AXIOM Approach for Modeling and Analysis of Complex Systems

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Abstract

AXIOM is an approach for modeling and analysis of complex systems based on expert-sourced data. It proposes a systems modeling language and a computational process to extract information of higher analytical value from a model built using the language. AXIOM can be placed in the family of cross-impact analysis techniques, and it proposes solutions for several practical problems associated with systems modeling using the established cross-impact techniques. This paper presents the AXIOM modeling language primitives, outlines the computational process and shows how the evaluated AXIOM model can be used for analysis of the modeled system.

1 Introduction

This paper describes the AXIOM approach for complex systems modeling and analysis. Modeling is generally defined as creating an approximation of the real world or a portion of reality (Sokolowski and Banks, 2009, 3). Such approximations or *models* can then be used in simulation of the real system they represent. Simulation is done to improve understanding of the real system, test how the system behaves under different conditions, and study the effects of changes, thus supporting planning and decision-making. International Council on Systems Engineering (INCOSE) (2017) defines a *system* as a "collection of elements that together produce results not obtainable by the elements alone". Systems thinking entails understanding the parts of reality under analysis as a set of components, that are logically connected abstractions of some real world objects or phenomena (Checkland, 1999, 99).

Systems modeling is therefore a process of representing reality using a *modeling language* which defines the available types components, and the logical connections and relationships between them for building the representation. The process of trying to capture the essential aspects of a system or a problem domain can as such be a useful learning experience (Checkland, 1999). However, as the components and their relationships and connections are modeling inputs, the *analysis* part of system modeling and analysis must derive its added value from a process of performing some kind of evaluation or simulation on the model. This process aims at revealing the emergent properties of the system itself, which can be difficult to observe by just looking at the system components at the lowest level of aggregation (Checkland, 1999, 99-103). The emergent properties are *systemic*, reflecting how the components and their connections function as a system.

Modeling a system involves abstracting the real system components and aspects into the model components and drafting the structure of their relationships, using the constructs available in the modeling language. The modeling process relies on a theory of the system, which guides the framing of the system, the inclusion or exclusion of components, and the aggregation level. Systems modeling does not necessarily go beyond this conceptual level representation of the system. In order to be able to perform evaluation of the model or use it for simulation, the relationships between the components should be *valuated* in some more formal logical or mathematical form. If such a form is not developed, the analysis that can be done on the basis of the system model remains more or less intuitive and heuristic: there is no transformation that can be done on the model to reveal the systemic "emergent properties".

The typical way of defining the relationships or connections between the components of the system model is to acquire a sufficient amount of quantitative data (Sokolowski and Banks, 2009) from statistics or empirical measurements on the modeled domain, and to derive a mathematical expression for the relationship on the basis of this available data. This data-driven approach obviously requires that the modeled components and relationships *a*) are quantifiable, and *b*) data about them is or can be made available for the modeling. The requirements of highly data-driven modeling can lead to omission of many important and interesting aspects of the system in modeling, or systems for which empirical data is not readily available not being subject to analysis with a systems modeling approach. For some systems, empirical data can be an impossibility: especially modeling employed in foresight and futures can deal with phenomena that do not yet exist and therefore can have no hard data available about them.

In lieu of empirical or statistical data, the system knowledge and understanding of knowledgeable people or *experts* of the modeled domain can be used in the process of formal valuation of the relationships between the model components. *Cross-impact analysis approaches* are a family of techniques to model systems based on expert-sourced information. The cross-impact approach goes beyond a conceptual-level model of a system, enabling, depending on the technique, some form of model evaluation or system simulation, aimed at extracting information about the emergent properties of the system. Tapping into expert-sourced data enables systems modeling in a theory-driven way, grounded in expert judgment and understanding: Cross-impact methods as modeling and analysis approaches fall in between empirical data-driven computational models and argumentative systems analysis, and they exhibit a high degree of disciplinary heterogeneity and focus on expert-sourced "soft" system knowledge (Weimer-Jehle, 2006). While a number of cross-impact techniques exist, there are barriers for adoption of the cross-impact approach, due to impracticalities in the modeling languages, intransparent documentation and lacking software implementations. AXIOM is a novel cross-impact approach, which proposes solutions for practical problems in existing cross-impact modeling techniques, with the aim of creating a clearly more feasible approach for systems modeling based on expert-sourced data. It also aims at providing output of more analytical value from the cross-impact modeling effort. AXIOM is transparently documented and implemented as free software, and is a ready-to-use tool for theory-driven systems modeling and simulation.

2 Cross-impact approaches

Cross-impact analysis has a long history in systems analysis and various foresight applications (Agami et al., 2010; Bañuls and Turoff, 2011; Bañuls et al., 2013; Blackman, 1973; Bloom, 1977; Brauers and Weber, 1988; Burns and Marcy, 1979; Choi et al., 2007; Dalkey, 1971; Godet et al., 1991; Godet, 1976; Godet et al., 1994; Gordon, 1969, 1994; Gordon and Hayward, 1968b; Ishikawa et al., 1980; Jackson and Lawton, 1976; Jeong and Kim, 1997; Kane, 1972; Kaya et al., 1979; Martino and Chen, 1978; Mitroff and Turoff, 1976; Nováky and Lóránt, 1978; Pagani, 2009; Thorleuchter et al., 2010; Turoff, 1971; Weimer-Jehle, 2006). The original motivation for the development of the approach was to complement the Delphi method by introducing analysis of interaction between elements of a given system (Godet et al., 1994; Gordon, 1969, 1994; Gordon and Hayward, 1968b). Recent research has focused mainly on application of the approach (Alizadeh et al., 2016; Blanning and Reinig, 1999; Chander et al., 2013; Choi et al., 2007; Gorane and Kant, 2013) and there has not been much methodological development. In spite of the methodological discussion, barriers exist for utilization of the cross-impact approach in modeling and research: many cross-impact techniques are not very transparent in their documentation and lack software tools and implementations.

