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The AXIOM approach for probabilistic and causal modeling with expert elicited inputs

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ABSTRACT

Expert informants can be used as the principal information source in the modeling of socio-techno-economic systems or problems to support planning, foresight and decision-making. Such modeling is theory-driven, grounded in expert judgment and understanding, and can be contrasted with data-driven modeling approaches. Several families of approaches exist to enable expert elicited systems modeling with varying input information requirements and analytical ambitions.

This paper proposes a novel modeling language and computational process, which combines aspects from various other approaches in an attempt to create a flexible and practical systems modeling approach based on expert elicitation. It is intended to have high fitness in modeling of systems that lack statistical data and exhibit low quantifiability of important system characteristics. AXIOM is positioned against Bayesian networks, cross-impact analysis, structural analysis, and morphological analysis. The modeling language and computational process are illustrated with a small example model. A software implementation is also presented.

1. Introduction

This paper proposes a novel modeling language and computational process, which combines aspects and analysis elements from various other approaches in an attempt to create a flexible and practical systems modeling approach based on expert elicitation. Modeling systems based on expert elicited inputs have potential in modeling systems that are difficult to model based on statistical data. Traditionally the modeling of systems has been strongly data-driven (Sokolowski and Banks, 2009), although a hybrid approach of augmenting data-driven models with expert information (Choy et al., 2009; Ford and Sterman, 1997; O'Hagan et al., 2006) has become more commonplace in modeling and decision support activities. The reliance on statistical data limits the use of models and modeling in research and decision-making, as *a)* only systems and problems with good statistical data availability will be modeled, *b)* only elements, aspects and properties of systems that are easily quantified and have good data availability will be included in the models, and *c)* generally, modeling will be considered as a possible approach only in domains where data availability is good. The methodological orientation of modeling towards easily quantifiable aspects of reality may cause models, and the decision support that they offer, become biased or limited in strategic scope and perspective. Involving expert informants in the modeling process as an alternative input source can help to account for critical considerations poorly covered by statistical data (Choy et al., 2009; Ford and Sterman, 1997; James et al., 2010; Kuhnert et al., 2010; O'Hagan et al., 2006).

A number of modeling and analysis techniques intended to be used in conjunction with expert elicitation have been proposed since the late 1960's, mainly in the futures studies and foresight domain, referred to by the original authors as techniques for cross-impact analysis (Godet, 1976; Gordon and Hayward, 1968; Honton et al., 1984), structural analysis (Godet et al., 1994; Linss and Fried, 2009), and morphological analysis (Ritchey, 2006; Weimer-Jehle, 2006). Bayesian networks and influence diagrams are a widely used decision support tool, and they are often augmented with expert elicited knowledge (James et al., 2010; Leonelli and Smith, 2015). While they are used in foresight applications (Bromley et al., 2005; Cinar and Kayakutlu, 2010; Culka, 2016), their use is less common, as their characteristics make them somewhat impractical in systems modeling with high abstraction level and high structural complexity.

AXIOM draws design features regarding the modeling language, computational process and inference from several existing modeling approaches with fitness to modeling systems or decision-making problems with expert-elicited model inputs. The design aim of AXIOM has been to identify the most viable design features of existing approaches for the expert elicited modeling niche, combine them in the same

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analytical framework, and expand on the ideas adopted in them to further elevate the fitness for expert informant processes. The most important of these approaches are Bayesian networks and influence diagrams, the cross-impact approach by Gordon and Hayward and its derivatives, and the BASICS approach. The characteristics of these techniques are discussed and appraised in terms of fitness for expert informant oriented modeling. AXIOM builds on and expands on many of the ideas introduced by existing approaches, as discussed in Section 3 with the aim of providing better tooling for modeling based on expert elicited inputs and making the formal modeling a more viable research and analysis approach in domains that are difficult to model using other modeling approaches.

This paper also discusses the main analytical orientations of expert informant based modeling approaches, identifying structural, morphological and probabilistic orientations. In terms of analytical output, AXIOM can produce outputs of all of the mentioned orientations, covering a great deal of the utilization area of other discussed modeling approaches. The use of AXIOM in decision support, probabilistic inference, as well as extraction of morphological and structural insights is illustrated in Section 4.3 with a small example model. A software implementation of the approach is freely available, and its development is ongoing. The further development of the AXIOM framework and its software implementation are discussed in Section 6.

2. Literature review

2.1. Systems thinking, modeling and simulation

System is defined (International Council on Systems Engineering (INCOSE), 2017) as a " collection of elements that together produce results not obtainable by the elements alone ": system parts work together. Systems thinking is geared towards understanding the systemic phenomena: The individual parts of the system are known and understood (they are "inputs" to the systems thinking), but their operation together, as a *system*, and the result of this operation are less understood and are the main object of interest in systems thinking (Checkland, 1999).

Systems thinking entails understanding a part of reality as a set of components, which are abstractions of real-world objects and phenomena (Checkland, 1999). The system as a whole can be described with these more atomic abstractions, which are logically connected by relationships of some kind. Practice of such systems thinking might lead to a more formal representation of the system, often called a system model (Sokolowski and Banks, 2009). The amount of information and detail in the system representation, or model, varies greatly. The information content of the model determines what kind of higher-order information can be extracted from the model, and what kind of insights can be made available.

Modeling is therefore creating an approximation or abstraction of the real world or a part of reality (Sokolowski and Banks, 2009). As abstractions, models attempt to capture the essential parts of reality. What is essential is determined dominantly by information needs, the questions the model is supposed to answer. Models representing the system with sufficient detail and formality can be used for simulation. If a model represents the system, simulation represents the *operation* of the system (Banks et al., 2005; Zeigler et al., 2000). Simulation has a temporal aspect. The representation of the operation can mean a continuous-time representation, if sufficient details are available in the model. On the other hand, the operation can also be represented as a starting point and an end point. In this two-step description of the operation of the system, a starting state is fed to a transformation and a transitioned state is output as the "end" result.

Systems modeling is said to be strongly data-driven, meaning that the formal descriptions or definitions of the relationships connecting the model components are extracted from statistical data. These formal descriptions are normally presented as mathematical equations relating the model variables. Often techniques such as regression analysis are used for parameterization of the relationships (Sokolowski and Banks, 2009). Even when the estimation of details of the relationships is based on data, such model is still considered "a formal representation of a theory" (Adèr, 2008). Data-driven modeling is fundamentally based on theoretical-level understanding of the system rather than 'hard' empirical evidence.

A common problem in systems modeling is data inavailability (Sokolowski and Banks, 2009), due to difficulties in quantifying the essential parts of the modeled system at the precision required by data-driven modeling approaches or the costs of data acquisition. Data inavailability limits modeling, both in application area of systems thinking and modeling (as only systems with good data availability will be modeled) and utility and reliability (as only system aspects for which data is available will be included in the models). These limitations might result in incomplete or biased models, which leave possibly crucial aspects of the system unmodeled and unaccounted for. The methodological limitations of modeling are reflected in the decision-making process using the modeling results, as their strategic and policy scope omits important considerations.

In some modeling domains, empirical data is an impossibility. For instance, foresight-oriented modeling of socio-techno-economic systems has to account for changing or emerging system characteristics that are not manifested in existing statistical data, as well as possible occurrence of singular historical events for which no frequentist-type data can exist. Historical data does not necessarily capture or reflect the way the modeled system is changing, even when the change and the dynamics involved might be well understood by experts of the modeled system.

Data-driven modeling is often called mathematical modeling, and thus contrasted with modeling approaches emphasizing an intuitive-logical way of describing the properties of the modeled systems. Underpinning the mathematicity of modeling can lead to a false impression of the model being based on a solid mathematical foundation: in the minds of the model users, the irrefutability of mathematics lends itself to the outputs of the model. However, in data-driven modeling, the fundamental choices about the model structure and logic are made not based on some axiomatic mathematical principles or empirical evidence but theory, expertise, intuition, or even guesswork. The theoretical foundation of models and simulations can sometimes be obscured by their claimed mathematicity. Often this theoretical foundation of the model is laid out in a rather informal and unstructured way, by a small modeling team or just one single person doing the modeling, and the foundation and theoretical choices made are not explicated. Given the high technical expertise requirement of data-driven modeling approaches the model-building team might consist of experts of the *modeling approach*, instead of experts of the modeled domain.

The theory-based structure of causalities and dependencies of models built using the data-driven approach is often nontransparent. Understanding the logical structure of the models might require good understanding of the underlying mathematics. Even with such expertise, understanding the structure might often be laborious. This cognitive cost of examining and understanding the model will often make the models "black boxes" whose output is used without good grasp of the logical structure underlying the model: from a user perspective, the general causal logic of the model might remain unclear.

The expert informant-based approach to modeling, or expert elicitation of model inputs, is an alternative to data-driven modeling (James et al., 2010; Pollino et al., 2007). Expert insight of the modeled system may cover domains or system aspects for which data in the required format for modeling does not exist, but which are still known at some level of detail, enough to base the modeling on (Choy et al., 2009; Ford and Sterman, 1997; Kuhnert et al., 2010). While the data-driven approaches can rely on expert inputs as well, expert elicitation is in secondary role, and not the methodological focus. The expert informant oriented modeling approaches typically attempt to provide a *modeling* *language* more suitable for modeling the system with expert-sourced information, rather than requiring the experts to directly specify mathematical equations which relate the system components to each other. This language should support the heuristic-logical mode of work, and be natural in use of an expert-oriented modeling process. A suitable modeling language relies on a less exact precision in description of the model component relationships than what is typical in a data-driven model, where the relationships can be parameterized on the basis of the available empirical data, using techniques like regression analysis. Several modeling languages aiming at enabling expert description of systems have been proposed alongside various analytical methods. These are discussed in Section 2.2. Specific methods of high relevance to and methodological overlap with AXIOM are detailed in Section 3. The modeling language will determine the level of detail and the nature of information in the expert informant sourced system model. Given a fairly simple modeling language, the system description may be relatively transparent, in comparison to the system description of the data-driven approaches. The nature of the information, in turn, determines what kind of transformations can be done on it to extract some kind of higher-order information from the model.

While modeling approaches with focus on expert informant sourced data do exist, there are important improvements to be made to increase both the fitness of the modeling language for expert elicitation working mode of modeling, and the inference procedures used to extract analytical value from the model. Section 2.2 discusses a number of systems modeling approaches with fitness for expert informant oriented modeling and identifies their analytical aims. Section 2.3 assesses the design options of these approaches, from the angle of fitness for modeling based on expert elicitation. Section 3 gives a description of modeling approaches with significant overlap with the AXIOM approach in some dimension, explains the similarities and differences and presents the argumentation for the design choices made in AXIOM. Several issues identified in the review justify further methodological development in the field. Section 4 presents the AXIOM modeling language and computational process, as well as possible analytical outputs. The contribution of this paper is methodological: it proposes a novel approach for a specific systems modeling and simulation niche, with an above-state-of-the-art fitness for the intended purpose. The language and the analysis process is illustrated with an example model previously (Honton et al., 1984; Weimer-Jehle, 2006) used to illustrate two other modeling approaches. A free software implementation of the approach is also presented.

