Spontaneous Focusing on Numerosity in preschool as a predictor of mathematical skills and knowledge in the fifth grade

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Abstract

Previous studies in a variety of countries have shown that there are substantial individual differences in children's Spontaneous Focusing On Numerosity (SFON), and these differences are positively related to the development of early numerical skills in preschool and primary school. A total of 74 five-year-old children participated in a seven year follow-up study, in which we explored whether SFON measured with very small numerosities at five years old predicts mathematical skills and knowledge, math motivation, and reading in fifth grade, at the age of 11 years. Results show that preschool SFON is a unique predictor of arithmetic fluency and number line estimation, but not rational number knowledge, mathematical achievement, math motivation, or reading. These results hold even after taking into account age, IQ, working memory, digit naming, and cardinality skills. The results of the present study further the understanding of how preschool SFON tendency plays a role in the development of different formal mathematical skills over an extended period of time.

Keywords: spontaneous focusing on numerosity, preschool, math achievement, early numerical skills; children; development

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Children´s early mathematical skills are important predictors of later mathematical achievement (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Jordan, Kaplan, Ramineni, & Locuniak, 2009). Among early number skills, verbal counting skills and visual digit naming are particularly important for the development of formal mathematical skills (Göbel, Watson, Lervåg, & Hulme, 2014). In addition, Hannula and Lehtinen (2005) demonstrated children´s Spontaneous Focusing On Numerosity (SFON) contributes to later mathematical skills. However, the exact nature of how preschool SFON with very small numerosities, digit naming, and cardinality skills together contribute to the long-term development of different aspects of mathematics has not previously been examined. The present study aims to explore the role of SFON on individual differences in performance on different components of fifth grade mathematical skills and knowledge, including whole number estimation, arithmetic fluency, and rational number knowledge. An investigation of the specific aspects of mathematical skills and knowledge that are related to early SFON tendency will clarify the role that SFON tendency may play in supporting mathematical development. In addition, the predictive specificity of these early number skills will be examined by exploring their relation to reading skills and math motivation.

Spontaneous focusing on numerosity

More than a decade ago, Hannula and Lehtinen (2005) showed that in a task in which children were asked to feed a bird with different colored glass berries only some children paid attention to the actual number of berries that were given to the bird. Many other children seemed to only pay attention to the way the berries were lifted to the mouth of the bird or to the feeding process itself – for example, feeding the bird all the available berries. Their conclusion was that children vary in spontaneously paying attention to the numerical dimension of this situation. Further evidence demonstrated that this variation was not entirely explained by their enumeration skills or other requisite skills needed for the activities, suggesting that these individual differences are a result of specific attentional processes. Based on this, SFON is defined as:

> a process of spontaneously (i.e., in a self-initiated way not prompted by others) focusing attention on the aspect of the exact number of a set of items or incidents and using of this information in one's action. SFON tendency indicates the amount of a child´s spontaneous practice in using exact enumeration in her or his natural surroundings. (Hannula, Lepola & Lehtinen, 2010, p. 395).

More recently, different instruments developed to capture this attentional tendency have constantly shown substantial inter-individual differences in children's SFON tendency (Batchelor, Inglis, & Gilmore, 2015; Gray & Reeve, 2016; Hannula & Lehtinen, 2005; Hannula et al., 2010, Hannula, Räsänen & Lehtinen, 2007; Hannula-Sormunen, Lehtinen & Räsänen, 2015). This tendency is relatively stable across different task contexts within individuals (Bojorque, Torbeyns, Hannula-Sormunen, van Nijlen, & Verschaffel, 2016; Hannula & Lehtinen, 2005; Hannula-Sormunen et al., 2015) and across different tasks using action-responses including feeding a bird, posting letters, stamping dinosaurs, selecting the right number of socks for different caterpillars, or finding a treasure hidden under similar hats arranged in a line forming a semicircle (Gray & Reeve, 2016; Hannula-Sormunen et al., 2015; Hannula & Lehtinen, 2005). In contrast, SFON tasks requiring verbal descriptions, such as time-unlimited picture description tasks, have shown less stability with other SFON measures (Batchelor et al., 2015; Rathé, Torbeyns, Hannula-Sormunen, & Verschaffel, 2016), though written picture description tasks with older children measuring spontaneous focusing on quantitative relations have been found to be relatively stable across time and task (McMullen, Hannula-Sormunen, & Lehtinen, 2017).