Cross-impact analysis could be described as an analytical technique for studying a system, and particularly interaction within it. A system is seen to consist of several components, states, events and forces that are partially dependent on each other and therefore have influence on each other. The objects are modeled as system descriptors. System descriptors are referred to by different terms by authors of different cross-impact techniques. Gordon (1994) uses the term *event*, Godet et al. (1994) speak of *hypotheses*, and Honton et al. (1984) use the term *descriptor*. The influence the objects of the modeled system have on each other are given a model representation as impacts. Impacts can be represented as conditional probabilities (Godet et al., 1994; Gordon, 1994), references to probability-adjusting functions (Honton et al., 1984; Luukkanen, 1994; Panula-Ontto, 2016), impact indices (Godet et al., 1994; Kane, 1972; Panula-Ontto and Piirainen, 2017), or simply a boolean indicator of interaction of some kind (Godet et al., 1994, 83).

The aim of cross-impact analysis is to extract information about the indirect and systemic interactions between the system components on the basis of the information on direct interactions. In a system with more than a few components, the indirect interactions can effectuate over a complex web of mediating components. Accounting for the effect of these interaction webs can reveal surprising and counter-intuitive relationships between the system components: seemingly unrelated components can be important for each other in a systemic way, and conversely an important direct impact of a component on another may be cancelled out or reversed by the systemic effects.

The inputs for cross-impact analysis include the system descriptors, their direct interactions and the valuations of properties for the descriptors and interactions. Typically this input data is provided by people with expertise considered relevant for the modeled system or topic. Technically one expert who supplies all the input data is enough to perform the analysis. Normally, however, there are several experts, and the facilitation of the expert process to supply the input data is of central importance for the cross-impact modeling exercise. The expert inputs can be collected in a Delphi-like expert panel, via a questionnaire, or some combination of these. This paper presents a cross-impact modeling language and a computational technique for processing the built cross-impact model and extracting information from it; it does not propose a particular solution for the use of experts in the cross-impact modeling. However, the questions of expert selection, model building, facilitating expert group work in model valuation and other processual details are very important for the modeling undertaking. For discussion of these aspects of cross-impact modeling see e.g. Alizadeh et al. (2016); Blanning and Reinig (1999); Enzer (1971); Godet et al. (1991, 1994); Linstone and Turoff (1977); Seker (2015).

The existing cross-impact techniques vary greatly in terms of their inputs, computational process and outputs, but they can be grouped into three categories based on the analytical output they produce. The categories and the specific techniques in these categories are

1. Structural orientation
    - MICMAC (Godet et al., 1991, 1994)
    - ADVIAN (Linss and Fried, 2010)
    - EXIT (Panula-Ontto and Piirainen, 2017)
    - KSIM (Kane, 1972)
2. Morphological orientation
    - Cross-impact balances approach (Weimer-Jehle, 2006)
    - BASICS (Honton et al., 1984)
    - JL-algorithm (Luukkanen, 1994)

3. Probability orientation
    - Gordon's technique (Gordon and Hayward, 1968a; Gordon, 1994)
    - SMIC (Godet et al., 1991, 1994)
    - AXIOM

The **structurally oriented approaches** focus on the impact network structure, and derive their analytical added value from revealing the indirect impacts between system descriptors and relating them to the direct impacts in some way. The most used technique in this category appears to be the MICMAC (Godet et al., 1994) approach, which is a computational approach based on matrix multiplication, and a part of a larger analytical approach Godet calls "structural analysis". A derivative of MICMAC has been also proposed Linss and Fried (2010). The KSIM approach (Kane, 1972) is quite different from the other cross-impact approaches listed, but can, with reservation, be placed in the structurally oriented group of approaches. The structurally oriented techniques require fewer inputs than the other approaches, and provide a faster and easier modeling process, but the analytical output is more abstract and less actionable.

The **morphological orientation** of cross-impact analysis enables identifying logical, probable or consistent states for the system. A system state can be understood to be the combination of particular states for the system components. It can also be thought of as a scenario. This utility of cross-impact analysis overlaps morphological analysis (see Ritchey, 2006). Some morphologically oriented cross-impact techniques deal with probabilities explicitly and some do not. Documented approaches in this category are the cross-impact balances approach (Weimer-Jehle, 2006), BASICS (Honton et al., 1984), and JL-algorithm (Luukkanen, 1994). BASICS and JL-algorithm could also be seen as probability-oriented techniques, but their implementations output only probabilities for system states, their added value being of the morphological type.

The **probability-oriented approaches** explicitly deal with probabilities and therefore require that the system descriptors or their possible states are assigned initial or *a priori* probabilities. Additionally, they require some expression of how the probabilities of the system descriptors are adjusted during the evaluation of the cross-impact model. This can mean defining a conditional probability matrix (Godet et al., 1994; Gordon, 1994) or referencing probability adjustment functions (Honton et al., 1984; Luukkanen, 1994; Panula-Ontto, 2016). The basic output of the model evaluation is a new set of probabilities for the system descriptors, the *a posteriori probabilities*, which are the probabilities when the emergent, systemic effects have been factored in. The modeling phase of the probability-oriented approach to cross-impact analysis is more difficult and time-consuming than in the other orientations, but this approach offers the greatest analytical possibilities. This approach is the most suited for simulation-type analysis with a cross-impact model, and can be used for testing effects of changes in or interventions to the system. The probability-oriented approaches can be also used for delivering similar analytical outputs as the structural approach and the morphological approach.

