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Spontaneous mathematical focusing tendencies in mathematical development

Jake McMullen[a], Lieven Verschaffel[b], and Minna M. Hannula-Sormunen[a]

[a]Department of Teacher Education, University of Turku, Turku, Finland; [b]Centre for Instructional Psychology and Technology, KU Leuven, Belgium

ABSTRACT
Children's own spontaneous mathematical activities are crucial for their mathematical development. Mathematical thinking and learning does not only occur in explicitly mathematical situations, such as the classroom. Those children with higher tendencies to recognize and use mathematical aspects of their everyday surroundings, both within the classroom and without, appear to have an advantage in learning formal mathematical skills and knowledge. In this introduction to the special issue, we provide an overview of the existing literature on spontaneous mathematical focusing tendencies. We then provide a brief overview of the contributions to the special issue.

KEYWORDS
Spontaneous mathematical focusing tendencies; SFON; mathematical development

Up until around 20 years ago, the vast majority of information that we had about children's mathematical abilities and processes was from mathematically explicit situations. In these investigations, either children were told that they should be thinking mathematically or there clearly was no other way to approach the task. This approach to investigations of mathematical abilities and processes makes sense in many ways; as researchers, we are mostly attuned to how children develop their mathematical skills in relation to formal schooling. However, children do not merely need mathematics for schoolwork. Life beckons, and life is full of math.

The past twenty years have seen an exponential growth in investigations into children's self-directed, spontaneous, mathematical activities. In particular, examinations of spontaneous mathematical focusing tendencies have led to an extensive accounting of how children attend to different mathematical aspects of non-explicitly mathematical situations (for recent reviews, see, McMullen, Chan, et al., 2019; Verschaffel et al., 2020). These investigations suggest that these tendencies may have important implications for mathematical thinking, learning, and instruction.

In this introduction to the special issue, "*Spontaneous mathematical focusing tendencies in mathematical development*", we provide a brief overview of the existing theoretical and empirical evidence in the field and introduce the contributions to the special issue.

We do not aim to provide a comprehensive overview of the field in this introduction text for two reasons. First, several contributors to this special issue, including the three guest editors, have recently produced two reviews that provide a comprehensive look at the current state of the field (McMullen, Chan, et al., 2019; Verschaffel et al., 2020). Second, the individual papers in the enclosed issue will provide a much more detailed account of the most relevant aspects of the literature for the particular contribution. Nonetheless, we proceed by pointing out some of the key foundations and major findings of the emerging field of spontaneous mathematical focusing tendencies especially for those who may be encountering it for the first time.

CONTACT Jake McMullen jamcmu@utu.fi Department of Teacher Education, Turun Yliopisto 20014, Finland

Theoretical and empirical foundations

The seminal investigation of spontaneous mathematical focusing tendencies was a set of studies examining the role of Spontaneous Focusing on Numerosity (SFON) in the development of early numeracy (Hannula & Lehtinen, 2005). In these studies, Hannula and Lehtinen (2005, pp. 240–241) claim that:

> within the frames of a child's existing mathematical competence it is possible to distinguish a separate mental process, one which refers to the child's tendency to spontaneously focus on the aspect of numerosity and utilize his or her enumeration skills in various activity situations.

The notion laid out by Hannula and Lehtinen (2005) is that paying attention to and use exact numerical information (e.g., the numerosity of a set) in everyday situations is not a totally automatic process and requires, among other things, explicitly focusing ones' attention on the numerical nature of a set. While this focusing of attention can occur through explicit external guidance, it is also possible that in situations that are not explicitly mathematical – that is, there are mathematical features embedded in the situation, but they are not explicitly referenced by an external indicator – focusing of attention toward numerical aspect must be initiated by the individual, spontaneously, without external guidance. Hannula and Lehtinen (2005) differentiated the more task or situation specific SFON from SFON tendency, the latter referring to more generalized spontaneous focusing on numerosity across different tasks and/or time.