Previous studies have shown a positive association between SFON and cardinality recognition, subitizing-based enumeration, object counting, and number sequence skills before school age (Hannula, 2005; Hannula & Lehtinen, 2005; Hannula et al., 2007, 2010). This association has not been explained by children's focusing on spatial aspects (Hannula et al., 2010), inhibition (Gray & Reeve, 2016), nonverbal IQ, or comprehension of instructions (Hannula et al., 2010; 2007). However, it is not yet clear which mechanisms are involved in developing SFON tendency or when it emerges. Sella, Berteletti, Lucangeli, and Zorzi (2016) documented the presence of SFON behavior among toddlers. By analyzing children's response patterns, the authors showed that children spontaneously deployed a nonverbal number recognition process to encode and reproduce numerosity. They suggested that this process relies on the approximate number system. In line with these results, Batchelor and colleagues (2015) demonstrated that the relation between SFON and symbolic number processing skills can be partly accounted for by individual differences in non-symbolic number skills and mapping skills among 4- to 5-year-old children. Also, Hannula and colleagues (2007) showed that SFON was indirectly related to object counting skills through its association with subitizing-based enumeration in a sample of 5-year-old children. Hannula and Lehtinen (2005) documented a mutually reinforcing relation between SFON and counting skills from the age of 3 to 6 years. The authors suggested that the positive relation between SFON and early mathematical skills might be explained by children's stronger tendency to focus on numerosity in their environment. Children with strong SFON tendency might have acquired more self-initiated practice with using of their exact number recognition skills in their everyday life, which benefit their mathematical skill development (Lehtinen, Hannula-Sormunen, McMullen, & Gruber, 2017).

Longitudinal studies covering the development of numerical skills from the age of 3 to 8 years showed that SFON in preschool is uniquely related to formal mathematical skills (Bojorque et al., 2016; Hannula & Lehtinen, 2001, 2005; Hannula et al., 2010). However, most longitudinal studies used academic achievement tests that captured formal mathematical skills globally without distinguishing between different mathematical dimensions. A recent six-year longitudinal study from the age of 6 to 12 years found that SFON predicts standardized mathematics achievement at the age of 12, when children were in the sixth grade, over subitizing-based enumeration and number sequence skills, even after controlling for non-verbal IQ (Hannula-Sormunen et al., 2015). However, the SFON tasks that were used in this study also covered counting-range numerosities (i.e. up to six items). Control measures of guided focusing on numerosity in the study of Hannula and Lehtinen (2005) showed that individual differences in children´s SFON are not explained by a lack of enumeration skills. In the present study, we use only very small numerosities as an even more stringent control for assuring that all participants have the enumeration skills needed in the tasks. This way, we aim to investigate whether variation in SFON performance with only very small numerosities will be completely separate from children’s individual differences in exact number recognition skills.

Counting skills and digit naming

Numerous studies show that counting skills are a significant predictor of later mathematical skills once children enter school (e.g., recent ones; Hannula-Sormunen et al., 2015; Koponen, Salmi, Eklund, & Aro, 2013; LeFevre et al., 2010; Martin, Cirino, Sharp, & Barnes, 2014). Before school age, children typically learn to produce a fairly long list of number words, are able to use the number word sequence for determining the cardinality of a set of items by counting objects one at a time, and can solve simple arithmetical tasks based on their understanding of numbers (e.g., Fuson, 1988). Five principles govern and define the counting of objects, as follows: one to one correspondence, stable-order, cardinality, abstraction, and order-irrelevance principles (Gelman & Gallistel, 1978). These object-counting skills form the basis for connecting numerical magnitudes with number words and are thus important for mathematical development. Previous studies documented significant associations between SFON and numerical skills (for a review, see Hannula-Sormunen, 2015).

In addition to verbal counting skills, the learning of written number symbols has been linked to later success in arithmetical skills (Baker et al., 2002; Bartelet, Vaessen, Blomert, & Ansari, 2014; Göbel et al., 2014; LeFevre et al., 2013; Purpura, Baroody, & Lonigan, 2013). Preschoolers seem to name Arabic digits by activating the lexical and syntactic structure of numbers without necessarily using magnitude information (Verguts & Fias, 2006). Developmentally, children traverse from first knowing the cardinal value of smallest number words to recognizing Arabic digits, learning their cardinal values, and ordinal positions (Knudsen, Fischer, Henning, & Aschersleben, 2015). Studies suggest that children with higher digit knowledge have a better understanding of place value and therefore are more successful in multi-digit number processing (Laski, Schiffman, Shen, & Vasilyeva, 2016; Moeller, Pixner, Zuber, Kaufmann, & Nuerk, 2011). Due to crucial role of symbolic number skills in numeracy it is important to investigate early digit naming skills in relation to SFON measures, which are indicators of self-initiated practice in the recognition and use of exact numerosity (Lehtinen et al., 2017). To this end, we now examine how digit naming and SFON, alone and together, predict later mathematical skills.