The best-known techniques in this group are Gordon's method (Gordon, 1969; Gordon and Hayward, 1968a; Gordon, 1994) and SMIC (Godet et al., 1991; Godet, 1976; Godet et al., 1994). AXIOM is also in this category.

3 Advantages of the AXIOM approach

As stated in Section 2, AXIOM is a probability-oriented cross-impact approach, and the probability-oriented approaches in general have the greatest analytical possibilities among the different varieties of cross-impact methods. What are the advantages of AXIOM in comparison to other probability-oriented approaches? AXIOM combines the strengths of several documented cross-impact techniques in order to create a general systems modeling tool, that is feasible, flexible and makes analytically powerful. The combination of the best features of various approaches makes AXIOM a recommendable method for use in cross-impact modeling. The advantages of AXIOM, in comparison to other probability-oriented cross-impact approaches, are the following:

1. **Model valuation in AXIOM is relatively easy.** For the probability-oriented cross-impact approaches, valuation refers to the task of assigning initial (a priori) probabilities for the system descriptors and defining conditional probabilities for them or expressing the interactions between the descriptors in some other way. The impact valuation phase in AXIOM is decisively easier when compared to cross-impact methods which represent interactions as conditional probabilities (such as Gordon's method or SMIC). The cognitive cost of providing a large number of conditional probabilities is very high for the expert valuators. The conditional probability valuations are needed for all ordered pairs of hypotheses in the model, even when the model valuators would conclude that there is no direct interaction between the hypotheses. For example, the conditional probability valuation $P(A|B) = P(A)$ might violate the probability axioms, so no "default" conditional probability value exists: all interactions have to be valuated. The valuations have to comply with the probability axioms, and as the number of hypotheses grows, simply finding a compliant valuation solution might become difficult (at least without a help of a computer program specifically designed for this purpose). In this difficult valuation process, the qualitative-nature understanding of the experts about the interactions in the modeled system might get distorted in the attempt to find an acceptable valuation solution, changing the focus from modeling the system in the best way possible on the basis of expert knowledge into a sudoku-like number-placement exercise.

2. **AXIOM is suited for cross-impact models with a large number of components.** Cross-impact techniques which represent the interactions as conditional probabilities are not well suited for constructing system models with a large number of components. The cognitively expensive valuation phase heavily limits the

practical number of components in the model. Godet et al. (1994, 149) actually recommend that the number of hypotheses should not exceed 6.

Modeling systems with such a small number of hypotheses is very limiting. In a system model with a handful of components represented by hypotheses, if those hypotheses are detailed and concrete, many relevant factors and driving forces are left outside the cross-impact model. Conversely, if the hypotheses are loaded with a lot of content so that each hypothesis represents many factors and driving forces simultaneously, the abstraction level of the hypotheses gets very high. This high abstraction level will make the model valuation difficult and ambiguous. The interpretation of results is likely to suffer from the high abstraction level and drawing concrete policy recommendations on the basis of the model might turn out difficult. Either way, practical and useful cross-impact modeling is very difficult if the nature of the cross-impact technique per se limits the number of model components.

As the object of interest in cross-impact modeling is the impact network of the modeled system, the limitations on the number of components in cross-impact models also limit the interestingness of the analysis. In a system model of few components, the impact chains cannot be very long. If the ability to investigate higher-order interactions, long impact chains and complex systemic effects is an important motivation to do cross-impact analysis, the cross-impact modeling technique should definitely support this aspiration.

3. **AXIOM primitives have comparatively high modeling power.** AXIOM statements have multiple possible values (called *options*), unlike Gordon's method or SMIC. It is easy to make the case that the multi-valued AXIOM statements are a better solution than separate boolean hypotheses for constructing useful and relevant cross-impact models. Boolean hypotheses can, to some degree, be used to model mutually exclusive system states akin to AXIOM options, but they are much less convenient and error-prone in modeling as they require the exclusiveness to be explicitly defined through conditional probabilities. Additionally, boolean hypotheses cannot model the exhaustiveness of AXIOM options: there is no mechanism to ensure that the probability distribution of a supposedly exclusive and exhaustive set of boolean hypotheses will remain valid during the model evaluation.

   AXIOM also has a statement property called *timestep*. The timestep property makes it easy to model passage of time in the cross-impact model. Incorporating temporal aspect to cross-impact modeling is a feature of AXIOM that greatly increases its power to model real systems compared to methods that do not offer a mechanism to model time. Providing a way to model time makes it easier to construct models from the perspective of modeling interventions: today's decisions can be modeled to take their effect on the future states of the system in a very convenient and natural way instead of providing means to only model a system with a single temporal space where events happen and system states take place without any temporal structure.

4. **AXIOM provides more analytical possibilities.** In Gordon's method and Godet's SMIC method, especially the process of studying the effect of interventions and policy actions on the modeled system is, compared to AXIOM, cumbersome (although this might be more dependent on the implementation than the method). Modeling interventions requires changes to the cross-impact model and possibly redefinition of the conditional probabilities. The AXIOM method offers tools to design the simulation of interventions cleanly in the model building phase, and the focus of the analytical outputs is from the start in the effects of the different intervention sets, which makes it easy to extract practical policy recommendations. In addition to this, a number of further analytical outputs can be easily extracted on the basis of the AXIOM computation.