2.2. Established expert informant oriented modeling approaches

Expert elicited systems modeling is practiced under several different names or labels. These include cross-impact analysis (Godet et al., 1991, 1994; Gordon and Hayward, 1968; Gordon, 1969, 1994; Honton et al., 1984; Huss and Honton, 1987), structural analysis (Godet et al., 1991, 1994; Linss and Fried, 2009, 2010; Panula-Ontto and Piirainen, 2018), morphological analysis (Ritchey, 2006; Weimer-Jehle, 2006), cross-consistency analysis (Johansen, 2018) and Bayesian belief networks and influence diagrams, Bayesian decision support systems, or Bayesian decision support systems (Baran and Jantunen, 2004; Bromley et al., 2005; Ceric, 2016; Cinar and Kayakutlu, 2010; Kristensen and Rasmussen, 2002; Kuikka and Varis, 1997; Lauría and Duchessi, 2006; Leonelli and Smith, 2015). There are several documented modeling approaches and associated computational processes within each mentioned branch of modeling. They have a great deal of conceptual and functional overlap, but also important differences. All approaches *(a)* utilize expert elicitation *(b)* in building a model representation of a real-world system or decision-making problem, *(c)* that can be represented as a graph, nodes as the system descriptors and edges describing their relationships, *(d)* to be used in analysis of the system, inference or decision support by means of a computational transformation on the model.

The expert informant driven modeling processes can result in conceptual models of low formality, for which there are no particular computational transformations or inference mechanisms available. Conceptual models, as well as the expert-driven process itself, can be very useful in understanding the system, and can yield processual benefits (Kelly et al., 2013) without any specific formal inference. However, when the model representation of the system is at a sufficient level of formality to analyze computationally, these approaches provide some process of computation and inference to facilitate the analysis of the models. The information content of the models determines what kind of further computational transformations are available to extract higher-order analytical information. Three distinct analytical orientations, not mutually exclusive, of expert informant oriented modeling can be identified.

1. In the *structural* orientation, the focus is on the structure of the relationship network. The aim is to form a picture of the *systemic relationships* of the model variables, inferred from the description of the direct relationships. The systemic relationship reflects the indirect or mediated influence between the variables, in addition to the direct influence: the inference mechanism aims at revealing the indirect relationships between the variables in some way. As the indirect relationships are discovered, on the basis of the direct relationships given as input, a new understanding of the relationships emerges.
   Analytical outputs of structural nature can be extracted from a graphical model where nodes represent system components, events, driving forces and trends, without necessarily having any additional information, and edges (directed or undirected) represent *direct* relationships of some kind, possibly having an indicator of magnitude representing the strength of the relationship (relative to other relationships in the model). Methods focusing on this utility are MICMAC (Godet et al., 1991, 1994), ADVIAN (Linss and Fried, 2010), and EXIT (Panula-Ontto et al., 2018; Panula-Ontto and Piirainen, 2018). Cognitive maps (Axelrod, 1976) and fuzzy cognitive maps (Kosko, 1986) also have similar analysis aims, although they are not typically identified as structural analysis approaches.
2. The *morphological* orientation aims at identifying logical, consistent or probable system states, or reducing the total 'problem space' into a smaller, internally consistent 'solution space' (Johansen, 2018). A system state is a specific combination of states of the system descriptors. A requirement for deriving morphological utility is that the model contains information about the 'agreement' of the system descriptors, so that system configurations where the states of system descriptors are 'harmonic' may be identified. Hence, the nodes should have state properties, such as a Boolean indicator of them being true or false, or a discrete state (a single state out of a set of possible states). Morphological information can also be inferred from probabilistic information about the relationships of the model components: Nodes may (Godet et al., 1994; Honton et al., 1984) or may not (Rhyne, 1974; Ritchey, 2006; Weimer-Jehle, 2006) have probability information about their possible states. The edges should, at a minimum level, contain Boolean information of "agreement" or consistency between the specific states of the model nodes. Methods oriented morphologically include BASICS (Honton et al., 1984), JL-algorithm (Luukkanen, 1994), general morphological analysis (Ritchey, 2006), Field Anomaly Relaxation (Rhyne, 1974)), SMIC (Godet et al., 1991, 1994), and the cross-impact balances approach (Weimer-Jehle, 2006).
3. The *probabilistic* orientation aims at probabilistic inference about the system, deriving the probability distributions for random variables in the model, given a set of variables with an assumed value. Probabilistic modeling orientation requires more input information than structural or morphological orientation, as probabilities are computed explicitly: The probabilistic conditionalities and dependencies need to be described in a more specific way. The

additional model information enables wider analytical possibilities. The obvious disadvantage is that the modeling is more costly in terms of time and effort. This can be a challenge for the modeling if the access to expert informants is limited.

The probabilistic orientation offers the greatest degree of direct decision support, as the effects of interventions can be observed from the probability distributions of random variables capturing some aspects of the system that are relevant for decision-making. The analytical utility comes from using the model for examining the systemic effects of events and developments, or strategic actions and interventions. Probabilistic information can be coupled with *utility functions*, which help in identifying the optimal intervention combination maximizing utility or minimizing harm by some criteria. Probabilistically oriented modeling techniques include Gordon-Hayward cross-impact analysis (Gordon and Hayward, 1968; Gordon, 1969, 1994), Bayesian networks and influence diagrams (Baran and Jantunen, 2004; Kristensen and Rasmussen, 2002; Kuikka and Varis, 1997; Lauría and Duchessi, 2006; Leonelli and Smith, 2015), and AXIOM.

The various alternatives differ in terms of *(a)* the information content of the descriptors, *(b)* the way (and in what detail) interactions are modeled, *(c)* the nature or interpretation of that interaction, *(d)* the possibility to model the temporal dimension, and *(e)* whether cyclical relationships are allowed. These features lead to the approaches being of a certain *(f)* difficulty level for the expert informants used as the information source, and a *(g)* focus on a specific analytical orientation. The next subsection discusses these key design options of the various established approaches, and considers their preferability and problems in the context of modeling relying strongly on expert elicitation for input data acquisition.

2.3. Motivation for further methodological development

Given the numerous documented approaches for creating graphical system models by means of expert elicitation, what is the motivation for developing new methods? A modeling approach with high fitness for this specific purpose should have a modeling language which is generic, but flexible and expressive, to enable model representation of all kinds of systems and heterogenous system features. This flexibility should be provided by a practical way for expressing the system characteristics, which takes into account the expert informant resources, which are, in practice, always limited. The ideal approach should also produce outputs from which all discussed analytical utilities can be extracted. Against this ideal of a modeling approach with optimal fitness for expert-elicited systems modeling, a number of problems, for which the AXIOM approach proposes solutions to, can be identified in the established approaches.

1. **Modeling power.** The modeling languages of Gordon-Hayward cross-impact analysis (Gordon, 1994), SMIC (Godet et al., 1994), MICMAC (Godet et al., 1994), and ADVIAN (Linss and Fried, 2010) only offer Boolean system descriptors, which represent events of or hypotheses about the modeled system. A modeling language with more modeling power allows system descriptors to have an arbitrary number of possible states. This makes it possible to clearly model system states that are mutually exclusive and exhaustive: such system properties cannot be reliably modeled with binary descriptors. System descriptors with an arbitrary number of possible values enable a flexible way of modeling a real system at arbitrary level of detail: Multivalued descriptors, used in Bayesian networks and influence diagrams, BASICS, and AXIOM, can model, in principle, any kind of system feature or property, from low-level and atomic detail such as a number or a share, to a high-level descriptor of the system, such as a subscenario describing a possible state of a subsystem, packaging a great deal of information.

   In most expert informant based systems modeling techniques discussed in Section 2.2, the modeling languages do not provide a way to represent the temporal dimension of the model, meaning that the system descriptors do not have a temporal position in relation to each other. All system descriptors are thought to exist in the same temporal space and are resolved "simultaneously" at the level of the computational transformation performed on the model, details varying by the specific technique. For several systems, the ability to model passage of time and the temporal relationship between the descriptors is highly desirable to create meaningful models. The Bayesian network representation of systems (Choy et al., 2009; Cowell et al., 2006; Kuhnert et al., 2010; Pollino et al., 2007) enables modeling a temporal aspect, but in a structurally deterministic way and with limited flexibility: the temporal logic of the model could be said to be coupled with the model structure. AXIOM descriptors have a *timestep* property that enables positioning the descriptors in the temporal dimension in relation to each other with arbitrary precision. A representation of time of relatively low precision is probably the best fit for expert informant oriented modeling, but any level of precision is made possible in a simple way with the timestep property.

   Bayesian networks as graphical system models impose structural limitations on the modeling of relationships, as Bayesian networks are directed acyclic graphs: cyclic interaction is not allowed in Bayesian network models. As the AXIOM transformation is based on a Monte Carlo process and bidirectional interaction is therefore nonproblematic, this limitation to modeling power is eliminated.
2. **Expression of interactions.** For all models of the approaches discussed in this paper, the description of interaction between the system descriptors is the most information-laden part of the model. In terms of valuating the models, the expert informants used as data source will spend most of their time describing the interactions. If the experts are understood to be the primary source of input information for the model, the amount of detail they need to give as input is a trade-off against the complexity of the model structure and the time the experts have available for contemplating the valuation of the interaction. The more information is needed to express the details of the relationships between the system descriptors, the smaller number of descriptors can be considered and the less time there is to consider the relationships carefully, assuming that the expert informant time is limited. In the approaches not dealing with probabilities explicitly, the interactions are expressed in a simple way, with Boolean indicators or magnitude indicators; this simplicity makes for easy model valuation, but analytically such models can provide only structural and morphological utility.

   Many approaches dealing with probabilities (Cowell et al., 2006; Godet et al., 1994; Gordon and Hayward, 1968; Pollino et al., 2007) require definition of a system conditional probabilities as a way to express their interdependencies. Conditional probabilities allow for expressing the descriptor interactions in a very detailed and exact way, but they require much more time and effort from the expert informants. In Gordon-Hayward cross-impact analysis (Gordon and Hayward, 1968) and SMIC (Godet et al., 1994), the model valuators define a conditional probability matrix which is in agreement with the probability axioms. This is often a considerable effort, and in the case of SMIC, Godet actually recommends (Godet et al., 1994) that the number of descriptors does not exceed six. This heavily limits the practical modeling power of cross-impact models of Gordon-Hayward cross-impact analysis and SMIC: the system model must, to remain feasible from valuation perspective, be very high-level and abstract, limiting its value as a decision support tool. In models based on Bayesian networks, conditional probabilities are expected for a descriptor for the Cartesian product of the possible states of its dependencies. If the probability distribution of the possible states of a system descriptor with four states is dependent on five other descriptors with four possible states, 4096 conditional probabilities should be defined for the dependent descriptor; in a complex system with possibly hundreds of descriptors, a case of a descriptor being dependent on ten other descriptors with four possible states each, 4,194,344 conditional probabilities would be required to fully define the dependence. In this sense, defining the model interactions as conditional probabilities is not a minor nuisance that places a requirement of more time being used in the modeling effort, but a hard practical limit to the complexity of the model of interactions of a system.