Mathematical skills and knowledge in late primary school

Most of the studies that explore how preschool numerical skills predict formal mathematical skills focus on the first wave of formal mathematical skills, such as basic arithmetic computation or solving word problems right after entering school. Currently there is little evidence on how very early number skills are related with later mathematical skills or with more advanced mathematical topics, such as fractions. These kinds of skills become a major component of the curriculum at the end of elementary school years. Examining how early numerical skills are related to later mathematical skills and knowledge will clarify the role these foundation skills have over the long term, besides their immediate impact upon school entry. Previous longitudinal studies showed that SFON was a domain-specific predictor of second graders' arithmetical skills (Hannula et al., 2010), sixth graders' rational number conceptual knowledge (McMullen, Hannula-Sormunen, & Lehtinen, 2015) and mathematical achievement (Hannula-Sormunen et al., 2015). The present study will bring novel evidence about the nature of SFON by exploring whether the long term effect of SFON can be generalized across different mathematical skills, including arithmetic fluency, whole number estimation, and rational number knowledge.

Whole number estimation skills have been found to be a strong underpinning of later mathematical development, including later rational number knowledge (Hansen et al., 2015; Ye et al., 2016). The ability to accurately locate the magnitude of a given number on the number line may be an indication of the exactness of the mental representation of numbers (Siegler & Booth, 2004) or an indication of strategy knowledge such as proportional judgement skills (Simms, Clayton, Cragg, Gilmore, & Johnson, 2016). Both of these mathematical skills would be expected to be supported by the more varied experiences with numerical aspects of the everyday environment proffered by SFON tendency. The quick, automatic procedural solution making that accompanies arithmetic fluency may be supported by the increased practice with numerical skills in and out of the classroom afforded by SFON tendency. Previous research indicates that SFON tendency predicts basic arithmetic competences (Hannula et al., 2010). Previous research also indicated a predictive association between rational number knowledge and SFON when only number sequence skills were controlled for (McMullen et al., 2015). The current study aims to investigate whether this association holds even after controlling for other early mathematical and cognitive skills.

The current study

The aim of the present 7-year longitudinal study is to investigate (a) the construct validity of preschool SFON measured with very small numerosities, (b) the unique contribution of preschool SFON on fifth grade mathematical skills and knowledge and (c) whether preschool SFON is a domain and content specific predictor of development. Thus, our research questions are as follows:

1. What is the construct validity of SFON when measured with very small numerosities?
2. What is the unique contribution of preschool SFON measured with very small numerosities on fifth grade children´s mathematical skills and knowledge after controlling for relevant cognitive and early number skills?
3. Is SFON a domain-specific predictor of mathematical skills, or is it a predictor of reading skills and math motivation as well? More specifically, does preschool SFON contribute to the development of reading skills and math motivation after controlling for relevant cognitive and early number skills?

Construct validity. All original SFON tasks were developed using number ranges that fell under the proficient enumeration range of the children (Hannula, 2005). Therefore, it is argued that the tasks do not merely capture enumeration accuracy, but actually measure SFON tendency. However, in guided SFON tasks, where children were aware of the mathematical content of the task, some children´s number recognition and production performance was not errorless (Hannula & Lehtinen, 2005). Thus, individual differences in the required cognitive skills might have been responsible for explaining some variance on these SFON measures. Moreover, as suggested by Batchelor et al. (2015), it may be possible that these SFON tasks are simply accuracy measures, since 85% of the SFON scores from these trials are based on the production of the "accurate" numerosity. In order to address these open questions, in the present study, we used a SFON measure which only includes very small numerosities (< 3), removing almost all possibilities of individual differences in enumeration skills on the SFON tasks. Thus, these measures are expected to only capture SFON tendency and not any enumeration skills. Additionally, we examined the nature of the responses when children did not produce the same number of items to determine whether different responses may be the result of enumeration errors or lack of focus on numerical features of the tasks. We expect that the response patterns in the number of produced items, which is the most common indicator of SFON, does not correspond to typical distribution of error patterns which would suggest difficulties with enumeration itself but rather differences in SFON tendency.

