# Relationship lending and switching costs <br> under asymmetric information about bank types 

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#### Abstract

This theoretical paper extends the pioneering articles on relationship lending (e.g., Sharpe 1990; Rajan 1992; von Thadden 2004) by examining relationship lending and hold-up problems in credit markets when borrowers are identical and banks are different. The results show that existing borrowers are informationally captured by good banks and yield profits to them, but new borrowers are unprofitable. In this market, short-term loan contracts and unsecured loans are optimal while loan commitments should not be used. Further, banks and borrowers have long-term relationships. This paper challenges the standard theories on product quality, reputation and experience goods by introducing scenarios in which good and bad banks can retain their existing borrowers. In the standard theories, consumers leave bad producers and search for good ones.


Keywords Financial intermediation • Relationship lending • Switching costs • Experience goods

- Reputation • Venture capital

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## 1 Introduction

The empirical evidence shows that switching costs may have dramatic consequences for credit markets. First, banks offer low interest rates to motivate borrowers to switch banks. After the switch, the new bank gradually raises the interest rate (e.g., Ioannidou and Ongena 2010; Barone et al. 2011). Second, lending rates often increase with the length of the relationship (e.g., Angelini et al. 1998; Degryse and Van Cayseele 2000; Hernandez-Canovas and Martinez-Solano 2010). Third, firms and banks have long lending relationships. For instance, Degryse and Ongena (2008) show that in Italy and France, these relationships last for over 15 years. In Germany, large firms have durations of more than 22 years. These findings mainly have a foundation built on the traditional models of relationship lending (e.g., Sharpe 1990; Rajan 1992; von Thadden 2004). According to these models, a bank that extends a loan to a firm learns more about the borrower's characteristics than other banks. A fundamental consequence of this asymmetric evaluation of information is the creation of ex-post market power. The bank has bargaining power over the firm's profits once projects have begun. Consequently, asymmetric information about the types of borrower generates switching costs to borrowers and excessive returns to banks. But competition drives banks to lend to new borrowers at interest rates that initially cause losses.

Do the traditional models of relationship lending provide the ultimate explanation for the observed structure of interest rates, loan maturities, switching costs, and lending relationships or is it possible to have an alternative explanation for these phenomena? The purpose of this paper is to construct an alternative model of switching costs and relationship lending. In the traditional models, there are various borrowers and identical banks in the credit markets. In this paper, the economy consists of identical borrowers and various banks (good and bad). Good banks grant relationship loans and thereby add value to the borrowers' projects. Bad banks extend transaction loans that have no value-added effects. More precisely, good banks (relationship lenders) improve
the probability of project success, while bad banks (transaction lenders) are unable to improve it. Banks are bad because their borrowers are worse off. Good banks and bad banks have the same probability of failure: they are riskless. Borrowers cannot observe whether a bank is good or bad, but the bank knows the characteristics of the identical borrowers. Borrowers learn the type of their bank during the loan period. Remarkably, this learning curve gives market power to the bank. If the initial bank is good, the borrower is reluctant to leave it. The borrower will retain the initial lending relationship even if outside banks offer better interest rates, because it cannot observe the types of the outside banks. The borrower does not want to take a risk of leaving a good bank for a bad one. The initial bank can take advantage of this captive borrower by increasing the interest rate. Competition motivates banks to lend to new borrowers at interest rates that initially cause losses.

Therefore, in the traditional models, the initial bank that has private information about the borrower causes a hold-up problem. The borrower has trouble switching banks because outside banks do not know the borrower's true quality. Owing to its information advantage, the initial bank can extract a surplus from the borrower. In our model, a borrower learns the bank's type by borrowing from the bank. If the bank is good, then the borrower is unwilling to switch to an outside bank due to the risk that is bad. Therefore, the borrower suffers a switching cost, because it does not know the true quality of the outside banks. This cost generates the hold-up problem. Thus, both models have rent extraction by the initial bank (the hold-up problem), but the mechanism is entirely different.

The paper relates to several strands of literature. First, researchers have made significant progress in understanding financial intermediation and delegated monitoring (e.g., Diamond 1984; Calomiris and Kahn 1991; Krasa and Villamil 1992,1993; Holmström and Tirole 1997; Bolton and Freixas 2000; Niinimäki 2004; Dinç 2006). Most of all, this paper builds on the research on relationship lending and switching costs in banking (e.g., Sharpe 1990; Rajan 1992; Chemmanur and Fulghieri 1994; Petersen and Rajan 1995; Dinç 2000; Boot and Thakor 2000; Egli
et al. 2006; Bolton et al. 2016). ${ }^{1}$ We add to this literature by suggesting a new model of relationship lending.

More precisely, Sharpe (1990) and Rajan (1992) established the traditional relationship lending models. The Sharpe (1990) model was later revised by von Thadden (2004). In these models, the setup includes two periods as in our model. Learning takes place in the process of lending during the first loan period. In these models, the economy has heterogenous borrowers and the initial bank (incumbent bank) learns what types its borrowers are during the first period. In our model, the economy has heterogenous banks and a borrower learns the type of the initial bank in the first period. Consider competition in the latter period. First, in our paper, a borrower knows the initial bank but does not know the types of other banks. The model has a pure strategy equilibrium in which good banks can retain their existing borrowers and bad banks lose their borrowers. In some other versions, bad banks can also retain their existing borrowers by lowering their interest rates to match the relatively poor quality of the bank. Second, in the traditional models, the initial bank knows the borrower's type, but the type is unknown to the outside banks in the second period. Due to the information advantage of the initial bank, the winner's curse impairs competition for borrowers. There is no equilibrium in pure strategies but an equilibrium in mixed strategies. A borrower may change banks in the equilibrium. Principally, a bad borrower switches banks whenever it gets an offer and a good borrower changes a bank with positive probability. Consequently, in our model and in the traditional models, banks informationally capture borrowers. In the latter, the captured borrowers are primarily good borrowers. In our model, good banks mainly

[^0]capture borrowers. Therefore, both models can be used to explain long-term lending relationships. The relationships consist of successive short-term loans. In both models, the expected return from an existing borrower is positive for the initial bank during the second period, and the return from a new borrower is negative. The models provide different recommendations. The traditional models, for example, emphasizes the transparency of borrowers while our model stresses the transparency of banks.

In Petersen and Rajan (1995), banks also bear losses in early stages that are offset by profit in the later stages. However, the mechanism is different than in the traditional models and in our model. Petersen and Rajan $(1995,440)$ summarize their theoretical findings:

Creditors in concentrated markets have an assurance of obtaining future surplus from the firm and consequently accept lower returns up front. This enables many more firms to be financed before the nominal interest rate rises to the point where the possibility of moral hazard forces the lender to cut off credit.

Petersen and Rajan (1995) find empirical support for their theoretical results. Young firms in concentrated markets receive more institutional finance than do firms in competitive markets. Creditors seem to smooth interest rates over the life cycle of the firm in a concentrated market by charging a rate that is lower than the competition when the firm is young and a rate that is higher than the competition when the firm is old.

In our model and in the traditional models, short-term loans are optimal. ${ }^{2}$ In the traditional models, short-term loans are useful, because a bank can acquire fresh borrower-specific information during the first loan period and use it to screen borrowers in the second period. Under short-term bank debt, the bank can easily terminate lending relationships with bad borrowers after the first period. In addition, a good borrower is free to look for a lower interest rate in the second

[^1]period. If a bank proves to be bad in our model, a borrower can leave it after the first period under short-term lending. The exit is difficult and expensive under long-term loans. Alternatively, under short-term debt, a borrower can threaten to leave the bad bank if it does not lower the interest rate to match its bad quality. That is, while the traditional models indicate borrower-based reasons for short-term loans, this paper brings forth a novel, lender-based theory for short loan maturities.

As discussed above, Rajan (1992) studies various borrower types. The problem of moral hazard also occurs in his model. Our model is similar in parts to Rajan (1992). However, there are important differences. In our model, a bank exerts effort in a project whose NPV is positive. In Rajan (1992), an entrepreneur exerts effort in their project and the NPV of the project may be negative. Then, liquidating the project is optimal. Under long-term bank debt, the bank must bribe the borrower to liquidate the bad project and terminate the loan. With short-term bank debt, the termination is easy. Hence, short-term bank loans are optimal. However, they are problematic in other scenarios. If the ongoing project has a positive NPV, the bank can threaten to interrupt it by denying to rollover the loan. To prevent the interruption, the borrower must bribe the bank. This negative effect is avoided with long-term bank debt. Consequently, both loan maturities have pros and cons. Under some scenarios, a borrower favors a short-term bank loan. Sometimes, a long-term bank loan is optimal. In addition, the bargaining power of the bank may reduce the entrepreneur's incentives to exert effort. To eliminate the excessive power of the bank, the borrower may prefer long-term arm's length debt to bank loans. As mentioned earlier, short-term bank loans are optimal in our model: a borrower can leave a bad bank after the first period under short-term lending. A borrower may favor arm's length debt (or a transaction loan) only in the second period if the initial bank proves to be bad.

In the traditional models, switching costs raise the interest rate in the second period and lower it in the first period. Both borrower types receive a loan in the first period but only good borrowers get a loan in the second period. Owing to the excessively low interest rate in the first
period, the bank shifts the allocation of capital to inferior firms that decreases production. The high interest rate of the second period incurs extra costs for good borrowers and may reduce their incentives to exert effort. We avoid these negative effects in our model because borrowers are identical and the borrowers' effort aversion problem is not present. The excessive interest rates in the second period are competed away via lower interest rates in the first period. In our model, the existence of bad banks and asymmetric information regarding the bank types decreases production, because a few borrowers choose a bad bank by accident. Under perfect information, only good banks can attract borrowers. Due to the unobservability of bank types, borrowers shift the allocation of capital to inferior banks. This negative effect on production is strongest in the first period, because a borrower may leave the initial bad bank in the second period.

Hauswald and Marquez (2006) advance the models of Sharpe (1990), Rajan (1992) and von Thadden (2004) by endogenizing the information acquisition in the first period. In Hauswald and Marquez (2006), banks are different because of specialization. In contrast to the traditional models, a bank does not learn the types of its borrowers at no cost during the first period. The information of the initial bank increases with the bank's investment in information acquisition. Banks overinvest in information to soften competition and extend their market share. ${ }^{3}$ Hauswald and Marquez (2006, p. 976) interpret their model as follows: "...every bank offers two types of loans in equilibrium: relationship lending based on information it has acquired about borrowers in its captive market and transactional lending outside this core segment." Consequently, the difference between relationship loans and transaction loans is based on the bank's information advantage. By contrast, in our model transaction loans have no effect on the project output while relationship lending boosts the output. The same idea, the value added effect of relationship lending, is present in a few papers (Chemmanur and Fulghieri 1994; Boot and Thakor 2000; Dinç 2000; Bolton et al.

[^2]2016). In these papers, the bank's role is different from the traditional models in which relationship banks focus on information acquisition, screening, and interim monitoring.

In detail, Chemmanur and Fulghieri (1994) assume that the firm's choice is between issuing bonds and borrowing from a given bank. If a firm is in financial distress, the lender (the bank or a bondholder) either liquidates the project or renegotiates the debt and allows time to overcome the problem. Under bond financing, mistakes in this reorganization process are common. Therefore, relatively risky firms favor banks. The bank can make a correct choice, liquidate, or renegotiate by acquiring information. The cost of information acquisition varies among bank types. Both bank types, low-cost and high-cost banks, aim to develop a reputation to make the correct choice in the reorganization process. The information structure of the model differs from our model in two ways. First, a high-cost bank (bad bank) can hide its true type by exerting effort and charging the same interest as the low-cost bank. Then, the borrower cannot discover the type of the initial bank in the process of lending. Second, the bank's choice in the first period (liquidate or renegotiate) becomes common knowledge before loans are made in the second period. Therefore, outsiders may learn the type of the high-cost bank if it does hide the type. These two features make hiding very profitable (the reputation effect). In our model, a bad bank can also hide its type by charging the same interest as the good bank. In contrast to Chemmanur and Fulghieri (1994), a bad bank cannot hide its type during the loan period: the borrower learns the type of the initial bank in the process of lending. However, the type does not ever become common knowledge as in Chemmanur and Fulghieri (1994), that is, outsiders do not learn the type in our model. Therefore, when a borrower leaves the initial bad bank and searches for a new bank, they make a random choice. In the Chemmanur and Fulghieri (1994) model, a firm cannot switch banks, because the economy has only one bank. However, the firm can replace the initial bank with bonds. This alternative constrains the interest rate of the bank loan. To be able to retain the borrower and charge a higher interest rate in the future, a bad bank hides its true type. Given the monopoly position, the bank makes excessive
profit. In our paper, the banking sector is fully competitive. Dinç (2000) and Bolton et al. (2016) elaborate on this research.

In Bolton et al. (2016), a bank is either a relationship bank or a transaction bank. A relationship bank can use its borrower-specific information during a financial crisis by offering greater security in the loan rollover for good firms than transaction banks. A borrower may select one bank type or favor a combination of loans. Since borrowers are different, the bank types are observable and all borrowers have one project, the model differs strongly from our model in which borrowers are identical and bank types are unobservable. Further, borrowers undertake two sequential projects and can switch banks.

In Dinç (2000), a bank can provide relationship loans or arm's length loans. A relationship loan includes a commitment to give a rescue loan to a distressed borrower. This loan is unprofitable to the bank but profitable to the borrower. In an infinitely repeated game, a bank's commitment to rescue a borrower can be enforced by its reputation. If the bank does not rescue the borrower, the future borrowers do not use that bank. Alternatively, banks can grant arm's length loans without the commitment. The model differs from our model in three important ways. First, all banks can offer relationship loans and transaction loans. Second, borrowers live for a period and undertake one project. Third, the bank's decision to keep its commitment is publicly observable. By contrast, in our model, banks are different, borrowers live for two periods and undertake two sequential projects, and the bank's action is private information. In Dinç (2000), a borrower does not leave the initial bank and start a random search for a better bank as in our model.

In Boot and Thakor (2000), banks are different, because they specialize in different sectors. A relationship lender has a value added effect on the borrower's project if the project represents the sector of specialization. The model differs from our model in three significant ways. First, each bank can extend relationship loans and transaction loans. Second, borrowers (many types) live for a period and undertake one project. Third, the bank's type (specialization) is
observable. In our model, first a few banks can provide only transaction loans and the other banks supply relationship loans. Second, identical borrowers live for two periods and undertake two sequential projects. Third, the bank's type (a relationship lender or a transaction lender) is unobservable. This difference in approach is important. In Boot and Thakor (2000), a borrower does not learn the type of the initial bank during the first project and does not begin a random search for a better bank in the second period as in our model.

