

Rate Operator Unraveling for Open Quantum System Dynamics

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Stochastic methods with quantum jumps are often used to solve open quantum system dynamics. Moreover, they provide insight into fundamental topics, such as the role of measurements in quantum mechanics and the description of non-Markovian memory effects. However, there is no unified framework to use quantum jumps to describe open-system dynamics in any regime. We solve this issue by developing the rate operator quantum jump (ROQJ) approach. The method not only applies to both Markovian and non-Markovian evolutions, but also allows us to unravel master equations for which previous methods do not work. In addition, ROQJ yields a rigorous measurement-scheme interpretation for a wide class of dynamics, including a set of master equations with negative decay rates, and sheds light on different types of memory effects which arise when using stochastic quantum jump methods.

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Introduction.—Any realistic description of a quantum system should take into account its interaction with the surrounding environment [1,2]. Many different approaches have thus been developed to characterize the evolution of open quantum systems, ideally covering different models and regimes, yet keeping the degree of complexity manageable [3–16].

Quantum unravelings yield a practically and conceptually useful tool, mapping a given master equation to one (of the infinitely many possible) pure-state stochastic evolution, which reproduces the given master equation on average [17,18]. On the one hand, this leads to a linear scaling of the simulation cost with the Hilbert space dimension of the open system, instead of the quadratic scaling which would affect the direct integration of the master equation. On the other hand, unravelings might provide us with a clear physical picture of the environmental influence on the open-system evolution. In particular, the stochastic pure-state evolution can be seen as the result of a continuous measurement operated on the open system, so that the master equation would correspond to the continuous action of a nonselective observer (the environment). This is the case, for example, in the well-known Monte Carlo wave function (MCWF) method [19,20], where the open-system pure state is subjected to a deterministic evolution interrupted by random and discontinuous jumps. Such piecewise deterministic evolutions under continuous monitoring have been observed in several experimental platforms [21–26].

Memory effects pose some relevant challenges to unraveling methods, so that novel strategies need to be developed to deal with non-Markovian dynamics [27–33]. Many nonequivalent definitions have been introduced [34–36], but broadly speaking we can say that non-Markovian dynamics are characterized by a twofold exchange of information between the open system and the environment, which leads to memory effects and, from the mathematical point of view, breaks relevant divisibility properties of the dynamical maps fixing the open-system evolution. The non-Markovian quantum jump (NMQJ) approach [29,37] accounts for the information flowing back to the open system by means of reversed jumps, which generalize the quantum jumps of the MCWF. However, it is not clear to what extent, if at all, the continuous-measurement interpretation can be extended to this and the other non-Markovian unravelings [38,39]. The basic intuition is that the (continuous) measurements would affect in a nontrivial way the backflow of information to the open system and hence the subsequent dynamics, thus generating an evolution which is not the same as the one given by the master equation to be unraveled [36,38].

Here, first we show that a fully consistent continuous-measurement interpretation [40] can be formulated for any positive- (P) divisible dynamics [41–43], via a jump unraveling approach which relies on the diagonalization of a proper rate operator, and is named rate operator quantum jump (ROQJ). The class of P-divisible dynamics includes master equations with negative rates and is larger than the

one where MCWF applies, thus highlighting the subtle border between Markovianity and non-Markovianity within the context of quantum unravelings. Furthermore, we extend ROQJ to deal with any open-system dynamics, including those where at least one master-equation coefficient is negative from the very beginning of the evolution [44–48], so that other non-Markovian techniques, such as NMQJ, cannot be used.