In the case of relying solely on expert informant valuation in modeling, one must ask what is the realistic upper limit of precision for expert informants when defining interactions of system descriptors as conditional probabilities. If the valuations of expert informants are assumed to be hazy, approximate quantifications, a compromise between the precision of definition of interactions and speedy and cognitively less expensive valuation process appears justified. An alternative approach to the use of conditional probability tables in description of probability-updating interactions between system descriptors is to use references to *probability updating functions*. They update the probabilities contextually, doing away with the need to define full conditional probability tables. This approach is discussed by Enzer (1972) and first adopted in the BASICS approach (Honton et al., 1984). Later it has been used in the JL-algorithm (Luukkanen, 1994). AXIOM also uses this basic idea of simplifying the description of probability updates, but expands on the idea. While describing the interactions in a complex system model with a large number of descriptors is still challenging, with the probability updating function reference approach the task becomes much more feasible: the most complex describable relationship in a case of a four-state descriptor dependent on ten other four-state descriptors, would require *at most* 204 valuations (normally less), contrasted to the 4,194,344 valuations required for description of the relationship using conditional probabilities. Often a smaller number of valuations would suffice in the updating function approach.

Providing a simplified way for expert informants to define the model interactions increases the modeling power of the modeling language in a very important way: it makes larger system models possible. If the modeling approach heavily limits the size and complexity of the model and the number of the descriptors, the model remains very high-level and abstract. Analysis of such models remains abstract as well. Modeling approach should support larger models as much as possible, to enable modeling that can produce the most policy-relevant outputs. Emphasizing the fitness of the modeling language to build large system models is also beneficial since systems modeling is often the most interesting and useful when models are more extensive: surprising, counter-intuitive and interesting systemic, emergent and higher-order interactions and long causal chains that would be difficult to analyze intuitively can only exist in models that have a relatively large number of system components represented.

3. **Inference and analytical output.** While the information available in the models of approaches discussed in this paper might enable, with changes to the computational process, the use of the model information to answer several different questions about the modeled system, many approaches do not discuss these ways of higher-order information extraction in their documentation or make them available in their software implementations. The versatility and usefulness of the analytical outputs of the expert informant based modeling approaches can thus be improved as well. The inference capabilities of most approaches are oriented either structurally, morphologically or probabilistically. As building system models are an extensive and work-intensive effort, it would be desirable that the approach could deliver outputs of all orientations, as is the case with AXIOM.

3. Methodological influences and the methodological contribution

As stated in the introduction, AXIOM builds upon the design choices introduced in existing modeling approaches. The most important influences of AXIOM approach are, in order of importance, *a)* Bayesian networks and influence diagrams (Cowell et al., 2006; Fienberg, 2006; Jensen and Nielsen, 2007), *b)* the Gordon-Hayward cross-impact analysis (Gordon and Hayward, 1968; Gordon, 1969), and its later derivative SMIC (Godet et al., 1991, 1994) and *c)* the BASICS approach (Honton et al., 1984; Huss and Honton, 1987). These techniques are discussed here in more detail to adequately position AXIOM against them, explain what are their problematic aspects in expert informant oriented modeling, and what is proposed in AXIOM to solve the identified issues.

From Bayesian networks and influence diagrams, AXIOM takes the basic inference principles and the model of decision support use. In comparison to Bayesian networks, the AXIOM modeling language provides more freedom to the modeler, allowing cyclic interaction in the modeling of causalities and a way to define the temporal structure of the model with the timestep property of the statements, decoupling the model temporal dimension from the structure of causal dependencies. AXIOM also proposes analytical processes which are not typically used in the case of Bayesian networks, but which can be found in the cross-impact analysis, structural analysis and morphological analysis tradition. From the Gordon-Hayward cross-impact analysis, AXIOM takes the idea of evaluating the model in a Monte Carlo process, but provides an easier and more feasible way for describing the knowledge base of the expert informants, by means of updating functions. The updating functions approach, in turn, is inspired by the BASICS approach (Honton et al., 1984; Huss and Honton, 1987), and its derivative JL-algorithm (Luukkanen, 1994). AXIOM significantly expands on the idea of BASICS updating functions. Other important influences are the above-discussed structural and morphological approaches, such as MICMAC (Godet et al., 1994), ADVIAN (Linss and Fried, 2010), general morphological analysis (Ritchey, 2006), and cross-impact balances approach (Weimer-Jehle, 2006). These approaches are technically quite far from AXIOM, but AXIOM design enables performing analysis that result in insights of structural and morphological nature, with a relatively low increase in conceptual complexity in the modeling.

The contribution of this paper is the proposal for a new expert informant based systems modeling approach. The design aim of the approach is to combine the best method design aspects of the older cross-impact analysis tradition, also expanding on these ideas, and use the hybrid approach for similar probabilistic inference and decision support as Bayesian networks and influence diagrams are used, with an eye on the feasibility of full expert elicitation in model parameterization, and flexible and expressive modeling language. The AXIOM modeling language and computational process are summarized in Section 4 and the analytical possibilities are illustrated with an example system model in Section 4.3.

3.1. Bayesian networks and influence diagrams

Bayesian belief networks are models for probabilistic causal reasoning (Cowell et al., 2006). They are widely used in scientific, industrial, and decision support applications. The basic use case for them in decision support is inferring the change in the probability distributions of the states of the node descriptors in the network, when other nodes are set to be in a known state, to represent a decision-making context, or a set of assumptions to be tested for their effect on the system. Alternatively changes can be made to the probability distributions of nodes of interest, to capture different assumptions about the distribution and to observe the effects of those assumptions. The probabilistic inference in a Bayesian network can be *predictive*, dealing with probability changes of effects given information about their causes, but also *diagnostic*, inferring the likely causes based on the observed effects (Lehikoinen, 2014).

The graphical representation of a Bayesian network is a directed acyclic graph, which describes causal relationships denoted by directed edges between variables or descriptors denoted by graph nodes. The Bayesian network nodes are probabilistic random variables and can represent almost any type of system properties. The random nodes can represent mutually exclusive discrete states, but also continuous quantitative system properties, and both types can be used in the same model. For influence diagrams, a special case of Bayesian belief network, also *decision nodes* and *utility nodes* are available as modeling primitives (Jensen and Nielsen, 2007), representing alternative decisions or policies. Decision nodes affect the probability distributions of the random nodes. Utility nodes receive information from random or decision nodes, and model the utility, harm, gain or cost of the states of their dependencies: they represent the decision making criteria, against which alternative decisions are assessed and compared against each other. In a model that holds several decision nodes, optimization of policy or interventions can be suggested by search of the combination of decision alternatives maximizing the expected utility or minimizing expected negative utility or harm (Jensen and Nielsen, 2007; Lehikoinen, 2014).

The graph edges represent causal dependency relationships of the head nodes on tail nodes, or as the Bayesian network is a directed acyclic graph, dependency of child nodes on their parent nodes. The relationships are numerically defined by populating the node-specific conditional probability tables with conditional probability distributions. The parent nodes are causes and their child nodes are effects, which can in turn be causes for other effects further down the causal hierarchy. This distribution contains information on the probability of a variable being in a certain state, dependent on the state of its causes. For defining the dependencies numerically, several methods can be applied: deterministic or probabilistic simulations (Dorner et al., 2007; Rahikainen et al., 2014), using learning algorithms on empirical or statistical data directly (Acid et al., 2004; Riggelsen, 2006), and expert elicitation (James et al., 2010; Kuikka and Varis, 1997; O'Hagan et al., 2006), or some combination of these. It is common to augment the model information with expert informant elicited knowledge.

Modeling using Bayesian networks is well supported by software implementations such as Netica (Norsys, Inc, 2004) and Hugin (Kjærulff and Madsen, 2013) that enable versatile analytical outputs, well beyond the basic output of Bayesian probability updating in a graph given some assumptions about the node states. Bayesian networks, however, specifically in systems modeling relying chiefly on expert elicited inputs, can be problematic. The number of required inputs, in cases of structurally complex models, easily becomes unmanageably high. As the structural complexity of the dependencies in the model increases, the amount of information required by the conditional probability table representation of the relationships grows exponentially. The number of conditional probabilities to be elicited for an effect e, in a case of n dependencies for e, is $\prod_{i=1}^{n} s(c_i) \times s(e)$, where $s(c_i)$ is the number of possible states a specific cause c_i can have, and $s(e)$ is the number of possible states of the dependent effect. An effect node with three possible states, and three dependencies, each also with three possible states, requires 81 conditional probabilities to have its relationship defined. While this number of values can be elicited from a determined expert group, it is laborious, as the 81 values will only define the relationship of *one* effect on its causes–and the model might have tens or hundreds of such effects. 4-state node with 5 dependencies having 4 possible states each would require elicitation of 4096 conditional probabilities. This is, with certainty, too much to ask even from the most dedicated expert panel. Such dependency structures are, based on the initial experiments of modeling with AXIOM, not uncommon in the way an expert group might want to model a system.

In expert elicitation of probability tables, the dependency structure of the model has to remain relatively simple to keep the number of elicited values manageable. The elicitation can aim at extracting parameters for probability distributions instead of the distributions directly, and this may reduce the work load, but this approach is normally applicable only for continuous variables, or discretized continuous variables. For discrete distributions without a logical ordering, probability updating signals implemented as updating functions as per the BASICS approach (Honton et al., 1984) or AXIOM approach are a possible, but apparently unutilized solution to reduce the elicitation work load.

Unlike other approaches discussed in this work, a Bayesian network graph is acyclic, thus the method does not allow modeling of cyclic interaction. The temporal aspect of the system, in cases where the system is modeled as a Bayesian network, is tightly coupled with the graph structure: no ambiguity about the cause-effect relationship between nodes is allowed, and structural inference loops are not normally possible. This imposes limitations on the expressive power of the model in higher abstraction level modeling exercises, such as modeling of societal, political or technological developments, typical in foresight.

AXIOM is, out of the approaches discussed in this chapter, conceptually and functionally, while not technically, closest to Bayesian networks. An AXIOM model could be approximated with a Bayesian network by *a)* allowing graph cycles in the Bayesian model, *b)* replacing the Bayesian updating logic with a Monte Carlo process, and *c)* describing the probabilistic effects of nodes on others by references to updating functions, akin to BASICS or AXIOM, instead of conditional probability tables. In this sense, AXIOM could be seen as a special case of a Bayesian network, or to generalize into one. While full implementation of AXIOM is not apparently possible with e.g. the Hugin (Kjærulff and Madsen, 2013) or Netica (Norsys, Inc, 2004) software, due to the limitations of the conditional statements that could be used to approximate the AXIOM updating functions, a relatively similar computation could, with great effort, be implemented within Hugin or Netica. To the best of the author's knowledge, such approach has never been used in the context of Bayesian networks. AXIOM explicitly aims at providing similar inference capabilities as Bayesian networks, making both predictive and diagnostic inference possible by means of the AXIOM iteration objects, discussed and illustrated in Section 4.3.

