New Results on Codes for Location in Graphs

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1 Introduction

Sensor networks consist of sensors monitoring various places and connections between these places. We model a sensor network as a simple and undirected graph $G = (V(G), E(G)) = (V, E)$. In this context, a sensor can be placed on a vertex v and its closed neighbourhood $N[v]$ represents the set of locations that the sensor monitors. Besides assuming that graphs are simple and undirected, we also assume that they are connected and have cardinality at least two. In the following, we present some terminology and notation. The *closed neighbourhood* of v is defined $N[v] = N(v) \cup \{v\}$, where $N(v)$ is the *open neighbourhood* of v, that is, the set of vertices adjacent to v. A *code* C is a nonempty subset of V and its elements are *codewords*. The codeword $c \in C$ *covers* a vertex $v \in V$ if $v \in N[c]$. We denote the set of codewords covering v in G by

$$I(G, C; v) = I(G; v) = I(C; v) = I(v) = N[v] \cap C.$$

The set $I(v)$ is called an *identifying set* or an *I-set*. We say that a code $C \subseteq V$ is *dominating* in G if $I(C; u) \neq \emptyset$ for all $u \in V$. If the sensors are placed at the locations corresponding to the codewords, then each vertex is monitored by the sensors located in $I(v)$. More explanation regarding location detection in the sensor networks can be found in [1, 8, 12].

Let us now define *identifying codes*, which were first introduced by Karpovsky *et al.* in [7]. For numerous papers regarding identifying codes and related topics, the interested reader is referred to the online bibliography [9].

Definition 1. A code $C \subseteq V$ is *identifying* in G if for all distinct $u, v \in V$ we have $I(C; u) \neq \emptyset$ and

$$I(C; u) \neq I(C; v).$$

An identifying code C in a finite graph G with the smallest cardinality is called *optimal* and the number of codewords in an optimal identifying code is denoted by $\gamma^{ID}(G)$.

Identifying codes require unique I-sets for codewords as well as for non-codewords. However, if we omit the requirement of unique I-sets for codewords, then we obtain the following definition of *locating-dominating codes*, which were first introduced by Slater in [10, 13, 14].

Definition 2. A code $C \subseteq V$ is *locating-dominating* in G if for all distinct $u, v \in V \setminus C$ we have $I(C; u) \neq \emptyset$ and

$$I(C; u) \neq I(C; v).$$

Notice that an identifying code in G is also locating-dominating (by the definitions). In [4], self-locating-dominating and solid-locating-dominating codes have been introduced and, in [5, 6], they have been further studied. The definitions of these codes are given as follows.

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Definition 3. Let $C \subseteq V$ be a code in G.

(i) We say that $C \subseteq V$ is *self-locating-dominating code* in G if for all $u \in V \setminus C$ we have $I(C;u) \neq \emptyset$ and

$$\bigcap_{c \in I(C;u)} N[c] = \{u\}.$$

(ii) We say that $C \subseteq V$ is *solid-locating-dominating code* in G if for all distinct $u, v \in V \setminus C$ we have

$$I(C;u) \setminus I(C;v) \neq \emptyset.$$

Observe that since G is a connected graph on at least two vertices, a self-locating-dominating and solid-locating-dominating code is always dominating. Analogously to identifying codes, in a finite graph G, we say that dominating, locating-dominating, self-locating-dominating and solid-locating-dominating codes with the smallest cardinalities are *optimal* and we denote the cardinality of an optimal code by $\gamma(G)$, $\gamma^{LD}(G), \gamma^{SLD}(G)$ and $\gamma^{DLD}(G)$, respectively.

In the following theorem, we offer characterizations of self-locating-dominating and solid-dominating codes for easier comparison of them.

Theorem 4 ([4])**.** *Let $G = (V, E)$ be a connected graph on at least two vertices:*

(i) A code $C \subseteq V$ is self-locating-dominating if and only if for all distinct $u \in V \setminus C$ and $v \in V$ we have

$$I(C;u) \setminus I(C;v) \neq \emptyset.$$

(ii) A code $C \subseteq V$ is solid-locating-dominating if and only if for all $u \in V \setminus C$ we have $I(C;u) \neq \emptyset$ and

$$\left(\bigcap_{c \in I(C;u)} N[c] \right) \setminus C = \{u\}.$$

Based on the previous theorem, we obtain the following corollary.