Quantum jumps for P-divisible dynamics.—As usual within the unraveling methods, we start from the master equation describing the dynamics of the open quantum system of interest. Any trace and Hermiticity preserving (time-local) master equation $d\rho(t)/dt = \mathcal{L}_t[\rho(t)]$ for the open-system state $\rho(t)$ can be written as [49]

$$\begin{aligned} \mathcal{L}_t[\rho(t)] = & -\frac{i}{\hbar} [H_S(t), \rho(t)] \\ & + \sum_{\alpha=1}^{n^2-1} c_\alpha(t) \left(L_\alpha(t) \rho(t) L_\alpha^\dagger(t) \right. \\ & \left. - \frac{1}{2} \{ L_\alpha^\dagger(t) L_\alpha(t), \rho(t) \} \right), \end{aligned} \quad (1)$$

where n is the finite dimension of the open system, $H_S(t) = H_S^\dagger(t)$ and $L(t)$ are possibly time-dependent operators on \mathbb{C}^n , and $c_\alpha(t)$ are real functions of time.

For now we restrict to P-divisible evolutions [41–43,46], i.e., the dynamical maps $\Lambda_t = T \exp[\int_0^t \mathcal{L}_s ds]$ (T is the time ordering operator) can be decomposed as $\Lambda_t = \Phi_{t,s} \circ \Lambda_s$, where $\Phi_{t,s}$ is positive (P), for any $t \geq s$. Let us stress that P divisibility is a weaker requirement than $c_\alpha(t) \geq 0$ for any α , which is precisely the condition guaranteeing that MCWF can be applied. In fact, the positivity of the coefficients coincides, under some regularity conditions, with the property of completely positive (CP) divisibility, i.e., that $\Phi_{t,s}$ in the decomposition above is CP [35,50]. The map $\Phi_{t,s}$ is CP when $(\Phi_{t,s} \otimes \mathbb{1}_n) \rho_{sa} \geq 0$, where $\mathbb{1}_n$ is the identity map on the ancillary Hilbert space \mathbb{C}^n and ρ_{sa} is any combined open system and ancilla state [1]. The basic observation, which we need to define the rate operator quantum jump unraveling, is that the evolution is P divisible if and only if the rate operator

$$\begin{aligned} W_{\psi(t)}^J = & \sum_{\alpha=1}^{n^2-1} c_\alpha(t) (L_\alpha(t) - \mathcal{L}_{\psi(t),\alpha}) |\psi(t)\rangle \langle \psi(t)| \\ & \times (L_\alpha(t) - \mathcal{L}_{\psi(t),\alpha})^\dagger, \end{aligned} \quad (2)$$

where $\mathcal{L}_{\psi(t),\alpha} = \langle \psi(t) | L_\alpha(t) | \psi(t) \rangle$ is a positive semidefinite operator for any fixed $|\psi(t)\rangle$ [51]. Then, the eigenvalues of W^J are non-negative and we define the jump operators

$$V_{\psi(t),j} = \sqrt{\lambda_{\psi(t),j}} |\varphi_{\psi(t),j}\rangle \langle \psi(t)|, \quad (3)$$

with $\lambda_{\psi(t),j}$ and $|\varphi_{\psi(t),j}\rangle$ eigenvalues and (orthonormal) eigenvectors of $W_{\psi(t)}^J$.

Now, consider the trajectories on the set of the open-system pure states, which are given by the deterministic evolution fixed by the non-Hermitian and nonlinear Hamiltonian,

$$\begin{aligned} H_{\psi(t)} = & H_S(t) - \frac{i\hbar}{2} \sum_{\alpha=1}^{n^2-1} c_\alpha(t) \\ & \times (L_\alpha^\dagger(t) L_\alpha(t) - 2\mathcal{L}_{\psi(t),\alpha}^* L_\alpha(t) + |\mathcal{L}_{\psi(t),\alpha}|^2) \end{aligned} \quad (4)$$

according to

$$|\psi(t)\rangle \mapsto |\psi(t+dt)\rangle = \frac{(1 - \frac{i}{\hbar} H_{\psi(t)} dt) |\psi(t)\rangle}{\|(1 - \frac{i}{\hbar} H_{\psi(t)} dt) |\psi(t)\rangle\|}, \quad (5)$$

interrupted by sudden jumps in the form

$$|\psi(t)\rangle \rightarrow \frac{V_{\psi(t),j} |\psi(t)\rangle}{\|V_{\psi(t),j} |\psi(t)\rangle\|} = |\varphi_{\psi(t),j}\rangle, \quad (6)$$

where the probability to have a jump j between t and $t+dt$ is

$$p_j(t) = \|V_{\psi(t),j} |\psi(t)\rangle\|^2 dt = \lambda_{\psi(t),j} dt. \quad (7)$$