AXIOM provides direct decision support use, similar to influence diagrams, by use of the intervention statements in lieu of the decision nodes of the influence diagrams, and using any AXIOM statements in the model in a similar way utility nodes are treated in influence diagrams. The main difference is the modeling language, allowing causal loops, the timestep property, and easier description of interactions. Building and expanding on the updating functions approach adopted in BASICS (Honton et al., 1984; Huss and Honton, 1987), AXIOM provides a more feasible way to describe the expert knowledge base on the probabilistic interactions between the states of the descriptors, as the conditional probability table based description is replaced by a hazier and more approximate, but dramatically easier description. Adopting this approach means that the number of inputs to be elicited grows only linearly as the dependency structure becomes more complex, whereas in a Bayesian network, the growth is exponential. Extraction of analytical outputs of structural or morphological nature can be performed with Bayesian networks to a degree, although the meaningfulness of such analysis is limited due to the acyclic nature of the Bayesian network. AXIOM approach supports structural and morphological analysis well, and the use of an AXIOM model for these purposes is illustrated in Section 4.3.

3.2. Gordon-Hayward cross-impact analysis and SMIC

The early experiments with modeling the causal relationships on the basis of expert elicited inputs in the context of futures studies and foresight were performed in the late 1960's (Gordon and Hayward, 1968; Gordon, 1969). The motivation for these modeling experiments was to be able to provide an auxiliary technique for forecasting and foresight work done utilizing expert panels, especially the Delphi technique. Gordon and Hayward (1968) called the approach augmenting the Delphi technique by incorporating consideration of the interaction between the future events *cross-impact analysis*.

The next two decades saw a lot of discussion (Blackman, 1973; Bloom, 1977; Brauers and Weber, 1988; Burns and Marcy, 1979; Dalkey, 1971; Godet, 1976; Gordon and Hayward, 1968; Gordon, 1969; Ishikawa et al., 1980; Jackson and Lawton, 1976; Kane, 1972; Kaya et al., 1979; Martino and Chen, 1978; Mitroff and Turoff, 1976; Nováky and Lóránt, 1978; Turoff, 1971) on the methodological details of foresight-oriented cross-impact techniques and applications of, incremental amendments to and methodological proposals inspired by the cross-impact technique have been published with lower frequency since (Agami et al., 2010; Bañuls and Turoff, 2011; Bañuls et al., 2013; Ceric, 2016; Choi et al., 2007; Godet et al., 1991, 1994; Gordon, 1994; Jeong and Kim, 1997; Medina et al., 2015; Pagani, 2009; Thorleuchter et al., 2010; Weimer-Jehle, 2006).

The techniques normally referred to as cross-impact analysis, and relatively widely used, are the Gordon-Hayward cross-impact analysis (Gordon and Hayward, 1968; Gordon, 1969, 1994), henceforth referred to as GHCIA, and the SMIC approach by Godet et al. (1994). GHCIA and SMIC are probabilistic binary descriptor cross-impact models. If they are represented as graphs, their graph nodes are system descriptors, presenting a hypothesis or a postulate about the state of the system in the future, also called an event by Gordon (1994). This state is assigned an initial or *a priori* probability of occurrence, which is the expert estimate of the probability of the hypothesis assuming no available information about the system, meaning that the states of the other descriptors are unknown.

Represented graphically, graphs for both approaches are cyclic, unlike a Bayesian network. The edges carry information about the occurrence probability of the head node hypothesis, conditional to the occurrence of the tail node hypothesis. In the SMIC approach, the edges additionally carry information about the occurrence probability of the head node hypothesis, conditional to the non-occurrence of the tail node hypothesis (Duperrin and Godet, 1975; Godet et al., 1994). In GHCIA, the probability of the head hypothesis conditional to the non-occurrence of tail hypothesis is inferred (Gordon, 1994).

The expert-elicited conditional probabilities are, in the case of GHCIA, checked for compliance with the standard probability axioms. The following conditions should be met:

1. $0 \leq P(i) \leq 1$
2. $0 \leq P(i \mid j) \leq 1$
3. $\frac{P(i) - 1 + P(j)}{P(j)} \leq P(i \mid j) \leq \frac{P(i)}{P(j)}$

If the initial conditional probabilities do not fall within permissible bounds, it is the task of the expert group to resolve the inconsistency by changing either the conditional probabilities or the initial probability valuations. In the case of SMIC, the software implementation features a linear optimization function (Godet et al., 1994), which corrects the initial expert-sourced valuations into permissible bounds, aiming to keep the corrected valuations as close to the original expert valuations as possible.

When the conditional probabilities have been defined, model evaluation can be performed. The evaluation process is a Monte Carlo process, where truth values are assigned to model descriptors in random order, according to the defined probabilities. When a descriptor is assigned a truth value, the probabilities of other descriptors are updated, using the odds ratio technique described by Gordon (1994). When all descriptors have been evaluated, the system of the model has a fully resolved state. This state can be thought of as a scenario. If a binary descriptor *occurs*, or is in the state true, in the scenario, a counter for its occurrences is incremented. The probabilities of the descriptors are reset to the initial values. The evaluation is repeated a large number of times.

The cross-impacted *posterior* probabilities are computed simply as the occurrence frequency of descriptors in the set of generated scenarios. The posterior probabilities reflect the influence of the impact network and aim at capturing the influence of longer impact chains. In GHCIA, the recommended analytical process is to test various assumptions with the model by changing the initial probability valuations, for instance to simulate interventions. Different initial setups are compared in terms of posterior probabilities. In the case of SMIC, the aim is to identify the most probable scenarios for further examination with other futures methods (Godet et al., 1994): the inference of SMIC is *morphological* in nature, although it could relatively easily be used for the same purpose as GHCIA. For a system model of *n* hypotheses, SMIC outputs the probabilities for 2^n scenarios, ordered by their probability. Godet also recommends deriving an elasticity matrix for the variables by means of performing sensitivity analysis on the initial probability valuations of the variables.

As the interactions between the system components are expressed as conditional probabilities, and these conditional probabilities need to meet the above-stated conditions, the complexity of the system, measured by the number of descriptors, is recommended to be kept low: Godet et al. (1994) recommend that the number of descriptors should not exceed 6. Any real systems modeling effort struggles to describe the system with such a limited number of descriptors, and the abstraction level in the model easily remains very high. The BASICS-like probability update strategy is a more viable solution for expert elicited modeling. As the descriptors are binary, mutually exclusive states for system components cannot be easily modeled, and an exhaustive state set cannot be modeled at all. GHCIA and SMIC also have no built-in way to express a time dimension in models: all the system descriptors exist in a single “temporal space”. These features limit the modeling power practicality, and usability of the approaches in systems modeling.

From the GHCIA and SMIC, AXIOM inherits the idea of performing the model evaluation as a Monte Carlo process. The Monte Carlo process of AXIOM is quite different from GHCIA and SMIC, as its logic is influenced by the temporal relationship of the descriptors expressed with the timestep properties, the use of intervention statements, and possibly the non-simple updating functions, discussed under the description of the BASICS approach. In AXIOM, all the model evaluation rounds can be saved in the *iteration* objects and used as the basis of inference when the aim is to enable more complex probabilistic inference similar to Bayesian networks and influence diagrams, or morphological outputs. Compared to GHCIA and SMIC, AXIOM also offers a more practical set of modeling primitives, as the AXIOM system descriptors are multivalued, and they have a built-in way of being temporally positioned against other descriptors with the *timestep* property.

3.3. The BASICS approach

An alternative approach to expressing the conditional probability effects in a cross-impact model is modeling them with *probability-updating signals* instead of plainly numerified conditional probabilities. This type of approach has been discussed by Enzer (1972) and implemented in the BASICS approach (Honton et al., 1984; Huss and Honton, 1987) and later in the JL-algorithm (Luukkanen, 1994) with incremental improvements.

In the BASICS modeling language, descriptors can have an arbitrary number (greater than one) of possible states, which are assigned prior probabilities, whose sum is equal to 1. The probability-changing interactions that the model components have on each other are expressed as references to probability updating functions. BASICS (Honton et al., 1984; Huss and Honton, 1987) updating functions take a probability to be updated as an argument and return an updated probability, altering the descriptors' probabilities *contextually*: update by the same function will result in a different amount of probability change in the influenced descriptor, depending on the value of the adjusted probability at the time of the update. This makes the description of probabilistic influences in the model hazier and approximate, but also dramatically reduces the difficulty and workload of describing the relationships between the system components. This is especially relevant in system models with a great number of descriptors and complex dependencies. Expressing the relationships of the system components as references to probability updating functions, such as '+ 3' to indicate a positive probability-changing impact, or '− 1' to indicate a smaller negative probability-changing impact, does away with the need to define conditional probabilities, and instead offers a way to express the interactions in an approximate way, but still keeping the quantified probabilities, central for decision support use, in the analysis.

Compared to the approach where full conditional probability tables, or GHCIA- or SMIC-like conditional probabilities satisfying the constraints defined for those approaches, are used to describe the causal dependency of an effect descriptor on a cause descriptor, the probability updating function approach is an approximate and 'hazy' way to quantitatively express the causal dependency. Similar dependency structures can be expressed, but a degree of accuracy is lost. What is gained is the easier way to describe the causal rules in the system, as the experts who are elicited can, instead of specifying conditional probabilities, invoke an appropriate probability update by referencing an updating function by the name of that function.

An example set of BASICS-like updating functions is graphed in Fig. 2. AXIOM also employs such updating functions. Updating functions of AXIOM are intended to be more versatile than the updating functions of BASICS, with the capability of both using other information in the model than the current probability for mapping it to an updated probability, and performing other updates to the model than simply updating probability values, such as immediately compelling a descriptor into a state and firing its updates instead of only updating probability distributions.

BASICS does not employ a Monte Carlo process in its model evaluation, and doesn't aim at producing a *posterior* probability distribution for the states of the system descriptors. In some applications of BASICS (Huss and Honton, 1987), posterior probabilities computed from the configurations produced by the model evaluation rounds are displayed, but it must be noted that the number of rounds performed in the BASICS approach is insufficient to compute posterior probabilities in the same sense as is done in Bayesian networks, GHCIA, or AXIOM. Instead, BASICS employs a deterministic process, where the model is evaluated twice for each possible state of all of its descriptors, assuming the state in question to "be true" or occur, and in a different iteration to be "false" or not occur. In the evaluation of descriptors, the most probable state is selected, making the model evaluation deterministic. Each model evaluation produces a set of descriptor states occurring in that evaluation, and this set can be interpreted as a scenario. A model with 10 descriptors, with 3 states each, results in $10 \times 3 \times 2 = 60$ scenarios (Honton et al., 1984; Huss and Honton, 1987).

