *Journal of Monetary Economics*, Vol. 51 (6), 1133–1153.
- Gali, J. (2008) *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. No. 1, Princeton University Press.
- Goodfriend, M. – King, R. G. (1997) The new neoclassical synthesis and the role of monetary policy. *NBER Macroeconomics Annual*, Vol. 12, 231–283.
- Greenwood, J. – Hercowitz, Z. – Huffman, G. W. (1988) Investment, capacity utilization, and the real business cycle. *The American Economic Review*, Vol. 78 (3), 402–417.
- Hall, R. E. (1988) Intertemporal substitution in consumption. *Journal of Political Economy*, Vol. 96 (2), 339–357.
- Hall, R. E. (2009) Reconciling cyclical movements in the marginal value of time and the marginal product of labor. *Journal of Political Economy*, Vol. 117 (2), 281–323.
- Hamermesh, D. S. (1984) Consumption during retirement: The missing link in the life cycle. *The Review of Economics and Statistics*, Vol. 66 (1), 1–7.
- Hamilton, J. D. (1994) *Time Series Analysis*. Princeton University Press.
- Hurst, E. () The retirement of a consumption puzzle, NBER Working Paper No. 13789.
- Jaimovich, N. – Rebelo, S. (2009) Can news about the future drive the business cycle? *The American Economic Review*, Vol. 99 (4), 1097–1118.
- Kiley, M. T. (2010) Habit persistence, nonseparability between consumption and leisure, or rule-of-thumb consumers: Which accounts for the predictability of consumption growth? *The Review of Economics and Statistics*, Vol. 92 (3), 679–683.
- Kilponen, J. (2012) Consumption, leisure and borrowing constraints. *The B.E. Journal of Macroeconomics*, Vol. 12 (1).
- Kimball, M. S. (1995) The quantitative analytics of the basic neomonetarist model. *Journal of Money, Credit and Banking*, Vol. 27 (4), 1241–1277.
- Kimball, M. S. – Shapiro, M. D. (2008) Labor supply: Are the income and

- substitution effects both large or both small? *National Bureau of Economic Research Working Paper Series*, Vol. No. 14208.
- King, R. G. – Plosser, C. I. – Rebelo, S. T. (1988) Production, growth and business cycles: I. the basic neoclassical model. *Journal of Monetary Economics*, Vol. 21 (2), 195–232.
- Magnusson, L. M. – Mavroeidis, S. (2010) Identification-robust minimum distance estimation of the new keynesian phillips curve. *Journal of Money, Credit and Banking*, Vol. 42 (2-3), 465–481.
- Smets, F. – Wouters, R. (2007) Shocks and frictions in us business cycles: A bayesian dsge approach. *American Economic Review*, Vol. 97 (3), 586–606.
- Stock, J. H. – Yogo, M. (2005) Testing for weak instruments in linear iv regression. In *Identification and Inference for Econometric Models: Essays in Honor of Thomas Rothenberg*, eds. J. H. Stock – D. W. K. Andrews, chap. 5, Cambridge University Press.
- Swanson, E. T. (2012) Risk aversion and the labor margin in dynamic equilibrium models. *American Economic Review*, Vol. 102 (4), 1663–1691.
- Walsh, C. (2010) *Monetary theory and policy*. The MIT Press, 3rd edn.
- Woodford, M. (2003) *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.
- Yogo, M. (2004) Estimating the elasticity of intertemporal substitution when instruments are weak. *The Review of Economics and Statistics*, Vol. 86 (3), 797–810.

Appendix A FRISCH ELASTICITY AND CONSUMPTION CONSTANT ELASTICITY OF LABOUR SUPPLY

Frisch elasticity of labour supply is defined as the elasticity of labour supply where the marginal utility of consumption is held fixed. Hence, we must have that

$$dU_C(C, N) = U_{CC}dC + U_{CN}dN = 0 \quad (29)$$

$$\frac{dc}{dn} \equiv \frac{d \log C}{d \log N} = -\frac{U_{CN} N}{U_{CC} C} = (1-s)\tau. \quad (30)$$

Furthermore, along constant marginal utility of consumption paths

$$c_\omega \equiv \frac{dc}{d\omega} = (1-s)\tau \frac{dn}{d\omega} = (1-s)\tau n_\omega, \quad (31)$$

where ω denotes the log of the real wage. From intratemporal condition for labour, we know that

$$n_\omega = \varphi^{-1}(1 - c_\omega) \quad (32)$$

where $\varphi \equiv \frac{v''(N)N}{v'(N)}$. Hence, substituting (31) into (32) and solving for n_ω gives:

$$\begin{aligned} n_\omega &= \varphi^{-1}(1 - (1-s)\tau n_\omega) \\ n_\omega &= \frac{1}{\varphi + (1-s)\tau} \end{aligned} \quad (33)$$

and where $n_\omega \equiv \xi$ is Frisch elasticity of labour supply.