Key to the examination of SFON was the design of a task in which children's responses could naturally include the relevant numerical feature, but did not need to do so in order to complete the task in a logical way. The iconic Elsi the bird task exemplifies such as task (Hannula & Lehtinen, 2005). In the task, a toy parrot is presented to the child along with a plate of red glass berries (2 cm in diameter). Hannula and Lehtinen describe the procedure as:

> The experimenter started the task by introducing the materials and then said: 'Watch carefully what I do, and then you do just like I did.' The experimenter put two berries one at a time into the parrot's mouth, and they disappeared with a bumping sound into the parrot's stomach. Then the child was told: 'Now you do exactly like I did'.

Since no hint or direction prior to the task suggested to the participant that the task involve numerosity, any actions, utterances, or behaviors that suggested the use of numerosity in reasoning or solving the task could be described spontaneous. Importantly, also, the numerosities involved were well within all children's enumeration range, so that if they focused on exact number of berries, they would be able to give exactly same number of berries to the bird. In addition, the probability of producing exactly same numerosity purely by accident is very low (one has 10 options for how many berries one can give to the bird), and all verbal or non-verbal indications of regarding exact numerosity or in the activity were scored as signs of spontaneous focusing on numerosity.

This process in and of itself is certainly interesting for those who aim to understand early numerical cognition and mathematical thinking. It also appears to have important implications for mathematical learning and instruction. Hannula and Lehtinen (2005) drew from research on the development of expertise more generally (e.g., Ericsson, 2006) in their argument as to why those children with a higher SFON tendency appear to gain an advantage in their early numerical development. They proposed that, akin to the early practice habits of future experts in other domains, those children with higher SFON tendency are likely to gain more self-initiated practice with skills and knowledge related to counting, which provides an advantage in learning these skills and knowledge. Such self-initiated practice, in which a child recognizes numerosities embedded in everyday situations (aided by a higher SFON tendency), would allow this child more opportunities to practice their budding skills than their peers who mainly engage in numerical acts only when externally guided to do so.

Hannula and Lehtinen (2005) provided evidence that SFON tendency is stable across different tasks and time (rank-order stability), that it is related to numeracy development, and that it is not explained by other potential factors such as non-verbal intelligence or the ability to recognize and use numerosity when explicitly guided to do so.

This set of findings has provided the foundation for the field that has expanded in many directions since this first publication. In particular, it has provided a model for the examination of various aspects of spontaneous mathematical focusing tendencies, which can be broken down into three main questions that can be raised for each of the spontaneous mathematical focusing tendencies that have been subsequently examined, most notably Spontaneous Focusing On quantitative Relations (SFOR), followed by Spontaneous Focusing on Number Symbols (SFONS), Patterns (SFOP), and Spatial relations (SFOS):

What is the nature of individual differences in the spontaneous mathematical focusing tendency?

What is the influence of contextual factors on the tendency?

How is the tendency related to mathematical development?

Using these three questions as guidelines, we outline below the most crucial empirical evidence pertaining to these questions and the associated publications that address these questions for the different spontaneous mathematical focusing tendencies.

The nature of individual differences

The first main question addresses how individuals differ in their spontaneous mathematical focusing. One major aim of this question is to determine whether such focusing can be described as a tendency. That is, is the spontaneous behavior that is captured something that is consistent within an individual across similar tasks? As well, is this behavior consistent relative to their peer group across time?

Consistency in mathematical focusing across tasks

In order for spontaneous mathematical focusing to be considered a tendency, Hannula and colleagues have argued that instances of focusing should be somewhat consistent across different tasks (Hannula & Lehtinen, 2005). For SFON tendency, there is a fairly consistent record of stability in instances of SFON across tasks, especially for tasks with similar modalities (Gray & Reeve, 2016; Hannula & Lehtinen, 2001, 2005; Hannula et al., 2007, 2010; Lepola & Hannula-Sormunen, 2019). That is, across tasks that involve behavioral imitation (e.g., feeding a stuffed animal food pellets) there is high levels of consistency in individual differences (e.g., Gray & Reeve, 2016). As well, across multiple verbal picture description tasks assessing SFON, there appears to be consistency (Rathé et al., 2018). However, different modalities of task type, such as verbal picture description tasks and behavioral imitation tasks, show less consistency (Batchelor et al., 2015; Nanu et al., 2020; Rathé et al., 2018), which suggests further research is needed regarding the validity of a single, multi-modal SFON tendency.