As shown in Sec. I of Ref. [52], this defines a legitimate unraveling; i.e., the state averaged over the different trajectories satisfies the master equation (1).

Such construction resembles the standard MCWF. But now, crucially, the different jump operators and their occurrence probabilities are fixed by the eigenvectors $|\varphi_{\psi(t),j}\rangle$ and eigenvalues $\lambda_{\psi(t),j}$ of the operator $W_{\psi(t)}^J$, rather than by the operators $L_\alpha(t)$ and coefficients $c_\alpha(t)$ as in MCWF. This is why we can have positive probabilities in Eq. (7) also for some dynamics with at least one negative rate $c_\alpha(t)$, where MCWF cannot be applied. Let us stress that jumplike unravelings for P semigroups (i.e., under the further assumption that $\Phi_{t,s} = \Lambda_{t-s}$) have been introduced in Refs. [53,54], while diffusive unravelings were defined in Refs. [54,55] and, for the more general case of P-divisible dynamics, in Ref. [51].

Continuous-measurement interpretation.—To introduce a proper continuous-measurement interpretation [40], let us consider the following setup. The open system of interest is surrounded by n measurement apparatuses, say, n counters, which monitor it continuously and are parametrized by the index j . In the current case, the n apparatuses correspond to the eigenstates of the rate operator in Eq. (2). If a given detector “clicks,” this means that the state of the system

jumps to the corresponding eigenstate; i.e., the detectors count the jumps to the eigenstates of the rate operator. In the case of no detection at a given moment of time, the evolution continues deterministically.

The type and instant of the counts up to time t define different sequences $\omega_t = (t_1, j_1; t_2, j_2; \dots t_m, j_m)$, with $t_1 \leq \dots \leq t_m \leq t$. So let $\mathcal{O} = \{\emptyset, j\}_{j=1, \dots, n}$ be the set of measurement outcomes, where j indicates that the counter j clicked, while \emptyset that no counter clicked. For any time t and sequence ω_t , we define the instrument [56] which maps any element of \mathcal{O} to an open-system operation, i.e., CP trace nonincreasing map, $\{\mathcal{I}_{\omega_t, \emptyset}, \mathcal{I}_{\omega_t, j}\}_{j=1, \dots, n}$. The latter fixes the state transformation $\rho \mapsto \mathcal{I}_{\omega_t, j(\emptyset)}\rho / \text{Tr}\{\mathcal{I}_{\omega_t, j(\emptyset)}\rho\}$ and probability $p_{j(\emptyset)}(t) = \text{Tr}\{\mathcal{I}_{\omega_t, j(\emptyset)}\rho\}$ associated with the outcome j (\emptyset); we restrict to purity-preserving transformations. As a result of the continuous measurement, the open system, initially in a pure state $|\psi(t_0)\rangle$, will follow the evolution $|\psi(\omega_t)\rangle$ obtained by applying every infinitesimal time dt one of the operations in $\{\mathcal{I}_{\omega_t, \emptyset}, \mathcal{I}_{\omega_t, j}\}_{j=1, \dots, n}$ (and normalizing the resulting state), according to the count sequence ω_t .