Above-stated strengths of the AXIOM approach, the freely available implementation, and the transparent documentation of the computation details make AXIOM a strong candidate for a general cross-impact modeling approach.

4 AXIOM modeling language, concepts and model evaluation

Any modeling approach has a modeling language associated with it, meaning a set of *modeling primitives* or building blocks to describe the characteristics of the real-world system that is being modeled. The building blocks for an AXIOM model are *statements* and their possible values called *options*, and *impacts* between the options. There are, however, a number of other important concepts that are also discussed in this section. Figure 1 presents an entity-relationship model of the important concepts of the AXIOM approach.

Statements represent system aspects or components that can have a state. They roughly correspond to what Gordon (1994) calls *events*, Godet et al. (1991) call hypotheses and Honton et al. (1984) call *descriptors*. AXIOM statements can have two to unlimited possible states (called *options*) whereas events or hypotheses in Gordon's or Godet et al.'s approaches only have a binary state (true or false) when evaluated or a state of being undetermined before evaluation. A statement should have *a*) a unique, identfying label *b*) a description detailing what they represent in the model *c*) a set of options *d*) a timestep value (explained in **timestep** definition), and *e*) a flag for whether the statement is to be treated as an intervention (explained in **intervention** definition). The options under a statement should be exclusive and exhaustive. Exclusiveness means that only one option can be evaluated to be the state of the statement (instead of more than one option being "true"). Exhaustiveness means that the options should cover the possible states of the component or aspect of the real system that corresponds to the AXIOM

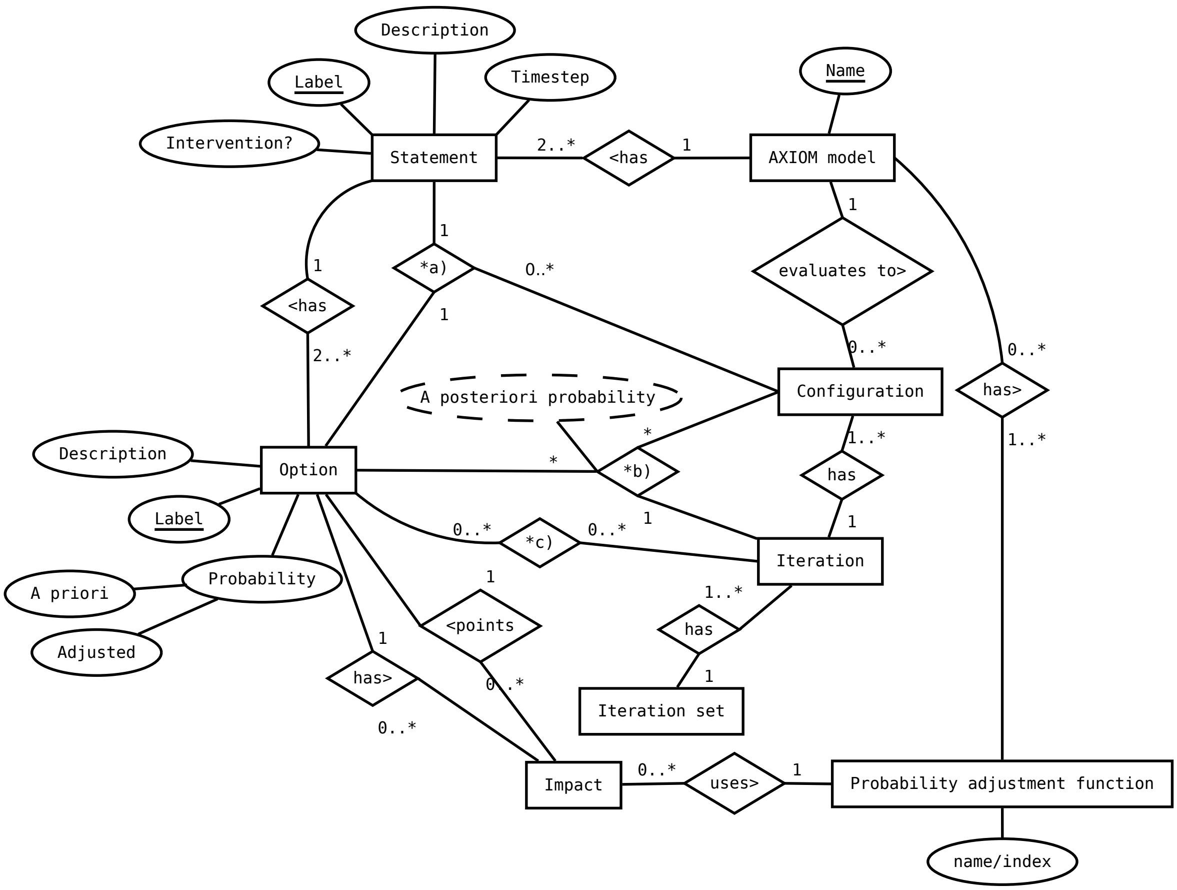

Figure 1: ER model of the AXIOM concepts

***a)** *Statement* is evaluated to an *option* in a single *configuration*
***b)** A *configuration* in an *iteration* has a single *option* for each *statement* in the model; the *a posteriori probability* of each option is the rate of occurrence of the option in configurations in the iteration.
***c)** An *iteration* can have *options* as active interventions

statement. This is rarely possible in practice. Selecting the most relevant possible states for an AXIOM statement is a part of framing of the model. Models in general can never cover all parts and details of the modeled system. They should focus on covering the pertinent and essential parts, aspects and features of the modeled system, in order to be useful (if the model is as complex as the reality, its usefulness is questionable).