In research on SFOR tendency, there appear to sufficient levels of consistency between written description tasks and drawing imitation tasks in measures of SFOR, when both tasks are paper-and-pencil based (McMullen et al., 2016). However, no existing study on SFOR has examined measures that use both behavioral imitation and description or pictorial tasks. As well, other potential focusing tendencies either have relied on a single measure or have not explicitly reported consistency across task (Perez & McCrink, 2019; Rathé et al., 2019; Wijns et al., 2019). Future studies of these potential tendencies will hopefully examine this crucial issue.

Relative consistency in performance across time

While not innate in nature, it is expected that individual differences in spontaneous mathematical focusing should be relatively stable over short to medium time ranges, without explicit intervention. More specifically, there should be rank-order stability in spontaneous mathematical focusing on a particular mathematical feature (e.g., numerosity) within populations, all other matters held constant. Importantly, children learn to focus on numerosity and other mathematical aspects in more and more demanding tasks and situations during their childhood years (Hannula, 2005). Thus socio-cultural mediation of numerical cognition develops children's ability to focus and use exact numerosity recognition and other cultural tools in their action. This enumeration practice, in turn, then develops enumeration skills further up to larger number ranges and more demanding enumeration contexts (Hannula & Lehtinen, 2005).

Thus, although there may be developmental effects that cause increases in instances of a particular tendency (such as increased formal knowledge) these should be relatively stable across individuals in a particular population. It is not surprising then to find that SFON and SFOR tendency have shown rank-order stability across both short (e.g., six weeks; Batchelor, 2014; McMullen, Hannula-Sormunen, et al., 2019) and medium intervals (e.g., one year; Hannula & Lehtinen, 2005; Hannula et al., 2007; McMullen et al., 2017). Importantly, this stability appears to remain even after taking into account the potential influences of formal development. Alongside this stability, in the same models there is evidence of reciprocal nature of the relation between SFON and cardinality related skills (Hannula & Lehtinen, 2005) and between SFOR and rational number knowledge (McMullen et al., 2017); formal mathematical skills predict spontaneous mathematical focusing, even after taking into account prior spontaneous mathematical focusing.

Contextual influence

While a tendency can only be described based on consistency in performance across modality or time, evidence suggests that spontaneous mathematical focusing tendencies are not fixed traits that are impervious to intervention. The second main question in spontaneous mathematical focusing research is the nature of contextual influences on spontaneous mathematical focusing. First, what is the effect of perceptual salience on attention to mathematical aspects? Second, what is the sustained effect of social interaction on individuals' spontaneous mathematical focusing tendencies, in both home environments and educational settings?

Effects of perceptual salience

Although within-individual variation should remain rather constant across similar tasks, differences in task design may nonetheless affect the frequency of focusing on mathematical aspects. In order for a task to measure a spontaneous mathematical focusing tendency there should be no explicit mention of the mathematical features of the task before nor during testing, there should be non-mathematical features of the task on which participants can also focus, and the task should be well within the range of participants' abilities (Hannula, 2005; McMullen, Chan, et al., 2019). In other words, the participant should be the one who figures out the task is about math and they should be able to do the math fairly easily. However, competing features of a task may interact with an individual's recognition of the mathematical features. Non-mathematical task features with strong salience or relevance may make individuals less likely to recognize the relevance of mathematical features, including numerosities (Chan & Mazzocco, 2017) and quantitative relations (Prather, 2020). Indeed, this may even extend to competing mathematical features, such as whole numbers being more salient than proportional relations (McMullen et al., 2013).