The motivation is to find scenarios that are "probable and consistent" (Honton et al., 1984), in the light of the supplied prior probabilities and interactions. The scenarios that emerge from multiple different evaluations are interpreted to be probable and consistent, warranting further study with other analytical techniques. In this sense, the output produced by BASICS is analytically serving a similar purpose as morphological analysis, discussed in Section 2.2. The information content of the BASICS model enables a wider range of outputs, but these possibilities are not documented or explored in the descriptions of the BASICS approach. JL-algorithm is derived from BASICS, and proposes changes to the model evaluation procedure to eliminate effects of the ordering of the descriptors in the user input, as they are significant in some of the BASICS approach implementations (Luukkanen, 1994).

BASICS and JL-algorithm make it possible to identify morphologically consistent scenarios. They do not support simulation-style use of the model for testing the effect of interventions or other changes to the system that can be observed from posterior probabilities. Posterior probabilities could be made available for BASICS if the evaluation process would be changed so that a sufficient number of evaluations would be performed and the evaluation process would be changed to probabilistic instead of the deterministic way. With these changes, the BASICS approach would be closer to the AXIOM approach. The analytical output, as the method is documented, is limited to the morphological output of identifying full system configurations that are probable with the given description of prior probabilities and interactions, inferred by the BASICS evaluation process.

From BASICS, AXIOM draws the basic idea of reducing the difficulty of the description of probabilistic rules of the system with contextual probability updates. AXIOM expands on the idea of updating functions used in the BASICS approach and JL-algorithm. The BASICS updating functions simply map a probability to an updated probability, and their only input is the old probability. AXIOM updating functions close over the entire model, and can use any information in it to map probabilities to updated probabilities. The probability updates can be made dependent on not only the occurrence of a single state in the model, but any set of states, or even the current probability distributions of a descriptor or a set of descriptors. This enables e.g. modeling of actor behavior, that can be dependent on how likely some event or outcome appears at a specific moment. This difference makes the AXIOM updating functions much more expressive: they can be used to describe more complicated dependencies than BASICS updating functions. Conditional logic, that is possible to describe using conditional probability tables akin to Bayesian networks, can be approximated with AXIOM updating functions. Additionally, the AXIOM updates can do more than simply change the probability distributions of the effect descriptors: The AXIOM updates can fire actions in the model, such as immediately setting a descriptor to a certain state, or some other change in the model, such as removing impacts, changing the updating functions of these impacts, or doing some other structural change in the model.

In terms of analytical outputs, AXIOM significantly widens the possibilities of BASICS. The BASICS output is morphological. AXIOM can deliver similar outputs, but it considerably expands the analysis of BASICS to the direction of probabilistic inference and decision support performed normally with Bayesian networks and influence diagrams. AXIOM approach also supports extraction of structural outputs akin to EXIT, MICMAC and ADVIAN, and fuzzy cognitive maps.

4. The AXIOM approach

AXIOM is a systems modeling approach designed for a specific niche of systems modeling, modeling of chiefly non-technical, non-deterministic systems with a complex interaction structure and with components of heterogenous nature, such as social, technological, economical, political or cultural components or driving forces. Components or system aspects of this nature often have relatively low quantifiability and data availability. Modeling such systems has to rely mostly on expert informants as the data source for definition of the relationships in the system, as there is not much statistical data to estimate the relationship in the form of a mathematical equation, using statistical modeling approaches such as regression analysis. The design of a modeling approach for this niche has to aim for a modeling language with high modeling power and fitness for use in expert elicitation, and a computational process enabling versatile analytical outputs and the use of the model to give as much information as possible of the modeled system, to compensate for the effort of constructing such a model. The modeling approaches discussed in Section 2.2 offer different solutions to the relevant design questions, and these solutions are assessed against the intended modeling use case requirements in Section 2.3 and Section 3. The design choices of AXIOM are based on this argumentation.

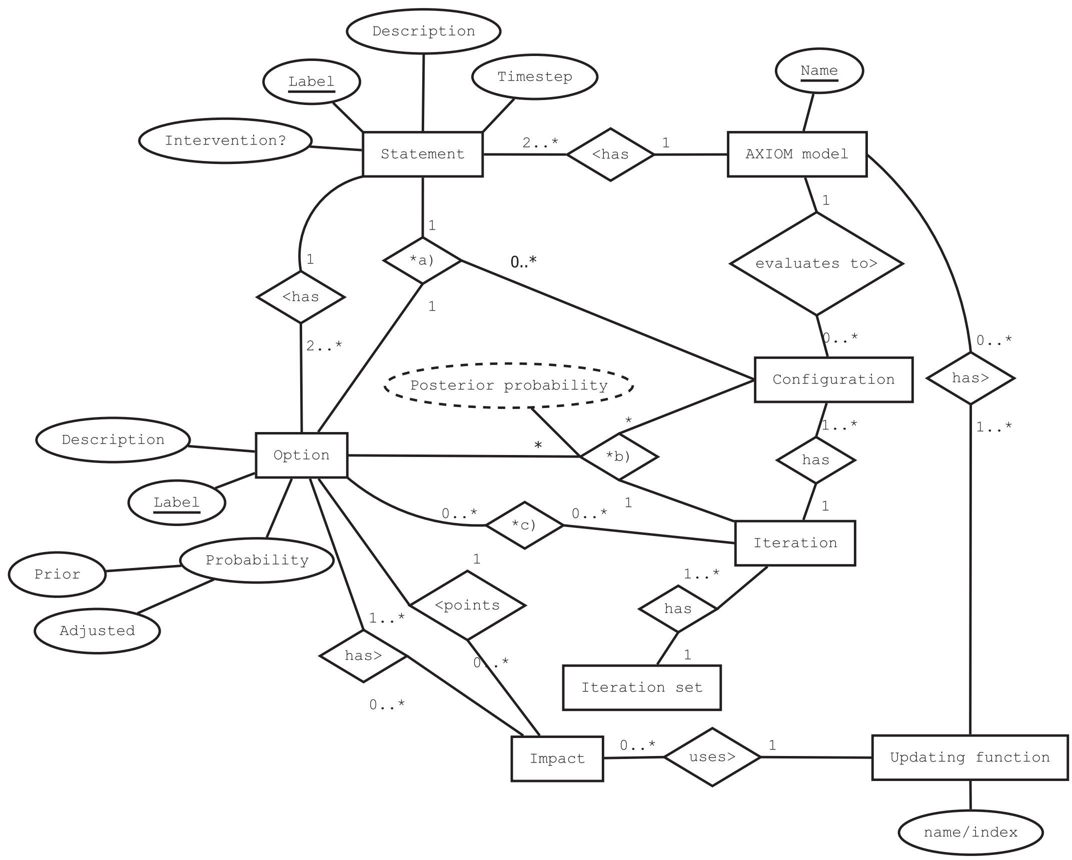


Fig. 1. Entity-relationship model of AXIOM concepts. ***a)** *Statement* is evaluated to an *option* in a single *configuration* ***b)** A *configuration* in an *iteration* has a single *option* for each *statement* in the model; the *a posteriori probability* of each option is the rate of occurrence of the option in configurations in the iteration. ***c)** An *iteration* can have *options* as active interventions

4.1. Modeling language

The modeling language, or the set of model building blocks of AXIOM, used to describe a system and its interactions, consists of three main primitives: statements, options and impacts. Fig. 1 presents an entity-relationship model (Chen, 1988) of the AXIOM concepts.

Statements represent components, driving forces and events of the modeled system. They have a temporal position (possibly equal) in relation to other statements in the model, called the *timestep* property. Statements also have a set of *options*, which are the possible values of the statement, or the (modeled) possible states of the system component the statement represents. The options have a probability value, indicating their likelihood to be assigned as the value of their respective statement. The initial probability is called the *prior* or *a priori* probability; the prior probability values of options under the same statement are estimated by the expert informants by assuming no available information about the system outside that particular statement. The options form a probability distribution, and the sum of probability values of all the options of a statement must equal 1: options are mutually exclusive, and thought to fully exhaust the range of possible states of the modeled system component and fully occupy the probability space.

Impacts represent probabilistic causal relationships between the system descriptors. Impacts are normally *simple impacts*, associated with two options in different statements, the cause option and the effect option. When the cause option is evaluated to be true, its effects are fired or 'take place', changing the probability of the effect option, as well as the probabilities of the complement options under the same statement as the effect option. Impacts can also be *non-simple*, modeling more complex dependencies, as discussed in Section 3.3, where several cause options influence the effect option. When the model, during its

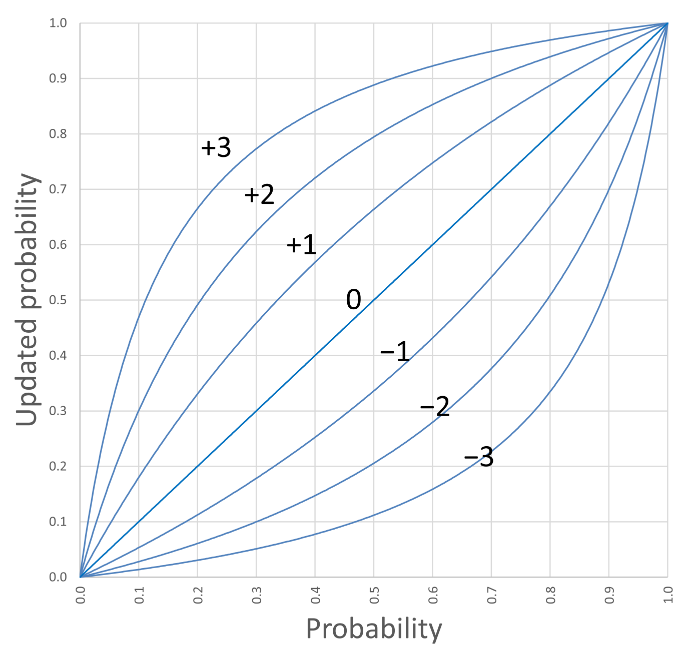


Fig. 2. Seven *simple* AXIOM probability updating functions graphed. See also Honton et al. (1984) for description of the BASICS updating functions.

evaluation, arrives at a state where the causes of an impact are true, a probability update (or some other update, such as a structural update) takes place. At the model level, the action of updating probabilities or doing other updates to the model is performed when the model state changes, meaning that new information about the system is available. This is analogous to " unfolding of the future" in reality: as events take place or system components assume a specific state, the outlook of what might happen next and with what likelihood changes as a result of causalities and new information.

In the initial state of the model, the statements do not have a state, only a probability distribution for their possible values, the options. The *evaluation of a statement* consists of selecting one of the options of the statement (according to the probability distribution), assigning it as the state or value of the statement, and executing the impacts the selected option has, if any. Each option has a likelihood of being selected equal to its current probability value. The selected option is now thought to 'occur' or 'be true'. It has a possibly empty set of impacts targeting other options in other statements in the model. The impacts are now realized and change the probabilities of their target options. The probabilities of other options under the same statement as the target option are also updated in order to preserve a valid probability distribution. If the model has non-simple impacts with several conditions, the occurrence of these conditions is checked when the model state changes, and the updates are performed if conditions are met.