Corollary 5. *If C is a self-locating-dominating or solid-locating-dominating code in G, then C is also solid-locating-dominating or locating-dominating in G, respectively. Furthermore, for a finite graph G, we have*

$$\gamma^{LD}(G) \leq \gamma^{DLD}(G) \leq \gamma^{SLD}(G).$$

The structure of the paper is described as follows. First, in Section 2, we give optimal locating-dominating, self-locating-dominating and solid-locating-dominating codes in the direct product $K_n \times K_m$ of complete graphs, where $2 \leq n \leq m$ as well as optimal solid-locating-dominating codes for graphs $K_q \square K_q \square K_q$ with $q \geq 2$. Then, in Section 3, we obtain optimal self-locating-dominating and solid-locating-dominating codes in infinite king and triangular grids, i.e., the smallest possible codes regarding their density.

2 Products of complete graphs

A graph is called a *complete graph* on q vertices, denoted by K_q, if each pair of vertices of the graph is adjacent. The vertex set $V(K_q)$ is denoted by $\{1, 2, \ldots, q\}$. The *Cartesian product* of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is defined as $G_1 \square G_2 = (V_1 \times V_2, E)$, where E is a set of edges such that $(u_1, u_2)(v_1, v_2) \in E$ if and only if $u_1 = v_1$ and $u_2v_2 \in E_2$, or $u_2 = v_2$ and $u_1v_1 \in E_1$. The *direct product* of two graphs G_1 and G_2 is defined as $G_1 \times G_2 = (V_1 \times V_2, E)$, where $E = \{(u_1, u_2)(v_v, v_2) \mid u_1v_1 \in E_1 \text{ and } u_2v_2 \in E_2\}$. A *complement* of a graph $G = (V, E)$ is the graph $\overline{G} = (V, E')$ with the edge set E' being such that $uv \in E'$ if and only if $uv \notin E$.

In this section, we first give optimal locating-dominating, self-locating-dominating and solid-locating-dominating codes in the direct product $K_n \times K_m$, where $2 \leq n \leq m$. For location-domination and solid-location-domination, the results heavily depend on the exact values of

$\gamma^{LD}(K_n \square K_m)$ and $\gamma^{DLD}(K_n \square K_m)$, which have been determined in [4]. In the graphs $K_n \times K_m$ and $K_n \square K_m$, the jth *row* (of $V(K_n) \times V(K_m)$) is denoted by R_j and it consists of the vertices $(1,j), (2,j), \dots, (n,j)$. Analogously, the ith *column* is denoted by P_i and it consists of the vertices $(i,1), (i,2), \dots, (i,m)$. Now we are ready to present the following observations:

- In the Cartesian product $K_n \square K_m$, the closed neighbourhood $N[(i,j)] = N[i,j]$ consists of the row R_j and the column P_i. Therefore, as the closed neighbourhood of a vertex resembles the movements of a rook in a chessboard, $K_n \square K_m$ is also sometimes called the *rook's graph*.
- In the direct product $K_n \times K_m$, we have $N((i,j)) = N(i,j) = V(K_n \square K_m) \setminus (R_j \cup P_i)$.

Due to the previous observations, we know that $\overline{K_n \square K_m} = K_n \times K_m$.

Recall that identification is a topic closely related to the various location-domination type problems. Previously, in [11], the identifying codes have been studied in the direct product $K_n \times K_m$ of complete graphs by Goddard and Wash. More precisely, they determined the exact values of $\gamma^{ID}(K_n \times K_m)$ for all m and n.

In what follows, we determine the exact values of $\gamma^{LD}(K_n \times K_m)$ for all m and n. For this purpose, we first present the following result concerning location-domination in the Cartesian product $K_n \square K_m$ of complete graphs given in [4].

