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Moving mathematics out of the classroom: Using mobile technology to enhance spontaneous focusing on quantitative relations

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Abstract

Spontaneous focusing on quantitative relations (SFOR) has been shown to be a strong predictor of rational number conceptual development in late primary school. The present study outlines a intervention program that examines the possibilities to enhance late primary school students' SFOR tendency. The intervention program harnessed mobile technology in order to allow students to explore and identify quantitative relations in their everyday environment, including situations outside of the classroom. A total of 38 thirteen-year-olds from two classrooms participated in the seven-week long quasi-experimental study. One classroom spent five lessons over five weeks participating in activities which involved uncovering, defining, and describing multiplicative relations in their everyday surroundings. In comparison to a business-as-usual control group, results show the intervention to be successful in enhancing SFOR tendency. These results suggest that it is possible to utilize mobile technologies to enhance students' awareness of the possibilities to use quantitative relations as explicit targets of focusing and reasoning in non-explicitly mathematical situations.

Moving mathematics out of the classroom: Using mobile technology to enhance spontaneous focusing on quantitative relations

One goal of mathematics education, especially with regard to the development of the number concept, is to harness the power of students' early informal quantitative skills and reasoning to provide the foundation for the culturally developed formal mathematical structures (Confrey, Maloney, Nguyen, Mojica, & Myers, 2009). However, these foundations are also potential barriers to development, and many individual differences that already exist in early childhood have been found to be exacerbated, and not eased, by instruction (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004). Understanding of both early (pre-)mathematical skills and learning and instruction in formal mathematical situations is under constant acceleration, due to increasing attention to mathematical development among researchers. However, this knowledge is highly concentrated in overly stylized and mathematically specific situations, where participants' attention is specifically and explicitly focused on the mathematics of the situation (cf. Lobato, Rhodehamel, & Hohensee, 2012). Over the past ten years a new body of research has introduced evidence that the relevance of mathematical features of the world is not equally apparent to all. Substantial individual differences in the tendency to pay attention to mathematics both in and out of the classroom have been found from early childhood to adulthood (for a review see Hannula-Sormunen, 2014). By examining, students' tendencies of spontaneous focusing on mathematical aspects of their environment, this new line of research has been able to uncover substantial contributors to individual differences in mathematical development from early childhood until late primary school.

The present study aims to further our understanding of one such spontaneous mathematical focusing tendency, Spontaneous Focusing On quantitative Relations (SFOR). In particular, the present study aims to examine if it is possible to use mobile technology to provide students with the opportunity to discover the multiplicative relations that exist in their everyday lives, both in and out of the classroom, and to test the effectiveness of an intervention involving these activities in enhancing students' SFOR tendency.

SFOR tendency

Previously, individual differences in students' tendency of Spontaneous Focusing On Numerosities (SFON) have been found to have substantial effects on the development of mathematical skills (Batchelor, Inglis, & Gilmore, 2015; Hannula & Lehtinen, 2005). It is hypothesised that those persons who have a higher SFON tendency in their everyday environment gain more practice with number skills and knowledge, leading to improvements in their formal mathematical knowledge (Hannula & Lehtinen, 2005; Lehtinen, Hannula-Sormunen, McMullen, & Gruber, 2017). However, in many situations in everyday life, exact number is not sufficient for reasoning about the situation – for example, evenly splitting two cookies with three people. As well, more advanced mathematical topics require more elaborate reasoning, beyond exact number.

Thus, more recent studies have extended this work to reveal that SFOR tendency with multiplicative relations is a component of mathematical development in primary school, especially with regard to learning about fraction and decimal concepts (McMullen, Hannula-Sormunen, Laakkonen, & Lehtinen, 2016). SFOR tendency is the spontaneous, (i.e. unguided) focusing of attention on quantitative relations and the use of these relations in situations which are not explicitly mathematical. SFOR tendency involves focusing on the quantifiable relation(s) between of two or more (sub-)sets of items or quantities, rather than only focusing on a single quantity (McMullen et al., 2016).

