## An Overview and Re-Interpretation of Paradoxes of Responsiveness

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## Abstract

One of the most obvious desiderata of democratic decision-making is that the political outcomes (policies, elected persons, legislation) be responsive to popular opinions. In representative forms of governance the responsiveness is not expected to pertain every single outcome, but the very idea of going to the people seems to presuppose some degree of responsiveness. In social choice theory several notions that aim to capture aspect of responsiveness have been introduced and related to other desiderata of social choice. We shall discuss the most common notions and discuss their relevance in democratic decision making. We shall also look at the paradoxes related to non-responsiveness from a novel angle, viz. we try to determine their significance to the multiple criteria decision making (MCDM). It turns out that some methods of aggregating criterion performances of policy alternatives can be ruled out because of their bizarre behavior under some decision settings.

## 1 Introduction

Responsiveness is one of the most obvious desiderata in democratic rule. At the very least unresponsive rules of governing are certainly not acceptable as the very idea of democracy presupposes that the ruled, the people, can, by expressing by their opinions in legitimate manner, bring about changes in the way public policies are formulated and executed. Elections are the normal institutions to transform the popular views into public policies or other electoral outcomes. The most common of the latter are, of course, those that pertain to composition of parliaments or offices of presidents. But what does unresponsiveness, then, mean? A clear example of an unresponsive voting rule is a constant one which results in a fixed outcome, say x, regardless of the distribution of the expressed opinions by the voters. I.e. no matter how the voters vote, x always wins. Clearly, constant rules make the act of voting meaningless in the instrumental sense, that is, as a way of influencing the way public policies are to be pursued. A similarly minimalistic way of defining responsiveness as the exclusion of constant rules is the condition on rules known as citizens' sovereignty. This requires that, given a set of alternatives A of k alternatives and any ranking R of those alternatives, there is a distribution of voter opinions over those alternatives so that R is the outcome resulting from the application of the rule to this distribution. This condition excludes blatant discriminations against some alternatives. This doesn't mean that rules that satisfy citizens' sovereignty are *eo ipso* intuitively responsive to the voter opinions. A case in point is the unanimity rule: the *status quo* alternative, say x is selected, unless all individuals prefer another alternative, say y, to x. This rule clearly satisfies citizens' sovereignty, but is extremely biased towards the *status quo*.

In what follows we shall investigate some intuitively natural forms of responsiveness of choice rules. The forms will be looked upon as invulnerability to certain kinds of paradoxes.Our primary focus is on variations in the choice sets resulting from rules under changes in individual opinions. Two types of settings are of interest: first, those where the changes in individual opinions happen in a fixed electorate, and second, those where the changes involve enlarging the electorate itself by including new voters in the voter set. The former settings will be called fixed electorate paradoxes and properties, while the latter will be called paradoxes and properties in variable electorates.

## 2 What is responsiveness?

A natural way of approaching the responsiveness problem is to start from comparing the opinions of the electorate to the result of the choice rule. The question then becomes, how well or accurately the latter represents the former. In any given choice situation we could argue that the better the choice result represents the voter opinions, the more responsive the rule. It turns out that nearly all voting rules that transform n- tuples of individual complete and transitive preference relations (rankings) (n being the number of voters) into collective rankings can be seen as the most optimal, i.e. most responsive, rules. What makes them different is their underlying idea of a consensus state, that is, a situation involving no disagreement as to the outcome and the distance metric used in measuring the distance of any preference profile from a consensus state. Such a consensus state can be one where all voters have identical rankings over the alternatives or one where all voters rank the same alternative first or one where a given alternative is the Condorcet winner. Similarly, the distance measure can be the inversion metric counting the number of binary inversions of adjacent alternatives needed to transform one ranking to another or a discrete metric that simply counts those rankings that differ in some respects from one another (see Meskanen and Nurmi 2006; Slinko et al.).

If the consensus state is one where all voters have an identical ranking over alternatives and if the distance between any two rankings is measured by the inversion metric, then the outcomes ensuing from the application of Kemeny's rule are optimal in the sense of minimizing the distance between the observed profile and the desired consensus state. Similarly, it has been shown by Nitzan () that the Borda count outcome represents best the voter opinions if the distance measure is the inversion metric and if the consensus state is one where all voters are unanimous about which alternative should be ranked first. Plurality voting, in turn, can be seen as the optimal representation of the voters' opinions if the distance measure is the discrete metric and the consensus state is the same as in the Borda count. Similarly, most voting systems can be defined as optimal distance minimizing rules (see Meskanen and Nurmi 2006; Eckert and Klamler 2011; Elkind et al. 2012).