Theorem 6 ([4], Theorem 14)**.** *Let m and n be integers such that $2 \leq n \leq m$. Now we have*

$$\gamma^{LD}(K_n \square K_m) = \begin{cases} m-1, & 2n \leq m, \\ \lceil \frac{2n+2m}{3} \rceil - 1, & n \leq m \leq 2n-1. \end{cases}$$

There is a strong connection between the values of $\gamma^{LD}(K_n \square K_m)$ and $\gamma^{LD}(K_n \times K_m)$ as explained in the following. In [3], it has been shown that $|\gamma^{LD}(G) - \gamma^{LD}(\overline{G})| \leq 1$. Therefore, as $\overline{K_n \times K_m} = K_n \square K_m$, we obtain that $\gamma^{LD}(K_n \square K_m) - 1 \leq \gamma^{LD}(K_n \times K_m) \leq \gamma^{LD}(K_n \square K_m) + 1$. This result is further sharpened in the following lemma.

Lemma 7. *For $2 \leq n \leq m$ and $(n,m) \neq (2,4)$, we have*

$$\gamma^{LD}(K_n \square K_m) - 1 \leq \gamma^{LD}(K_n \times K_m) \leq \gamma^{LD}(K_n \square K_m).$$

If $\gamma^{LD}(K_n \times K_m) = \gamma^{LD}(K_n \square K_m) - 1$, then the optimal locating-dominating code C in $K_n \times K_m$ has a non-codeword v such that $I(v) = C$.

Proof. First denote $G = K_n \square K_m$ and $H = K_n \times K_m$. The lower bound of the claim is immediate by the result preceding the lemma. For the upper bound, let C be an optimal locating-dominating code in G. The code C can also be viewed as a code in H. If we have $I(H;u) = I(H;v)$ for some non-codewords u and v, then a contradiction follows since $I(G;u) = C \setminus I(H;u) = C \setminus I(H;v) = I(G;v)$. Hence, we have $I(H;u) \neq I(H;v)$ for all distinct non-codewords u and v. Moreover, if $I(G;v) \neq C$ for each non-codeword v, then we also have $I(H;v) \neq \emptyset$, and the upper bound follows since C is a locating-dominating code in H.

Hence, we may assume that $I(G;v) = C$ for some non-codeword v. This implies that $C \subseteq P_i \cup R_j$ for some i, j. There exists at most one non-codeword in $P_i \setminus \{v\}$ since otherwise there are at least two non-codewords with the same I-set. Similarly, there exists at most one non-codeword in $R_j \setminus \{v\}$. Furthermore, if both $P_i \setminus \{v\}$ and $R_j \setminus \{v\}$ contain a non-codeword, then there exists a vertex with an empty I-set. Thus, in conclusion, there exists at most two non-codewords in $P_i \cup R_j$ and, hence, we have $|C| \geq n + m - 3$. Dividing into the following cases depending on n and m, we next show that $|C| \geq n + m - 3 > \gamma^{LD}(G)$ in majority of the cases of the lemma:

- If $n \geq 3$ and $m \geq 2n$, then we have $\gamma^{LD}(G) = m - 1 < n + m - 3 \leq |C|$ (by Theorem 6).
- If $n \geq 4$, $n \leq m \leq 2n-1$ and $(n,m) \neq (4,4)$, then $\gamma^{LD}(G) = \lceil 2(n+m)/3 \rceil - 1 < n+m-3 \leq |C|$ (by Theorem 6).

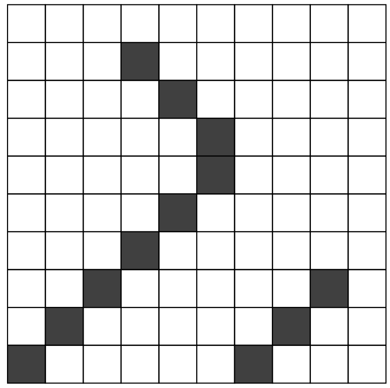

Figure 1: Optimal locating-dominating code for $K_{10} \times K_{10}$. Dark boxes are codewords.

Thus, if $n \geq 3$ and $m \geq 2n$, or $n \geq 4$, $n \leq m \leq 2n-1$ and $(n,m) \neq (4,4)$, then a contradiction with the optimality of C follows. Hence, in these cases, we have $\gamma^{LD}(H) \leq \gamma^{LD}(G)$. The rest of the cases are mostly small special cases which can be verified one by one (the details are omitted).