In particular, for any time t and sequence ω_t , we define the operation associated to the count j between t and $t + dt$ as

$$\mathcal{I}_{\omega_t, j}\rho = V_{\omega_t, j}\rho V_{\omega_t, j}^\dagger dt, \quad j = 1, \dots, n, \quad (8)$$

which is indeed CP and trace nonincreasing; here $V_{\omega_t, j}$ is a shorthand notation for $V_{\psi(\omega_t), j}$, which is defined as in Eq. (3), with respect to the state $|\psi(\omega_t)\rangle$ (see also Sec. II of Ref. [52]). Moreover, let

$$\mathcal{I}_{\omega_t, \emptyset}\rho = F_{\omega_t, \emptyset}\rho F_{\omega_t, \emptyset}^\dagger \quad (9)$$

be the operation associated with the “null count.” As a defining property of any instrument, the overall probability has to be 1. By virtue of Eqs. (2)–(4), one can see that this is achieved by defining

$$F_{\omega_t, \emptyset} = \left(\mathbb{1} - \frac{i}{\hbar} H_{\omega_t} dt \right) \Pi_{\omega_t}, \quad (10)$$

where $\Pi_{\omega_t} = |\psi(\omega_t)\rangle\langle\psi(\omega_t)|$, and introducing an auxiliary event a , associated with $\mathcal{I}_{\omega_t, a}\rho = (\mathbb{1} - \Pi_{\omega_t})\rho(\mathbb{1} - \Pi_{\omega_t})$, so that $\sum_{j=1}^n p_j(t) + p_\emptyset(t) + p_a(t) = 1$ for any ρ .

Now, when applied to the pure state $\rho = |\psi(\omega_t)\rangle\langle\psi(\omega_t)|$, the state transformation and occurrence probability fixed by Eq. (8) coincide with, respectively, Eqs. (6) and (7), while the state transformation due to Eqs. (9) and (10) coincides with the deterministic one in Eq. (5); indeed, the auxiliary event a happens with probability 0. We can thus conclude that, for any sequence of counts ω_t , the open-system state obtained by applying $\mathcal{I}_{\omega_t, j}$ and $\mathcal{I}_{\omega_t, \emptyset}$ every infinitesimal time dt and resulting in $|\psi(\omega_t)\rangle$ provides us with the same

trajectories and associated probabilities as the unraveling described in the previous paragraph (identifying ω_t with the sequence of jumps). In Sec. I of Ref. [52] we also give the description of the above continuous-measurement evolution in terms of a stochastic differential equation [40].

Let us stress that Eqs. (8) and (9) define a family of instruments, one for every time t and sequence ω_t . In the standard approaches [19,20,40] the probabilities to have a certain count j given the sequence ω_t do depend on ω_t , i.e., they are to be understood as conditional probabilities [40]; in rate operator jumps, in addition to this, the instrument itself, and then the resulting state transformation, becomes an object conditioned on ω_t , in principle different for any count sequence. This is the key feature which allows us to introduce consistently and systematically a measurement interpretation for a class of dynamics, the P -divisible ones, which is strictly larger than the set of CP -divisible dynamics, where the standard scheme applies. Later on, we will discuss the meaning of this dependence of the instrument on ω_t in terms of memory effects in the unraveling.

General open quantum system dynamics: Reverse quantum jumps.—We now move on to the second main purpose of the Letter, that is, introducing a general version of the rate operator quantum jump method, able to deal also with non- P -divisible dynamics.

When P divisibility is broken, the rate operator W^J in Eq. (2) is not positive definite, but it is still Hermitian and we can thus write its spectral decomposition as

$$W_{\psi(t)}^J = \sum_{j^+} \lambda_{\psi(t), j^+} |\varphi_{\psi(t), j^+}\rangle\langle\varphi_{\psi(t), j^+}| - \sum_{j^-} |\lambda_{\psi(t), j^-}| |\varphi_{\psi(t), j^-}\rangle\langle\varphi_{\psi(t), j^-}|, \quad (11)$$