Both Boot and Tharkor (2000) and Dinç (2000) assume imperfect competition and explore how intensified competition changes the incentives to create banking relationships. This is different from the problem we study, in which banks operate under perfect competition. In Boot and Thakor (2000) and Chemmanur and Fulghieri (1994) relatively good borrowers favor bonds or transaction loans, while Dinç (2000) finds the opposite result. In our model, a borrower may favor these instruments in the second period if the initial bank proves to be bad. Our model can also be used to study the effects of intensified competition in the capital market. If transaction loans and arm's length lending become relatively more inexpensive compared to relationship loans, borrowers replace bank loans with arm's length financing (e.g., bonds). Consequently, Boot and Thakor (2000), Dinç (2000) and Bolton et al. (2016) use the same idea as we do: the value added effect of banks. However, in neither paper is the bank type unobservable, which is the main feature we are interested in. Intertemporal taxes and subsides in the traditional models - new borrowers are unprofitable and existing borrowers profitable - have no role in these three models. This is different from the problem we study. We link the value added role of banks (Chemmanur and Fulghieri 1994; Boot and Thakor 2000; Dinç 2000; Bolton et al. 2016) and the intertemporal subsides of the traditional models. Our main results can be derived in the Boot and Thakor (2000) setup if we extend it by assuming two sequential projects and unobservable bank types.

Reputation plays a crucial role in our model. Borrowers learn the type of the initial bank, and the bank represents the same type in the second period. Both a bank and its borrowers
know the type of bank. Reputation formation is important also in Chemmanur and Fulghieri (1994), Dinç (2000), and Egli et al. (2006). Egli et al. (2006) explore the choice between relationship loans and arm's length lending. In their model, strategic defaults are likely. Relationship lending allows an entrepreneur to build a private reputation for repayment that reduces the cost of financing. However, in an environment where the risk of strategic defaults is low, the benefits of reputation building are outweighed by the rents extractable by the initial bank. Now borrowers favor arm's length loans. ${ }^{4}$

Furthermore, our paper contributes to the literature on product quality, reputation, switching costs, and experience goods (e.g., Shapiro 1983; Klemperer 1987, 1995; Villas-Boas 2006). Borrowers cannot verify the quality of an experience good prior to the initial purchase. The bank type is now an experience good, since a borrower can only evaluate the bank after the start of the loan period. Two effects occur if a bank can increase the probability of a project's success. First, the borrower's expected profit grows. Second, the expected repayment income of the bank increases. The latter effect is new because it does not occur in standard models on experience goods. In these models, only the customer benefits from high quality. In banking, both the producer (a bank) and the customer (a borrower) benefit from high quality (the bank increases the probability of a project's success). Therefore, in contrast to the standard models of experience goods, a relationship lending bank is motivated to exert effort to improve quality even under perfect competition and in a model of one period (i.e., in the absence of reputation effects). Our contribution to this literature is to explore experience goods in banking. Above all, we propose an interesting extension to the model of switching costs and experience goods. In the standard models of experience goods, customers abandon a bad firm when they discover the poor quality of its products and search for a new producer. In our model, poor quality service might lead a borrower to switch banks. However, we introduce scenarios in which a bad bank can retain existing borrowers. First, if borrowers do not

[^3]know the average quality of banks, then poor quality service will lead them to lower their estimate of the average quality of banks and this estimate will lead to sticking with the current bad bank. Second, if the borrower's expected lifespan is long (in period 1), they will want to look for a good bank, because they will enjoy a good customer-bank relationship for a long time. If the expected lifetime is short (in period 2), then looking for a good bank is uneconomical and retaining the existing relationship with the bad bank is optimal. Therefore, both bad banks and good banks could retain their existing borrowers. These findings provide new explanations for remarkably long lending relationships. Most of all, the findings can be used generally in the analysis of experience goods and switching costs.

Finally, collateral has a positive role in lending (see, e.g., Bester 1985; Niinimäki, 2018), because it mitigates the borrower's incentive problems. By contrast, in our model a lender has an effort aversion problem. The borrower's collateral assets alleviate the lender's incentives to exert effort, and unsecured lending is optimal. Borrowers can use loan commitments, which fix future interest rates, to protect themselves against the market power of initial banks. Maybe surprisingly, loan commitments are not optimal in our model. They increase the interest income of bad banks in the first period, and their borrowers pay the increase.

The paper proceeds as follows: In Section 2, we outline the model, and Section 3 conveys the main results. Sections 4 and 5 contain models in which both bank types can retain existing borrowers. In Section 6, we survey the evidence on how banks can increase the probability of a project's success. Then, we discuss the conclusions of this study in Section 7.

## 2 Economy

Consider a risk-neutral economy with two periods and a risk-free gross interest $r, r=1$. The economy has three players: entrepreneurs, firms (borrowers) and banks (lenders). Firms are run by entrepreneurs who maximize their expected business return, while banks maximize their expected profits. Each firm can undertake a similar short-term project in both periods. It requires a unit of input at the start of the period and produces stochastic output at the end of the period. A successful project produces output $Y$, and an unsuccessful project has no value. If a project is unsuccessful in period 1 and the firm fails, the entrepreneur can set up a new firm in period 2. Consequently, the difference between firms and entrepreneurs is that firms may fail in period 1 while entrepreneurs continue to period 2. Entrepreneurs have no money of their own, and they raise the investment capital by selling bonds or borrowing the amount needed from a bank. Entrepreneurs are protected by limited liability: if a firm fails in period 1 , the entrepreneur does not bear the costs of the failure in period 2.

The financial system consists of bond and bank markets. Under bond financing, a project is successful with probability $p$ and has a positive NPV, $p Y>r$. The interest rate is $R_{T}, R_{T}=r / p$. In the bank market, banks compete for borrowers and pay gross interest $r$ on deposits. A bank is either a relationship lending bank (i.e., a good bank) or a transaction bank (i.e., a bad bank). The bank's type is private information and a bank cannot change its type. It remains a good bank or a bad bank in both periods. Bad banks grant transaction loans and bank formation incurs a fixed cost $C_{B}$ at the start of period 1. A project succeeds with probability $p$. Since the probability of success is the same under bond financing but transaction loans are more expensive (cost $C_{B}$ ), transaction loans represent a bad financial instrument.

Good banks grant relationship loans and can increase the probability of project success. A project is successful with certainty under relationship lending. A relationship loan is relatively expensive, because expertise in relationship lending entails $\operatorname{cost} C_{G}, C_{G}>C_{B}$, per a loan in both periods. This cost can be viewed as the bank's investment in the physical infrastructure of branches, employees, and related expenses. A good bank can make one new relationship loan in a period. This restriction limits the maximum size of a good bank: one loan in period 1 and two loans in period 2. Relationship lending incurs a non-verifiable cost $c_{v}$ per loan if the bank uses its expertise. Thus, good banks have a potential incentive problem. The bank can use its expertise only if the project is about to fail (probability $1-p$ ). Therefore, the expected costs of the relationship loan totals $C_{G}+(1-p) c_{v}$ in both periods. The breakeven gross interest rate for a relationship loan is:

$$
\begin{equation*}
R=r+C_{G}+(1-p) c_{v} . \tag{1}
\end{equation*}
$$

The output covers the costs of relationship lending, $Y>R$. We make the following assumption.

Assumption $1(1-p) Y>C_{G}+(1-p) c_{v}$.

Assumption 1 means that relationship lending has a positive NPV. A good bank increases the expected output by $(1-p) Y$. That is, a borrower encounters financial distress with probability $1-$ $p$ when the project is about to fail. A good banker identifies the financial distress and can help the firm avoid the failure. The bank is motivated to exert effort in the project, because the cost of effort is lower than the increase in the bank's repayment income, $c_{v}<R$ (an assumption). Owing to

Assumption 1, borrowers prefer relationship loans to bonds and transaction loans. A bad bank can attract borrowers only if it misrepresents its type and mimics good banks by selling loans that are de facto transaction loans. Borrowers are unable to screen bank types and may choose a bad bank by mistake. Assumption 2 ensures that a transaction loan yields a profit for a bad bank.

Assumption $2 p R>r+C_{B}$.

A good bank has an external option. Owing to its expertise in relationship lending, it can operate either in the bank market or in the external market. If the interest rate in the bank market is insufficient, a good bank moves to the external market in which it can always gain a breakeven return on the investment in expertise. This external option ensures that relationship banks do not compete too aggressively in the bank market under the sunk costs of expertise, $C_{G}$. Therefore, the external option determines a lower boundary for the interest rate $R$. In addition, banks have other incomes, and they do not go into bankruptcy. These outcomes mean that banks do not benefit from limited liability.

We make three assumptions that are standard in the relationship lending theory:

A1 Loans are short term and last for a period.

A2 Banks cannot pre-commit to future interest rates. The interest rate in period 1 is set at its beginning and the interest rate in period 2 is set at its beginning.

A3 If projects are successful in period 1, borrowers consume the profit at the end of the period. That is, they cannot invest the profit in the firm or pledge the profit as loan collateral.

The number of good bankers is the same as the number of borrowers. Good bankers arrive first to the market. Bad banks enter later. Bad banks see the number of banks and enter the market sequentially until the expected profit from the entry is zero. Let $N$ label the total number of banks (very large), and $n$ indicates the number of borrowers. The proportion of bad banks is $b$. The rest of the banks, $g=1-b$, are good. The proportions are public information. Now, borrowers arrive to the market. Figure 1 portrays the borrowers' choices in equilibrium.


Fig. 1 Uncertainty structure and project payoff

With probability $g$, a firm contacts a good bank at the start of period 1 . The top line in Figure 1 illustrates this scenario. The firm learns the bank's type during period 1. In equilibrium, the good bank offers an interest rate for period 2 such that the borrower retains the initial lending relationship. With probability $b$ the initial bank is bad (the lower knot in Figure 1). Again, the firm learns the bank's type during period 1 . Whether the first project is successful or not, the borrower leaves the initial bank after period 1 and contacts a new bank. That bank is good with probability $g$ and bad with probability $b$.

## 3 Good banks retain existing borrowers

This section presents the model in which good banks can retain existing borrowers after period 1. New borrowers are unprofitable to good banks, and existing borrowers yield profits. In equilibrium, the proportion of bad banks is such that both bank types make zero lifetime profit.

Since a bad bank can attract borrowers only by misrepresenting its type and mimicking good banks, it must supply loans at the same interest rate as good banks.

Assumption $3 \beta(Y-R)>p\left(Y-R_{T}\right), \quad \beta=(1-b)+b p$.

The proportion of bad banks $b$ and the probability of project success without relationship lending $p$ are public information, $R_{T}=r / p ; R$ is defined in equation (1). The left-hand side of Assumption 3 of the inequality shows the borrower's expected profit if they apply for a relationship loan. With probability $1-b$ they contact a good bank that provides a relationship loan. With probability $b$ the bank is bad and grants a loan that is de facto a transaction loan and the project succeeds with probability $p$. The right-hand side indicates the expected profit if the borrower chooses bonds (or a transaction loan). Further, the borrower benefits from limited liability. Assumption 3 expresses that the proportion of good banks $1-b$ is relatively high in the economy and thus, the quality of the average relationship loan is quite high. Therefore, a borrower prefers an expensive relationship loan, $R>R_{T}$, to inexpensive financial instruments. Assumption 4 ensures that the proportions of good and bad banks are fixed.

Assumption 4 If a loan yields a breakeven return to a bank, the bank finances the project.

This kind of game is solved by backward induction. We first need to characterize competition in period 2 ; then we examine period 1 .

### 3.1 Bank market in period 2

Since the bank's type is unobservable, a borrower is ignorant about whether their initial bank is good or bad at the start of period 1 and an evaluation of the bank can only occur after a period as a borrower. That is, a borrower learns the type of the initial bank during period 1 but remains uninformed about the types of other banks. In period 2, a bank can set different interest rates to its existing borrower (who knows the type of the bank) and to the existing borrowers of other banks (who do not know the type).

The sequence of events is as follows in period 2. (i) Bond markets bid interest $R_{T}=$ $r / p$. Steps (ii) and (iii) are the same as in Sharpe (1990, 1076): "A bank makes offers to its old customers first. Then outside banks offer contracts, behaving like Bertrand competitors with respect to each other." Outside banks observe the offer of the initial bank in our model. (iv) Borrowers choose the source of financing. (v) Banks attract deposits and grant loans. (vi) The project outputs mature at the end of the period and the borrower settles the claims. In equilibrium, borrowers never choose bonds.

More precisely, borrowers treat price as a signal of quality. We restrict our attention to a perfect Bayesian equilibrium in which off-the-equilibrium-path loan offers that undercut the breakeven offers on the interest rates of good banks are interpreted by borrowers as a signal that the
bank is bad. The interpretation is correct because of the external option of good banks. Therefore, $R$ is the equilibrium short-term interest rate that outside banks offer in period 2.

The initial bank is good (the top line of Figure 1 after period 1). If a borrower switches banks after period 1 and contacts an outside bank for a relationship loan, their expected return in period 2 is:

$$
\begin{equation*}
\pi_{2}^{\text {out }}=\beta(Y-R), \quad \beta=g+(1-g) p . \tag{2}
\end{equation*}
$$

Here $R$ is the offer of outside banks, that is, the breakeven interest rate in short-term lending. A new bank is good with probability $g$ and bad with probability $1-g$. Since the initial bank is good, the borrower favors it. The initial bank knows this and can extract a surplus from its existing borrower. The bank offers interest $R_{2}^{G}$ for period 2 such that the existing borrower stays with the initial lending relationship. Under $R_{2}^{G}$ the borrower's expected return is:

$$
\begin{equation*}
\pi_{2}^{G}=Y-R_{2}^{G} \tag{3}
\end{equation*}
$$

because the initial bank increases the probability of project success. The initial bank sets $R_{2}^{G}$ such that the existing borrower is indifferent to the initial bank or the outside banks: $\pi_{2}^{G} \geq \pi_{2}^{\text {out }}$, or

$$
\begin{equation*}
R_{2}^{G}=R+(1-\beta)(Y-R), \tag{4}
\end{equation*}
$$

or $R_{2}^{G}=R+b(1-p)(Y-R)$. Here $R_{2}^{G}$ rises with $1-p$ and $b$. Since $R_{2}^{G}>R$, the initial bank can take advantage of the existing borrower and charge a higher interest rate than outside banks, $R$. Again, the borrower knows that the initial bank is good. If the firm switches banks it takes a risk that the new bank is bad. The risk is significant if the proportion of bad banks, $b$, is high. This proportion creates market power for the initial bank and the existing borrower yields profit, $R_{2}^{G}>$ $R$. The market power strengthens with the value of the relationship lending services, $(1-p)(Y-$ $R)$. Therefore, we propose:

Proposition 1 Good banks keep existing borrowers in period 2. Existing borrowers yield profits to good banks.