Options represent the different possible states a system component modeled as an AXIOM statement can have. Every option in an AXIOM model has *a*) one statement the fall under, *b*) identifying label, *c*) a description of what they represent, *d*) an immutable a priori probability *e*) a mutable, adjusted probability valuation, and *f*) a (possibly empty) set of impacts directed to other options in the model. The immutable a priori probability is the initial, expert-sourced probability valuation of an option. The a priori probability is interpreted as the probability of the option to become true, as estimated when no other information about the system or its state is available; the a priori probability valuation is given in a context where the states of the other statements are unknown. The mutable probability valuation might change during model evaluation, as impacts in the model are realized or take place. The set of a priori probabilities and the set of mutable probabilities under a statement both form a probability distribution, meaning that the sum of values in both sets of probabilities must equal 1 at all times. The AXIOM options are flexible and can model the possibilities of the modeled system in various ways. It is possible that the different options under a statement embody a very clear and atomic value or fact about the system, such as a number or a percentage, or a single boolean fact. It is also possible that the options represent a big group of connected details, or a mini-scenario. These different uses can be combined in the same model unproblematically.

Impacts are probability-changing influences between options. Impacts have an owning option and a target option. Impacts are realized when their owning option is evaluated to be the state of the statement it falls under; when an option is known to be *true*, its impacts ensue.

Impacts, when realized, change the probability of their target option in some way. In AXIOM, the exact amount of probability adjustment is determined with probability adjustment functions (defined later). Any option o in an AXIOM model can have zero to $n_m - n_o$ impacts, where n_m is the number of options in the model and n_o is the number of options under the statement the option o falls under. This is because there can be no need to adjust the probability of options that are under the same statement as the owning option of the impacts; the owning option of an impact is already evaluated to be the state of its statement upon the time of realization of any impact.

An impact points to a probability adjustment function, that map the mutable, adjusted probability of the target option to a new adjusted probability value. The new probability value of the target option now reflects the valuation of the target option's probability when new information has become available (as the owning option is now known to be true). The probability adjustment functions have names, which can be indices: a set of names of probability adjustment functions could be $\{"-3","-2","-1","+1","+2","+3"\}$. Probability adjustment function $"-3"$ could refer to a significant negative change in probability, while $"-1"$ could refer to a slight negative change in probability. $"+3"$ could refer to a significant positive change in probability. What is a significant positive change in probability means different things in different contexts. A very improbable event or descriptor state might see its probability going from 0.00001 to 0.00100, making it a hundred times more probable but still having a very low probability. On the other hand, a probable event or descriptor state might have a probability of 0.8; Its probability cannot see a hundredfold increase. A strong positive change in its probability means a reduction in its uncertainty, and the adjustment must be no bigger than a part of the remaining 0.2 probability, that the probable descriptor will not be true. This kind of contextual probability adjustment is achieved by using probability adjustment functions: In AXIOM, the impact an option (when true) will have on the probability of some other option is expressed as a reference to (or as a name of) a probability adjustment function. This approach avoids the need to define a conditional probability matrix. The difficulties of using a conditional probability matrix in expressing the interactions in the cross-impact model has been discussed in Section 3. The probability adjustment function approach is an easier and more flexible way to express the cross-impact interactions.

Probability adjustment functions map probability values into new, adjusted probability values. In AXIOM, the named probability adjustment functions are used to contextually change the probabilities of options. They can be freely defined by the analyst. The adjustment functions need to have a domain and range of $[0, 1]$. Figure 2 presents the graphs of four probability adjustment functions.

However, the probability adjustment functions should adjust probabilities in a way that is easy to understand and coherent from the perspective of the model valuators. For this reason, there are recommended properties the functions should have, in order for them to provide a clear and understandable way for the expert valuators to express interactions within the AXIOM model.

- They should be symmetric about the line $y = -x + 1$
- Should have the property $y(x_0) < y(x_1)$ when $x_0 < x_1$
- Should have the property $y(x) > x$ if the name of the function implies positive (probability-increasing) impact, and the property $y(x) < x$ if the name of the function implies negative (probability-decreasing) impact.

The probability adjustment functions in Figure 2 have this property. These recommendations can be disregarded by the modeler(s) if some functions not conforming