A simple impact is defined by its cause or source option, its effect or target option and an updating function reference, similar to the approach adopted in BASICS (Honton et al., 1984). An AXIOM model has a set of updating functions, that are referenced by impacts to describe how the impact is meant to update the probability of the effect or target option, or what other updates to perform, in the case of non-simple impacts. Simple functions update the effect option probabilities contextually, mapping the current probability value to an updated value, reflecting the probabilistic influence the impact has on them. The probability updating functions have a domain of [0,1] and a codomain of [0,1]. Additionally, simple updating functions are recommended to *(a)* be symmetric about the line $y = -x + 1$, *(b)* have the property $y(x_0) < y(x_1)$ when $x_0 < x_1$, and *(c)* have the property $y(x) > x$ if the name of the function implies positive (probability-increasing) impact, and the property $y(x) < x$ if the name of the function implies negative (probability-decreasing) impact. The purpose of describing relationships in the model with updating functions is to circumvent the need to define conditional probability tables (the rationale for this was discussed in Section 2.3). Instead, the effects of knowing that a specific model descriptor is 'true' or that a part of the system is in a certain state are delegated to a specific updating function. Seven simple probability updating functions named '0', '+ 1', '+ 2', '+ 3', '− 1', '− 2', '− 3', are graphed in Fig. 2. Function "0" does not map any change to probability, representing a neutral relationship; "+ 3" represents the greatest positive change to probability out of the presented functions; "− 1" represents a modest negative probability change. This updating function set enables modeling of probability effects as per the BASICS (Honton et al., 1984) approach. Unlike BASICS, an AXIOM model can have as many updating functions as are seen necessary to describe the relationships in the model.

Compared to the updating functions in BASICS, an AXIOM updating function can, instead of a probability update, force a statement immediately into a state, consequently firing all the updates linked to that state. Such an update would represent a deterministic relationship of a model state on some other state. AXIOM updating functions also close over the entire model and can use all information in the model, such as current probabilities of any option in it, to determine the amount of probability update. Such updating functions can, for instance, be used to model actor behavior: The decision or behavior of an actor would be represented by a statement or a set of statements in the model, and the likelihood of an actor to make a specific decision can be made dependent on the current probabilities of specific model states at the time the decision is made. An important use for the non-simple updating functions is modeling more complex dependencies than what can be modeled by binary updating functions with a cause option and an effect option. A probability update can be made conditional to several facts in the system, such as the occurrence of a set of states instead of a single state. In Bayesian networks, this type of dependency is expressed with conditional probability tables, and in some cases modelers might want to model such complex probabilistic dependencies. An AXIOM updating function can be made dependent of several facts, making modeling such more complex dependencies possible. Currently the AXIOM implementation supports only simple impacts and simple updating functions directly from input, but non-simple impacts and non-simple updating functions can be implemented by any user by accessing the freely available source code. The user interface will be expanded to support the use of non-simple updates as the development of the implementation proceeds.

4.2. Model evaluation

Inference in AXIOM is based on a Monte Carlo process of repeated model evaluations. As illustrated by Fig. 1, a single model evaluation results in a *configuration*, which is saved to an *iteration* object, and several iterations with different initial setups make an *iteration set*. A pseudocode description of the AXIOM model evaluation process, as well as the computation of the *iterations* and *iteration sets* are presented in Appendix A.

A model evaluation means resolving the state of the model, by performing the evaluation of all the statements in the model. The order of evaluation is, firstly dictated by the timestep values of statements, and secondly random: statements with a lower timestep value are always evaluated before statements with a higher timestep value, and statements with equal timestep value are evaluated in random order. After the evaluation of all model statements, every descriptor has a state and therefore the whole model has a state (the combination of descriptor states emerged as a result of the evaluation): all options are either true or false. This result is saved. The probabilities of options are re-initialized to their prior probability values and a new model evaluation process is performed, again saving the result. The evaluation is performed a large number of times: the default number of evaluations in the AXIOM implementation is 10^6, but a higher number may be necessary in complex models to yield accurate estimates of the posterior probabilities. Each evaluation produces a *configuration*, a model state as a combination of descriptor states, which can be thought of as a scenario. The collection of configurations resulting from the Monte Carlo process is called an *iteration*. It captures the system states that result from a specific set of initial conditions.

From the iteration, it is possible to compute the *a posteriori* or *posterior* probabilities of the options in the model by simply counting the frequency of occurrence for each option. This posterior probability value takes into account the systemic, emergent higher-order interactions in the model. From the information content of the iteration, it is also possible to compute probabilities for *morphologies*, partial system states described by a specific set of options, by counting the frequency of those option sets in the iteration. The iteration is a dataset in the association rule learning sense, so the association rule learning concepts and operations can be used in its analysis. The posterior probabilities of single options, or option combinations (morphologies), are computed as their *support*. Other association rule learning operations like *confidence*, *lift* and *conviction* can also be computed from an iteration.

A major motivation for building system models is to gain the ability to test the behavior of the system under different assumptions, and simulate effects of changes to the system. Such changes can be prior probability valuations, strengths of model impacts, the structure of impacts, or structure of the model in terms of statements and options. Once these changes to the model have been made, the Monte Carlo process can be performed again, resulting in a new iteration. An *iteration set* is a collection of iterations under different initial setups of the model. The iterations in the iteration set are compared against each other to reveal the effect of the changes made, or the differences of outcomes between the setups.

AXIOM provides an analytical convenience mechanism called *intervention statements* to test the systemic effects of particular interventions. Statements can be flagged as intervention statements, which will then be treated specially in the model evaluation: intervention statements will not be evaluated in the normal probabilistic way, but will rather have a predefined state, already determined when the model evaluation commences. The states of the intervention statements change only between different iterations. Other details of the model evaluation are the same: the impacts of the predetermined options of the intervention statements take place when the intervention statement is taken up for evaluation. When the model has flagged intervention statements, an iteration set will automatically be generated with a single iteration capturing the normal model evaluation results without interventions and the rest of the iterations capturing each possible combination of the options of the flagged intervention statements. The function of intervention statements is that they can model policy actions, strategic options available to actors in the system or some other aspect of the system which the analyst wants to test in different states: intervention statements have the same function as *decision nodes* in influence diagrams.

4.3. Example model and analysis of results

The AXIOM approach is illustrated with a system model presented by Weimer-Jehle for demonstration of the cross-impact balances (Weimer-Jehle, 2006) (CIB) approach, which in turn is amended from a BASICS cross-impact model presented by Honton et al. (1984). The CIB model describes a limited set of drivers for oil price and the interactions between these forces and the oil price. The interactions are of direct causal nature, so the original CIB model is suited to be transformed into an AXIOM model. AXIOM model requires additional information of initial probabilities of options, timestep property values for statements, and probability updating functions, which have been added to the model, based on the judgment of the author. As described by Weimer-Jehle (2006), the model does not attempt to comprehensively represent the system, but is meant to provide "an illustrative and manageable frame for description of the method". The model consists of five statements, having 3 to 4 options each and 16 altogether, and their directed probability-changing interactions, whose magnitudes are expressed with an integer in the range $[-3, +3]$. The amended AXIOM model with its statements and their timesteps, options and their initial probabilities, and impact valuations in an impact matrix format, is presented in Table 1. The impact magnitude indicators reference to the simple updating functions presented in Fig. 2: during the model evaluation, the probabilities of options are adjusted according to the function referenced in the impact matrix. In the table, row descriptors are the impactors and the column descriptors the impacted items: the impact valuation of option "$< 2\%/\text{yr}$" of statement "World GDP growth" is $+2$ and can be read from row 1, column 4 of the impact matrix of Table 1.

To illustrate the modeling of temporal dimension, the statement "Oil price" has been placed in temporal category 2, whereas all the other statements are in category 1, and therefore resolved before the state of the oil price. As a result, the impacts that the "Oil price" statement is modeled to have never taken place in the example model, as all the other statements have already been evaluated before oil price. If additional statements with timestep 2 or higher would be added to the model, oil price could influence them. Similarly, if the timestep property of the oil price statement would be changed to 1, it would be evaluated "simultaneously" with the other statements and would influence them.

The repeated model evaluation process described in Section 4.2 results in a set of model states, where each statement has a value (one of its options), or *configurations*. This set of configurations is called an *iteration*. Table 2 presents an iteration with 50 configurations, which are displayed in columns, so that the value (option) of the statement in that configuration is represented by a shaded cell. Each statement has been evaluated into one of its options in each configuration. The posterior probability for each of the options is calculated as the frequency of occurrence in the iteration, and presented in the last column. The four last rows of the table display the computation of probabilities of *morphologies*, combinations of options.

By computing occurrence frequencies of options and morphologies, possibly conditional to occurrence of other options and morphologies, various questions related to morphological, structural and probabilistic information needs can be posed to the model. These include the following:

- What is the probability of atomic subscenarios (options) after the systemic effects have been accounted for?
- What are the probabilities of morphologies (specific combinations of system states)?
- Which system states are logical, compatible and consistent, judged by their frequent co-occurrence?
- What are, on the basis of the modeled direct relationships, the indirect, systemic relationships of the system descriptors?
- How will the system behave under a specific intervention or other change? (predictive probabilistic inference)
- What are the likely causes of an effect? (diagnostic probabilistic inference)
- What are the effects of combinations of interventions or changes?
- What are the strongest antecedents to specific system states?
- What are the outcomes of policies or strategies?
- What is the most preferable system state against some criteria?
- What are the most effective interventions to perform on the system to reach that preferable state?

The morphology $1a \wedge 3a$, meaning the combination of low world GDP growth and strong world tensions, has a probability of 0.12. The probability for morphology $5c \vee 5d$, where oil price is higher than 35$, has a probability of 0.62. The third presented morphology $(\neg 1c \wedge \neg 2a)$, a scenario where GDP growth is at most 3% annually and borrowing policy of industrial countries is not high, has a probability of 0.42, and the probability of that morphology occurring together with $5c \vee 5d$ is 0.28. Used in this way, AXIOM delivers analytical outputs of the *morphological* nature, comparable to the cross-impact balances approach (Weimer-Jehle, 2006), SMIC approach (Godet et al., 1994), and BASICS and JL-algorithm approaches (Honton et al., 1984; Luukkanen, 1994). The 50 configurations presented Table 2 are obviously insufficient to compute posterior probabilities accurately, and the table is presented for illustration of how analytical outputs are derived from AXIOM iteration objects. From a sufficiently large set of configurations, the emergent, systemic characteristics of the model captured by the posterior probabilities can be estimated accurately (or to the degree of accuracy of the elicited inputs).