In their examinations of SFON tendency, Hannula-Sormunen and Lehtinen (Hannula & Lehtinen, 2005; Hannula-Sormunen, 2014) have outlined four key requirements for satisfactory evidence of a spontaneous mathematical focusing tendency such as SFOR. Three of these four requirements have been found in previous SFOR studies. First, in multiple samples both in early and late primary school, substantial inter-individual differences have been found in SFOR tendency, that are not entirely explained by differences in the requisite skills needed to solve the tasks (McMullen et al., 2016; Van Hoof et al., 2016). This indicates that SFOR is at least a partially distinct aspect of a person’s existing mathematical competences. Second, SFOR has been found to be a stable trait across multiple tasks, even over a year long period, and is thus not entirely dependent on task context (McMullen, Hannula-Sormunen, & Lehtinen, revised). In this way, SFOR can be said to be a more generalizable tendency and is therefore expected to be related to individuals' everyday activities. Third, SFOR has been shown to be related to the development of relevant mathematical skills, specifically the development of rational number conceptual knowledge (McMullen et al., 2016; Van Hoof et al., 2016). SFOR is shown to have a unique impact on the development of rational number knowledge even after taking into account a number of mathematical and other cognitive factors, such as non-verbal intelligence and mathematical achievement. The present study aims to address the fourth proposed type of evidence needed to suggest the relevance of SFOR tendency for mathematical development – the possibility to enhance SFOR tendency through social interaction.

Towards the use of a mobile application to enhance SFOR tendency

The expectation is that SFOR tendency, in being malleable to instruction through social interaction, is not a static, ingrained trait that is fixed within an individual, but is instead a socially developed tendency. While previous research suggests that in younger children it is possible enhance their SFON tendency in their everyday environment (Hannula, Mattinen, & Lehtinen, 2005), such a hypothesis has not been tested with SFOR tendency. Since it was young children who participated in the SFON enhancement study, it was necessary to have adults support and modelling the focusing of attention on number. However, in the present study among early adolescents, both mobile technology and peer interaction are used to support focusing on quantitative relations in everyday situations.

There has been growing interest in the use of mobile and digital tools in mathematics education, based on their potential for the practice and acquisition of non-routine mathematical skills through learning by doing in an open manner (Gee, 2007). Moeller and colleagues (2015) have argued that the latest technological developments have opened up new directions for the training of the mathematical competencies. In particular, computer technology provides possibilities to increase the interactivity of training approaches, experiencing mathematics concepts in real life context, and in an embodied fashion. Embedded embodied cognition draws together research on embedded cognition and embodied cognition in order to guide the design of learning manipulatives (Pouw, van Gog, & Paas, 2014). In fact, several sensor technologies, such as Kinect™ (Link, Moeller, Huber, Fischer, & Nuerk, 2013), dance mats (Moeller et al., 2015), and accelerometers of mobile devices (Ninaus, Kiili, McMullen, & Moeller, 2017) have been used to implement embodied number line trainings, with natural and rational numbers. However, so far the embodied approaches have been restricted to single user environments (Moeller et al., 2015) and thus the present study opens up a new research approach focusing on shared attention in embodied mathematics learning mediated by mobile technology.

In order to support the integration of the theoretical foundation of SFOR tendency and available technologies in the design of intervention activities, we defined five key requirements for the mobile-based learning environment: 1) The activities should be experienced outside the classroom and support an embodied cognition approach as well as physical activities. 2) The creation and management of the learning tasks should be so easy that students can also produce mathematical content in the activities. 3) The activities should support the use of different kinds of tasks with quantitative relations, utilizing photos, videos, and physical distances. 4) It should be possible to track and record students' activities in the activities. 5) The activities should enable collaborative learning. Overall the chosen technology, ActionTrack (Holm & Laurila, 2014), was found to meet these needs and the standards of a quality model that distinguishes technical aspects for mobile learning solutions (Sarrab, Elbasir, & Alnaeli, 2016).