SFON predicting mathematical skills and knowledge. Previous research investigating the unique contribution of SFON to later arithmetic skills demonstrated a significant association of early SFON with formal mathematical skills (Hannula et al., 2010; Hannula-Sormunen et al., 2015; McMullen et al., 2015). However, previous SFON tasks from longitudinal studies mostly used items that fell both into subitizing and counting ranges. It is not clear if SFON tendency measured by very small numbers (< 3), when variance in exact number recognition skills is not present, would also uniquely predict formal mathematical skills. Hence we will investigate whether five-year-olds' SFON tendency, measured by using very small numerosities, predicts mathematical outcomes seven years later after controlling for early mathematical and cognitive skills. Particularly since a previous study showed that verbal working memory was associated with mathematical skill profiles when children were five years old (Hannula-Sormunen et al., 2017), the effect of it, along with age and IQ needs to be controlled for when predicting fifth grade mathematical skills.

Domain-specificity of SFON. To explore the domain specificity of SFON, we will investigate its predictive value on both math motivation and reading skills. Previous studies have suggested that SFON is a cognitive rather than an affective or motivational component of mathematical development (e.g. Edens & Potter, 2013). Although motivation is related to mathematical skills (Wigfield & Cambria, 2010), children who more frequently spontaneously focus on numerosities may not be better motivated toward math (Edens & Potter, 2013). However, it might be that SFON tendency would, over time, contribute to a positive attitude toward math-related content, or to an interest in math (Garon-Carrier et al., 2016) since for young children, there is more evidence for achievement being a determinant of motivation rather than vice versa (Marsh & Martin, 2011). In line with this, we expect SFON to be related to later math motivation. However, based on previous studies (Hannula et al., 2010; Hannula-Sormunen et al., 2015) no relation is expected between SFON and reading skills.

Material and methods

Participants

A total of 74 Finnish speaking, typically-developing children were followed from preschool to the fifth grade. They were part of the control sample of a regional, multidisciplinary longitudinal research project called PIPARI Study (“Development and functioning of very low birth weight infants from infancy to school age”). The study was approved by the Ethics review committee of the Hospital District of Southwest Finland. Research permissions and informed consents were obtained from the parents after the birth of children and again before the follow-up. Children gave informed assents in fifth grade as they verbally agreed to participate in the study. Schools gave permission for conducting the research study.

Procedures

The children were assessed twice: first at 5 years of age (+0–2 months) as part of a psychologist’s developmental assessment at the University Hospital research unit and for the second time in the fifth grade at the age of 11 (M = 11.76, SD = .29) in a separate room of the child’s own school by a trained research assistant. Children’s SFON, cardinality skills, digit naming, verbal working memory and IQ were assessed individually. IQ was assessed in a separate session. In between SFON tasks, children completed two verbal tasks. In the fifth grade, children were tested individually on arithmetic fluency, number line estimation, rational number knowledge, mathematical achievement, math motivation, and reading skills. Short breaks were provided between tasks as necessary.

Predictors at 5 years of age

Spontaneous focusing on numerosity. Two imitation tasks, the Parrot and the Backpack task (modified from Hannula and Lehtinen, 2005) were used to measure preschool children´s

SFON tendency. The SFON assessment was based on structured observations. Videotaped test sessions before the assessment and subsequent checking sessions throughout the testing period were used to train the experimenter in testing and strategy observation procedures. Before, or during, the presentation of the SFON tasks the experimenter did not utilize any phrase that could have suggested that the tasks were mathematical or quantitative. The tasks included only very small numerosities (< 3), which all children were able to recognize and produce.

All the child's (a) utterances including number words and referring to task materials (e.g., "I'll give him two berries"), (b) use of fingers to express numerosities, (c) counting acts, such as a whispered number word sequence and indicating acts by fingers, (d) other comments referring either to exact quantities or counting (e.g., "Oh, I miscounted them") or (e) interpretation of the task's goal as quantitative (e.g., "I gave an exactly accurate number of them") were identified. For each trial, the child was given a SFON score of 1 if she or he produced the same numerosity as the experimenter and/or the child was observed presenting any of the mentioned (a–e) exact number recognition acts. The scoring was based on analyses of video-recorded task situations. Inter-rater reliability varied from 0.95 to 1.00. The maximum score for both SFON tasks was four. Due to the high correlation of the two tasks (Spearmans correlation of 0.79), the sum score of all eight items was counted, and it was highly reliable with intra class correlation of 0.95.