Why does an outside bank not take a mixed strategy of offering an interest rate below that of the initial good bank $R_{2}^{G}$ with a positive probability? To answer this question, consider a good outside bank that has an option to operate in the external market. There, a good bank can always gain a breakeven return $R$. Hence, a good outside bank charges at least $R$ in the credit market if it aims to attract borrowers from other banks. To mimic good outside banks and to be able to attract borrowers, a bad outside bank must bid at least $R$. However, the initial good bank charges $R_{2}^{G}>R$. Why can an outside bank not bid $\left.R_{Z}, R_{Z} \in\right] R, R_{2}^{G}$ [ to attract borrowers? Consider the outside offer $R_{Z}$. The borrower of the initial good bank does not know the type of the offering bank. With probability $g$, the outside bank is good and with probability $1-g$ it is bad. Therefore, the expected quality of the outside loan is worse than the known quality of the inside loan (good quality). The borrower will keep the existing relationship with the initial good bank (and interest rate $R_{2}^{G}$ ) even if the outside bank offers a lower interest rate $R_{Z}$ because the type of the outside bank is unobservable and the expected quality of the outside bank is relatively poor. The borrower is ready
to pay more for high quality (the initial good bank) than for an average quality (a random outside bank), More precisely, the borrower is indifferent between the inside rate $R_{2}^{G}$ and outside offer $R$, and outside banks charge at least $R$. Owing to the switching costs, the initial bank can exploit the existing borrower by charging a higher rate that is just low enough to ensure that the borrower is still better off by not switching.

The initial bank is bad (the lower knot of Figure 1 after period 1). If a borrower switches banks after period 1 and applies for a new relationship loan, the expected profit emerges in equation (2). If the borrower retains the initial relationship, the bad bank grants a transaction loan, and the new project succeeds with probability $p$ in period 2 . The probability is the same as under bond financing. Since the interest rate is $R_{T}$ in the bond market, the initial bank can charge at most $R_{T}$. It covers the variable costs of the transaction loan but does not cover the sunk cost $C_{B}$. Under this interest rate, the borrower's expected profit, $p\left(Y-R_{T}\right)$, is lower than the expected profit from the relationship loan, $\beta(Y-R)$, according to Assumption 3. Therefore, the borrower leaves the initial bank after period 1 for the relationship loan offers of outside banks.

Proposition 2 Bad banks lose their existing borrowers after period 1.

When the borrowers contact new banks in period 2, each bank receives a new borrower with the same probability. This probability means that each bank receives zero or one new borrower. Since new borrowers pay interest $R$ in period 2 , they are profitable for bad banks and yield zero profit for good banks.

Competition. Alternative I. We make the same assumption as Sharpe (1990, 1076): "A bank makes offers to its old customers first. Then outside banks offer contracts, behaving like

Bertrand competitors with respect to each other." Now the interest offers are observable, and a borrower is indifferent between inside bid $R_{2}^{G}$ (the good initial bank) and outside bid $R$. If the bid of the initial good bank exceeds $R_{2}^{G}$, an outside bank can later offer $R$ and grab the borrower. Obviously, the initial bank bids $R_{2}^{G}$ to deter outside banks from making profitable counteroffers. To hide its true type, a bad initial bank also makes public offer $R_{2}^{G}$ to its existing borrower. Competition for switching borrowers decreases the interest rate to the lower limit $R$ among outside banks. Moreover, a borrower accepts offer $R_{2}^{G}$ if the initial bank is good and contacts a random outside bank with offer $R$ if the initial bank is bad. ${ }^{5}$ We assume this type of competition (Alternative I).

Many forms of competition imply the same results. Consider Alternative II: a descending-bid auction. Each bank declares an interest rate for its existing borrower and a different rate for outside borrowers. Each bank observes the competitors' offers and can decrease its offers. Banks undercut prices until the interest rates for outside borrowers decline to the lower limit $R$. Good initial banks offer $R_{2}^{G}$ to their existing borrowers. To hide the bad type and to be able to attract new borrowers, bad banks make public offer $R_{2}^{G}$ to their existing borrowers. Further, a borrower accepts offer $R_{2}^{G}$ if the initial bank is good and switches banks if the initial bank is bad.

In Alternative III, banks declare interest offers in the free market. Competition is perfect. Then, after observing the interest offers, the initial bank can make the final offer to its existing borrower in secret. The justification for this secrecy is that any borrower can use the threat

[^4]of leaving and obtaining the free market rate to bargain for a lower rate from the initial bank. ${ }^{6}$ Competition decreases the interest rates to the lower limit $R$ in the free market. Thereafter, good initial banks can bid $R_{2}^{G}$ to their existing borrowers and retain these borrowers. The borrowers of bad banks switch banks.

In these three alternatives, perfect competition decreases the interest rate of outside loans to $R$ and initial good banks can retain the existing borrowers by setting $R_{2}^{G}$. Consequently, the form of competition is rather irrelevant. In sections 4 and 5, however, we must assume Alternative III. It is important that a bad initial bank can make its offer in secret. A public offer would reveal the bank type (bad).

Nash equilibrium. Good banks offer $R_{2}^{G}$ to their existing borrowers and keep these borrowers. Borrowers leave bad banks. Outside banks attract these borrowers by setting the interest rate $R$. This clearly is a Nash equilibrium. Consider a good initial bank that offers $R_{2}^{G}$ to its existing borrower. The borrower does not observe the types of outside banks but knows that an outside bank is good with probability $g$ and bad with probability $1-g$. The borrower is indifferent to inside bid $R_{2}^{G}$ or outside bid $R$ and retains the initial bank relationship. If the bank lowers the offer, it reduces its profits, because the borrower is profitable and willing to pay $R_{2}^{G}$. If the bank raises the offer, it reduces profits, because the bank loses this profitable borrower to an outside bank. For an initial bad bank, it loses the existing borrower after the first period and cannot prevent the loss by charging its breakeven rate $R_{T}=r / p$.

Good outside banks attract switching borrowers by bidding $R$. The bank does not lower the bid, because good banks can earn reservation return $R$ in the external market. If the bank

[^5]raises the bid, it cannot attract new borrowers because other outside banks charge $R$. However, a good bank will get a new borrower (Assumption 4). A bad outside bank attracts new borrowers by bidding $R$. This rate yields profits to the bank. If it raises the interest, the bank reduces its profits: the bank cannot attract profitable new borrowers, because other outside banks offer $R$. If the bank decreases the interest rate, the bank cannot attract borrowers. The decrease displays the bank type (bad) to borrowers, and borrowers avoid bad banks. A good bank does not offer these low interest rates, because it can make higher returns in the external market.

### 3.2 Bank market in period 1

This subsection examines the interest rate in period 1. The timeline is the following: (i) banks make public offers of relationship loans, and (ii) borrowers accept the best offer. Since borrowers prefer relationship loans to other loan types in period 1 (Assumption 3), we can limit ourselves to relationship loans. A good bank anticipates correctly that it can retain a lending relationship in period 2. A borrower will yield profit $(1-p) b(Y-R)$ to the bank in period 2. To achieve this future profit, good banks compete fiercely for new borrowers in period 1. Under perfect competition, a lending relationship yields zero profit in its lifetime.
$R_{1}-R+\delta(1-p) b(Y-R)=0$.

The first two terms together show the expected return in period 1. Here the first (second) term indicates the repayment income (costs of lending). The third term refers to the profit in period 2. Given the profit, the return in period 1 must be negative. A lending relationship yields a negative return in period 1 and a positive return in period 2. Equation 5 shows the interest rate in period 1, $R_{1}$. To be able to attract borrowers, bad banks mimic good ones and bid interest rate $R_{1}$.

Proposition 3 In period 1, each bank bids $R_{1}=R-(1-p) b(Y-R)$. The interest is lower in period 1 than in period 2 . Since $R_{1}<R$, a loan is unprofitable for a good bank in period 1 .

### 3.3 Noisy signal

In Sharpe (1990), a bank receives a noisy signal for the borrower type. What is the effect of this noisy signal in our model? This is examined in Appendix A. If the quality of the signal improves (that is, the probability that a good signal is revealed by a good bank increases), the transparency of the banking system enhances: it is easier to separate good and bad banks. The enhanced transparency erodes the market power of the initial bank in period 2 . This erosion lowers the interest rate in period 2 and raises the interest rate in period 1 . However, if the signal has no value, the interest rates are the same as above: $R_{1}$ in period 1 and $R_{2}^{G}$ in period 2. If the signal is perfect, the interest rate is $R$ in both periods.

In Chemmanur and Fulghieri (1994), a borrower cannot learn the bank type in the process of lending if there is no informational event to judge the bank's behavior. This kind of event arises with probability $1-p$ in our model. Assume that a borrower learns the type of the initial bank with probability $1-p$. Outsiders cannot observe the bank type. The change erodes the market power of the initial bank in period 2: it has market power with probability $1-p$. This power lowers the expected interest rate in period 2 and raises the interest in period 1.

### 3.4 The proportion of bad banks in equilibrium

This subsection analyzes the entry of bad banks. The number of bad banks is such that their expected profit is zero.

A good bank makes zero return if it operates entirely in period 2 or in both periods. The return is negative if it operates only in period 1 . Therefore, a good bank is ready to operate in both periods. For a bad bank, the expected return is positive, $p R-r>0$, in period 2 , and positive or negative, $p R_{1}-r$, in period 1. If the return is negative in period 1 , a bad bank maximizes its return by operating in period 2 . We assume that this is impossible. Firms observe the life cycles of the banks and are unwilling to borrow if a bank operates only in period 2 . This strategy is a negative signal on bank quality. Therefore, each bank operates in both periods.

As discussed, the number of good banks is the same as the number of borrowers, $n$. First, good bankers open banks. Thereafter, bad bankers open banks successively until the expected return from the entry is zero. The total number of banks is $N$. The proportion of bad (good) banks is $b(g=1-b)$, and the number of banks exceeds the number of borrowers, $N>n$. Therefore, the probability that a bank receives a borrower in period 1 is $n / N=1-b$. A bank can go on operating in period 2 only if it receives a borrower in period 1 . As seen earlier, a good bank can retain its existing borrower. The borrowers of bad banks leave their initial banks after period 1 and search for new banks but are unable to separate good and bad ones. The number of departing borrowers totals $b(1-b) N$. These borrowers are shared equally between $(1-b) N$ operating banks, and each bank receives either zero or one new borrower at the start of period 2. A bank receives a new borrower in period 2 with probability $b$. Since a bad bank loses its existing borrower after period 1 and receives a new borrower with probability $b$, it has zero or one borrower in period 2. Since a good bank can retain its existing borrower and receives a new borrower with probability $b$, it has one or two borrowers in period 2. A new, arriving borrower pays interest rate $R$ in period
2. This rate covers the cost of lending if the new bank is good. Therefore, the appearance of new borrowers in period 2 does not increase the returns of good banks. On the contrary, if a new borrower contacts a bad bank in period 2, the borrower yields an expected profit to the bank, $p R>r$.

For a good bank, if it does not receive a borrower, it can use its expertise in external markets and make the same profit as in the banking sector. The good bank does not lose anything if it develops expertise but is unable to attract a borrower. Consequently, the good bank is willing to establish expertise in period 1.

For a bad bank, the cost structure is different. If a bad banker establishes a bank, he bears cost $C_{B}$. The banker cannot use this know-how elsewhere and bears the cost of establishment even if the bank cannot attract a borrower. The expected profit of the bad bank is:
$\pi_{B}(b)=(1-b)\left(p R_{1}-r\right)+\delta(1-b) b(p R-r)-C_{B}$.

Note that $r=1=\delta$. With probability $1-b$, the bank can attract a borrower in period 1 and makes the expected return $p R_{1}-r$. The bank can go on in business in period 2 only if it has a borrower in period 1 , that is, with probability $1-b$. It loses the existing borrower after period 1 and receives a new one with probability $b$. The borrower yields an expected return $p R-r$. The bank bears the sunk cost of expertise, $C_{B}$, with certainty. Appendix B proves the following result:

Lemma 1 There is a $b_{\text {max }}, 0<b_{\text {max }}<1$, such that $\pi_{B}\left(b_{\text {max }}\right)=0$.

When $b<b_{\text {max }}$, we have $\pi_{B}(b)>0$ and $d \pi_{B} / d b<0$. The expected profit of a bad bank is positive. This profit drives their entry. The entry reduces the expected profit and continues until $b=b_{\max }$. Then, the expected profit of bad banks is zero, and the entry stops. If the proportion of bad banks exceeds (is lower than) $b_{\max }$, their returns are negative (positive).

In equilibrium, the banking sector is fully competitive. Both bank types make zero profit. As a result, a bank does not want to change its type. It is rather hard to construct a model that meets these requirements. Therefore, the model may seem to be overly complicated.