As the information content of an AXIOM iteration is like an association rule learning dataset, the association rule learning operations can be utilized in its analysis (Hahsler et al., 2007; Piatetsky-Shapiro, 1991). Computing the a posteriori probability of a single option or a more complex morphology is identical to computing the support of an itemset. *Confidence* can be used to compute the conditional probabilities of morphologies, given antecedent morphologies (Hahsler et al., 2007). For instance, the confidence $(\neg 1c \wedge \neg 2a \Rightarrow 5c \vee 5d)$ is the conditional probability of high oil prices given non-high GDP growth and non-high borrowing scenario. It is calculated as $\frac{\text{SUPPORT}((\neg 1c \wedge \neg 2a) \wedge (5c \vee 5d))}{\text{SUPPORT}(\neg 1c \wedge \neg 2a)} = \frac{0.28}{0.42} = 0.67$. Other association rule learning operations, such as *lift* and *conviction* (Hahsler et al., 2007) can be used to discover interesting and important relationships from the iteration objects' data content.

By examining the subset of configurations where a specific option of interest is "true", or the evaluated state of its statement, it is possible to

Table 1
Example AXIOM model, adapted from Weimer-Jehle (2006).

Statement	Timestep	Option	A priori probability	World GDP growth			Borrowing, industrial countries			World tensions			OPEC cohesion			Oil price			
				<2%/yr	2-3%/yr	>3%/yr	high	medium	low	strong	moderate	weak	strong	moderate	weak	<20$	20-35$	35-50$	>50$
World GDP growth	1	<2%/yr	0.40				+2		−2	+2		−2				+2	+1	−1	−2
		2-3%/yr	0.35				−1	+2	−1							−1	+1	+1	−1
		>3%/yr	0.25				−2	+1	+1	−1		+1				−2	−1	+1	+2
Borrowing, industrial countries	1	high	0.25	+1		−1				+1		−1							
		medium	0.50																
		low	0.25	−1		+1				−1		+1							
World tensions	1	strong	0.10	+1		−1	+1		−1				+1		−1	−3	−2	+3	+2
		moderate	0.55																
		weak	0.35	−1		+1	−1		+1				−1		+1	+1	+2	−1	−2
OPEC cohesion	1	strong	0.20													−3	−2	+3	+2
		moderate	0.35													−1	+1	+1	−1
		weak	0.45													+1	+1		−2
Oil price	2	<20$	0.10	−2		+2	−1		+1				−2		+2				
		20-35$	0.30	−1		+1							+2	−1	−1				
		35-50$	0.35																
		>50$	0.25	+1		−1				+1		−1	−1		+1				

compute the posterior probabilities of other model options, conditional to the presence of the system descriptor option of interest. By comparing these probabilities to the posterior probabilities computed from the total set of configurations, the magnitude of systemic impacts of the option of interest can be estimated as the difference. In a complex system model with complicated interdependencies and extant long causal impact chains, this systemic relationship might turn out to be very different to the modeled direct relationship, as it also accounts for all the indirect, mediated interaction of a system descriptor on another. Table 3 shows an impact matrix, reporting the systemic effect of row options on column options as the change in posterior probability conditional to the guaranteed realization of the row option. The changes that are within a margin of ± 0.015 are highlighted in gray, as these small differences result from the random component of the Monte carlo process.

The posterior probability of option "Oil price: 35$–50$" is 0.434 overall, looking at the total set of configurations, but conditional to the presence or actualization of option "OPEC cohesion: strong", the probability is elevated to 0.772, and the difference + 0.34 is presented in the impact matrix of Table 3 as the amount of probability change the systemic relationship of the impactor (row) descriptor has on the impacted (column) descriptor. This systemic relationship might not be directly observable from the model input data describing the direct interactions. Obviously this tabulation could also be multidimensional, showing the model options' posterior probabilities conditional to several antecedent options simultaneously. Used in this way, AXIOM can deliver analytical value of *structural* nature, comparable to MICMAC (Godet et al., 1994), ADVIAN (Linss and Fried, 2010) and EXIT (Panula-Ontto et al., 2018; Panula-Ontto and Piirainen, 2018) approaches.

Table 4 illustrates the use of AXIOM intervention statements and presents the posterior probabilities of the model options in ten different iterations, representing different assumptions about the system. The

Table 2
An AXIOM *iteration* consisting of 50 *configurations*.

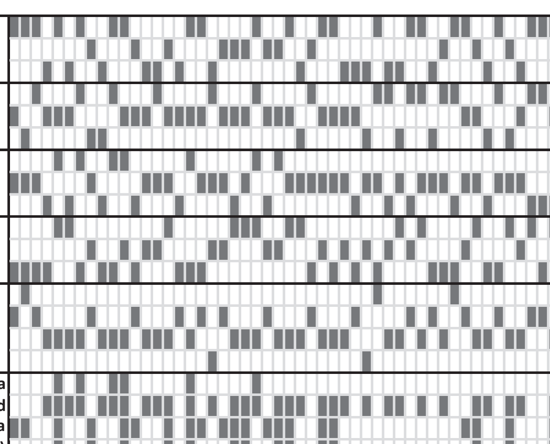
Statement	Option		Configurations	Probability
World GDP growth	<2%/yr	1a		0.440
	2-3%/yr	1b		0.240
	>3%/yr	1c		0.320
Borrowing, industrial countries	high	2a		0.320
	medium	2b		0.500
	low	2c		0.180
World tensions	strong	3a		0.140
	moderate	3b		0.560
	weak	3c		0.300
OPEC cohesion	strong	4a		0.280
	moderate	4b		0.300
	weak	4c		0.420
Oil price	<20$	5a		0.060
	20-35$	5b		0.320
	35-50$	5c		0.580
	>50$	5d		0.040
		1a∧3a		0.120
		5c∨5d		0.620
		¬1c∧¬2a		0.420
		(¬1c∧¬2a)∧(5c∨5d)		0.280

Table 3
Structural analysis using the AXIOM approach: Probability changes of options conditional to other model options.

Effects of \ Effects on		World GDP growth			Borrowing, industrial countries			World tensions			OPEC cohesion			Oil price			
		<2%/yr	2-3%/yr	>3%/yr	high	medium	low	strong	moderate	weak	strong	moderate	weak	<20$	20-35$	35-50$	>50$
World GDP growth	<2%/yr				+0.18	−0.13	−0.05	+0.10	0	−0.11	0	0	0	+0.08	+0.02	−0.07	−0.03
	2-3%/yr				−0.11	+0.15	−0.04	−0.06	+0.03	+0.03	0	0	0	−0.05	+0.08	0	−0.04
	>3%/yr				−0.12	0	+0.11	−0.09	−0.04	+0.13	0	0	+0.02	−0.04	−0.10	+0.05	+0.10
Borrowing, industrial countries	high	+0.11	−0.02	−0.10				+0.07	+0.02	−0.08	0	0	0	0	−0.03	+0.02	0
	medium	0	0	0				0	+0.02	0	0	0	0	0	0	0	0
	low	−0.10	0	+0.10				−0.06	−0.04	+0.10	0	0	0	0	+0.03	−0.03	0
World tensions	strong	+0.14	−0.03	−0.11	+0.10	−0.03	−0.07				+0.11	+0.02	−0.13	−0.06	−0.38	+0.43	0
	moderate	+0.03	0	−0.03	0	0	−0.02				0	0	−0.02	+0.03	−0.05	0	+0.02
	weak	−0.08	0	+0.08	−0.06	0	+0.05				−0.06	−0.03	+0.09	0	+0.26	−0.23	−0.03
OPEC cohesion	strong	0	0	0	0	0	0	0	0	0				−0.06	−0.36	+0.34	+0.09
	moderate	0	0	0	0	0	0	0	0	0				−0.04	+0.06	0	0
	weak	0	0	0	0	0	0	0	0	0				+0.05	+0.09	−0.12	−0.02
Oil price	<20$	0	0	0	0	0	0	0	0	0	0	0	0				
	20-35$	0	0	0	0	0	0	0	0	0	0	0	0				
	35-50$	0	0	0	0	0	0	0	0	0	0	0	0				
	>50$	0	0	0	0	0	0	0	0	0	0	0	0				

intervention statements are functionally similar to *decision nodes* of influence diagrams. Statements 'borrowing' and 'OPEC cohesion' have been flagged as intervention statements, so iterations in columns 6–14 display the posterior probabilities of model options under specific combinations of options of the intervention statements. The initial *prior* probabilities are presented in the fourth column ('A priori'). The fifth column ('No intervention') presents the cross-impacted *a posteriori* or *posterior* probabilities in an iteration without active interventions.

The remaining columns present the posterior probabilities under different combinations of options of the flagged intervention statements: each of them captures the systemic effects of a specific combination of a borrowing subscenario and a OPEC cohesion subscenario. Column 6 ("Borrowing:high + OPEC:strong") presents the posterior probabilities assuming a high borrowing policy and a strong OPEC cohesion; the last column presents the same information assuming a low borrowing policy and a weakly cohesive OPEC. The modeling results would seem to suggest, for instance, that the likelihood of high global GDP growth is maximized by observing a policy of low borrowing, and OPEC cohesion is insignificant for GDP growth (this might be considered obvious already by looking at the input data of the miniaturish example model, but observations of this nature are much less obvious in a more complex model). In this way, AXIOM can be used

Table 4
Probabilities of model options under different preconditions.

		Utility valuation	A priori	No intervention	Borrowing: high + OPEC: strong	Borrowing: medium + OPEC: strong	Borrowing: low + OPEC: strong	Borrowing: high + OPEC: moderate	Borrowing: medium + OPEC: moderate	Borrowing: low + OPEC: moderate	Borrowing: high + OPEC: weak	Borrowing: medium + OPEC: weak	Borrowing: low + OPEC: weak
World GDP growth	<2%/yr	−2	0.40	0.378	0.492	0.375	0.275	0.492	0.375	0.276	0.491	0.377	0.276
	2-3%/yr	+1	0.35	0.330	0.313	0.344	0.328	0.313	0.344	0.328	0.314	0.342	0.327
	>3%/yr	+3	0.25	0.293	0.195	0.281	0.397	0.195	0.281	0.396	0.195	0.282	0.397
Borrowing, industrial countries	high	−1	0.25	0.258	1	0	0	1	0	0	1	0	0
	medium	+0	0.50	0.537	0	1	0	0	1	0	0	1	0
	low	+1	0.25	0.205	0	0	1	0	0	1	0	0	1
World tensions	strong	−4	0.10	0.162	0.226	0.147	0.099	0.225	0.148	0.099	0.225	0.148	0.1
	moderate	−2	0.55	0.513	0.531	0.531	0.482	0.532	0.53	0.481	0.531	0.532	0.481
	weak	+2	0.35	0.325	0.243	0.322	0.419	0.244	0.322	0.42	0.244	0.321	0.419
OPEC cohesion	strong	+0	0.20	0.194	1	1	1	0	0	0	0	0	0
	moderate	+1	0.35	0.336	0	0	0	1	1	1	0	0	0
	weak	+0	0.45	0.470	0	0	0	0	0	0	1	1	1
Oil price	<20$	+3	0.10	0.070	0.011	0.01	0.009	0.039	0.035	0.029	0.138	0.124	0.107
	20-35$	+1	0.30	0.430	0.07	0.071	0.07	0.469	0.494	0.507	0.49	0.527	0.554
	35-50$	+0	0.35	0.435	0.786	0.77	0.751	0.446	0.419	0.403	0.338	0.308	0.291
	>50$	−4	0.25	0.065	0.134	0.149	0.17	0.046	0.052	0.061	0.034	0.04	0.047
Utility score			−0.6	+0.1	−3.0	−1.1	+0.9	−1.2	+0.8	+2.8	−1.8	+0.2	+2.1

for predictive probabilistic inference, comparable to Bayesian networks and influence diagrams (Cowell et al., 2006; Lauría and Duchessi, 2006), or Gordon-Hayward cross-impact analysis (Gordon and Hayward, 1968; Gordon, 1969), testing the system under different conditions and policies, and comparing the results to other sets of conditions.