In the Parrot imitation task (Hannula & Lehtinen, 2005), a blue toy parrot, placed on the table, in front of the child and a plate of red glass berries placed in front of the parrot were used. The experimenter introduced the materials and said, "Watch carefully what I do, and then you do just like I did". The experimenter lifted with a large hand movement two berries, one at a time, into the parrot's peak, and these dropped with a bumping sound into the parrot's stomach, so that nobody could see them. Next, the child was told, “Now you do exactly like I did”. One, two, and one berries were used in the following trials.

In the Backpack imitation task (modified from Hannula & Lehtinen, 2005), an empty blue backpack and a basket of eight plastic, natural-sized oranges and eight pears were used. The tester sat opposite the child and held the backpack open on his or her lap. The basket of fruits was on the table next to the child and the tester. The tester said “Let’s play going outdoors and packing the backpack. Look carefully what I do. Then, you do it just like I did.” and took two pears one at a time and put them into the backpack making sure that the child did not see inside the backpack while saying “Look, I do it. Now, please do it just like I did”. After the child had imitated the tester, no feedback was given, and all fruits were returned into the basket. In the second trial, the tester put one orange into the backpack. In the third trial, a red backpack was used together with eight tomatoes and eight lemons placed in a different basket on the table. The following trials included two tomatoes (third trial) and one lemon (fourth trial). The maximum score for the task was four. The scoring was based on the matching numerosity and any indications of SFON.

Digit naming. Preschool children´s recognition of number symbols was measured with the Digit naming subscale from the Test of Early Mathematics Ability - TEMA 3 (Ginsburg & Baroody, 2003). Each child had to name 15 different visually-presented Arabic numerals, varying from one to four digits. One point was received for each correctly named digit, providing a maximum score of 15. Cronbach’s alpha for these items was 0.87.

Cardinality skills. Children’s cardinality-related skills were tested with a “give-a-number” task (Wynn, 1990). The child was asked to place certain number of items from a box on the table. Materials were 7 different sets of wooden painted flat figures. The numbers requested were 3, 5, 7, 9, 13, 19 and 23 respectively. If the child gave a wrong number of items, the same trial was repeated once. The task was discontinued after two mistakes with the same number. The correct number of items given was given a score of 1 point, and success on the second trial of the same numerosity after a failed first attempt was given a score of 0.5 points. Sum score of all trials was used for further analyses, maximum being 7. Cronbach alpha for the task was .69.

Verbal working memory. The word list interference task from the standardization edition of the Finnish NEPSY-II (Korkman, Kirk, & Kemp, 2008) was used to measure verbal working memory. The child listened to pairs of word series, progressing from short to longer series. For each pair, the child had to repeat the first series of words immediately after it was presented, and then the second series of words after it was presented. Finally, both series of words had to be repeated in the order of the presentation. This test used the total recall score.

IQ. Children´s general cognitive development was assessed with a Finnish translation of the short version of Wechsler Primary and Preschool Scales of Intelligence- Revised (WPPSI-R) test (Wechsler, 1995).

Outcome measures at fifth grade

Arithmetic fluency. Arithmetic fluency was measured using the Woodcock-Johnson standardized mathematics fluency sub-test (Woodcock, McGrew, & Mather, 2001). Participants were asked to complete as many single-digit basic arithmetic problems (addition, subtraction, and multiplication) out of a two-page set of 160 as they could in 3 minutes. Reliability and validity of this instrument are reported in Schrank, Woodcock and McGrew (2001).

Number line estimation. A number line estimation task was used to measure the accuracy of placing numbers on a number line ranging from 0 to 1000 (Siegler & Opfer, 2003). The child was shown where the middle number of the line was and he or she had to estimate 22 numbers ranging from 2 to 938. For each estimate, we calculated the percentage absolute error (PAE) that quantifies the difference between the individual´s estimate and the actual position of the number (Siegler & Booth, 2004).

Rational number knowledge. The rational number test (RNT) consisted of 12 multiple choice and short answer items modified from Martinie (2007) and Stafylidou and Vosniadou (2004). The items measured participants' knowledge of the size of fractions and decimals. Students were asked to compare three pairs of fractions (e.g. "Circle the largest fraction: 5/8; 4/3") and three pairs of decimals (e.g. "Circle the largest decimal number: 0.36; 0.5") and order three sets of fractions (e.g. "Put the numbers in order from smallest to largest: 6/8; 2/2; 1/3") and three sets of decimals (e.g. "Put the numbers in order from smallest to largest: 6.79; 6.786; 6.4"). Each item was scored as correct or incorrect with a maximum score of 12. Cronbach's alpha across all 12 items was .66.