### 3.5 Example 1

Values in all scenarios: $p=0.93, Y=1.36, C_{G}=0.055, c_{v}=0.5, r=1$.

| $C_{B}$ | 0.0091575 | 0.0107 | 0.0117 | 0.01368 | 0.0137 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $b$ | $27.513 \%$ | $17.764 \%$ | $11.68 \%$ | $0.1138136 \%$ | $0 \%$ |
| $R$ | 1.09 | 1.09 | 1.09 | 1.09 | 1.09 |
| $R_{2}^{G}$ | 1.0952 | 1.093357 | 1.0922 | 1.090002 | 1.09 |
| $R_{1}$ | 1.0848 | 1.0866 | 1.0878 | 1.089998 | 1.09 |
| $\beta$ | $98.07 \%$ | $98.7565 \%$ | $99.18 \%$ | $99.992 \%$ | $100 \%$ |
| $p\left(Y-R_{T}\right)$ | 0.2648 | 0.2648 | 0.2648 | 0.2648 | 0.2648 |
| $\beta(Y-R)$ | 0.2648 | 0.2666 | 0.2678 | 0.27 | 0.27 |
| $Y-R_{2}^{G}$ | 0.2648 | 0.2666 | 0.2678 | 0.27 | 0.27 |

Table 1 Borrowers leave bad banks after period 1

In Table 1, the probability of success without relationship lending $p$, the output of a successful project $Y$, the costs of relationship lending $C_{G}$ and $c_{v}$, and the interest rate of the economy $r$ are fixed. Table 1 presents the economy under five different cost levels of bad banks $C_{B}$. Inserting $R_{1}$
from Proposition 3 into eq. (6) gives $13.7-17.577 b+3.877 b^{2}=1000 C_{B}$ that shows the proportion of bad banks $b$ declines with $C_{B}$. Table 1 displays the same information. Again, $R$ is the breakeven interest rate for a short-term relationship loan. Outside banks bid $R=1.09$ in period 2 to attract new borrowers from other banks. The realized $b$ values induce the rest of the results. A good bank can retain the existing borrower by charging $R_{2}^{G}$ in period 2 . We observe that $R_{2}^{G}$ exceeds the cost of a relationship loan, $R=1.09$, due to the hold-up problem; and $R_{2}^{G}$ increases with $b$. When the proportion of bad banks $b$ is high, a borrower is unwilling to leave the initial good bank for a random outside bank even if the initial bank charges a higher interest rate. Since a lending relationship yields zero profit, a high interest rate in period $2, R_{2}^{G}$, is associated with a low interest rate in period $1, R_{1}$. Both bank types bid $R_{1}$ in period 1 . Since the breakeven interest rate for good banks is $R=1.09$, existing borrowers are profitable for good banks, $R_{2}^{G}>R=1.09$, but new borrowers are unprofitable, $R_{1}<R=1.09$. The expected probability of project success under a random bank relationship $\beta$ decreases with the proportion of bad banks $b$.

The first column, $C_{B}=0.0091575$, shows a threshold scenario: a borrower is indifferent to a short-term loan of a random bank or bonds, $\beta(Y-R)=p\left(Y-R_{T}\right)$. If $C_{B}$ is smaller, the economy has more bad banks. This scenario means that $b$ is higher, $\beta$ is lower, and borrowers prefer bonds to short-term loans $\beta(Y-R)<p\left(Y-R_{T}\right)$. Assumption 3 abolishes this alternative. Consequently, Assumption 3 defines implicitly that the cost of bad banks $C_{B}$ is sufficiently high (exceeds 0.0091575 ) so that the realizing proportion of bad banks $b$ is sufficiently low. As a result, the quality of a random bank loan $\beta$ is relatively good, and borrowers prefer short-term loans to bonds, $\beta(Y-R)>p\left(Y-R_{T}\right)$. This is important in period 2. If the initial bank is bad, the borrower chooses a new bank and a new relationship loan.

In the other columns, the cost of bad banks $C_{B}$ is higher. As a result, the economy has fewer bad banks; that is, $b$ is lower. Therefore, the quality of a random bank loan $\beta$ is higher and a
borrower strictly prefers a random short-term bank loan to bonds, $\beta(Y-R)>p\left(Y-R_{T}\right)$. As a result, in each scenario (in each column), a borrower applies for a bank loan in both periods. If the first bank proves to be bad, the borrower contacts a new one. If the first bank is good, the bank retains the initial lending relationship and makes profit $Y-R_{2}^{G}$ in the second period. Further, $Y-$ $R_{2}^{G}=\beta(Y-R)$ under each $b$. Logically, the initial good bank raises interest rate $R_{2}^{G}$ to so high a level in period 2 that the existing borrower is indifferent to the initial bank or a random outside bank. Under interest rates $R_{1}$ and $R_{2}^{G}$, each lending relationship yields zero profit to a good bank. Under each cost level $C_{B}$, the realizing proportion of bad banks $b$ is so high that the expected profit of bad banks is zero.

The fourth and fifth columns display the latter threshold. The exact threshold is $C_{B}=$ 0.0137. For clarity, we also analyze $C_{B}=0.01368$. Now the cost of bad banks $C_{B}$ rises to the upper limit so that the proportion of bad banks approaches zero, $b=0.1138136 \%$. The quality of bank loans is high. A project is successful almost with certainty under a random lending relationship, $\beta=$ $99.992 \%$. This success erodes the information advantage of the initial good bank and mitigates the hold-up problem. The existing borrower is willing to leave the initial good bank for a random bank, because the risk of contacting a bad bank is very low. The initial bank can retain the existing borrower only if its interest rate offer is relatively low, $R_{2}^{G}=1.09002$. Therefore, the bank's surplus from the existing borrower is small, $R_{2}^{G}-R=0.00002$. On the contrary, the borrower's expected profit under a random bank relationship approaches the upper limit, $\beta(Y-R)=0.27$. If the cost of bad banks $C_{B}$ exceeds the threshold of 0.0137 , bad banks do not enter the market. The economy has only good banks, the interest rate is $R$ in both periods, and each borrower makes a risk-free profit $Y-R=0.27$ in both periods. However, we have eliminated this alternative by making Assumption 2. Therefore, the proportion of bad banks is positive. Consequently, the thresholds define the upper limit and the lower limit for the cost of bad banks, $C_{B}$. If the cost for bad banks exceeds the upper limit, the economy has no bad banks. If the cost is lower than the lower limit, the proportion of bad
banks is so high that borrowers prefer bonds to poor-quality bank loans in short-time lending. Assumptions 2 and 3 ensure that the economy operates between the thresholds. This condition means that the proportion of bad banks is positive, but it is not excessive (over $27.513 \%$ ). As a result, borrowers favor relationship loans in both periods and the borrowers of bad banks leave their initial banks looking for a new bank.

Consider, for instance, the third column $C_{B}=0.0117$. In period 1, banks charge $R_{1}=$ 1.0878 , and a borrower's expected profit under a random bank relationship is $\beta\left(Y-R_{1}\right)$ or $0.9918(1.36-1.0878) \approx 0.27$. Since it exceeds the expected return under bond financing, $p\left(Y-R_{T}\right)=0.2648$, the borrower chooses a bank loan. In period 2, outside banks offer shortterm loans and charge $R=1.09$. The borrower's expected return under a random outside bank relationship is $\beta(Y-R)$ or $0.9918(1.36-1.09)=0.2678$, which exceeds the expected return under bond financing, 0.2648 . If the initial bank is good, it raises interest rate $R_{2}^{G}$ to so high a level that the borrower is indifferent between the initial bank and a random outside bank, $Y-R_{2}^{G}=$ 0.2678. This indifference means $R_{2}^{G}=1.0922$. Given $1.0878<R=1.09<1.0922$, new borrowers are unprofitable to good banks, but old borrowers yield profits. Finally, assume that the initial bank is bad. The borrower's expected return under a random outside bank relationship is $\beta(Y-R)=0.2678$ that exceeds the expected return under bond financing and under the initial bad bank relationship $p\left(Y-R_{T}\right)=0.2648$. Therefore, the borrower switches banks. Fig. 2 portrays the borrower's profit under bank financing and compares it with the profit under bond financing.


Fig. 2 The borrower's profit under bank financing and bonds. If the costs of bad banks $C_{B}$ and the output Y are relatively high, then borrowers prefer bank loans to bonds.

The grey rectangular sheet shows the difference in the profits $\beta(Y-R)-p\left(Y-R_{T}\right)$. It is positive or negative. We observe that if $\beta(Y-R)>p\left(Y-R_{T}\right)$, borrowers prefer bank loans to bonds, as shown in the top left corner of the rectangular sheet (above the red curve). In the lower part of the rectangular sheet, bank loans are less profitable than bonds (below the red curve on the right). Fig. 2 also emphasizes the degree to which the profits depend on $C_{B}$ and $Y$. Bank loans are valuable if the cost of bad banks $C_{B}$ and the project output $Y$ are high. If $C_{B}$ is high, the proportion of bad banks $b$ is low and the quality of the average bank loan is good. The higher the project output $Y$, the more important is the probability of project success. A high project output increases the value of bank loans because good banks can raise the probability of success.

Fig. 3 illustrates the interest margin of the initial good bank $R_{2}^{G}-R$ if borrowers prefer bank loans to bonds $\beta(Y-R)>p\left(Y-R_{T}\right)$.


Fig. 3 The excessive interest rate under the hold-up problem in period 2. The interest margin is large if the project output Y is high and the cost of bad banks $\mathrm{C}_{\mathrm{B}}$ is low.

Owing to the hold-up problem in period 2, the initial good bank can extract excessive interest on loans. This surplus $R_{2}^{G}-R$ increases with output $Y$. Logically, if $Y$ is high, the borrower is unwilling to leave the existing good bank for a random bank. If the new bank is bad, the expected decrease in the borrower's return is $(1-p) Y$. The higher the output $Y$, the larger the expected decrease is. In addition, $R_{2}^{G}-R$ declines with the cost of bad banks, $C_{B}$. If $C_{B}$ is small, the economy has plenty of bad banks. This quantity boosts the market power of the initial good bank. A borrower is unwilling to leave the initial good bank if a new bank is bad with a high probability.

### 3.6 Short-term loans are optimal

So far, banks have provided short-term loans. This is a standard assumption in the relationship lending theory (e.g., Sharpe 1990). This subsection shows that short-term loans are optimal. Longterm loans increase the returns of bad banks and impose costs on their borrowers.

First, let $\theta_{1}$ be the interest rate in period 1 and $\theta_{2}$ in period 2. The interest payments must cover the costs of lending, $\theta_{1}+\theta_{2}=2 R, \theta_{2} \geq R .{ }^{7}$ It is possible, for instance, that the interest rates are $\theta_{1}=\theta_{2}=R$, or the same as under short-term lending, $\theta_{1}=R_{1}$ and $\theta_{2}=R_{2}^{G}$.

Assume that the initial bank is bad. The borrower pays $\theta_{1}$ in period 1 . If the borrower and the bad bank retain their relationship, then the bank makes the expected profit $p \theta_{2}-r$ in period 2, and the borrower earns $p\left(Y-\theta_{2}\right), p Y-r$ in all. If the borrower switches banks, the initial bank gets no return from this lending relationship in period 2, and the borrower's expected profit is $\beta(Y-R)$. Since $\beta(Y-R)>p Y-r$, their total returns are maximal if the borrower switches banks. The switch creates surplus $\Delta=\beta(Y-R)-(p Y-r)$. The initial bank and the borrower bargain for the surplus. The borrower receives $p\left(Y-\theta_{2}\right)+\alpha \Delta$, and the bank gets $p \theta_{2}-$ $r+(1-\alpha) \Delta$. Here $\alpha, 0 \leq \alpha \leq 1$, is the bargaining power. The larger the $\alpha$, the stronger the borrower's bargaining power. If their bargaining power is maximal, $\alpha=1$, then the borrower's expected return is $\beta(Y-R)-\left(p \theta_{2}-r\right)$. It consists of two parts. First, the borrower pays $p \theta_{2}-$ $r$ to the bank to cancel the long-term loan. Thereafter, the borrower can look for a new bank. This new bank relationship generates expected return $\beta(Y-R)$. The borrower maximizes their expected profit by leaving the bad bank after period 1 if:
$\beta(Y-R)-\left(p \theta_{2}-r\right) \geq p\left(Y-\theta_{2}\right)$.

[^6]This is true with certainty (Assumption 3). Borrowers leave bad banks after period 1.

Now consider the scenario in which a borrower finds a bad bank in period 1 and makes a long-term loan contract. The bank's expected profit from this lending relationship is $\left(p \theta_{1}-r\right)+$ $\left(p \theta_{2}-r\right)$. It includes two parts. First, the expected return in period 1 . Second, the borrower pays the bank to cancel the long-term loan contract. Under the short-term contract, the bank makes the expected return $p R_{1}-r$ in period 1 . Under both contracts, the borrower leaves the bank after period 1. The difference in profits is $p\left(\theta_{1}+\theta_{2}-R_{1}\right)-r$ or $p\left(2 R-R_{1}\right)-r>0$. Consequently, longterm loans are more profitable for bad banks than short-term loans. These extra profits are paid by their borrowers who must pay their initial bad banks to cancel the long-term contracts. The result is independent of the allocation of repayments $\left(\theta_{1}, \theta_{2}\right)$ between the periods.

Proposition 4 A long-term loan generates an extra profit to bad banks and an extra cost to their borrowers because they must pay the initial bank to cancel the long-term contract after period 1 .

Further, a borrower will not renegotiate the long-term contract if the initial bank is good, and the bank and the borrower will retain the lending relationship in period 2.

### 3.7 Loan commitments are not optimal

So far, banks have been unable to commit to future interest rates. This is a standard assumption in the relationship lending theory (e.g., Sharpe 1990). In this subsection, we study loan commitments. They increase the returns of bad banks, because their borrowers must pay more interest in period 1.

We have observed that good banks have market power over their existing borrowers in period 2 and the interest rate exceeds the breakeven level, $R_{2}^{G}>R$. However, existing borrowers can protect themselves against their banks. In period 1, a bank supplies a loan commitment for period 2. It gives an option to borrow a unit at interest rate $R$. If the borrower uses the loan commitment in period 2, the bank can accept or reject the application, but it cannot reprice the loan. Thus, the bank will accept the application. Given interest rate $R$ in period 2 and competition for borrowers, the interest rate of period 1 is $R$. Is the loan commitment beneficial?

First, consider a borrower and a good bank. We know that their relationship lasts for two periods. As seen earlier, the borrower pays $R_{1}$ in period 1 and $R_{2}^{G}$ in period 2 without the loan commitment. With the commitment, the borrower pays $R$ in both periods. We know $2 R=R_{1}+$ $R_{2}^{G}$. The borrower and the bank are indifferent between the alternatives. The commitment changes the allocation of the repayments between the periods but has no effect on total repayments.

Second, if the initial bank is bad, the borrowers switch banks after period 1 with or without the loan commitment. In both cases, they pay interest rate $R$ to the new bank in period 2 . The interest rate of period 1 is lower without the loan commitment, $R_{1}<R$. The loan commitment raises the borrower's interest rate.

Third, consider the scenario with a bad bank. Under the loan commitment, the bank receives repayment $R$ in both periods (The bank has a new borrower with probability $b$ in period 2). In absence of the commitment, the bank receives $R_{1}$ in period 1 and $R$ in period 2 (The bank gets a new borrower with probability $b$ in period 2 ). Since $R_{1}<R$, the loan commitment increases the repayment income of the bad bank. More commonly, if a loan commitment decreases the interest payment by a dollar in period 2 , the borrower must pay a dollar more in period 1 . The change is profitable to bad banks that lose borrowers after period 1.

Proposition 5 Loan commitments increase the expected interest income of bad banks in period 1. Their borrowers suffer the costs by paying an higher interest rate in period 1 .