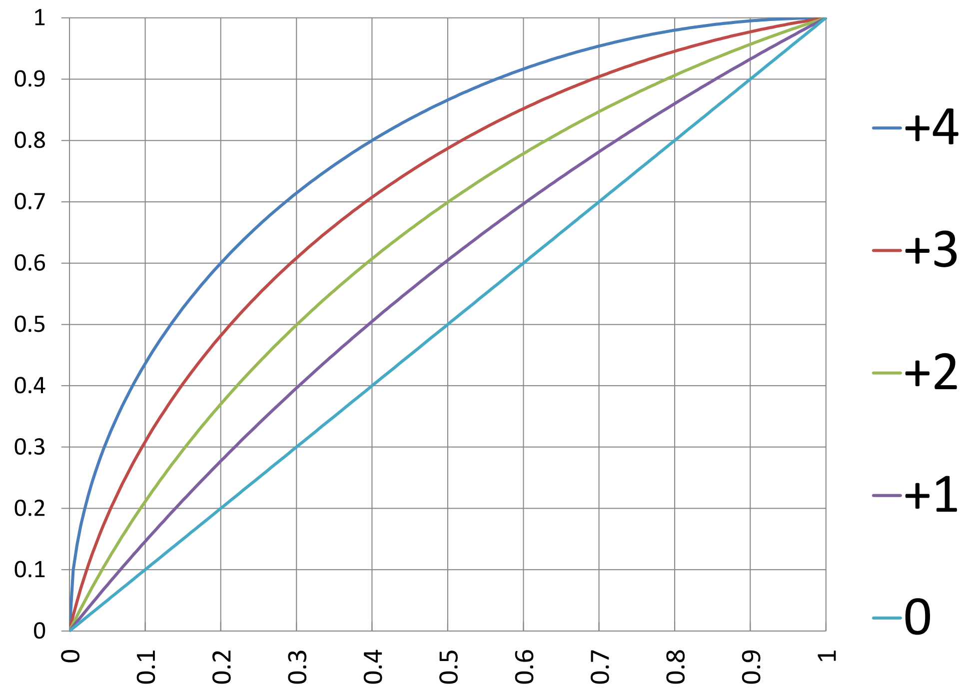

Figure 2: Examples of AXIOM probability adjustment functions

to these requirements is seen as useful in the model valuation.

When the adjusted probability of an option of an AXIOM statement is changed according to the used probability-adjusting function, the probabilities of the other options under the impacted statement must be adjusted too. We can call the probability adjustment of the option that is the target of the impact *primary probability adjustment* and the probability adjustment of all the other options under the impacted statement *secondary probability adjustment*. The secondary probability adjustment is necessary because the probabilities of the options of a statement form a probability distribution and the sum of the probabilities of the options must always be equal to 1.

The primary and secondary probability adjustment are performed so that the probability of the impacted option is changed according to the probability adjustment function pointed by the impact, *(primary adjustment)* and the probabilities of the other options change so that their summed probability is equal to the complement of the new adjusted probability of the impacted option and the probability share each of these other options gets out of that summed probability is equal to their share of their summed probability before the probability adjustment *(secondary adjustment)*. When the other options under the same statement as the impacted option is have their probabilities adjusted in this way, the total sum of the probabilities of all the statement's options remains equal to 1.

Timestep is a property of an AXIOM statement. It defines the temporal position of a statement in relation to other statements in the model. In model evaluation, the statements with the lowest timestep are evaluated before statements with a higher timestep value. Statements that have the same timestep value are evaluated in random order. In other words, in AXIOM model evaluation, statements are evaluated in groups of statements that share a timestep value. This makes it possible to simulate a system with temporal depth: events or descriptor states to take place in the near future can influence the descriptors that lie further in the future. A policy implemented in the next four years might have an impact of a particular economic scenario being true in the next four year period. Timesteps can be years, but they can also simply be ordinal numbers of the time categories (however they are defined in the actual model building), only their ordering as numbers is significant from the point of view of model evaluation.

Statement evaluation means assigning a state for an AXIOM statement, or setting one of the statement's options as its value. This is done probabilistically, with each option of the statement having a probability equal to its current adjusted (mutable) probability of being selected as the state of the statement. When a statement is evaluated to a state (one of its options) all the impacts of the state option ensue or "take place". The probabilities of the target options of the ensuing impacts are adjusted according to the probability adjustment functions associated with the ensuing impacts. After this, the statement is evaluated and has a known state.

Model evaluation means evaluating all of its statements. As explained in the *timestep* definition, the statements are evaluated in time categories, from lowest(earliest) to highest(latest), and statements in the same time category are evaluated in random order. During the model evaluation process, as more information about the state of the system becomes available, the probabilities of options in yet unevaluated statements are adjusted to reflect the effect of the newly available information. After evaluation of every statement, the model now has a state, as it now has a value for each of its statements. This combination of values is called a configuration: Model evaluation produces a configuration. For full details on the model evaluation, AXIOM algorithms pseudocode and a full example of an AXIOM model evaluation are presented in Panula-Ontto (2016).

Configuration is the result of the model evaluation. The information content of a configuration is a set of options, one option for each statement in the model. The options in the configuration are the options evaluated to be the states of each of the statements in a single model evaluation. A configuration can be understood as a scenario for the modeled system. As the model is evaluated multiple times, the resulting sets of configurations or *iterations* are used to derive a posteriori probabilities for the model options and other higher-order information. This is discussed in Section 5.

Intervention statements are treated in a special way in the model evaluation. Any statement can be flagged as an intervention statement in AXIOM model construction. They are not evaluated in the normal probabilistic way as non-intervention statements. In a single model evaluation, an intervention statement will have a predefined state; their state is determined when the model evaluation commences. Other details of the model evaluation are the same: the impacts of the predetermined options of the intervention statements take place when the intervention statement is taken up for evaluation. The states of the intervention statements change only between different iterations.

The function of intervention statements is that they can model policy actions, strategic options available to actors in the system or some other intervention-type aspect of the system. They provide an easy way to study the impacts and systemic effects of the different options available for the real-world component or aspect that the intervention statement represents.

Iteration is a list of configurations. A single model evaluation produces a configuration and several consecutive model evaluations produce an iteration. The utility of iterations is to be able to calculate the frequencies of different model options from a set of configurations with identical characteristics. Identical characteristics means same interventions, same model valuations and same model components. The frequence of occurrence of each option in an iteration is the *a posteriori* probability of that option.