Let then C' be a locating-dominating code in H. Similarly as above, we get that if $I(H;v) \neq C'$ for each non-codeword v, then C' is also a locating-dominating code in G. Therefore, if $\gamma^{LD}(H) = \gamma^{LD}(G) - 1$, then there exist a non-codeword v such that $I(H;v) = C'$. Thus, the last claim of the lemma follows. □

Now with the help of the previous lemma and Theorem 6, we determine the exact values of $\gamma^{LD}(K_m \times K_n)$ in the following theorem. The rather long and technical proof is omitted.

Theorem 8. *For $2 \leq n \leq m$ we have*

$$\gamma^{LD}(K_n \times K_m) = \begin{cases} m-1, & 2n \leq m \text{ and } (n,m) \neq (2,4), \\ \lceil \frac{2n+2m-1}{3} \rceil - 1, & 2 < n \leq m < 2n \text{ and } (m,n) \neq (4,4), \\ m, & n = 2, m \leq 4, \\ 5, & n = 4, m = 4. \end{cases}$$

Let us next briefly consider solid-location-domination. The following result has been shown in [4].

Theorem 9 ([4])**.** *For all integers m and n such that $m \geq n \geq 1$, we have*

$$\gamma^{DLD}(K_n \Box K_m) = \begin{cases} m, & 4 \leq 2n \leq m \text{ or } n = 2, \\ 2n, & 2 < n < m < 2n, \\ 2n-1, & 2 < m = n. \end{cases}$$

In the following theorem, we show that the cardinalities of optimal solid-locating-dominating codes are same for $K_n \times K_m$ and $K_n \Box K_m$.

Theorem 10. *For all integers m and n such that $m \geq n \geq 2$, we have*

$$\gamma^{DLD}(K_n \times K_m) = \gamma^{DLD}(K_n \Box K_m).$$

Proof. By [6, Theorem 21], we have $\gamma^{DLD}(G) = \gamma^{DLD}(\overline{G})$ if G is not a discrete or a complete graph. Therefore, as this is the case for $G = K_n \times K_m$, we have $\gamma^{DLD}(K_n \times K_m) = \gamma^{DLD}(\overline{K_n \times K_m}) = \gamma^{DLD}(K_n \Box K_m)$. □

Let us then consider self-location-domination. Unlike location-domination [3, Theorem 7] and solid-location-domination [6, Theorem 21], the optimal cardinality of a self-locating-dominating code in G does not depend on the one of the complement graph $\overline{G}$. In the following theorem, we first give the result presented in [4] regarding $\gamma^{SLD}(K_n \Box K_m)$.

Theorem 11 ([4]). *For all integers m and n such that $m \geq n \geq 2$, we have*

$$\gamma^{SLD}(K_n \Box K_m) = \begin{cases} m, & 2n \leq m, \\ 2n, & 2 \leq n < m < 2n, \\ 2n-1, & 2 < m = n, \\ 4, & n = m = 2. \end{cases}$$

In the following theorem, we determine the exact values of $\gamma^{SLD}(K_n \times K_m)$ for all values of m and n. Notice that $\gamma^{SLD}(K_n \Box K_m) = \gamma^{SLD}(K_n \times K_m)$ if and only if $n = m$, $m = n + 1 > 3$, or $n = 2$ and $m \geq 4$ (the proof is omitted).

Theorem 12. *For all integers m and n such that $m \geq n \geq 2$, we have*

$$\gamma^{SLD}(K_n \times K_m) = \begin{cases} m + n - 1, & n > 2, \\ m, & n = 2, m > 2, \\ 4, & n = m = 2. \end{cases}$$

Previously, in [5], an optimal self-locating-dominating code in $K_q \Box K_q \Box K_q$ has been presented as well as some upper and lower bounds for $\gamma^{ID}(K_q \Box K_q \Box K_q)$. In the following theorem, we present the optimal value for $\gamma^{DLD}(K_q \Box K_q \Box K_q)$, the proof is omitted.