In the traditional models of relationship lending, a bank learns what types its borrowers are during period 1, and (primarily) good borrowers get a loan in period 2. Both borrower types receive a loan in period 1. The loan commitments, which raise the interest rate in period 1 and lower it in period 2 , reduce the profits of bad borrowers, who receive a loan in period 1 . Good borrowers benefit from the loan commitments. Now loan commitments are optimal.

### 3.8 Unsecured lending is optimal

So far, loans have been unsecured. This is a standard assumption in the relationship lending literature (e.g., Sharpe 1990). A firm, however, makes a profit in period 1 and could pledge the profit as loan collateral in period 2 . This is not optimal. First, assume that the value of the collateral asset is low so that a good bank is motivated to exert costly effort. A project and the loan will succeed with certainty if the bank is good and with probability $p$ if it is bad. Therefore, the collateral is valuable only for bad banks, and unsecured lending is optimal. Second, assume that the high value of the collateral asset makes the new loan risk free. A good bank receives the same repayment whether the project succeeds or not and is not motivated to exert effort in the project. The collateral gives negative incentives to the bank.

In the traditional models of relationship lending, identical banks have low-risk and high-risk borrowers. In these models, collateral entails costs first and foremost to high-risk borrowers and secured lending is optimal.

## 4 Both bank types retain existing borrowers 1

The preceding analysis shows how good banks can retain existing borrowers after period 1 . This is a well-known result in the literature on product quality; consumers leave bad firms and search for a new firm, while good firms can keep existing customers. This section introduces scenarios in which both bank types supply relationship loans in period 1. In period 2, good banks can retain existing borrowers by supplying relationship loans, and bad banks retain existing borrowers by issuing transaction loans. Since borrowers do not leave initial banks, banks do not receive new borrowers in period 2.

More precisely, Assumption 3 is met in period 1 and borrowers prefer relationship loans to bonds and transaction loans. In period 2, borrowers have opposite preferences. If a borrower must leave the initial bank after period 1, he prefers bond markets to the relationship loan offers of outside banks. For now, we simply assume that borrowers have this kind of preferences. Later, we analyze reasons for the preferences.

Assume that the initial bank is bad. In this scenario, the borrower learns the bank's type, the bank can only offer a transaction loan, and the project succeeds with probability $p$ in period 2. Since the borrowers reject the offers of relationship loans from outside banks in period 2, they compare the transaction loans to bonds. In the bond markets, the interest rate is $R_{T}$ and the project succeeds with probability $p$. Therefore, the initial bad bank can retain the lending relationship by charging $R_{T}$ that covers the variable costs of the transaction loan but does not cover the sunk cost $C_{B}$. The initial bank offers a transaction loan and the existing borrower accepts the offer.

Now, assume that the initial bank is good. In this scenario, the borrower learns the bank's type and keeps the lending relationship. Even if the borrowers prefer bonds to the offers of
relationship loans from outside banks in period 2, they favor the existing lending relationships with the good banks. A good bank can offer a valuable relationship loan with certainty. The offers from outside banks are less valuable because banks of both types make these offers. The initial bank knows the borrower's preferences: if the borrower leaves the bank, they will contact bond markets. To avoid this, the initial bank sets the interest rate at $\hat{R}_{2}^{G}$ so that the existing borrower is indifferent to the bank market or the bond market:

$$
\begin{equation*}
Y-\hat{R}_{2}^{G} \geq p\left(Y-R_{T}\right) \tag{8}
\end{equation*}
$$

The left-hand side shows the borrower's expected profit under the existing good bank relationship. The right-hand side indicates the expected profit with bond financing. The inequality gives the maximal interest rate $\hat{R}_{2}^{G}=r+(1-p) Y$. The interest rate in period $1, \hat{R}_{1}$, is such that a lending relationship yields zero profit in its lifetime:
$\hat{R}_{1}-R+\delta\left(\hat{R}_{2}^{G}-R\right)=0$.

Since bad banks must mimic good banks to be able to attract borrowers in period 1, both bank types bid $\hat{R}_{1}$ in period 1. If a bad bank receives a borrower in period 1 , then the bank receives the lifetime return of $p \hat{R}_{1}-r-C_{B}$. It consists of the expected profit in period 1 , because the transaction loan yields zero profit in period 2. To ensure the existence of bad banks, we assume:

Assumption $5 p \hat{R}_{1}-r-C_{B}>0$.

First, good bankers established banks. Their number is the same as the number of borrowers, $n$. Thereafter, a few bad bankers set up banks. The total number of banks exceeds the number of borrowers, $N>n$. As before, $b$ denotes the proportion of bad banks. A bank receives a borrower with probability $1-b$ in period 1 . A bad bank makes the expected lifetime return:
$\hat{\pi}_{B}(b)=(1-b)\left(p \hat{R}_{1}-r\right)-C_{B}$.

Here $\hat{\pi}_{B}(b)$ declines with $b, \hat{\pi}_{B}(0)>0$ and $\hat{\pi}_{B}(1)<0$. There is a proportion of bad banks for which $\hat{\pi}_{B}=0$. Since bad bankers establish banks sequentially and can oversee this process, the entry of bad banks goes on until $\hat{\pi}_{B}(b)=0$. In sum, banks bid interest rate $\hat{R}_{1}$ in period 1 . In period 2, good banks charge $\hat{R}_{2}^{G}$, and the interest rate of bad banks is $R_{T}$. Both bank types retain existing borrowers and make zero profit in their lifetime.

So far, we have simply assumed that borrowers prefer bonds to the relationship loan offers of outside banks in period 2 and have opposite preferences in period 1. Subsection 4.1 analyzes this kind of preferences in detail. In period 1, the timeline is the same as in subsection 3.2 and in period 2, the timeline the same as in subsection 3.1.

### 4.1 Relationship loans are entirely profitable in a long-term relationship

Subsection 4.1 shows how both bank types retain existing borrowers in period 2, because an offer of a relationship loan from an average bank is profitable only in a long-term relationship. In section 3, Assumption 3 was met, and a borrower preferred an offer of a relationship loan from an average bank over bonds even in a short-term lending relationship:

$$
\begin{equation*}
\beta(Y-R)>p\left(Y-R_{T}\right), \quad \beta=b p+(1-b) . \tag{11}
\end{equation*}
$$

Bad banks misrepresent their types and supply relationship loans even if they are de facto transaction loans. In eq. (11), $\beta$ measures the quality of the relationship loans that decreases with the proportion of bad banks $b$. If $b$ is large enough, then the inequality in eq. (11) is not true; a high proportion of these loans is de facto transaction loans. Due to the poor quality of expensive relationship loans, a firm favors other loan types for the short term. We assume this kind of economy:
$\beta\left(Y-\hat{R}_{1}\right)>p\left(Y-R_{T}\right)>\beta(Y-R)$.

Equation (9) defines $\hat{R}_{1}$ as the interest rate in period 1 for long-term relationship lending. The $R$ is the interest rate in a short-term relationship lending and the cost of a relationship loan, $\hat{R}_{1}<R$, and $R_{T}=1 / p$ is the interest rate on bonds and transaction loans, $R_{T}<R$. Outside banks offer shortterm relationship loans to the existing borrowers of other banks in period 2 and bid interest rate $R$. Given the latter inequality in eq. (12), a borrower prefers bonds and transaction loans to the offers of relationship loans from outside banks in period 2 due to the poor quality and high price of the short-term relationship loans. Put differently, the latter equality states that Assumption 3 is not met. Given the first inequality in eq. (12), a borrower prefers a relationship loan to other loan types in period 1 because of the low interest $\hat{R}_{1}$. Consequently, relationship lending is profitable in a longterm relationship and unprofitable in a short-term relationship. Borrowers favor relationship loans in period 1 and other loan types in period 2. Since firms avert the offers of outside banks in period 2, good (bad) banks can retain their existing borrowers by charging interest $\hat{R}_{2}^{G}\left(R_{T}\right)$ in period 2 . Both banks bid $\hat{R}_{1}$ in period 1.

Proposition 6 Both bank types can retain existing borrowers after period 1 if borrowers find that searching for a good bank is only profitable for long-term relationship lending.

More precisely, since $\hat{R}_{1}<R$ in eq. (12), we know that $\beta\left(Y-\hat{R}_{1}\right)>\beta(Y-R)$. However, $\beta\left(Y-\hat{R}_{1}\right)$ may be larger or smaller than $p\left(Y-R_{T}\right)$. In addition, $p\left(Y-R_{T}\right)$ may be larger or smaller than $\beta(Y-R)$. To ensure that the inequalities in eq. (12) are met, we define two thresholds.

The second threshold in eq. (12) is $p\left(Y-R_{T}\right)>\beta(b)(Y-R)$, where $\beta(b)=$ $b p+1-b$. If $b=1$, then $p\left(Y-R_{T}\right)>\beta(1)(Y-R)$, because $\beta(1)=p$ and $R_{T}<R$. If $b=$ 0 , then $p\left(Y-R_{T}\right)<\beta(0)(Y-R)$, because $\beta(0)(Y-R)-p\left(Y-R_{T}\right)$ can be rewritten as $(1-p) Y-C_{G}-(1-p) c_{v}>0$. This is true, because relationship lending has a positive NPV (Assumption 1). In sum, since $\beta(0)(Y-R)>p\left(Y-R_{T}\right)>\beta(1)(Y-R)$, there is a $b^{*}$ such that $p\left(Y-R_{T}\right)=\beta\left(b^{*}\right)(Y-R)$. If $b>b^{*}$, then borrowers prefer bonds to short-term loans. If $b<b^{*}$, then borrowers favor short-term loans to bonds. We must have $b>b^{*}$ so that the second inequality in eq. (12) is met.

The first threshold is $\beta(b)\left(Y-\hat{R}_{1}\right)>p\left(Y-R_{T}\right)$. If $b=0$, then $\beta(0)(Y-$ $\left.\hat{R}_{1}\right)>p\left(Y-R_{T}\right)$. The reasoning is that $p\left(Y-R_{T}\right)=\beta\left(b^{*}\right)(Y-R)$. Since $\beta(b)\left(Y-\hat{R}_{1}\right)>$ $\beta(b)(Y-R)$ with each $\beta(b)$, we have $\beta\left(b^{*}\right)\left(Y-\hat{R}_{1}\right)>p\left(Y-R_{T}\right)$. Since $\beta(b)$ decreases with $b$, we have $\beta(0)\left(Y-\hat{R}_{1}\right)>p\left(Y-R_{T}\right)$. If $b=1$, we obtain $\beta(1)\left(Y-\hat{R}_{1}\right)<p\left(Y-R_{T}\right)$, because $\beta(1)=p$ and $\hat{R}_{1}>R_{T}$. In sum, since $\beta(0)\left(Y-\hat{R}_{1}\right)>p\left(Y-R_{T}\right)>\beta(1)\left(Y-\hat{R}_{1}\right)$, there is a $b^{* *}>b^{*}$ such that $\beta\left(b^{* *}\right)\left(Y-\hat{R}_{1}\right)=p\left(Y-R_{T}\right)$. If $b>b^{* *}$, then borrowers favor bonds to poor quality bank loans in period 1. If $b<b^{* *}$, then they prefer loans to bonds in period

1. We must have $b<b^{* *}$ so that the first inequality in eq. (12) is met. Both inequalities are met between the thresholds, $b^{*}<b<b^{* *}$.

### 4.2 Example 2

Given the thresholds, the inequalities in eq. (12) are met if $b^{*}<b<b^{* *}$. We now address the thresholds in our example: $p=0.93, Y=1.36, C_{G}=0.055, c_{v}=0.5, r=1$ and $R=1.09$. The second threshold can be expressed as $0.2648>\beta(b) 0.27$. This expression gives $\beta(b)<0.9807$ and $b>27.51 \%$. The first threshold is $\beta(b)(1.36-1.0848)>0.2648$. This threshold implies that $\beta>0.9622$ and $b<53.99 \%$. The inequalities of eq. (12) are met if $27.51 \%<b<53.99 \%$ : a borrower prefers a random relationship loan to bonds in period 1 but the preferences are the opposite in period 2 . Table 2 offers an example.

Values in all scenarios: $p=0.93, Y=1.36, C_{G}=0.055, c_{v}=0.5, r=1$.

| $b$ | $27.51 \%$ | $35 \%$ | $45 \%$ | $50 \%$ | $53.99 \%$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\beta$ | $98.07 \%$ | $97.55 \%$ | $96.85 \%$ | $96.5 \%$ | $96.22 \%$ |
| $R$ | 1.09 | 1.09 | 1.09 | 1.09 | 1.09 |
| $p\left(Y-R_{T}\right)$ | 0.2648 | 0.2648 | 0.2648 | 0.2648 | 0.2648 |
| $\beta(Y-R)$ | 0.2648 | 0.2634 | 0.2615 | 0.2606 | 0.2598 |
| $\hat{R}_{1}$ | 1.0848 | 1.0848 | 1.0848 | 1.0848 | 1.0848 |
| $\hat{R}_{2}^{G}$ | 1.0952 | 1.0952 | 1.0952 | 1.0952 | 1.0952 |
| $Y-\hat{R}_{2}^{G}$ | 0.2648 | 0.2648 | 0.2648 | 0.2648 | 0.2648 |
| $\beta\left(Y-\hat{R}_{1}\right)$ | 0.2699 | 0.2685 | 0.2665 | 0.2656 | 0.2648 |

Table 2 Both bank types retain existing borrowers. Here $b$ is the proportion of bad banks and $\beta(p)$ is the expected probability of project success under a random relationship loan (without a relationship loan). Outside banks bid $R=1.09$ in period 2 to attract borrowers from other banks. Each bank bids $\hat{R}_{1}$ in period 1 and the existing borrowers of good banks pay $\hat{R}_{2}^{G}$ in period 2. Further, $R_{T}$ is the interest rate on transaction loans and bonds.

In the first column of Table 2, the proportion of bad banks is $b=27.51 \%$ that shows the second threshold. The borrower is indifferent to a short-term bank loan or bonds in period 2, $\beta(Y-R)=$ $p\left(Y-r_{T}\right)=0.2648$. If $b$ is smaller, then the borrower strictly prefers a short-term bank loan to bonds in period 2. This scenario was examined in Section 3 and in Table 1.