Furthermore, as a part of the design of the intervention tasks, we tried to identify the possible risks that a technological implementation involving physically activating aspects can generate. In the risk analysis we applied a design framework for educational exergames (Kiili & Perttula, 2013). Educational exergames combine gameplay elements from educational games (cognitively challenging games) and exergames (physically challenging games), which align closely with the goals of the present study. We identified two risks in our design. First, the surroundings or the environment where the game is played creates a risk of injury. For example, the activities can take so much attention that the player stumbles or even hits something while watching the screen or when reasoning with mathematics and moving at the same time. Second, the intensity of the physical activities (navigating in the physical game world) may make it more difficult for students to focus their attention on the quantitative relations (Tenenbaum & Connolly, 2008), the main objective of the game. These identified risks were taken into account in the instructions of the intervention, in determining the rhythm of the activities.

The present study

The main aim of the present study is to examine the effectiveness of an intervention to enhance SFOR tendency in late primary school students. Through the use of activities using a mobile application, the intervention is expected to have a significant effect on students SFOR tendency in comparison to normal classroom activities. As well, previously, Hannula and colleagues (2005) have found that an intervention among young children in promoting their SFON tendency to be successful among those students who had some initial SFON tendency. Thus, the present study aims to determine if there are similar constraints in the effectiveness of the intervention on SFOR tendency.

Methods

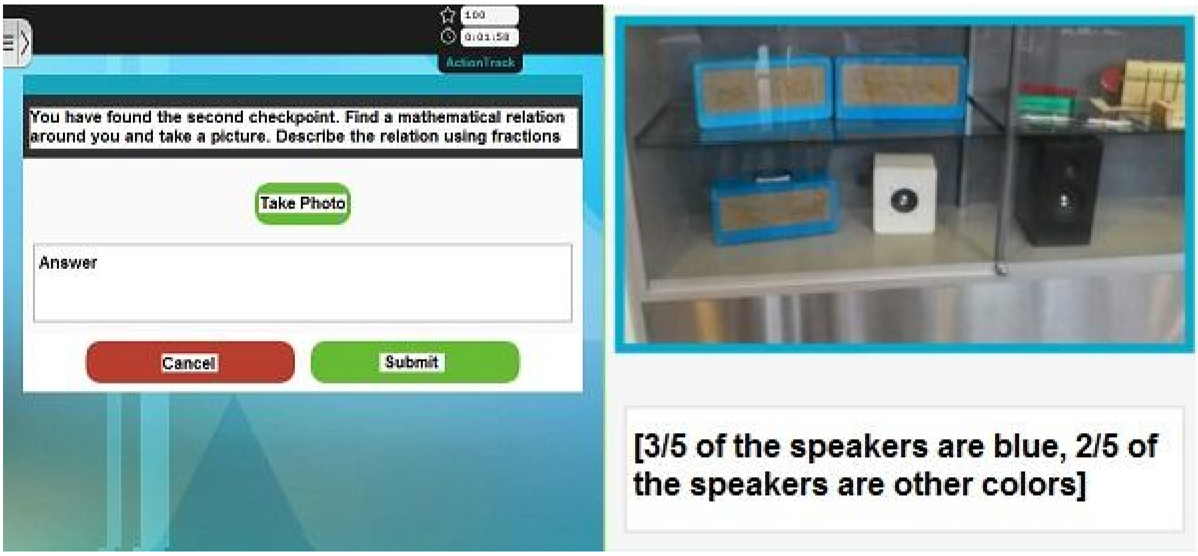
Students from two 6th grade classrooms (N = 38; M_{age} = 13;0 years) took part in a pre-/post-test quasi-experimental study over a seven-week period during the spring semester. One classroom was assigned to the experimental condition, in which students participated, once a week, for five weeks, in activities aimed at enhancing their SFOR tendency during their normal math lessons. The other classroom continued with their normal mathematics instruction, which was about rational numbers. There is no ability grouping in Finnish comprehensive schools up through 9th grade.