where $\lambda_{\psi(t), j^+}$ ($|\varphi_{\psi(t), j^+}\rangle$) and $\lambda_{\psi(t), j^-}$ ($|\varphi_{\psi(t), j^-}\rangle$) are the positive and negative eigenvalues (eigenvectors) of $W_{\psi(t)}^J$, respectively. Once again, we define the rate operator jump unraveling as the deterministic evolution fixed by Eqs. (4) and (5) interrupted by sudden jumps, associated to the spectral decomposition of W^J . For the positive eigenvalues $\lambda_{\psi(t), j^+}$, we can proceed exactly as in the P -divisible case, introducing the operators $V_{\psi(t), j^+}$ as in Eq. (3), which induce the jump in Eq. (6) with probability as in Eq. (7). On the other hand, for the negative eigenvalues $\lambda_{\psi(t), j^-}$ we cannot proceed in the same way, as we would get negative probabilities (analogously to what happens in MCWF for negative coefficients in the master equation). A possible way out is obtained by relating the different trajectories of the unraveling to each other [29]. Hence, let us consider the ensemble $\Psi(t) = \{|\psi_i(t)\rangle\}_{i=1, \dots, N}$ of the pure states generated by the N trajectories of the unraveling at time t . We define a second kind of jump operator, given by

$$B_{\psi_k(t), \psi_{k'}(t), j^-} = \sqrt{|\lambda_{\psi_{k'}(t), j^-}|} |\psi_{k'}(t)\rangle \langle \psi_k(t)|, \quad (12)$$

and we postulate that it acts only if the source and target states are related by

$$|\psi_k(t)\rangle = |\varphi_{\psi_{k'}(t), j^-}\rangle, \quad (13)$$

inducing the state transformation $|\psi_k(t)\rangle \mapsto |\psi_{k'}(t)\rangle$, with probability

$$p_{j^-}^{(k \rightarrow k')}(t) = \frac{N_{k'}(t)}{N_k(t)} |\lambda_{\psi_{k'}(t), j^-}| dt, \quad (14)$$

where $N_i(t)$ is the number of elements $|\psi_i(t)\rangle$ in $\Psi(t)$. In Sec. III of Ref. [52], we show that the trajectories described above do provide a valid unraveling; i.e., the average state $\sum_i N_i(t) |\psi_i(t)\rangle \langle \psi_i(t)| / N$ satisfies the master equation (1).

Differently from the jumps in Eq. (3), each of the jumps in Eq. (12) connects couples of states $[|\psi_k(t)\rangle \text{ and } |\psi_{k'}(t)\rangle]$ which must be both in the ensemble $\Psi(t)$ before the jump, and the associated probability depends on the number of corresponding ensemble members $[N_k(t) \text{ and } N_{k'}(t)]$; see Eq. (14). The crucial point is that only if the source state $|\psi_k(t)\rangle$ is related to the target state $|\psi_{k'}(t)\rangle$ by the relation in Eq. (13) they will be connected by a jump $B_{\psi_k(t), \psi_{k'}(t), j^-}$. Note that this also means that this kind of jump can be interpreted as a reverse jump, with respect to the “standard” ones. The extension of the rate operator quantum jump method to non-P-divisible dynamics is in fact inspired by the reverse quantum jumps of the NMQJ method [29,37], but, as will be shown explicitly below, ROQJ has a wider range of applicability.

Two case studies.—First, we consider a master equation where one of the decay rates is negative for all times $t > 0$ —while the corresponding dynamical Λ_t is CP, not CP divisible, and still P divisible for all $t > 0$. In this case, one cannot use MCWF nor NMQJ methods. Take the dynamics of a two-level system fixed by the master equation [45,48,57,58],

$$\frac{d}{dt} \rho(t) = \frac{1}{2} \sum_{k=1}^3 \gamma_k(t) [\sigma_k \rho(t) \sigma_k - \rho(t)], \quad (15)$$

where the $\{\sigma_k\}_{k=1,2,3}$ are the Pauli matrices. Equation (15) is exactly solvable and the P divisibility of the corresponding evolution is equivalent to the conditions [48,58] $\gamma_i(t) + \gamma_j(t) \geq 0$, $i \neq j$; the controlled transition between P-divisible and non-P divisible evolutions for the dynamics in Eq. (15) has been realized experimentally in Ref. [46].