This subsection focuses on cases in which the proportion of bad banks is higher, $b>$ $27.51 \%$. This proportion means that a borrower prefers bonds to a short-term bank loan (a relationship loan) from a random outside bank, $p\left(Y-R_{T}\right)>\beta(Y-R)$ due to the poor quality of an average outside bank (high proportion of bad banks b) as Table 2 reflects. The borrower's expected return under bond financing is fixed, $p\left(Y-R_{T}\right)=0.2648$, and exceeds the return under
short-term bank financing: $0.2634(b=35 \%), 0.2615(b=45 \%)$, and so on. This is important in period 2. Borrowers do not switch banks. They choose bonds or retain the initial bank relationship. The latter alternative is true in the model. If the initial bank is bad, it grants a transaction loan, charges interest rate $R_{T}$, and the borrower's expected return is $p\left(Y-R_{T}\right)=0.2648$. If the initial bank is good, it grants a relationship loan, charges interest rate $\hat{R}_{2}^{G}$, and the borrower gets return $Y-\hat{R}_{2}^{G}=0.2648$. In both cases, the borrower's return is the same as financing with bonds. The initial bank, good or bad, raises the interest rate to the upper limit so that the existing borrower is indifferent to the initial bank or bonds in period 2. In period 1, borrowers favor bank loans (that is, relationship loans) to bonds: $\beta(b)\left(Y-\widehat{R}_{1}\right)>p\left(Y-R_{T}\right)$ if $27.51 \% \leq b<53.99 \%$. For instance, if $b=35 \%(45 \%)$, then a random bank loan yields an expected return of $0.2685(0.2665)$ to the borrower in period 1 , which exceeds the expected return of 0.2648 when using bonds. Therefore, borrowers choose bank loans in period 1 and retain these lending relationships in period 2. For example, if $b=35 \%(45 \%)$, then this strategy yields an expected lifetime return of $0.2685+0.2648=0.5333(0.2665+0.2648=0.5313)$ to the borrower. This lifetime return is higher than the lifetime return $2 \times 0.2648=0.5296$ when using bonds. A good bank bears losses $1.0848-1.09=-0.0052$ in period 1 , makes profit $1.0952-1.09=0.0052$ in period 2, and earns a zero lifetime profit. The proportion of bad banks $b$ and their costs decrease the expected lifetime profits of bad banks to zero.

For the first threshold, column $\mathrm{b}=53.99 \%$ in which a borrower is indifferent between bank loans and bonds in period 1: $\beta\left(Y-\hat{R}_{1}\right)=p\left(Y-R_{T}\right)=0.2648$. Furthermore, $\beta(b)\left(Y-\hat{R}_{1}\right)$ declines with $b$. In period 2, a borrower favors bonds over the loan offers of outside banks, $p\left(Y-R_{T}\right)>\beta(Y-R)$ or $0.2648>0.2598$; and the initial bank, good or bad, can retain the lending relationship by granting a new loan to the borrower. The interest rate ( $\hat{R}_{2}^{G}$, if the initial bank is good and $R_{T}$ if it is bad) is such that the borrower is indifferent to the new bank loan or to
bonds in period 2, that is, the borrower gets the expected return 0.2648 under bank lending. Consequently, in the first threshold borrowers are indifferent to loans or bonds in both periods. Both alternatives yield the expected return of 0.2648 in both periods. If the proportion of bad banks is higher and exceeds $b=53.99 \%$, then borrowers strictly prefer bonds to loans due to the poor quality of bank loans. Asymmetric information eliminates banks. In sum, between these two thresholds, $27.51 \%<b<53.99 \%$, each borrower applies for a bank loan in period 1 and retains the lending relationships with the initial bank (good or bad) in period 2.

Finally, it is not essential that the interest rate is low in period $1, \hat{R}_{1}<R$. Assume that $b=35 \%$, banks commit to upper limit $R=1.09$ for both periods, and that a borrower applies for a relationship loan in period 1 . The borrower gets the expected profit $\beta(Y-R)=0.2634$ in period 1. If the initial bank is good, then the borrower retains the lending relationship and earns the profit $Y-R=0.27$ in period 2 . If the initial bank is bad, it can retain the lending relationship by charging $R_{T}$. Then, the borrower makes the expected profit, $p\left(Y-R_{T}\right)=0.2648$ in period 2 . The expected profit totals $0.2634+0.65 \times 0.27+0.35 \times 0.2648=0.53158$ in the two periods. In an alternative strategy, the borrower chooses bonds (or transaction loans) in both periods that yield the lifetime profit of $2 \times 0.2648=0.5296$. A relationship loan is the optimal choice in period 1 $(0.53158>0.5296)$ even if the borrower prefers a transaction loan or bonds to the offers of shortterm relationship loans from outside banks in period $2(0.2648>0.2634)$. It is essential that a borrower can enjoy a good lending relationship for two periods if the initial bank is good!

## 5 Both bank types retain existing borrowers 2

This section presents the second scenario in which both bank types can retain borrowers in period 2. The proportion of good banks, $g$, is stochastic. As a result, the quality of a relationship loan, $\beta$, is stochastic. In period 1, a borrower applies for a relationship loan. If the initial bank proves to be good, the borrower will retain the lending relationship in period 2 . If the initial bank is bad, the borrower will become pessimistic regarding the realized quality of relationship loans (= realized proportion of good banks in the economy) and favors a transaction loan over the offers of relationship loans from outside bank in period 2. The initial bank can offer a transaction loan and retain the lending relationship.

The realized proportion of good banks is unknown, but borrowers know the prior probabilities. With probability $h$, the proportion of good banks is high, $g_{H}$, and with probability $1-h$, it is low, $g_{L}$. Assumption 3 is satisfied at the start of period 1:
$\beta(Y-R)>p\left(Y-R_{T}\right), \quad \beta=p+(1-p) g$.

Earlier $g$ denotes the fixed proportion of good banks. Now, $g$ denotes the expected proportion of good banks at the start of period 1. Assumption 3 states that the expected proportion of good banks is quite high. Therefore, the expected quality of relationship loans is high, and borrowers choose them in period 1. During period 1, borrowers learn the type of the initial bank and can use this information to update their prior belief on the realized proportion of good banks.

First, consider a borrower whose initial bank is good. Given this information, the posterior probability for the high realized proportion of good banks in the economy is:
$h_{g}=h g_{H} /\left[h g_{H}+(1-h) g_{L}\right]$.

In detail, $h_{g}$ is a conditional probability; the likelihood of the event "high proportion of good banks" occurring given that "the initial bank is good" is true. Because $h_{g}>h$, the posterior probability for the high proportion of good banks is greater than the prior probability. Given $h_{g}$, the borrower expects that the proportion of good banks in period 2 is:
$E\left(g_{2} \mid g_{1}\right)=h_{g} g_{H}+\left(1-h_{g}\right) g_{L}$,
if the initial bank is good in period 1. Because $E\left(g_{2} \mid g_{1}\right)>g$, the expected proportion of good banks is higher in period 2 than in period 1. In period 2, the borrower is more optimistic that a random bank represents the good type than in period 1, because the borrower's initial bank in period 1 was good. Second, consider a borrower whose bank in period 1 was bad. The posterior probability for a high realized proportion of good banks is:
$h_{b}=h\left(1-g_{H}\right) /\left[h\left(1-g_{H}\right)+(1-h)\left(1-g_{L}\right)\right]$.

More precisely, $h_{b}$ is a conditional probability; the likelihood of the event "high proportion of good banks" occurring given that "the initial bank is bad" is true. Given eq. (15), the borrower expects that the proportion of good banks in period 2 is:
$E\left(g_{2} \mid b_{1}\right)=h_{b} g_{H}+\left(1-h_{b}\right) g_{L}$,
when the initial bank is bad. We know $h_{g}>h>h_{b}$ and $E\left(g_{2} \mid g_{1}\right)>g>E\left(g_{2} \mid b_{1}\right)$. Here $g>$ $E\left(g_{2} \mid b_{1}\right)$ means that the expected proportion of good banks is lower in period 2 than in period 1 if the borrower's initial bank is bad. A borrower who has a good (bad) bank in period 1 is optimistic (pessimistic) that a random bank is good in period 2. The borrower uses this updated information at the start of period 2 when they decide whether to switch banks.

Consider a borrower whose initial bank is good. They are optimistic about the realized proportion of good banks and their expected profit from a relationship loan with an outside bank is:

$$
\begin{equation*}
\left[p+(1-p) E\left(g_{2} \mid g_{1}\right)\right](Y-R) \tag{17}
\end{equation*}
$$

In period 1, the borrower favors a relationship loan. Given $E\left(g_{2} \mid g_{1}\right)>g$, the expected return from a relationship loan is higher in period 2 than in period 1 . The initial bank charges $\bar{R}_{2}$ such that the borrower will retain the existing lending relationship:

$$
\begin{equation*}
Y-\bar{R}_{2} \geq\left[p+(1-p) E\left(g_{2} \mid g_{1}\right)\right](Y-R) \tag{18}
\end{equation*}
$$

or

$$
\begin{equation*}
\bar{R}_{2}=Y-\left[p+(1-p) E\left(g_{2} \mid g_{1}\right)\right](Y-R) . \tag{19}
\end{equation*}
$$

Given this, the breakeven interest rate in period 1 is:
$\bar{R}_{1}=2 R-\bar{R}_{2}$.

Consider a borrower whose initial bank is bad. They are pessimistic about the realized proportion of good banks, $E\left(g_{2} \mid b_{1}\right)<g$. In period 2, a relationship loan from an outside bank yields the expected profit:
$\left[p+(1-p) E\left(g_{2} \mid b_{1}\right)\right](Y-R)$.

Owing to $E\left(g_{2} \mid b_{1}\right)<g$, the expected profit from an outside relationship loan is lower in period 2 than in period 1. Therefore, a borrower might prefer other loan types to the offers of relationship loans from outside banks in period 2 even if the borrower chooses a relationship loan in period 1. In this case, the initial bad bank can offer a transaction loan and retain the lending relationship in period 2. The interest rate is $R_{T}$.

Proposition 7 When the proportion of good banks is stochastic, a borrower might favor a relationship loan in period 1. If the initial bank is good, the borrower chooses a relationship loan from it in period 2. If the initial bank is bad, the borrower becomes pessimistic about the average quality of the relationship loans in the economy and may prefer a transaction loan to a relationship loan in period 2. The initial bad bank can supply a transaction loan. Both bank types can retain existing borrowers in period 2 .

We now examine in detail a scenario in which borrowers prefer short-term bank loans to bonds or a transaction loan in period 1 . However, a pessimistic borrower replaces a bank loan (a relationship loan) with bonds or a transaction loan after period 1 . This scenario arises if:
$\beta(Y-R)>p\left(Y-R_{T}\right)>\left[p+(1-p) E\left(g_{2} \mid b_{1}\right)\right](Y-R)$,
in which $\beta=p+(1-p) g$ and $g=h g_{H}+(1-h) g_{L}$. The first inequality states that borrowers prefer short-term relationship loans to bonds and transaction loans in period 1 under the expected proportion of good banks. The second inequality means that a borrower prefers bonds (or a transaction loan) to a relationship loan in period 2 if the initial bank is bad. We label
$\Delta_{1}=\beta(Y-R)-p\left(Y-R_{T}\right)$,
$\Delta_{2}=\beta(Y-R)-\left[p+(1-p) E\left(g_{2} \mid b_{1}\right)\right](Y-R)$.

Here $\Delta_{1}$ repeats the first inequality in eq. (22), $\Delta_{1}>0$, and $\Delta_{2}$ compares the expected returns from a short-term bank loan in periods 1 and 2 if the initial bank proves to be bad. A borrower prefers bonds (or a transaction loan) to a relationship loan in period 2 only if $-\Delta_{1}+\Delta_{2}>0$. This is true, if $\Delta_{1}, \Delta_{1}>0$, is smaller than $\Delta_{2}$. We can express $\Delta_{2}$ as:
$\Delta_{2}=\frac{(1-p)(Y-R)\left(g_{H}-g_{L}\right)^{2} h(1-h)}{1-g}$.

Moreover, $\Delta_{2}$ is high if $h=0.5$ and $g_{H}-g_{L}$ is large. Consequently, a borrower is likely to favor bonds (or a transaction loan) after period 1 ; if the initial bank is bad, $\Delta_{1}$ is small, $h=0.5$, and $g_{H}-$ $g_{L}$ is large. We analyze two examples. In Table 3, $\Delta_{1}$ is small; and in Table 4, it is large. Further, $\Delta_{1}$ is big if $\beta$ is large, and $\beta$ is large if the expected proportion of good banks $g$ is high.

In all scenarios: $p=0.93, Y=1.36, C_{G}=0.055, c_{v}=0.5, r=1, h=0.5$.

| $g$ | $73 \% \pm 15 \%$ | $73 \% \pm 10 \%$ | $73 \% \pm 5 \%$ | $73 \% \pm 1 \%$ |
| :--- | :---: | :--- | :--- | :--- |
| $\beta$ | $98.11 \%$ | $98.11 \%$ | $98.11 \%$ | $98.11 \%$ |
| $p\left(Y-R_{T}\right)$ | 0.2648 | 0.2648 | 0.2648 | 0.2648 |
| $\beta(Y-R)$ | 0.264897 | 0.264897 | 0.264897 | 0.264897 |
| $h_{b}$ | $22.22 \%$ | $31.48 \%$ | $40.74 \%$ | $48.15 \%$ |
| $E\left(g_{2} \mid b_{1}\right)$ | 0.6467 | 0.693 | 0.7207 | 0.7296 |
| $\pi\left(b_{1}\right)$ | 0.2633 | 0.2642 | 0.2647 | 0.26489 |
| $\bar{R}_{1}$ | 1.0855 | 1.0852 | 1.085 | 1.0849 |
| $\beta\left(Y-\bar{R}_{1}\right)$ | 0.2693 | 0.2696 | 0.2698 | 0.2699 |

Table 3 The expected proportion of good banks is $\mathbf{7 3} \%$, that is, $\Delta_{\mathbf{1}}$ is small. Here $\beta$ is the expected probability of project success in period 1 under a random bank relationship (a relationship loan), $\mathrm{h}_{\mathrm{b}}$ is the likelihood of the event "high proportion of good banks" occurring if the initial bank is bad, and $\mathrm{E}\left(\mathrm{g}_{2} \mid \mathrm{b}_{1}\right)$ indicates the expected proportion of good banks if the initial bank is bad. Further, $p\left(Y-R_{T}\right)$ is the expected profit under bond financing or a transaction loan. A random short-term bank loan (relationship loan) yields expected profit $\beta(Y-R)$ in period 1 and $\pi\left(b_{1}\right)$ in period 2 if the initial bank is bad. Each bank bids $\bar{R}_{1}$ in period 1 and $\beta\left(Y-\bar{R}_{1}\right)$ is the borrower's expected profit in period 1 if it chooses a relationship loan.