Intervention

The SFOR intervention activities are mainly carried out using ActionTrack, a commerciallyavailable mobile application, in which “scavenger hunt”-like activities can be created for participants to complete (Holm & Laurila, 2014). In the first of five sessions, a trained researcher provided instructions to the students on the idea of finding quantitative relations in everyday situations and some basic instructions on the use of ActionTrack (taking photos and videos, typing in responses). Examples of using quantitative relations to describe everyday situations were presented by the researcher and students were asked to provide their own examples in whole-class discussions led by the researcher.

Students had extensive prior instruction with rational numbers and multiplicative and proportional reasoning. According to the national core curriculum, fractions and decimal numbers and basic arithmetic with them (addition and subtraction) are introduced starting in 3rd grade and this instruction continues up through 6th grade. So that they had experience reasoning about the mathematical features that were discussed in this session, though the exact language that was used (i.e. “quantitative/mathematical relations”) was new to them.

In the second, third, and fifth session students completed the core activity of the intervention, namely the Find-the-Relation scavenger hunt, which included two parts: (a) following routes which were described using quantitative relational terms to define distances and locations and (b) finding and describing examples of quantitative relations in the surroundings. Students were in groups of two or three, which shared one tablet computer and completed the tasks with only minimal practical support from researchers so that the mathematics of the tasks were left open for the students to interpret within their groups:. In the fourth session students designed their own routes, which were to be completed by their classmates the following week.


You have found the second checkpoint. Find a mathematical relation around you and take a picture. Describe the relation using fractions

[3/5 of the speakers are blue, 2/5 of the speakers are other colors]

Figure 1.Screenshot from one checkpoint (translated) and example of photo taken and translated description written by students.

Routes were designed by the researchers by specifying the start, middle, and end points of a route using printed QR-codes that were placed at specific locations. When scanned by the students the QR-codes provided feedback on whether the group was in the right location and also provided a task to complete and/or directions to the next checkpoint. Directions used quantitative relations to describe where the next QR-checkpoint was (e.g. “The task can be found halfway between the starting and ending point.” “You have now gone 1/3 of the way to the end-point, guess where the end is.”). Thus, students were able to experience relational distances and get practice with considering distances and journeys in relative terms (key requirement 1). Multiple routes were setup around the school’s physical environment and groups began with different routes as to not overlap in their scavenging. When creating routes for their classmates, students needed to follow a similar structure by defining a starting and ending point, and then a mid-point with a multiplicative relation.

When the groups scanned the QR code at the end of each route, they were given one of three versions of a task of finding and describing quantitative relations in their immediate surroundings using their tablets, allowing them to create mathematically rich content (key requirements 2 and 3). In the first version of the task, students were asked to take a photo of an example of a multiplicative relation and provide a written description of it using either a fraction, a decimal, or a whole number ratio (for example, Figure 1). The second version asked students to film a video and verbally describe at least three quantitative relations in the same location. Finally, in bonus tasks, student groups were asked to describe the quantitative relations of a given photo. All three tasks were completed within the mobile application and students' responses were recorded in an accompanying online environment (key requirement 4).

Students were given points for all answers and there was no criteria for correctness for responses. However, at the start of each session good examples of the quantitative relations were described by the researcher and preliminary evidence from video observational data collected during the sessions suggests that there was meaningful collaboration within the groups in identifying and defining the quantitative relations that were described in these tasks (Hilppö, Rajala, & McMullen, 2017; key requirement 5). After it became apparent that most examples described by the students were discrete relations (e.g. two out of three jackets are red), emphasis was placed on providing students examples of continuous relations (e.g. the glass is half full) in everyday situations. Nonetheless, most relational examples described by the students were discrete or discretized.

SFOR Measures

One week before and two weeks after the intervention, participants completed measures of their SFOR tendency. Crucial to the measurement of SFOR tendency, participants cannot be aware of the mathematical nature of the tasks, therefore no mention of mathematics is made either before or during testing. Likewise, SFOR testing cannot be conducted during normal mathematics lessons. For the Post-Test a new tester was brought in who the students had never met and the tasks were modified to be novel contexts.