Mathematical achievement. Participants mathematical achievement was assessed with the fifth grade version of LUKILASSE (Häyrinen, Serenius-Sirve & Korkman, 1999), a curriculum based standardized test for students in grades 1-6. It consists of arithmetic problems (addition, multiplication, subtraction, and division), transcoding between verbal and Arabic digit numbers, and converting problems between units of length and volume. For the fifth grade version, most of the problems (80%) involve fractions and decimals, as this is one of the main topics of the Finnish fifth grade curriculum.

Math motivation. Motivation towards mathematics was measured with the shortened version of the Fennema-Sherman test (Mathematics Attitude Scales, Metsämuuronen, 2009). The measurement contains three dimensions with five items in each: Liking math (e.g. I like Mathematics lessons), Self-concept in math (e.g. I think I'm good in Mathematics), and Utility in math (e.g. Mathematical knowledge and skills are important in everyday-life situations). Cronbach alpha was .83 for the first dimension, and .73 and .80, respectively, for the second and third dimensions.

Reading skills. Reading skills were assessed with the word recognition subtest from a Finnish Standardized Reading test for Primary School “ALLU – Ala-asteen Lukutesti” (Lindeman, 2000). Participants had to read chains of two to four words and mark as many as possible word limits with a line as they could in three-minutes and 30 seconds.

Results

Construct validity of SFON imitation tasks

The distribution of responses on the individual trials in the Parrot and Backpack imitation tasks was found to be bimodal (Figure 1). On approximately half of the trials, children responded by giving exactly the same number of berries or fruits as the experimenter gave. As well, on approximately 40% of the trials, children fed the bird with all available berries or packed all the remaining fruits in the backpack. Less than 5% of children made what could be described as an accuracy mistake and inserted, for example, two berries instead of one or three fruits instead of two. The Kolmogorov-Smirnof’s test of normality was significant for each trial in both the Parrot Imitation Task (e.g. KS (74) = .32, $p < .001$) and the Backpack Imitation Task (e.g. KS (74) = .35, $p < .001$), rejecting the hypothesis of a normal distribution. The sum scores of all SFON trials showed that 31% of children did not focus on numbers in any of the trials while 37% of children always focused on numbers. Because of the bimodal shape of the distribution of the sum scores across all eight trials on the two tasks, a categorical variable was created from these scores with the following three categories: “Low SFON” ($n = 23$), when children showed no sign of focusing on numerosity or they focused on numerosity only on one trial; “Some SFON” ($n =$

23) , when children focused on numerosity on 2 to 6 trials; and “High SFON” (*n* = 28) when children focused on numerosity on 7 or 8 trials.