Theorem 13. *We have for $q \geq 2$*

$$\gamma^{DLD}(K_q \Box K_q \Box K_q) = q^2.$$

3 Grids

In this section, we consider solid-location-domination and self-location-domination in the so called infinite king and triangular grids. Previously, for finite graphs, the optimality of a code has been defined using the minimum cardinality. However, this method is not valid for the infinite graphs of this section. Hence, we need to define the concept of *density* of a code. Let us first consider the *infinite king grid*.

Definition 14. Let $G = (V, E)$ be a graph with $V = \mathbb{Z}^2$ and for the vertices $v = (v_1, v_2) \in V$ and $u = (u_1, u_2) \in V$ we have $vu \in E$ if and only if $|v_1 - u_1| \leq 1$ and $|v_2 - u_2| \leq 1$. The obtained graph G is called the *infinite king grid*. Further let V_n be a subset of V such that $V_n = \{(x, y) \mid |x| \leq n, |y| \leq n\}$. The *density* of a code $C \subseteq V = \mathbb{Z}^2$ is now defined as

$$D(C) = \limsup_{n \to \infty} \frac{|C \cap V_n|}{|V_n|}.$$

We say that a code is *optimal* if there exists no other code with smaller density.

In what follows, we first consider solid-location-domination in the king grid. In the following theorem, we present a solid-locating-dominating code in the king grid with density $1/3$. The code is illustrated in Figure 2. Later, in Theorem 17, it is shown that the code is optimal.

Theorem 15. *Let $G = (V, E)$ be the king grid. The code*

$$C = \left\{(x, y) \in \mathbb{Z}^2 \mid |x| + |y| \equiv 0 \pmod 3\right\}$$

is solid-locating-dominating in G and its density is $1/3$.

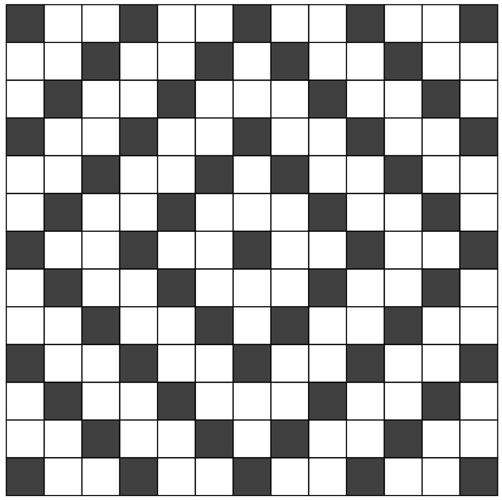

Figure 2: Solid-locating-dominating code of density $\frac{1}{3}$ in the king grid. The darkened squares are codewords.

In order to prove that the solid-locating-dominating code of the previous theorem is optimal, we first present the following lemma on forbidden patterns of non-codewords.

Lemma 16. *Let $G = (V, E)$ be the king grid and $C \subseteq V$ be a solid-locating-dominating code in G. Then $T = \{(i, j), (i, j + 1), (i, j + 2), (i + 1, j + 2), (i - 1, j + 2)\}$ and any formation obtained from T by a rotation of $\pi/2$, π or $3\pi/2$ radians around the origo contains a codeword of C.*

In the following theorem, we prove that the solid-locating-dominating code of Theorem 15 is optimal, i.e., there is no code with density smaller than 1/3.

Theorem 17. *If $G = (V, E)$ is the king grid and $C \subseteq V$ is a solid-locating-dominating code in G, then the density $D(C) \geq \frac{1}{3}$.*

Proof. Let S^j be a subgraph of G induced by the vertex set $V'_j = \{(x, y) \mid 1 \leq x \leq 3, 1 \leq y \leq j\}$. Recall first the definition $V_n = \{(x, y) \mid |x| \leq n, |y| \leq n\}$. Observe now that we may fit into the first quadrant $\{(x, y) \mid 1 \leq x \leq n, 1 \leq y \leq n\}$ of V_n $\lfloor n/3 \rfloor$ graphs isomorphic to S^n. Similarly, the other three quadrants of V_n can each contain $\lfloor n/3 \rfloor$ graphs isomorphic to S^n. Thus, in total, $4\lfloor n/3 \rfloor$ graphs isomorphic to S_n can be fitted into V_n.