All the SFOR tasks were paper-and-pencil picture description tasks (for more detail about SFOR measures, see McMullen et al., 2016, Supplementary Material A), in which the students were asked to describe how objects had changed during a transformation (Teleportation task/Magic task), or how two meals were different from each other (Plate/Picnic tasks). Images (Figure S1) were projected on the screen in the front of the class and were printed on the paper. Since participants were not guided to focus on the mathematical features of the tasks, it is claimed that if they used multiplicative relations in their responses, they needed to first pay attention to the multiplicative relations in the tasks on their own. In other words, using multiplicative relations in a response on these tasks requires the participant to spontaneously focus on quantitative relations on that trial

Each descriptive phrase used by participants on the SFOR tasks was classified as either involving multiplicative relations or other aspects. Explicit multiplicative relations were considered the most mathematically advanced way to describe the relations in the task, as opposed to separate additive relations. Compare “everything was multiplied by three” with “there were six more cans, eight more drinks, and two more baskets of fruit”. For the drawing items, participants were given two points for drawing the correct number of items based on the multiplicative relation shown on the previous item and one point for partially correct responses.

The total number of each type of descriptive phrase was calculated for each student for all eight trials of the SFOR tasks. Two independent coders scored 10 participants’ Pre-Test written responses and were in agreement on 97% of all coding. In order to control for overall descriptiveness, the proportion of multiplicative relations descriptions to the total number of descriptions was calculated for each trial. In order to measure students’ SFOR tendency, we separately calculated a standardized score of students’ proportion of multiplicative relation descriptions and a standardized score of their drawing sum scores and added together these standardized scores at each time point. Test-retest reliability among students in the control group was sufficiently high for the overall measure of SFOR tendency (Spearman’s $\rho = .80$)

Rational Number Test

At the post-test[1], participants also completed a 16-item test measuring their conceptual knowledge of rational number sizes (McMullen, Laakkonen, Hannula-Sormunen, & Lehtinen, 2015), with 3 fraction comparison items (e.g. “Which is larger 2/3 or 4/9”), 3 fraction ordering items (e.g. “Put 4/7, 2/6, and 5/10 in order from smallest to largest”), 3 decimal comparison items (e.g. “Which is larger .36 or .5”), 3 decimal ordering items (e.g. “Put 3.682, 3.2, and 3.84 in order from smallest to largest”), and 4 fraction and decimal comparison items (e.g. Which is larger, 1/8 or .8). Reliability for the measures was sufficient to good (Fraction Size: α = .72; Decimal Size: α = .88; Fraction and Decimal Size: α = .81).

Results

Table 1 presents descriptive statistics for SFOR tasks at both pre- and post-tests and the rational number test at post-test for the experimental and control groups separately.

Table 1: Means and Standard Deviations for SFOR tasks and rational number test.

	SFOR tendency (standardized sum score)		Rational number size knowledge		
	Pre-test	Post-test	Fractions	Decimals	Fraction vs. Decimals
Experimental group	.31 (3.78)	1.16 (4.14)	4.84 (1.42)	4.94 (2.12)	2.21 (1.51)
Control group	-.35 (3.25)	-.97 (2.42)	4.66 (1.60)	5.29 (1.26)	2.67 (1.52)

In order to examine the effect of the intervention on students SFOR tendency a repeated measures ANOVA was run with Pre- and Post-Test SFOR scores as within-subject variables and condition as between-subject variable. In order to control for overall mathematical ability, students’ rational number size knowledge scores were included as a covariate in the repeated measures

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[1] The rational number test was also administered at the pre-test, but time constraints in the classroom did not allow students adequate time to complete the test and is therefore not considered in the present study.

ANOVA.