Appendix B STANDARD MODEL

This appendix replicates the key equations and the parameter definitions of the standard sticky price monetary policy model (adapted from Gali, 2008, ch. 3).

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t \quad (34)$$

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - s(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \quad (35)$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_{\tilde{y}} \tilde{y}_t + v_t \quad (36)$$

where $r_t^n \equiv \rho + \frac{1}{s} \psi_{ya}^n \mathbb{E}_t \Delta a_{t+1}$ and

$$\begin{aligned} \lambda &\equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \Theta, \quad \Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon} \\ \kappa &\equiv \lambda \left(\frac{1}{s} + \frac{\xi+\alpha}{1-\alpha} \right), \quad \tilde{y}_t \equiv (y_t - y_t^n), \\ y_t^n &= a_t + \vartheta_y^n, \quad \vartheta_y^n \equiv \frac{(1-\alpha)(\ln(1-\alpha) - \mu)}{\frac{1}{s}(1-\alpha) + \xi + \alpha} \\ \psi_{ya}^n &\equiv \frac{1+\xi}{\frac{1}{s}(1-\alpha) + \xi + \alpha} \end{aligned}$$

Appendix C MODEL WITH HABIT PERSISTENCE

This appendix shows the key log linearised equations and the parameter definitions of the sticky price monetary policy model with external habit persistence and King-Plosser-Rebelo preferences. Detailed derivation of the model is available by request from the authors.

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t + \lambda \frac{b}{1-b} \Delta \tilde{y}_t \quad (37)$$

$$\tilde{y}_t = \omega_1 \tilde{y}_{t-1} + \omega_2 \mathbb{E}_t \tilde{y}_{t+1} - \psi(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \quad (38)$$

$$r_t^n = \rho + \frac{1}{s} \left(\frac{(1-s)\tau}{(1-b)(1-\alpha)} E_t (\Delta a_{t+1} - \Delta y_{t+1}^n) - \frac{b}{(1-b)} y_{t+1}^n + \frac{b}{(1-b)} y_{t-1}^n + \Delta y_{t+1}^n \right) \quad (39)$$

$$y_t^n = \frac{(1-\alpha)b}{(1+\varphi)(1-b) + (1-\alpha)b} y_{t-1}^n + \frac{(1+\varphi)(1-b)}{(1+\varphi)(1-b) + (1-\alpha)b} a_t \quad (40)$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_{\tilde{y}} \tilde{y}_t + v_t \quad (41)$$

where

$$\begin{aligned}\lambda &\equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta, \Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon} \\ \kappa &\equiv \lambda \frac{(1+\varphi)}{1-\alpha}, \omega_1 = \frac{b(1-\alpha)}{(1+b)(1-\alpha)-(1-s)\tau} \\ \omega_2 &= \frac{(1-\alpha)-(1-s)\tau}{(1+b)(1-\alpha)-(1-s)\tau}, \psi = \frac{(1-b)(1-\alpha)}{(1+b)(1-\alpha)-(1-s)\tau} \\ \tilde{y}_t &\equiv (y_t - y_t^n)\end{aligned}$$

Furthermore, it can easily be shown that the relationship between Frisch elasticity of labour supply, ξ_h , and consumption constant elasticity of labour supply in the presence of external habits is given by $\xi_h = 1/(\varphi + \frac{1-s}{1-b}\tau)$.

Annales Universitatis Turkuensis



Turun yliopisto
University of Turku

ISBN 978-951-29-7280-7 (PRINT)
ISBN 978-951-29-7281-4 (PDF)

ISSN 2343-3159 (Painettu/Print) | ISSN 2343-3167 (Verkkajulkaisu/Online)