Let us fix in particular $\gamma_i(t) = \mu_i(t) - \mu_j(t) - \mu_k(t)$, with $\mu_i(t) = -(x_j + x_k)/(x_j + x_k + e^{2t}x_i)$, for $i \neq j \neq k = 1, 2, 3$ and with x_1, x_2, x_3 non-negative numbers summing up to 1. P divisibility holds at any time, but the rates $\gamma_i(t)$ can be negative; even more, there are choices of the x_i 's such that

one of the $\gamma_i(t)$ is negative for any time $t > 0$; i.e., CP divisibility is broken at any $t > 0$. Dynamics with a perpetually negative master-equation coefficient have been extensively studied in the literature [44–48,59,60] and are usually referred to as eternal non-Markovian. This kind of master equations cannot be unraveled by the standard MCWF [19] since a negative decay rate leads to a negative quantum jump probability. NMQJ [29,37], in turn, is based on canceling previously occurred quantum jumps when the decay rate turns negative. Having a negative rate since the very beginning of the time evolution implies that one should cancel something that never happened—which leads to the mathematical problems in addressing such jumps. On the other hand, rate operator quantum jumps can easily treat such a situation, as shown in Fig. 1. By choosing $x_1 = x_2 = 1/2$ and $x_3 = 0$, the corresponding decay rates in master equation (15) are $\gamma_1(t) = 1$, $\gamma_2(t) = 1$, and $\gamma_3 = -\tanh(t) < 0$ for $t > 0$. We report in Fig. 1(a) the evolution of the two-level system population averaged over $N = 10^3$ realizations. The excellent

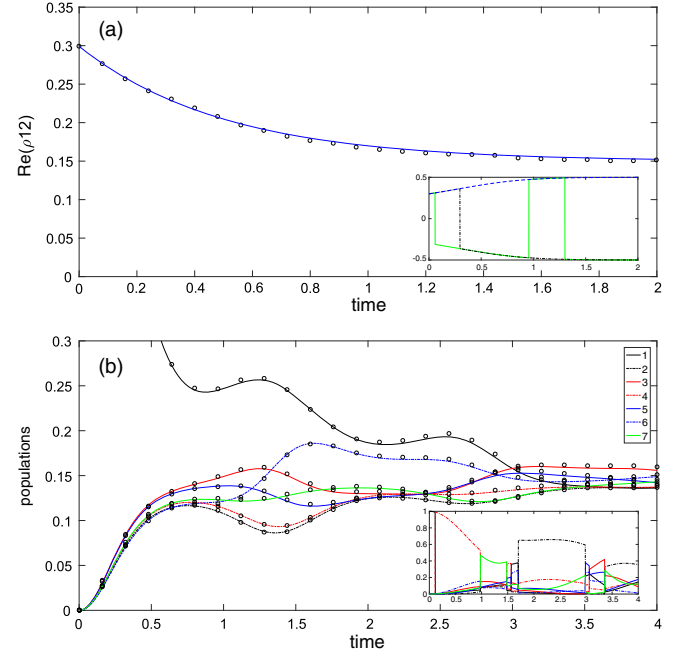


FIG. 1. (a) Evolution of the real part of the two-level system coherence, $\text{Re}(\rho_{12}(t))$, according to the master equation in Eq. (15) (solid line), and as average of 10^4 trajectories (circles) with $dt = 0.002$. The decay rates are $\gamma_1(t) = \gamma_2(t) = 1$ and $\gamma_3 = -\tanh(t) < 0$ for $t > 0$. Inset: Three examples of realizations: Evolution of $\text{Re}(\alpha(t)\beta^*(t))$ with $|\psi(t)\rangle = \alpha(t)|1\rangle + \beta(t)|0\rangle$. (b) Dissipative seven-coupled-site system (for more details, see the text). The simulation results for the site populations (circles) show excellent match with the analytical results (solid lines). The system is initially in the pure state $|\psi\rangle = |1\rangle$. We have used ensemble size 3×10^4 and time-step size $dt = 0.005$. Inset: An example realization: Evolution of the site populations. In both cases, the error bars of the simulation results are smaller than the circles.