Experimental condition was found to have an effect on the proportion of multiplicative responses from pre- to post-tests, $F(1, 33) = 6.59$, $p = .02$, $\eta_p^2 = .17$. Figure 2 indicates that participants in the experimental condition had a larger increase in the proportion of multiplicative responses from pre- to post-test than those in the control condition.

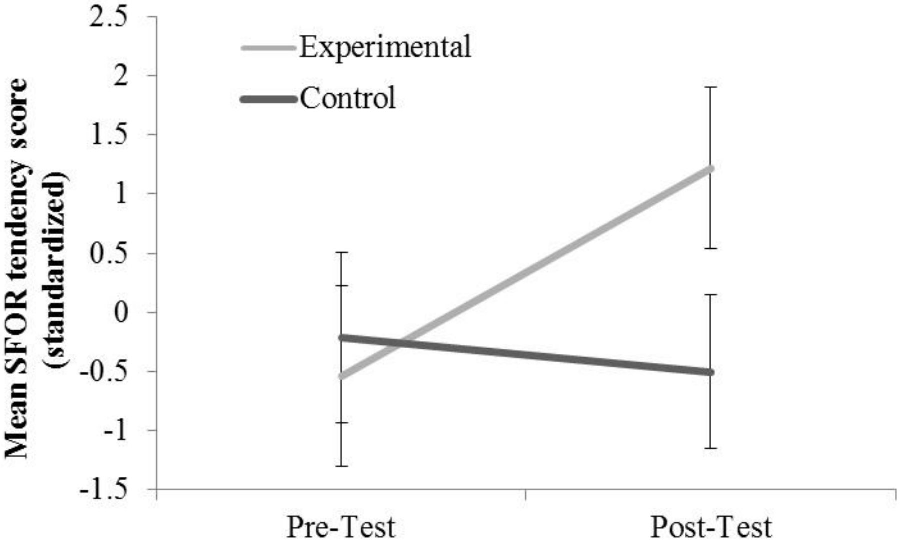
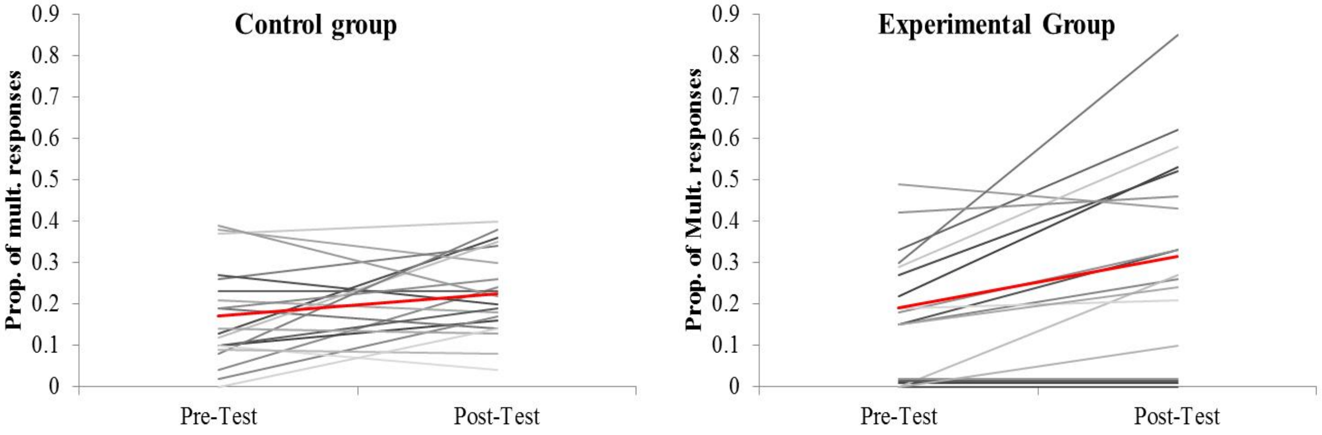


Figure 2. Mean standardized SFOR tendency sum scores on Pre- an Post-Tests for experimental and control conditions, controlling for rational number size knowledge. Error bars represent standard error for the means.