In each scenario (column), the proportion of good banks is stochastic with an average of $73 \%$. In the first column, $73 \% \pm 15 \%$ means that the proportion of good banks is $g_{L}=58 \%$ with
probability 0.5 and $g_{H}=88 \%$ with probability 0.5 . The variance is high. In the second column, the proportion of good banks is either $g_{H}=83 \%$ or $g_{L}=63 \%$ with the same probability 0.5 . A borrower knows the characteristics of the economy, that is, they know "the number of the column" and the realized variance. They do not know whether the realized proportion of good banks is high or low. They know, for instance, that the proportion of good banks is either $58 \%$ or $88 \%$ if they operate in the economy of the first column. They do not know whether the realized proportion is $58 \%$ or $88 \%$. Since the average proportion of good banks is the same in each column, $73 \%$, the expected probability of project success $\beta$ is fixed in period 1 . Since $\beta\left(Y-\bar{R}_{1}\right)>p\left(Y-R_{T}\right)$, borrowers prefer relationship loans to bonds and transaction loans in period 1 and choose relationship loans. Do borrowers leave bad banks after the period and search for a new short-term relationship loan? Since $\beta(Y-R)>p\left(Y-R_{T}\right)$ in each column, borrowers prefer short-term relationship loans to bonds and transaction loans in both periods without updating. ${ }^{8}$ That is, borrowers leave bad banks without updating. However, borrowers learn the type of the initial bank and use this information to update their prior beliefs. Now $\pi\left(b_{1}\right)=\left[p+(1-p) E\left(g_{2} \mid b_{1}\right)\right](Y-$ $R$ ) indicates the borrower's expected return from a random outside loan (a relationship loan) in period 2 if the initial bank is bad. Since $\pi\left(b_{1}\right)<p\left(Y-R_{T}\right)$ in the first three columns, a borrower prefers bonds or a transaction loan over a random outside loan (a relationship loan) in period 2 if the initial bank is bad. Then, the borrower does not switch banks. The fourth column offers the opposite result. The borrower prefers an outside relationship loan to bonds or a transaction loan if the initial bank is bad, $0.26489>0.2648$, and switches banks. We can draw the following conclusions: When the initial bank is bad, the borrower becomes very pessimistic regarding the quality of the relationship loans if the variance is sufficient, that is, $g_{H}-g_{L}$ is large. As a result, the

[^7]borrower prefers bonds or a transaction loan to the relationship loan offers of outside banks and chooses a transaction loan from the initial bad bank. In Table $4, \Delta_{1}$ is large.

In all scenarios: $p=0.93, Y=1.36, C_{G}=0.055, c_{v}=0.5, r=1, h=0.5$.

| $g$ | $83 \% \pm 15 \%$ | $83 \% \pm 10 \%$ | $83 \% \pm 5 \%$ | $83 \% \pm 1 \%$ |
| :--- | :---: | :--- | :--- | :--- |
| $\beta$ | $98.81 \%$ | $98.81 \%$ | $98.81 \%$ | $98.81 \%$ |
| $p\left(Y-R_{T}\right)$ | 0.2648 | 0.2648 | 0.2648 | 0.2648 |
| $\beta(Y-R)$ | 0.2668 | 0.2668 | 0.2668 | 0.2668 |
| $h_{b}$ | $5.88 \%$ | $20.59 \%$ | $35.29 \%$ | $47.06 \%$ |
| $E\left(g_{2} \mid b_{1}\right)$ | 0.6977 | 0.7712 | 0.8153 | 0.8294 |
| $\pi\left(b_{1}\right)$ | 0.2643 | 0.2657 | 0.2665 | 0.26678 |
| $\bar{R}_{1}$ | 1.0873 | 1.087 | 1.0868 | 1.08679 |
| $\beta\left(Y-\bar{R}_{1}\right)$ | 0.2695 | 0.2697 | 0.2699 | 0.26996 |

Table 4 The expected proportion of bad banks is $83 \%$, that is, $\Delta_{1}$ is large.

Since $\beta\left(Y-\bar{R}_{1}\right)>p\left(Y-R_{T}\right)$ in each column, borrowers prefer relationship loans to bonds and transaction loans in period 1 and choose relationship loans. Do borrowers leave bad banks after the period and search for a new short-term relationship loan? Since $\beta(Y-R)>p\left(Y-R_{T}\right)$ or $0.2668>0.2648$ in each column, borrowers prefer short-term relationship loans to bonds and transaction loans in both periods without updating. However, borrowers learn the type of the initial bank and use this information to update their prior beliefs. Recall that $\pi\left(b_{1}\right)$ indicates the borrower's expected return from a random outside loan (a relationship loan) in period 2 if the initial bank is bad. For period 2, the economy in the first column has high variance in $g$, and we have $p\left(Y-R_{T}\right)>\pi\left(b_{1}\right)$, or $0.2648>0.2643$ : a borrower prefers bonds or a transaction loan over a
random outside loan (a relationship loan) if the initial bank is bad. Then, the borrower does not switch banks. In the economies of the other columns for moderate and low variance, a borrower prefers a random outside relationship loan to bonds or a transaction loan if the initial bank is bad, $\pi\left(b_{1}\right)>p\left(Y-R_{T}\right)$, and switches banks. Just as in Table 3, a borrower becomes very pessimistic toward banks after period 1 and replaces a relationship loan with a transaction loan only if the variance is sufficient ( $g_{H}-g_{L}$ is large). In Table 3, the borrowers of bad banks replace relationship loans with a transaction loan in the economies of the first three columns. In Table 4, the replacement is optimal in the economy of the first column. Therefore, the expected proportion of good banks ( $g=73 \%$ in Table 3 and $g=83 \%$ in Table 4) has a strong effect on the borrowers' choice. In sum, Tables 3 and 4 confirm our findings that the borrowers of bad banks become very pessimistic toward banks and prefer bonds or a transaction loan to the relationship loan offers of outside banks if $\Delta_{1}$ is low and $g_{H}-g_{L}$ is high. This kind of borrower chooses a transaction loan from the initial bad bank in period 2.

Now we examine the economy of the first column (Table 4). Consider a borrower whose initial bank is bad. They are pessimistic and prefer bonds or a transaction loan to the offers of relationship loans from outside banks in period $2,0.2648>0.2643$. The initial bad bank can retain the borrower by granting a transaction loan and setting interest rate $R_{T}=1 / p=1.0753$. The borrower receives the same expected return as when using bond, 0.2648 . Consider a borrower whose initial bank is good. Now, eq. (19) expresses the interest rate such that the initial good bank can retain the relationship, $\bar{R}_{2}=1.0927$. The borrower pays $\bar{R}_{2}=1.0927$ in period 2 and earns $Y-\bar{R}_{2}=0.2673$, which exceeds the expected profit under bond financing 0.2648 . Given equation (20), the interest rate is $\bar{R}_{1}=1.0873$ in period 1. At the start of period 1, the expected proportion of good banks is $83 \%$ and the borrower's expected profit from a relationship loan is $\beta\left(Y-\bar{R}_{1}\right)=$ 0.2695 . This profit exceeds their expected profit under bond financing, 0.2648 , and they choose a relationship loan in period 1. If the initial bank is good, the borrower takes a new relationship loan
from the bank in period 2, pays $\bar{R}_{2}=1.0927$ and makes a profit of $Y-\bar{R}_{2}=0.2673$. If the initial bank is bad, it can retain the lending relationship by making a transaction loan and setting $R_{T}=$ 1.0753. Now the borrower's expected profit is $p\left(Y-R_{T}\right)=0.2648$.

Why does the proportion of bad banks fluctuate (column 1, Table 4)? The cost of expertise has two alternative levels. With probability $50 \%$, it is low, $C_{B}=0.00760852$; and with probability $50 \%$, it is high, $C_{B}=0.01096522$. The breakeven constraint of a bad bank is now
$(1-b)\left(p \bar{R}_{1}-1\right)-C_{B}$.

The low cost generates a bank market with plenty of bad banks ( $32 \%$ ), and the high cost creates markets with few bad banks (2\%). Equation (25) reflects these markets.

We have illustrated two scenarios in which both bank types can retain borrowers in period 2 (Sections 4 and 5). Another scenario is de novo firms that have a high risk of failure (e.g., de Meza and Southey, 1996). When a firm matures, the probability of success increases. If the probability of project success is higher in period $2\left(p_{2}\right)$ than in period $1(p), p_{2}>p$, then a relationship loan might be optimal in period 1, $[g+(1-g) p](Y-R)>p\left(Y-R_{T}\right)$; but borrowers prefer other loan types to the offers of relationship loans from outside banks in period 2 , [ $\left.g+(1-g) p_{2}\right](Y-R)<p_{2}\left(Y-R_{T}\right)$. Using these assumptions, we can construct the third example in which both bank types can retain borrowers in period 2 . Borrowers choose relationship loans in period 1. If the initial bank is good, it offers a new relationship loan for period 2 . If it is bad, it can retain the borrower by providing a transaction loan.

## 6 How can relationship lenders help borrowers?

We assume that a relationship bank can increase the probability of project success. In the theoretical literature, relationship banks acquire information and use it to screen borrowers (e.g., Sharpe 1990; Rajan 1992; von Thadden 2004; Hauswald and Marquez 2006) or to mitigate the moral hazard of borrowers (e.g., Rajan 1992) and thereby improve the availability of credit. In Chemmanur and Fulghieri (1994), Boot and Thakor (2000), Dinç (2000), and Bolton et al. (2016), relationship lenders have an even more obvious role in increasing the probability of project success and helping borrowers (during financial distresses). Our paper is more related to the latter literature. What does this kind of help mean in reality? This section reviews various ways that relationship banks can help borrowers.

A bank can offer information and advice to borrowers who may be young and lack business expertise. In this role, banks can use their experience, contacts, and reputation. Banks may also help entrepreneurs find new business contacts. Scott (1986, 948-949) stresses this kind of help:

Many commercial banks, for example, routinely provide both financial and managerial advice to business firms ... banks indicated that they made special efforts to accommodate small business borrowers by providing financial counseling, and referrals to technical and management assistance as non-fee services. As part of their cash management services, most commercial banks now offer comprehensive analysis of customer receipts and disbursements, as well as credit information, market analysis, financial management assistance and production advice.

Nichols et al. (2017) give more examples on this kind of services to borrowers.

Second, a relationship lending bank can waive unnecessary loan covenants after a careful case-by-case judgement. The bank knows the borrower's financial condition very well
owing to their close-knit contact. Consequently, relationship banks pursue the socially optimal flexible covenant policy while transaction banks and bond markets follow an inflexible policy that is injurious to firms. The former policy increases the probability of project success. In their empirical research, Denis and Wang (2014) show that debt covenants are frequently renegotiated. Denis and Wang (2014, p.349) summarize their findings:

Nevertheless, when covenant variables evolve close to their contractual limit, renegotiations of covenant limits are far from automatic. In the majority of these cases, covenants are not relaxed. Consistent with a state-contingent exercise of control rights, our evidence indicates that creditors take into account the borrower's specific operating conditions and prospects when making the decision. We find, for example, that among those cases in which planned capital expenditures would exceed current covenant limits, changes in those limits are positively associated with a measure of quality of the borrower's investment opportunities.

Third, a relationship bank can help financially healthy firms overcome temporary financial difficulties by lending more to them during economic downturns. This lending increases the borrowers' probability of success. Transaction lenders and bond markets cannot offer this kind of help because they do not know the borrowers' true financial conditions. This idea is supported by impressive empirical evidence (e.g., DeYoung et al. 2015; Bolton et al. 2016; Puri et al. 2017; Beck et al. 2018; Karolyi 2018). For instance, Puri et al. $(2017,557)$ discover:

Further, while affected banks significantly reduce their acceptance rates during the financial crisis, we find relationships help mitigate the supply side effects on bank lending. Customers with relationships with the affected bank are less likely to have their loans rejected as compared with new customers.

Karolyi (2018, p.5) reports more supporting evidence:

Personal lending relationships benefit firms across loan terms, especially during macroeconomic downturns. Increased financial flexibility from personal relationships insulated firms from financial shocks during the recent financial crisis ..

Fourth, banks tailor financial products according to the borrowers' needs. Nichols et al. (2017, 27) describe these differences between banks:

As a bank, you have limited resources to drive marketing and bring on new products. Because of this fact, banks are not equipped to meet the needs of every customer. Sometimes it is best for a customer to go to a different bank that is positioned to meet those needs.

Nichols et al. $(2017,28)$ go on: "Managing a customer that does not fit the goals and capabilities of the bank, often serves to drive up cost and frustrates the customer." Consequently, banks have different products, resources, goals, and capabilities. Bank $W$ may be a good bank for borrower $w$ and a bad bank for borrower $z$, while Bank $Z$ may be a good bank for borrower $z$ and a bad bank for borrower $w$.

## 7 Conclusions

In this paper, we introduce a novel model of lending relationships and switching costs. In the credit market, we assume that all borrowers are identical, but banks are different. The type of a bank is unobservable to outsiders and a firm can learn it only by borrowing from the bank. In the key model, good banks can retain existing borrowers in period 2, while the borrowers of bad banks leave their initial banks and look for a new bank. New borrowers are unprofitable for good banks and existing borrowers are profitable. Short-term loans and unsecured loans are optimal. Loan commitments should not be used. In two scenarios, both bank types can retain existing borrowers in period 2 even if each borrower searches for a good bank in period 1. First, it may be profitable to search for a good bank and for a valuable relationship loan only if the lending relationship is long. If the relationship is short, it is unprofitable to look for a good bank, and a borrower chooses a transaction loan from the current bad bank. Second, if the realized proportion of good banks is stochastic (the average quality of banks is unknown), poor quality service leads borrowers to lower their estimate of the average quality of banks, and they will stick with the current bad bank.