The number of configurations in an iteration is not defined in the AXIOM method. The more model evaluations (and resulting configurations) the less the randomness of the Monte Carlo process effects the option frequencies. This is why a high number of configurations is recommended. For iterations that will be used for extracting final results to be analysed, at least 10^6 configurations is recommended. This recommendation is for calculating the a posteriori probabilities of individual options. If the idea is to compute a probability for a morphology or "partial scenario", i.e. the frequency of configurations that contain a specific set of options, the number of configurations should be even higher.

Iteration set is simply a set of iterations. The utility of an iteration set is to enable comparisons between the outputs of different model setups. The different model setups most commonly mean a different intervention combination (see **intervention statements**) but can also mean different a priori probability and impact valuations and inclusion and exclusion of different statements and options. When the model has flagged intervention statements, the AXIOM software implementation will automatically create an iteration set containing an iteration for each intervention combination derivable from the flagged intervention statements. This facility makes it straightforward to investigate how alternative policy actions modeled by the options of intervention statements affect the a posteriori probabilities of other model options.

The definitions of the concepts of the AXIOM approach outline the AXIOM modeling language and the computation process of AXIOM. The full description of the process is detailed in Panula-Ontto (2016), with pseudocode for the algorithms and a step-by-step computation example provided.

5 Output and analysis

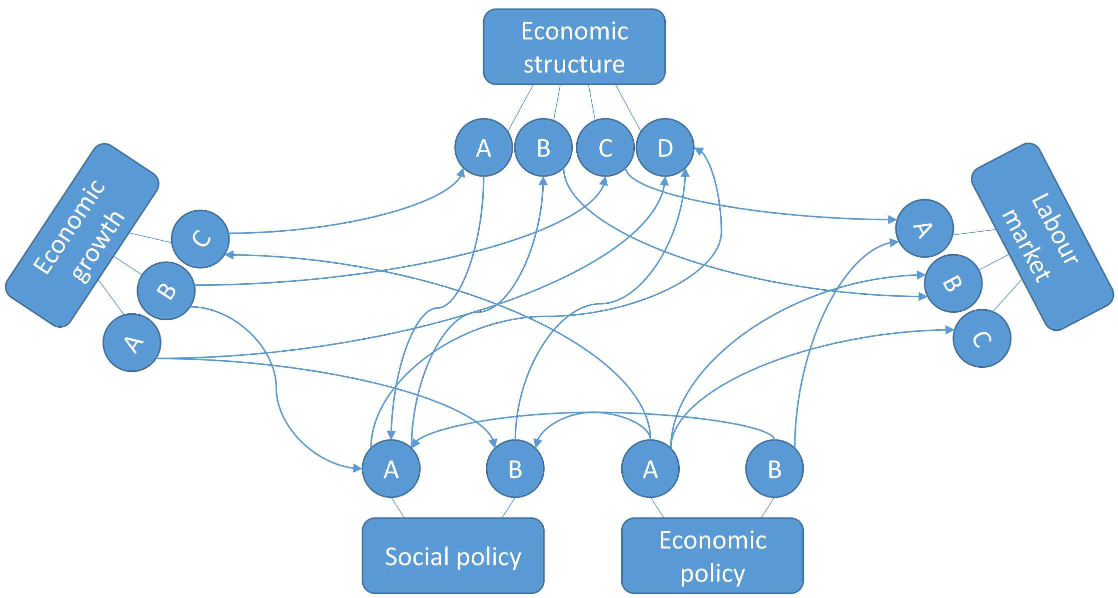

Figure 3: A conceptual system model

Figure 3 presents a conceptual model of an economic system and a strategic decision-making problem framework. The (hypothetical) real system is modeled using AXIOM primitives: Statements, representing social policy, economic policy, economic growth, economic structure, and labour market aspects in the system; Options, describing possible states for the modeled system aspects, or *subscenarios* for the system aspects; and impacts, representing the influences that different subscenarios (AXIOM options), if realized, have on the likelihood of other subscenarios being realized. In the conceptual system model of Figure 3, economic policy subscenario A influences the probability of labour market subscenarios B and C, while economic policy subscenario B influences the probability of labour market subscenario A. The model valuations (a priori probabilities and impact valuations) are not presented in the conceptual model. For purposes of the example, let's assume an expert group has performed the model valuation and the model can be evaluated with the AXIOM computation process.

Statements "Social policy" and "Economic policy" represent policy interventions to the system. Both statements have two options. Assuming these two statements are flagged as intervention statements, there are four possible combinations of interventions to investigate. Table 1 presents an example of the basic output of AXIOM. The table presents the a posteriori probabilities for the different model options of the conceptual system model presented in Figure 3. The second column displays the a priori probabilities. The third column presents the a posteriori probabilities in a case where no statement is treated as an intervention statement. These probabilities have the systemic interactions and higher-order impacts factored into them.

Table 1: AXIOM iteration set consisting of five iterations, one without active interventions and four iterations with different intervention combinations