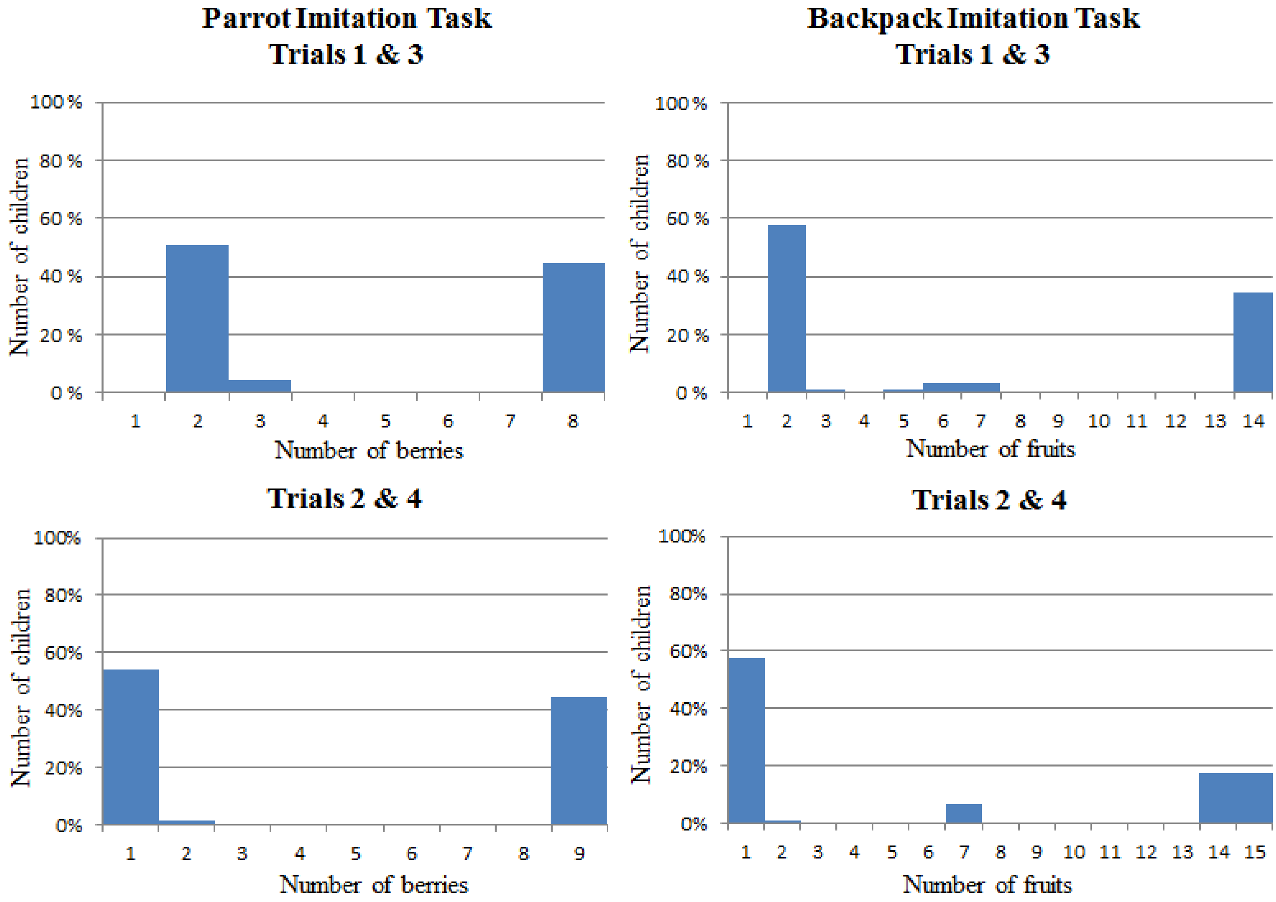

Fig. 1 Distribution of responses on individual trials in the Parrot and Backpack imitation tasks.

Preschool SFON predicting mathematical achievement in fifth grade

Descriptive statistics are presented in Table 1. Results of the tasks show considerable individual differences in children’s SFON as well as in other skills.

Table 1 Descriptive statistics of the variables**.**

Variables	*M*	*SD*	Skewness	Kurtosis	[Min, Max]
Preschool					
Age (months)	60.41	0.41	0.443	2.848	[58.94, 61.77]
IQ	111.54	12.87	-0.270	-0.031	[75, 136]
Verbal working memory	9.82	2.62	-0.546	3.858	[1, 19]
SFON sum scores	4.27	3.53	-0.138	-1.797	[0, 8]
Digit naming[1]	4.77	3.20	0.003	-1.098	[0, 11]
Cardinality skills	4.64	1.90	-0.880	-0.104	[0, 7]
5th grade					
Arithmetic fluency	70.88	15.81	0.083	-0.178	[34, 108]
Number line estimation	4.63	1.33	0.827	1.485	[1.6, 8.8]
Rational number knowledge	8.50	3.01	-0.735	-0.314	[1, 12]
Mathematical achievement	10.38	4.04	0.139	-0.101	[2, 20]
Liking math	3.66	0.88	-0.295	-0.729	[1.4, 5.0]
Self-concept in math	3.83	0.72	-0.359	-0.471	[1.8, 5.0]
Utility in math	4.40	0.57	-1.065	1.007	[2.4, 5.0]
Reading	132.24	34.55	0.122	0.405	[51, 212]

Note. *N* = 74. [1] Three cases have missing data for this measure.

To examine the relations between early numerical skills and fifth grade skills and knowledge, correlations between all variables were calculated (Table 2). SFON correlated with concurrent measures of cardinality skills and with fifth grade arithmetic fluency, number line estimation, and rational number knowledge. No relations were found between SFON and concurrent digit naming or mathematical achievement, motivation measures, or reading skills in fifth grade. Both digit naming and cardinality skills were related with all later formal mathematical skills, as well as with math self-concept and reading skills.

Table 2 Spearman correlations between all variables.