agreement with the exact solution can be seen on the whole timescale. Moreover, some illustrative trajectories are reported in the inset of Fig. 1. Indeed, the jumps can be read as the action of the operations defined in Eq. (8), associated with the “click” of a detector which is continuously monitoring the two-level system.

As a second example, we consider a seven-site system including Hamiltonian interaction between the sites and also dissipative jumps between them. The open system Hamiltonian is $H_S = \sum_{i \neq j} \Omega_{i,j} |i\rangle\langle j|$, where $\Omega_{i,j} = \Omega_{j,i}$, and the values for each are chosen uniformly random so that $0 \leq \Omega_{i,j} \leq 0.6$. In other words, all of the sites are coupled unitarily to all other sites with random coupling strength. In the dissipator, jumps can happen between any pair of sites i and j ; i.e., we have 49 jump operators given by $|i\rangle\langle j|$ for any combination of i and j . Here, for simplicity, we use for all operators equal rate, which we choose as $c(t) = 0.5[(1 - e^{-0.5t})0.3 + e^{-0.3t} \sin(4.5t)]$. This also guarantees CP of the dynamical map since the time integral of the rate remains positive. The rate oscillates between positive and negative values, and P divisibility is broken whenever the rate is negative. By exploiting the properties of the ROQJ method, we have in the simulation only $\sqrt{49} = 7$ decay channels. Figure 1(b) shows the excellent match between the analytical and simulation results while the inset displays an example realization.

In between Markovian and non-Markovian.—We now clarify the different degrees of memory effects present in rate operator quantum jumps, also in comparison with other (jump) unraveling approaches. Let us start from CP -divisible dynamics, which have been identified with quantum Markovian dynamics in Ref. [35]. Here, MCWF can be applied and the resulting unraveling is built up via the same non-Hermitian Hamiltonian and Lindblad operators for any sequence of jumps ω_t . On the other hand, the probability to have a jump j at a time t depends on the state before the jump, $|\psi(\omega_t)\rangle$, and then on all the previous sequence of types and instants of jumps which led to that state. We conclude that the jump probabilities do carry some memory [61], though the averaged state can follow, e.g., semigroup dynamics.

If we now move to the P -divisible case and the unraveling provided by the ROQJ method, we see that the memory described above gets amplified, since now not only the probabilities but also the kind of jump at a given time depend on the previous sequence of jumps. In terms of the measurement interpretation, this means that not only the outcome at time t but also the measurement apparatus used to realize a certain instrument will have to depend on the past outcomes.

The strongest form of memory for the unravelings is certainly the one characterizing the reversed jumps, both in NMQJ and in the non- P -divisible version of the ROQJ method. Here, the jump probabilities and operators connect different trajectories, in a way that the event at a time t on a given trajectory will depend on the previous events also on

all the other trajectories. No measurement interpretation is possible in this case.

Conclusions.—In this Letter, we have introduced a quantum jump unraveling, named rate operator quantum jumps, which allowed us to define a consistent measurement interpretation for a wider class of dynamics than those where the standard MCWF interpretation applies. This includes the case where the master equation contains negative rates and the corresponding dynamical map is not CP divisible. Our approach is able to deal with any open quantum-system dynamics—including dynamical regions where neither MCWF nor NMQJ can be used—and provides a unified framework for using quantum jumps to deal with open-system dynamics. Moreover, our results highlight the different kinds of memory effects which arise within the context of quantum unravelings and will hopefully help further clarify the difference between Markovianity and non-Markovianity in the quantum realm.

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