	a priori	no policy	SP_A + EP_A	SP_A + EP_B	SP_B + EP_A	SP_B + EP_B
Social policy A ($\mathbf{SP}_A$)	0.825	0.860	1	1	0	0
Social policy B ($\mathbf{SP}_B$)	0.175	0.140	0	0	1	1
Economic policy A ($\mathbf{EP}_A$)	0.258	0.260	1	0	1	0
Economic policy B ($\mathbf{EP}_B$)	0.742	0.740	0	1	0	1
Economic growth A ($\mathbf{G}_A$)	0.028	0.045	0.137	0.480	0.331	0.613
Economic growth B ($\mathbf{G}_B$)	0.715	0.717	0.425	0.041	0.581	0.192
Economic growth C ($\mathbf{G}_C$)	0.258	0.237	0.438	0.479	0.088	0.195
Economic structure A ($\mathbf{S}_A$)	0.144	0.176	0.357	0.438	0.027	0.469
Economic structure B ($\mathbf{S}_B$)	0.242	0.289	0.028	0.157	0.314	0.204
Economic structure C ($\mathbf{S}_C$)	0.152	0.185	0.118	0.046	0.413	0.232
Economic structure D ($\mathbf{S}_D$)	0.461	0.350	0.497	0.359	0.246	0.094
Labour market A ($\mathbf{L}_A$)	0.439	0.485	0.322	0.235	0.457	0.643
Labour market B ($\mathbf{L}_B$)	0.328	0.361	0.230	0.526	0.008	0.329
Labour market C ($\mathbf{L}_C$)	0.233	0.154	0.448	0.239	0.536	0.027

Columns 4–7 in Table 1 present the a posteriori probabilities of the model options under different intervention combinations. Column 4 presents the a posteriori probabilities assuming a combination of social policy A and economic policy A; column 6 the a posteriori probabilities under a combination of social policy B and economic policy A. If the subscenario A for economic growth would be desirable, its probability would be maximized under a policy combination of social policy B and economic policy B (column 7). Similarly, if labour market subscenario B would be particularly undesirable, its probability would be minimized under social policy B combined with economic policy A (column 6).

A posteriori probabilities for individual options under different assumptions and policy combinations are easy to read from this output. The analyst may however be interested in more complex questions, such as what kind of policy mix would maximize (or minimize) the likelihood of a particular system morphology. Such morphologies in the case of the example model could be "Economic growth A" and "Labour market B" ($G_A \wedge L_B$) or "Economic growth B" and not "Labour market C" ($G_B \wedge \neg L_C$), or perhaps something more complicated such as $(\mathrm{G}_B \vee \mathrm{G}_C) \wedge (\mathrm{S}_A \vee (\mathrm{S}_B \wedge \mathrm{L}_C))$. The AXIOM iteration object is suited for calculating probabilities of such morphologies and performing various frequent itemset mining operations that might be of use for the analyst.

Table 2: Using AXIOM iteration object to compute probabilities of system states as frequencies of morphologies

Morphology	p	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
SP_A	0.80	1	0	1	1	0	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1
SP_B	0.20	0	1	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0
EP_A	0.35	0	0	1	0	0	1	1	0	1	0	1	0	0	0	0	0	0	1	1	0
EP_B	0.65	1	1	0	1	1	0	0	1	0	1	0	1	1	1	1	1	1	0	0	1
G_A	0.25	1	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1
G_B	0.50	0	0	0	1	1	0	1	1	0	0	1	1	0	0	1	1	1	0	1	0
G_C	0.25	0	1	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	1	0	0
S_A	0.25	1	0	0	0	1	0	1	0	0	0	1	0	0	0	1	0	0	0	0	0
S_B	0.25	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	1	1	0	0
S_C	0.30	0	1	1	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0	1	0
S_D	0.20	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1
L_A	0.55	1	1	1	1	0	0	1	0	1	0	0	1	0	1	0	0	1	0	1	1
L_B	0.35	0	0	0	0	1	1	0	1	0	1	0	0	1	0	1	1	0	0	0	0
L_C	0.10	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0
$\mathrm{G}_A \wedge \mathrm{L}_B$	0.05	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
$\mathrm{G}_B \wedge \neg \mathrm{L}_C$	0.45	0	0	0	1	1	0	1	1	0	0	0	1	0	0	1	1	1	0	1	0
$(\mathrm{G}_B \vee \mathrm{G}_C) \wedge (\mathrm{S}_A \vee (\mathrm{S}_B \wedge \mathrm{L}_C))$	0.25	0	0	0	0	1	0	1	0	0	0	1	0	0	0	1	0	0	1	0	0

Table 2 displays an iteration of 20 configurations, frequencies of different individual options within these 20 configurations, and frequencies of three example morphologies. 20 configurations is obviously insufficient to derive the a posteriori probabilities from the occurrence frequencies, but the principle is the same in an iteration of any number of configurations.

6 Software implementation

AXIOM is implemented as a Java program, and it can be downloaded from `https://github.com/jmpaon/AXIOM`. The GitHub page provides basic instructions for use and links to more resources on AXIOM.

7 Discussion

The AXIOM approach proposes a new modeling language and a computational process to extract information of higher added value from a system model built with that language. The rationale for a new probability-oriented or simulation-oriented cross-impact approach is illustrated in Section 3 by pointing out practical difficulties of modeling systems with the primitives available in Gordon's cross-impact approach and SMIC and proposing improvements, that have been incorporated into AXIOM approach. Section 4 discussed the AXIOM modeling language primitives, the computational process and how the evaluated AXIOM model can be used for analysis of the modeled system.

As a general systems modeling approach, the best fitness of AXIOM lies in high-level systems modeling where expert understanding of the system could be seen as the best source of information. For some modeling domains, the approach of using expert-sourced data is obviously not the best approach. For instance, technical systems with well-known limits and clearly measurable relationships and characteristics, the AXIOM modeling language, while a possible approach, is not a natural fit. Approaches like AXIOM can, however, offer tools to model systems from a very different perspective and attempt to incorporate aspects of the system that would be difficult or impossible to model using a more traditional data-driven approach. Combining theory-driven modeling and data-driven modeling in the same modeling framework provides interesting possibilities and warrants further experimentation, study and methodological development.

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