Let C be a solid-locating-dominating code in G. In the final part of the proof, which is omitted, we show that any subgraph of G isomorphic to S^n contains at least $n - 3$ codewords. Assuming this is the case, the density of C can be estimated as follows:

$$D(C) = \limsup_{n\to\infty} \frac{|C \cap V_n|}{|V_n|} \geq \limsup_{n\to\infty} \frac{4\lfloor \frac{n}{3} \rfloor \cdot (n-3)}{(2n+1)^2} \geq \limsup_{n\to\infty} \frac{4(n-3)^2}{3(2n+1)^2} = \frac{1}{3}.$$

It remains to be shown that any subgraph of G isomorphic to S^n contains at least $n - 3$ codewords. By symmetry, it is enough to show that $|C \cap V'_n| \geq n - 3$. □

Above, we have shown that the density of an optimal solid-locating-dominating code in the king grid is 1/3. Recall that a self-locating-dominating code is always solid-locating-dominating. Hence, by the previous lower bound, we also know that there exists no self-locating-dominating code in the king grid with density smaller than 1/3. However, the construction given for the solid-location-domination does not work for self-location-domination. For example, we have $I(2, 0) = \{(2, -1), (2, 1), (3, 0)\}$ and $N[(2, -1)] \cap N[(2, 1)] \cap N[(3, 0)] = \{(2, 0), (3, 0)\}$ contradicting with the definition of self-locating-dominating codes (see Figure 2). In the following theorem, we present a self-locating-dominating code in the king grid with the density 1/3. Notice that this code is also solid-locating-dominating.

Theorem 18. *Let $G = (V, E)$ be the king grid. The code*

$$C = \{(x, y) \in \mathbb{Z}^2 \mid x - y \equiv 0 \pmod 3\}$$

is self-locating-dominating in G and its density is $1/3$.

Proof. The density $D(C) = 1/3$ since in each row every third vertex is a codeword. Furthermore, C is a self-locating-dominating code since each non-codeword v is covered either by the set of three codewords $\{v + (1, 0), v + (0, -1), v + (-1, 1)\}$ or $\{v + (-1, 0), v + (0, 1), v + (1, -1)\}$, and in both cases the closed neighbourhoods of the codewords intersect uniquely in the vertex v. □

In conclusion, we have shown that the density of an optimal self-locating-dominating code in the king grid is $1/3$. Next we consider self-locating-dominating and solid-locating-dominating codes in the infinite triangular grid.

Definition 19. Let $G = (V, E)$ be a graph with the vertex set

$$V = \left\{ i(1, 0) + j\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \mid i, j \in \mathbb{Z} \right\}$$

and two vertices are defined to be adjacent if their Euclidean distance is equal to one. The obtained graph G is called the *infinite triangular grid* and it is illustrated in Figure 3. We further denote $v(i, j) = i(1, 0) + j\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Let R_n be the subgraph of G induced by the vertex set $V_n = \{v(i, j) \mid |i|, |j| \leq n\}$. The density of a code in G is now defined as follows:

$$D(C) = \limsup_{n\to\infty} \frac{|C \cap V_n|}{|V_n|}$$

We say that a code is *optimal* if there exists no other code with smaller density.

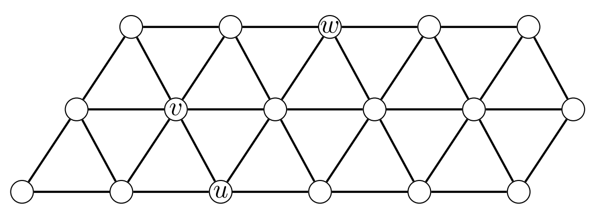

Figure 3: Triangular grid with the vertices $v = v(0, 0)$, $u = v(1, -1)$ and $w = v(1, 1)$.

In the following theorem, optimal self-locating-dominating and solid-locating-dominating codes are given in the triangular grid. We omit the proof.

Theorem 20. *Let $G = (V, E)$ be the triangular grid. The code*

$$C = \{v(i, j) \mid i, j \equiv 0 \pmod 2\}$$

is self-locating-dominating in G and, therefore, also solid-locating-dominating. The density of the code C is equal to $1/4$ and there exists no self-locating-dominating or solid-locating-dominating code with smaller density, i.e., the code is optimal in both cases.

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