Effects of social interaction and educational interventions

To date, there is a paucity of evidence regarding the origins of individual differences in spontaneous mathematical focusing tendencies. While related to formal mathematical skills (e.g., early numeracy) and other factors (e.g., non-verbal intelligence), these cognitive factors do not appear to explain a great deal of variance in the focusing tendencies (Hannula & Lehtinen, 2005; Hannula et al., 2010; McMullen et al., 2016; Perez & McCrink, 2019). Likewise, there is not clear evidence of how other social factors, such as the mathematical home environment or school setting, affect individual differences in spontaneous mathematical focusing tendencies.

Current evidence is mixed on the role of the home environment in explaining individual differences in spontaneous mathematical focusing tendencies, though there is little, if any, support for spontaneous mathematical focusing tendencies to be innate traits. Some evidence from young children in Finland suggests that the home environment may be related to SFON tendency (Mattinen, 2006) and parent and child use of mathematical language has been found to be correlated in other samples (Chan & Mazzocco, 2017). However, other evidence shows no relation between children's home mathematics environment (measured by a home numeracy questionnaire completed by children's parents) and children's SFON or SFONS scores (Rathé et al., 2020).

Nonetheless, there is evidence that these tendencies may be malleable to explicit intervention, especially through social interaction. Interventions aimed at increasing SFON tendency have been found to elicit immediate and sustained effects in young children (Braham et al., 2018; Hannula et al., 2005). The main objective in these interventions is to make the mathematical aspect a clearer and regularly recognized target of focusing through guided activities, with the hope that they transfer to children's spontaneous behavior. Even brief interventions in informal settings have led to increased SFON tendency (Braham et al., 2018). Sustained social interaction in early childhood settings showed immediate and sustained improvements in SFON tendency (Hannula et al., 2005; Mattinen, 2006). As well, a classroom intervention using mobile technology to promote recognition and use of multiplicative relations in everyday situations appeared to improve students' SFOR tendency (McMullen, Hannula-Sormunen, et al., 2019).

Relation to mathematical development

While the first two questions mainly deal with the nature of spontaneous mathematical focusing tendencies, the final question is perhaps most crucial: is there a (positive) relation between a particular tendency and the development of mathematical knowledge and skills? Given the correlational nature of most research on spontaneous mathematical focusing tendencies, an important feature of this question is examining potential alternative explanations for any relation, often in the form of statistical controls. These include examining if any correlational effect is concentrated on the most relevant mathematical skills and knowledge and not explained by potential confounds. Ideally, this research culminates in evidence for or against a causal relation.

Domain and topic specificity

These issues pertain to the construct validity in the proposed effect a particular focusing tendency would have on mathematical development. As explained above, the benefit of a particular focusing tendency is proposed to be that an individual gains more self-initiated practice with the corresponding mathematical knowledge and skills that such focusing would promote. On a general level, this suggests that the effect of spontaneous mathematical focusing tendencies should be specific to mathematics, and not related to, for example, reading ability (Hannula et al., 2010). On a more specific level, this would suggest that each focusing tendency should be more closely related to relevant mathematical knowledge and skills than to other mathematical abilities. SFON tendency is more strongly related to whole number numerical skills than other mathematical skills, including general mathematical achievement (Nanu et al., 2018). As well, spontaneous focusing on patterns (SFOP) is more strongly related to patterning skill than to general mathematical ability (Perez & McCrink, 2019). Finally, spontaneous focusing on number symbols (SFONS) was particularly strongly related to symbolic number skills (Rathé et al., 2019).