A utility function can be defined to help identify preferable combinations of interventions or preferable scenarios overall. In this capacity, AXIOM can deliver similar outputs as an influence diagram. Any AXIOM node can function akin to a *utility node* in an influence diagram, with an appropriate utility function. A simple utility function can be defined by assigning a utility valuation for all model options, as is done in column 3 ("Utility valuation") of Table 4. The unpreferability of an option is expressed with negative utility valuation and preferability with positive valuation. The utility score (in the last row of Table 4) is then computed by multiplying the probability of an option with its index and summing the values. The utility function could also be based on probabilities of more complicated morphologies. Based on this simple utility function, and the very subjective utility valuation of subscenarios represented by the model options, the intervention combination of low borrowing and moderate OPEC cohesion appears the most optimal scenario.

The same information could be derived with association rule learning operations, by only examining configurations where the intervention statements have the desired option as their state, and computing the posterior probabilities for other options from that subset of configurations. The intervention statement functionality, however, limits the number of required evaluations and enables easy comparison of model outputs under different assumptions about the system, this assumed, by the author, to be the typical use case for higher-order information extraction from an AXIOM model.

The *inverse logic* or *diagnostic inference*, or inferring the likely causes given some observed effects, typical in Bayesian networks, can be performed with AXIOM as well. The process is simply to generate a sufficient number of configurations, select from those configurations the ones where the observed effects under investigation occur, and compute the posterior probabilities of causes from that set of configurations. Computationally this is inefficient in comparison to the diagnostic inference of Bayesian networks, but still completely feasible.

As the AXIOM iteration objects are itemsets, they can be used as input for algorithms that learn Bayesian networks from such inputs: a Bayesian network can be derived from AXIOM output. The resulting Bayesian network can then be augmented with other Bayesian network model components based on empirical or statistical data. This enables combining expert-elicited modeling results and data-based modeling results in the same analytical framework.

4.4. Software implementation

The software capable of performing the AXIOM transformation described in Section 4.2 is freely available (Panula-Ontto, 2017). The current implementation does not feature advanced association rule learning functionalities, but can output data that can easily be analyzed with, for example, free tools available for R environment, such as the `arules` package (R Core Team, 2014). The main analysis functionalities, iteration sets and intervention statements, are available in the AXIOM implementation. As mentioned in Section 4.1, the implementation does not yet support addition of non-simple updating functions directly from input, but this functionality will be added in the future.

5. Discussion

This paper gave a review of the various modeling approaches based on expert inputs, used in high abstraction level modeling of systems with modeling challenges related to lack of statistical data and exhibiting low quantifiability of important system characteristics. Against the background of this review, the design choices of these approaches were assessed with their fitness to expert informant elicited modeling process in mind. The identified design features with relatively high fitness for this purpose have been the outset for the design of AXIOM as a systems modeling approach. AXIOM proposes a combination of inference practices familiar from Bayesian networks and influence diagrams, and the best aspects of several techniques in the cross impact analysis tradition. The aim is to provide a flexible and expressive modeling language suitable for use in modeling using expert informants as the primary data source and versatile analysis facilities covering probabilistic, structural and morphological analytical outputs and insights.

Providing tools and techniques suitable for expert informant oriented systems modeling is important as it brings systems thinking and enables modeling based research in study of systems that would be difficult to model otherwise, using more traditional data driven techniques. Having approaches for modeling of such systems and system aspects adds important tools to modelers' toolbox and to decision support and planning activities. Expert informants as a data source enable adding important considerations to models, possibly improving decision-making by expanding the scope of foresight, strategy and policy-making. With suitable tools, systems modeling based on expert elicited inputs is, from the technical expertise requirement standpoint, easier than data-driven modeling. This may lower the threshold of using modeling as a research approach in fields where modeling is less used. Probabilistic models are also often more accessible and understandable from a model user standpoint: The logical and causal structure and the theoretical foundation of the model is very transparent compared to many data-driven models.

Composing formal representations of real systems is challenging regardless of what the used tools and approaches are, but the process is useful at multiple levels when dealing with complex systems and 'diabolical' decision-making contexts. The modeling itself, without any computational techniques aimed at discovery of higher-order information from the system model, partitions the expert-laden understanding of the system and the theory of its internal dynamics into an abstracted representation, useful in understanding the system and discussing its features. The formality of the model enables generic computational transformations that can reveal systemic and emergent properties of the model which are difficult to observe intuitively, without inference procedures.

6. Conclusions and future work

The development of the AXIOM approach and the software implementation is ongoing. A high priority update to the implementation is to add support for defining non-simple updating functions directly in the user input. Currently the implementation has several sets of updating functions, but defining new updating functions requires changes to the source code.

The modeling language can be expanded in a number of ways. The introduction of system descriptors representing continuous values is a possibility: such modeling primitives are available in software implementations of Bayesian networks and influence diagrams. While continuous value descriptors would increase the modeling power marginally, they are not strictly needed, as the same information can be represented with discrete state descriptors, and they are easier for expert valuators of the model, as the probability changes, modeled 'hazily' using the updating function approach, could be argued to be more predictable in their case.

Introducing ways to parameterize parts of an AXIOM model on the basis of statistical data instead of expert elicitation is an interesting idea and might widen the use sphere of the approach considerably, but such parameterization might prove challenging to do in a justified way. A more feasible approach to combine expert elicited modeling and data-driven approaches in the same framework is to perform the heavily expert informant based parts of a systems modeling process with the AXIOM approach, and use the AXIOM output in parameterization of a Bayesian network, which can then be augmented with statistical data.

The development of the software implementation from an ease-of-use perspective is probably more important for the adoption of the method than incremental modeling language expansions. Currently the implementation has no graphical user interface: the model is fed to the computer program in a text file. While the current implementation is completely sufficient to perform the analyses presented in this paper, creating a graphical user interface would lower the adoption barrier considerably.

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Appendix A. Pseudocode description of the AXIOM model evaluation

This appendix presents the pseudocode detailing the computational procedure of evaluating an AXIOM model and generating iterations and iteration sets. Algorithm 1 presents the process of model evaluation.

Algorithm 1. AXIOM model evaluation.

```
function evaluateModel(AXIOM Model m) : Configuration c
    for all unique timestep values t in m from lowest to highest t do
        ss ← statements in m that have timestep t
        shuffle(ss)                                   ▷ Place statements in random order
        for all Statement s in ss do
            evaluateStatement(s)
            add s.state to c
            ns ← non-simple impacts of m whose conditions are true
            while ns.count > 0 do
                n ← random element from ns
                n.updatingFunction(m)
                check validity of m
                update c if new statements have been resolved
                remove n
                ns ← non-simple impacts of m whose conditions are true
    return c
```

The model statements are evaluated in the order determined by their timestep properties. Statements with equal timestep property values are evaluated in random order. Simple probability updates, tied to a single cause, are performed in the statement evaluation procedure. When the model state changes, the non-simple impacts whose conditions are true are executed in random order, and removed after their execution. After a non-simple update, the model validity (mainly the probability distributions of option sets of statements) is checked. A non-simple update may resolve several statements of the model, so these state changes are updated to the configuration being created in the model evaluation. The non-simple update may also make conditions of other non-simple impacts true, so the list of non-simple impacts is repopulated after a state change.

Algorithm 2. AXIOM statement evaluation.

```
procedure evaluateStatement(Statement s)
    if s is an intervention statement then
        s.state ← s.model.activeIntervention(s)
    else
        r ← random real from the interval [0,1]
        sum ← 0
        for all Option o in s.options do
            sum ← sum + o.currentProbability
            if r ≤ sum then
                s.state ← o
    is ← shuffle(s.state.impacts)
    for all Impact i in is do
        probabilityUpdate(i)
```

The statement evaluation procedure (Algorithm 2) *a*) assigns a state for the statement, and *b*) for each *simple* impact the assigned state has, calls the procedure to effectuate the impact. The intervention statements have a predefined state in the model being evaluated, so they are simply assigned that predefined state; other statements are evaluated to one of their possible options according to the adjusted probability distribution of the statement's options. Impacts are placed in random order (shuffled) before being executed; this is to eliminate the effect the impact order might have on model evaluation results over the course of multiple model evaluations.

Algorithm 3. AXIOM simple probability update.

```
procedure probabilityUpdate(Impact i)
    P_new ← i.UpdatingFunction(i.effect.currentProbability)
    P_complement ← 1 − P_new
    for all Option o in i.effect.statement.options do
        if o is i.effect then
            o.currentProbability ← P_new
        else
            os ← i.effect.statement.options where option is not i.effect
            share ← o.currentProbability / (sum of current probabilities of Options in os)
            o.currentProbability ← P_complement × share
```

The procedure of a simple impact execution is presented in Algorithm 3. The probability of the effect option of the impact is updated according to the probability updating function pointed by the impact. The probabilities of the other options under the same statement as the targeted option are updated as well, to ensure the sum of the probabilities of the option set remains equal to 1. The complement probability of the updated probability of the effect option is divided to the other options so that each option's share of the new complement probability remains equal to their share of the old complement probability.

Algorithm 4. AXIOM iteration computation.

```
function computeIteration(Model m, iterationCount) : Iteration i
    for 1 to iterationCount do
        Configuration c ← evaluateModel(m)
        add c to Iteration i
        reset m to its initial state
    return i
```

The computation of an iteration (Algorithm 4) simply consists of performing the model evaluation multiple times and saving the resulting configurations to the iteration. The model structure, valuation and its active interventions are reset before each model evaluation during the computation of an iteration.

Algorithm 5. AXIOM iteration set computation.

```
function computeIterationSet(Model m, iterationCount): IterationSet is
    is.add computeIteration(m, iterationCount)      ▹ Single iteration without interventions
    repeat
        m.nextInterventionCombination
        add computeIteration(m, iterationCount)
    until all possible intervention combinations of m have been processed
```

The iteration set computation (Algorithm 5) consists of computing a single iteration without active interventions, and an iteration for each possible combination of options of the intervention statements.

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