The paper has many empirical conclusions. In the traditional models, Sharpe (1990), Rajan (1992), and von Thadden (2004), new borrowers are unprofitable for the initial bank and existing borrowers yield profits. This finding means the primary result is that the interest rate rises during the relationship. However, the opposite result is possible in these models. Since the initial bank discovers the types of the borrowers during the first period and some information spreads to outside banks, banks can allocate loans to good firms in the second period and the breakeven loan interest is lower in the second period than in the first period. If the market power of the initial bank is small (large), the loan interest is lower (higher) in the second period than in the first. We also find that new borrowers are unprofitable for the initial bank and existing borrowers yield profits
(Propositions 1 and 3). This finding generates the same primary result as in the traditional models: the interest rate is higher in the latter period. However, Sections 4 and 5 introduce scenarios in which bad banks can retain their existing borrowers by decreasing interest rates. As a result, the interest rate of bad banks declines during the relationship. Hence, our study provides the same result as the traditional models: the interest rate rises or declines during the relationship. In the introduction, we reviewed evidence that loan rates rise as the relationship matures (e.g., Petersen and Rajan 1995; Angelini et al. 1998; Degryse and Van Cayseele 2000; Hernandez-Canovas and Martinez-Solano 2010; Ioannidou and Ongena 2010; Hasan et al. 2019). By contrast, Berger and Udell (1995) and Bharath et al. (2011) identify decreases in rates over time. For a more extensive study, we use the survey of Degryse et al. (2009). They conclude that the empirical findings are mixed. Most US studies report that interest rates decline in each relationship year, while many European studies find that loan rates rise or are unaffected each year.

We predict long-term bank-borrower relationships. In the key model, good banks can retain existing borrowers. In the complementary models, both bank types can retain existing borrowers. This is consistent with the available empirical evidence that long-term lending relationships are common (e.g., Elsas and Krahnen 1998; Degryse and Ongena 2008; Degryse et al. 2009). The traditional models also predict long-term lending relationships: a few good borrowers retain the initial lending relationship. ${ }^{9}$

Our prediction that relationship loans are relatively unsecured is consistent with the empirical evidence (e.g., Berger and Udell 1995; Degryse and Van Cayseele 2000; Bharath et al. (2011). Degryse et al. (2009) draw the same conclusions in their survey. By contrast, the traditional models of relationship lending favor secured lending, because collateral imposes costs on bad borrowers the most of all.

[^8]According to our theory, relationship lenders provide valuable help to borrowers. This is consistent with the evidence. Hombert and Matray (2017) present fresh evidence that relationship lending has a positive effect on innovative firms and innovations. Longer relationships enhance access to loans (e.g., Petersen and Rajan 1994; Hernandez-Canovas and Martinez-Solano 2010; Bharath et al., 2011). Most of all, Elsas and Krahnen (1998), Iyer et all (2014), Bolton et al. (2016), Puri et al. (2017), Beck et al. (2018), and Karolyi (2018) report strong evidence that relationship lending alleviates financial constraints during a downturn. ${ }^{10}$ The same prediction is found in the theoretical literature (Chemmanur and Fulghieri 1994; Dinç 2000; Boot and Thakor 2000; Bolton et al. 2015) but not in the traditional models on relationship lending. It is, however, possible to interpret the traditional models more generally. A bank acquires soft information during a lending relationship. This information makes it relatively easy to analyze and control the borrower. As a result, long-term relationships enhance access to loans during financial distresses. This is consistent with the empirical evidence.

In the traditional models of relationship lending, banks banish bad borrowers after the first period. Our study predicts that borrowers leave bad banks. This prediction is supported by empirical evidence (e.g., Gan 2007; Jiménez et al. 2012; Iyer et al. 2014; Degryse et al. 2019). They discover that banks face negative shocks and the impact of the shocks varies strongly between banks. A few banks have to cut back lending and their borrowers must search for new banks. The likelihood of the termination of an existing lending relationship is higher for banks with larger shocks. Consequently, borrowers leave bad banks. Howorth et al. (2003, 305) find supporting empirical evidence: "There was strong evidence that the main drives of the decision to switch or consider switching banks were difficulties obtaining finance and dissatisfaction with the service provided." Gopalan et al. (2011) find evidence that firms form new banking relationships to overcome borrowing constraints. These switching borrowers experience an increase in capital expenditure,

[^9]leverage, and sales growth. The impact of the switch is positive. In the traditional models of relationship lending, bad borrowers switch banks and their new projects are relatively unproductive. In our model, borrowers leave bad banks and search for better banks. Therefore, new projects are relatively productive. Consequently, the findings of Gopalan et al. (2011) support our predictions.

As to the future research, our model indicates that empirical studies should analyze the sources of the switching costs. In addition, we assume that relationship loans are more expensive than transaction loans. The assumption is supported by empirical evidence (Bolton et al. 2016). More empirical research on this field could be fruitful.

As to the policy implications of the model, we mentioned in the introduction that this paper stresses the transparency of banks. Appendix A points out how improved bank transparency erodes the market power of the initial bank in period 2 and thereby lowers the interest rate in period 2. To improve bank transparency and mitigate the hold-up problem associated with heterogenous banks, it might be useful to create a new type of credit registry. It would be a platform for borrowers to share information about banks: bank-specific information about foreclosures, bankruptcies, debt renegotiations, interest rates, etc. This kind of registry might provide statistical information and a platform for informal communication between borrowers. Does bank X pursue a flexible or inflexible policy regarding loan covenants? Does it waive unnecessary covenants after a case-bycase analysis and provide help during temporary financial difficulties? Do credit rationing problems vary among banks? Banks supply different products. Are the differences important or cosmetic? A few firms may have many bank relationships, and they can compare banks. This kind of credit registry might help evaluate banks and avoid bad banks. This avoidance would alleviate asymmetric information and boost production in our model setup. The registry differs strongly from standard credit registries in which banks share information about borrowers' repayment behavior (e.g., Pagano and Jappelli 1993). In our credit registry, borrowers share information about bank lending behavior.

## Appendix A

Firms that did not borrow from Bank X in the first period, which are labeled outside firms with respect to Bank X, observe a signal of the bank type in period 1. This signal has a conditional distribution function:
$\operatorname{prob}(\tilde{y}=\tilde{G} \mid G)=\operatorname{prob}(\tilde{\gamma}=\tilde{B} \mid B)=(1+\emptyset) / 2$,
$\operatorname{prob}(\tilde{y}=\tilde{G} \mid B)=\operatorname{prob}(\tilde{\gamma}=\tilde{B} \mid G)=(1-\emptyset) / 2$,
$0 \leq \varnothing \leq 1$. Here $\tilde{\gamma}(X) \in\{\tilde{G}, \tilde{B}\}$ is a noisy signal regarding Bank X. The signal is either good, $\tilde{G}$, or bad, $\tilde{B}$. All outside firms and Bank X observe the same signal, and they do so without cost. Thus, if $\varnothing=0 \quad(\emptyset=1)$, then outside firms learn nothing (have perfect information) about the bank. Consider a bank with a good signal. It represents the good type with probability:
$\operatorname{prob}(G \mid \tilde{G})=\frac{g(1+\emptyset)}{g(1+\emptyset)+(1-g)(1-\emptyset)}$.

Here $\operatorname{prob}(G \mid \tilde{G})$ increases with $\emptyset$. If $\emptyset=0$, we have $\operatorname{prob}(G \mid \tilde{G})=g$. If $\emptyset=1$, then we get $\operatorname{prob}(G \mid \tilde{G})=1$. A loan succeeds with certainty if the bank is good, and with probability $p$, if it is bad. If a borrower contacts an outside bank with a good signal, their expected return is:
$[1-\operatorname{prob}(G \mid \widetilde{G})] p(Y-R)+\operatorname{prob}(G \mid \widetilde{G})(Y-R)$.

The first (second) term shows expected repayments if the new bank proves to be bad (good). Recall from eq. (3) in subsection 3.1 the firm's expected return if it retains the initial loan relationship, $\pi_{2}^{G}=Y-R_{2}^{G}$. The initial bank can retain the lending relationship if $\pi_{2}^{G}=Y-R_{2}^{G}$ is at least (A.3), that is, the interest rate of the initial bank is at most:

$$
\begin{equation*}
R_{2}^{G}=R+[1-\operatorname{prob}(G \mid \widetilde{G})](1-p)(Y-R) . \tag{A.4}
\end{equation*}
$$

If the signal has no value, $\varnothing=0$, then we have $\operatorname{prob}(G \mid \widetilde{G})=g$ and $R_{2}^{G}$ is the same as in eq. (4) in subsection 3.1. The initial bank has a great information advantage in period 2, and it can set a high interest rate. If the signal is perfect, $\varnothing=1$, then we have $\operatorname{prob}(G \mid \widetilde{G})=1$ and $R_{2}^{G}=R$ : banks operate under perfect competition in period 2, and the information advantage of the initial bank disappears. If $\varnothing$ increases, then the transparency of the banking system improves, the information advantage of the initial bank shrinks and $R_{2}^{G}$, declines. Since a lending relationship yields zero expected profit, the improved transparency rises the loan interest of period 1. Finally, we have implicitly assumed that the noisy signal does not change bank competition. The signal may give market power to banks if the number of banks with a good signal is small compared to the number of firms.

## Appendix B

This appendix shows that there is a proportion of bad banks to good banks where all bad banks make zero profit. To begin, recall the expected profit of a bad bank from eq. (6)
$\pi_{B}(b)=(1-b)\left(p R_{1}-r\right)+\delta(1-b) b(p R-r)-C_{B}$.

First, we show that if the proportion of bad banks is very small, they are profitable. Assume that the proportion of bad banks is $b_{\varepsilon}>0$. In period 1, the interest rate is $R_{1}=R-(1-p) b_{\varepsilon}(Y-$ $R)$. Here $b_{\varepsilon}$ can be chosen so that $p R_{1}>p R-\varepsilon$ with each $\varepsilon$. Therefore, it is possible to have $b_{\varepsilon}$ such that a loan is profitable and covers the cost of bank formation: $p R_{1}-r-C_{B}>p R-$ $\varepsilon-r-C_{B}>0$. The latter inequality is based on Assumption 2, $p R-r-C_{B}>0$, and the fact that $\varepsilon$ is small enough. We know that $\pi_{B}\left(b_{\varepsilon}\right)>0$.

We aim to find out the optimal proportion of bad banks. Now eq. (B.1) becomes:
$\frac{d \pi_{B}}{d b}=-2 b(p R-r)-(1-2 b) p(1-p)(Y-R)$.

Two scenarios occur: Scenario 1: If $2(\mathrm{pR}-\mathrm{r})>\mathrm{p}(1-\mathrm{p})(\mathrm{Y}-\mathrm{R})$ in eq. (B.2), we have $d \pi_{B} / d b<0$ with each $b$ and the expected return minimizes if $b=1$. If $b=1$, then eq. (B.1) shows $\pi_{B}(1)=-C_{B}<0$. Given this, $\pi_{B}\left(b_{\varepsilon}\right)>0$ and $d \pi_{B} / d b<0$, there is $b^{*}$ such that the
expected profit is zero, $\pi_{B}\left(b^{*}\right)=0$. Scenario 2: If $2(\mathrm{pR}-\mathrm{r})<\mathrm{p}(1-\mathrm{p})(\mathrm{Y}-\mathrm{R})$ in eq. (B.2), we get $d \pi_{B} / d b=0$ if $b=b_{\text {min }}$,

$$
\begin{equation*}
b_{\text {min }}=\frac{1}{2} * \frac{p(1-p)(Y-R)}{p(1-p)(Y-R)-(p R-r)} . \tag{B.3}
\end{equation*}
$$

Here we have $0.5<b_{\min }<1$. When $b=b_{\min }$, the bank profit achieves the minimum value. The minimum bank profit without the cost of bank formation, $C_{B}$, is:

$$
\begin{gathered}
\pi_{B}\left(b_{\text {min }}\right)=\Phi[(1-p) p(Y-R)-(p R-r)][(1-p) p(Y-R)-2(p R-r)] . \\
\text { where } \\
\Phi=\frac{-b_{\min }\left(1-b_{\min }\right)}{p(1-p)(Y-R)} .
\end{gathered}
$$

The terms in both square brackets are positive owing to the definition of Scenario 2. Since $\Phi<0$, (B.4) is negative. The bank gets a negative return if $b=b_{\min }$ even without $-C_{B}$. When $b_{\min } \leq$ $b<1$, we have $d \pi_{B} / d b>0$ and $\pi_{B}(b)<0$ even without $-C_{B}$. Therefore, the optimal proportion of bad borrowers is lower than $b_{\text {min }}$. When $0 \leq b \leq b_{\text {min }}$, we have $\pi_{B}\left(b_{\varepsilon}\right)>0$, $d \pi_{B} / d b<0$, and $\pi_{B}\left(b_{\text {min }}\right)<0$. There is a proportion of bad banks, $b^{* *}, 0<b^{* *}<b_{\text {min }}$, such that a bad bank makes zero profit, $\pi_{B}\left(b^{* *}\right)=0$.

In sum, let $b_{\max }=b^{*}$ if $2(p R-r)>p(1-p)(Y-R)$ in eq. (B.2) and $b_{\max }=$ $b^{* *}$ if $2(p R-r)<p(1-p)(Y-R)$ in (B.2). Now, $b_{\max }$ gives the maximal proportion of bad banks such that each of them has zero profit.

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[^0]:    ${ }^{1}$ See also Dell'Ariccia et al. (1999), Dell'Arricia (2001), Berger and Udell (2002, 2006), Hauswald and Marquez (2003, 2006), von Thadden (2004), Dell'Ariccia and Marquez (2004), DeYoung et al. (2004), Bolton and Freixas (2006), Black (2011), Puri et al. (2011, 2017), DeYoung et al. (2015), and Karolyi (2018). Freixas and Rochet (2008), Degryse and Ongena (2008) and Degryse et al. (2009) survey the literature.

[^1]:    ${ }^{2}$ In some scenarios of Rajan (1992), long-term loans are optimal.

[^2]:    ${ }^{3}$ For empirical evidence, see Agarwal and Hauswald (2010).

[^3]:    ${ }^{4}$ For supporting evidence, see Puri et al. (2017).

[^4]:    ${ }^{5}$ The existing borrower leaves the initial bad bank with certainty. In this regard the offer of the initial bad bank $R_{2}^{G}$ is insignificant. However, the bad bank will attract profitable new borrowers. To hide its true type, the bad bank mimics good initial banks and makes public offer $R_{2}^{G}$ to its existing borrower.

[^5]:    ${ }^{6}$ Dell'Ariccia et al. (1999), Dell'Ariccia (2001), and Dell'Ariccia and Marquez (2004), for instance, assume this kind of competition in their relationship lending models.

[^6]:    ${ }^{7}$ Short-term loans are optimal even if $\theta_{2}<R$.

[^7]:    ${ }^{8}$ This difference is important in compared with subsection 4.1.

[^8]:    ${ }^{9}$ In some model versions of Sharpe (1990), all firms retain the initial lending relationships.

[^9]:    ${ }^{10}$ See also Degryse et al. (2009, 115-117) for an extensive survey on the literature.