Previous interventions involving spontaneous focusing on mathematical aspects have found that the strongest effects seem to appear with those students who have an initial tendency to spontaneously pay attention to the mathematics (Hannula et al., 2005). Thus, in order to examine more closely the effect of the intervention on individual students, students' written multiplicative responses at pre-test were compared with their multiplicative responses at post-test. Drawing responses were discarded for this portion of the analysis as they were less discriminant and showed a ceiling effect among participants at the pre-test (mean = 3.44, median = 4.0, standard deviation = 1.16, maximum = 4).



Figures 3a and 3b. Proportion of pre- and post-test multiplicative responses by condition. Grey scale lines represent one participant, red line represents means for condition.

Figures 3a and 3b indicates that much of the difference between conditions could be located in a few students who had fairly large increases in the proportion of multiplicative responses. However, there was only one student in the experimental condition who provided multiplicative responses at the pre-test who did not increase in their proportion of multiplicative responses, and this student was already fairly comprehensive with their multiplicative descriptions at pre-test. This figure also shows that, while there was some movement among those participants with zero multiplicative responses on the pre-test, there seems to be a mixed effect on the intervention with those who had no multiplicative responses on the pre-test. This result (further analysis can be found in supplementary material) is in line with previous findings that enhancing individual’s tendency to pay attention to mathematical aspects of their environment may be most effective with those who already display an initial tendency to do so (Hannula et al., 2005).

Discussion

The present study presents the first attempt to enhance SFOR tendency in primary school students. Results suggest that the targeted intervention using the ActionTrack mobile application was moderately successful in increasing student’s SFOR tendency. Considering the small sample and low dosage of the intervention, these results are extremely encouraging for the further development and investigation of SFOR enhancement programs. However, these results are also preliminary and a full scale randomized experimental design is necessary to properly validate these procedures as relevant for use in the classroom. Nonetheless, it appears possible to support primary school students’ tendency to recognize quantitative relations as explicit targets of focusing both in and out of the classroom.

This finding is promising for the further consideration of SFOR tendency as a relevant component of mathematical development, as it fulfils the final type of evidence needed to support the existence of a spontaneous mathematical focusing tendency (Hannula, 2005). This finding points to the shared nature of foci of attention in everyday activities, and the need for modelling of how to interact with the mathematical aspects of situations (Gunderson & Levine, 2011). The goal of the SFOR intervention was to make multiplicative relations explicit targets of focusing in students’ everyday activities. In this way, they come to understand (some even for the first time) that it is possible to use these multiplicative relations in their reasoning about their everyday surroundings. Just as in family life, everyday activities can emphasize or deemphasize the mathematical concepts in everyday life (Mix, 2002). Since the design of activities can have an impact on how salient mathematical features are to students (Lobato et al., 2012), it is important to help students understand that the mathematics they use their classroom can also be used in situations all around them. The present study indicates that mobile technology was a useful tool to provide students with collaborative opportunities to create common foci of attention on multiplicative relations. Providing students with these opportunities through the use of the ActionTrack, to experience, find, and describe multiplicative relations outside of the classroom environment was able to increase SFOR tendency in these students.

In previous instances of attempts to enhance spontaneous focusing on mathematical aspects of their environment (Hannula et al., 2005), more experienced individuals (in this case early childhood educators) provided the children with models for how to focus on numerosities of their everyday environment. In a school setting this is not possible, given the high number of students per educator. Thus, through the use of a mobile application, the present study is the first to provide students with an avenue for collaboration among peers in noticing, defining, and describing instances of mathematics in their everyday surroundings (cf. Moeller et al., 2015).

Mobile technologies can be used very effectively as learning tools by a surprisingly broad range of learners in a variety of settings (Kukulska-Hulme, 2010). However, there are mixed findings on the effect of mobile environments on learning outcomes (Zydney & Warner, 2015). This brings up the fact that it is not only the technology, but the pedagogical implementation that matters. For example, Zydney and Warner (2015) stated that researchers should make more explicit connections between the instructional principles and the design features of their mobile learning environment in order to better integrate theory with practice. Others have emphasized that the use of computer-supported learning environments per se is not the key to effective mathematical training, but the benefits seem to depend on how technological solutions are actually implemented (Moeller et al., 2015).