Potential confounds (Guided focusing and other factors)

Of extreme importance to the field, given the over-reliance so far on correlational studies, is the extent to which relations between spontaneous mathematical focusing tendencies and mathematical development are explained by other potential confounds. First, central to the theory of spontaneous focusing is that the relation between the focusing tendency and mathematical skills are not due to differences in the ability of individuals to handle the mathematical aspects embedded in the spontaneous tasks. In other words, the ability to use the mathematical aspects embedded in the spontaneous task (e.g., to enumerate the numerosities used in SFON tasks) should not fully explain the relation between the focusing tendency and mathematical skills. This has been explicitly tested mainly in SFOR studies, in which the ability to describe or use the multiplicative relation embedded in the spontaneous tasks has been statistically controlled for in examining the relation between SFOR tendency and mathematical development (McMullen et al., 2016, 2017). This has also been implicitly examined in SFON studies, by showing that all children in a sample are capable of focusing on numerosities when explicitly guided to do so (Hannula & Lehtinen, 2005). SFONS studies have found that symbolic number knowledge was only partially related to SFONS responses (Rathé et al., 2019). Not all children with high patterning ability (based on their performance on a test measuring their ability to extend, generalize and memorize given patterns on demand) were also found to spontaneously create a pattern in an unguided tower construction situation), showing also that differences in SFOP cannot be fully explained by differences in patterning ability (Wijns et al., 2019).

Along with this particular need for examining confounds is the more general concern that other mathematical, cognitive, and non-cognitive factors confound the relation between a spontaneous mathematical focusing tendency and mathematical skills and knowledge. Most generally, this would include prior knowledge and general mathematical achievement. These mathematical factors have been found to not entirely explain the relation between SFON and numerical development (Hannula & Lehtinen, 2005; Hannula et al., 2010), SFONS and numerical development (Rathé et al., 2019), SFOR and rational number development (McMullen et al., 2016; Van Hoof et al., 2016)

As for cognitive factors, SFON tendency has been found to be related to numerical development even after taking into account listening comprehension, non-verbal intelligence, rapid naming, comprehension of instructions, visio-motoric skills, working memory, and motivational orientations (Batchelor et al., 2015; Hannula & Lehtinen, 2005; Hannula et al., 2010; Kucian et al., 2012; Lepola & Hannula-Sormunen, 2019). As well, SFOR tendency has been found to be related to mathematical development even after taking into account non-verbal intelligence, written descriptiveness, and spatial reasoning (McMullen et al., 2016; Van Hoof et al., 2016). SFONS has been found to predict numerical ability after controlling for spatial ability and language ability (Rathé et al., 2019). Finally, SFOP tendency and mathematical ability remained related after taking into account visio-spatial working memory and spatial skills (Wijns et al., 2019).

Non-cognitive factors also appear to not entirely explain the relation between spontaneous mathematical focusing tendencies and mathematical development. SFON tendency and arithmetic skills remain related after taking into account children's motivational orientations (Lepola & Hannula-Sormunen, 2019).

Causal evidence

The brass ring of behavioral sciences, evidence of a causal relation, is a clear end goal of most research examining spontaneous mathematical focusing tendencies. Importantly, most of the positive effects from increases in a mathematical focusing tendency should be delayed effects, which should appear after the increased mathematical focusing in everyday life leads to extra self-initiated practice with the formal mathematical skills and knowledge (Lehtinen et al., 2017). Thus, while there may be immediate increases in the focusing tendency, the transfer to mathematical skills and knowledge should mainly at a later time point.

The gold standard (not to mix metaphors) of a large-scale randomized control trial is still lacking in the field. Nonetheless, some evidence of mathematical focusing tendencies having a positive effect on mathematical development is causal in nature. Most notably, several quasi-experimental studies have found that increasing SFON tendency leads to long-term gains in numerical skills and knowledge

(Hannula et al., 2005; Mattinen, 2006). A quasi-experimental study has also found that an intervention aimed at improving SFOR tendency led to gains in knowledge about multiplicative relations and fractions (Määttä et al., 2019).

Special issue contributions

The contributions of this special issue aim to address a number of open questions regarding the nature, development, and role of spontaneous mathematical focusing tendencies.

First, Mazzocco and colleagues continue their exploration of the effects of perceptual salience on attention to number. Extending their previous work, their contribution investigated the differing effects of low and high salience features on attention to number. As well, they aim to examine, for the first time, the relation between attention to number and mathematical achievement in adults. Their findings support the notion that attention to number is affected by the perceptual salience of competing features. Yet, as in SFON research, clear individual differences in the tendency to pay attention to exact number are present in both children and adults.

Second, Rathé and colleagues continue their work in exploring the nature of individual differences in SFONS, especially in relation to early mathematical knowledge and skills. They provide the first evidence of how different spontaneous mathematical focusing tendencies are related to each other, by looking at the factor structure of SFON and SFONS across picture description tasks. As well, they continue the exploration of the nature of SFONS by examining its relation to performance on a variety of early numeracy tasks. They find that SFON and SFONS may be best represented by two different constructs and further validate the relevance of SFONS for early numerical development.

In the third article, Silver and colleagues continue the examination of the relation between spontaneous mathematical focusing tendencies and mathematical development. Crucially, they examine the relation between SFON and early numerical skills also taking into account parental factors, such as home mathematical environment, along with domain general and specific cognitive skills. Their results suggest that the relation between SFON and mathematical ability is not entirely explained by parental factors, nor approximate number system acuity.

The fourth article by Hannula-Sormunen and colleagues provides a comprehensive examination of educational interventions on SFON tendency. In their contribution, they detail two SFON tendency and numeracy intervention programs for early childhood education, providing a framework and best practices for naturalistic, early childhood interventions on integrating spontaneous mathematical focusing tendencies training with training relevant early numeracy skills. The quasi-experimental studies with active control group found positive long-term effects on early numeracy skills for both interventions.

In the fifth article, Alexander and colleagues turn their attention toward relations, both mathematical and otherwise. Their contribution introduces a novel approach to examining performance on spontaneous mathematical focusing tasks, by examining the nature of alternative responses to the task. Notably, they found that non-relational response patterns followed discernable patterns, suggesting that participants were deliberate in their choices on the spontaneous focusing task. Finally, there was a relation between performance on the spontaneous mathematical focusing task and a measure of relational reasoning, especially analogous reasoning.

Finally, the sixth contribution by McMullen and Siegler extends work on SFOR tendency and its relation to fraction knowledge to examine how SFOR tendency may exactly contribute to understanding fraction magnitudes. In this short report, they explore different aspects of fraction magnitude knowledge that SFOR tendency may be related to, including a task in which whole number part-whole relations embedded in narrative vignettes must be translated into fraction magnitudes. Their results suggest that SFOR tendency contributes in particular to increases in the precision with which students translate fractional relations.

A critical discussion of these papers and the field at large is provided in the commentary by Matthew Inglis.

Notes on contributors

Jake McMullen is a postdoctoral researcher and adjunct professor (Docent) at the Department of Teacher Education, University of Turku, Finland. His main research interests are mathematical learning and instruction, especially examining the development of adaptive expertise in mathematics.

Lieven Verschaffel obtained in 1984 the degree of Doctor in Educational Sciences at the University of Leuven, Belgium. After having been a research fellow and a post-doc researcher of the Scientific Research Foundation – Flanders (FWO), he became professor in educational sciences of that same university, with a main interest in educational psychology and more particularly learning and teaching in specific content domains, particularly mathematics. His main research topics are early mathematics education, whole number arithmetic, estimation and number sense, and word problem solving. He (co-)authored about 300 articles in international peer-reviewed journals and also many (edited) books and book chapters. Lieven Verschaffel is a member of the editorial board of various international journals in the domain of (mathematics) learning and teaching.

Minna Hannula-Sormunen is a Professor of Education, particularly early mathematical development and learning environments, at the Department of Teacher Education, University of Turku, Finland. Her main research interests are particularly early but also later mathematical development and everyday and digital learning environments supporting it as well as broader aspects of children's learning and early education.

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ORCID

Jake McMullen http://orcid.org/0000-0002-7841-7880

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