Limitations

In the present study one particular issue that needs to be further addressed is how collaboration using mobile technologies was related to individual differences in the effects of the intervention. In Hannula and colleagues' (2005) previous study on enhancing SFON tendency, the effect of the intervention was found to be strongest among those students who began with some initial SFON tendency. The results of the present study suggest that while the effects of the intervention were stronger among these students, there was not a substantial difference between excluding those with no initial written SFOR responses and not excluding these students. One potential cause for this difference in outcomes between the two interventions was that in the present study the students completed the activities in groups. Preliminary evidence suggests that there was positive collaboration among group members in identifying, defining, and describing the quantitative relations embedded in the checkpoint tasks and relational directions (Hilppö et al., 2017). However, a more in-depth analysis of this process-oriented data is needed to better understand the reasons the present activities were able to increase students’ SFOR tendency, especially in examining exactly how physically activating mobile applications (e.g. ActionTrack) may be fruitful avenues for encourage students to use their formal mathematical skills both in and out of the classroom.

Another limitation of the present study is that it was not possible to test the causal relation between SFOR tendency and rational number conceptual knowledge. Given the longitudinal relation between SFOR tendency and rational number development (McMullen et al., 2016; Van Hoof et al., 2016), this proposed causal relation should be examined. A large scale randomized-control experimental design with delayed post-tests for rational number knowledge would be needed in order to determine if increasing SFOR tendency would also have a positive effect on rational numbers knowledge over the long term, as short term gains in rational number knowledge would not be expected (Hannula et al., 2005).

Educational implications and Conclusions

The most important implication of the present study is that it shows the possibility of enhancing SFOR tendency in late primary school students. The fact that this possibility exists and that SFOR tendency is related to the development of rational number knowledge do not alone warrant a recommendation for the inclusion of SFOR activities in the classroom. However, this does suggest that students’ SFOR tendency is not a fixed feature that is immune to instruction. This suggests that students’ social interactions may play a role in determining their SFOR tendency, be it at home, in the classroom, or among their peers. Providing students the opportunity to model this behaviour potentially opens up more students to the possibilities to pay attention to mathematical relations in more situations. The activities described here provide one mechanism for this.

The ubiquitous nature of mobile technologies provides new avenues to extend mathematical thinking beyond the classroom and into the real world. Working together with educators, instructional design professionals, and software designers, it may be possible to extend a broader bridge from those topics that students are encountering in their formal mathematical learning to situations in their everyday life in which they can be applied. Those students with a higher SFOR tendency are expected to gain a larger amount of self-initiated practice in reasoning about multiplicative relations in their everyday life (e.g. McMullen et al., 2016). This form of deliberate practice (Lehtinen et al., 2017) gives these students an advantage by allowing them more opportunities to practice their newly acquired formal mathematical knowledge in a diverse number of situations. Being able to flexibly apply their new knowledge in novel situations is a crucial part of the development of adaptive expertise with mathematics (Hatano & Oura, 2003), one of the main goals outlined in mathematics education (National Council Of Teachers Of Mathematics, 2000). Providing all students these opportunities may be crucial for developing adaptive expertise among a larger number of students, not just those who already gain a large amount of self-initiated practice with mathematical relations due to a high SFOR tendency.

Statements on open data, ethics and conflict of interest

The authors are pleased to provide the data of this study upon request. All identifying information was immediately scrubbed after matching pre- and post-test scores, which were coded without awareness of condition. Institutional protocol is followed to protect any identifying information, which is only available to project members. Subjects were informed that participation was voluntary, would not affect their course grade, and that they could quit the study at any time without consequence. The authors would like to state that they have no potential conflicts of interest.

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