Variables	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Preschool														
1. Age	-													
2. IQ	.22	-												
3. Verbal working memory	.22	.44**	-											
4. SFON	.08	.16	.21	-										
5. Digit naming	.20	.50**	.34**	.17	-									
6. Cardinality skills	.13	.51**	.33**	.24*	.61**	-								
5th grade														
7. Arithmetic fluency	.13	.28*	.35**	.38**	.42**	.39**	-							
8. Number line estimation	-.13	-.38**	-.23	-.35**	-.47**	-.37**	-.39**	-						
9. Rational number knowledge	.16	.39**	.30*	.27*	.53**	.50**	.54**	-.42**	-					
10. Mathematical achievement	.21	.46**	.35**	.17	.42**	.49**	.64**	-.43**	.59**	-				
11. Liking math	.10	.09	.09	.11	.09	.18	.44**	-.32**	.31**	.40**	-			
12. Self-concept in Math	.06	.38**	.18	.08	.27*	.34**	.46**	-.19	.59**	.54**	.54**	-		
13. Utility in math	-.03	.03	.02	-.08	.20	.30**	.08	-.18	.26*	.21	.22	.18	-	
14. Reading	.06	.22	.14	.07	.41**	.38**	.44**	-.19	.47**	.39**	.15	.22	.19	-

Note. $N = 74$.

*$p < .05$.

**$p < .01$.

Next, we studied the unique contribution of preschool SFON on fifth grade children´s mathematical skills and knowledge and motivation by running a series of hierarchical regression analyses. The domain specificity of SFON was additionally investigated by exploring the predictive relation of SFON on later reading skills and math motivation. Predictor variables were entered in three blocks: First, age, IQ, and verbal working memory; second, traditional early mathematical skills; and third, categorical SFON. Incremental R^2 values and standardized beta coefficients are displayed to show both the unique contribution of each predictor and the amount of variance explained by the subset of predictors.

As shown in Table 3, for mathematical skills and knowledge, the total amount of variance explained by the predictor variables ranged from 28% for number line estimation to 35% for rational number knowledge. SFON at preschool age was a unique predictor for arithmetic fluency and number line estimation, but not for rational number knowledge and mathematical achievement. Although SFON was associated with rational number knowledge, this longitudinal association was fully accounted for by age, IQ, verbal working memory, and digit naming. For reading (Table 4), the model explained 18% of the variance but none of the investigated predictors was significant. In predicting dimensions of motivation, the explained variance was much smaller varying from 14% for math self-concept to 3% for liking math. SFON was not a significant predictor of math motivation.

Table 3 Hierarchical regression analyses predicting unique contribution of SFON for the fifth grade mathematical skills and knowledge after controlling for age and preschool cognitive and mathematical skills

	Arithmetic fluency		Number line estimation		Rational number knowledge		Mathematical achievement	
	b	ΔR^2	b	ΔR^2	b	ΔR^2	b	ΔR^2
Step 1		.18**		.13*		.19**		.26**
Age	.11		-.07		.10		.11	
IQ	.13		-.29*		.32**		.43**	
Verbal working memory	.32*		-.08		.13		.09	
Step 2		.06		.08*		.14**		.04
Digit naming	.32*		-.37*		.35*		.15	
Cardinality skills	-.04		.06		.18		.14	
Step 3		.06*		.07*		.02		.00
SFON	.25*		-.28*		.14		.02	
Total R^2		.30		.28		.35		.30

Note. $N = 74$.
*$p < .05$.
**$p < .01$.

Table 4 Hierarchical regression analyses predicting unique contribution of SFON for the fifth grade motivation and reading after controlling for age and preschool cognitive and mathematical skills

	Liking math		Self-concept in math		Utility in math		Reading	
	b	ΔR^2	b	ΔR^2	b	ΔR^2	b	ΔR^2
Step 1		.01		.15*		.03		.06
Age	.00		-.01		-.11		.00	
IQ	.10		.39*		.07		.22	
Verbal working memory	-.04		-.01		.12		.07	
Step 2		.01		.02		.08		.11*
Digit naming	-.04		.14		.13		.28	
Cardinality skills	.11		.02		.26		.20	
Step 3		.01		.00		.03		.01
SFON	.09		.01		-.18		-.07	
Total R^2		.03		.17		.14		.18

Note. $N = 74$.

*$p < .05$.

**$p < .01$.

Discussion

A large body of research now indicates that SFON tendency is related to both early numeracy and basic arithmetical skills (for reviews, see Hannula-Sormunen et al., 2015; Rathé, et al. 2016), as well as being a predictor of mathematical skills even in late primary school (Hannula et al., 2015). In the current study, three aspects of SFON tendency and its role in mathematical development were investigated. First, in order to further confirm that SFON tendency is distinguishable from enumeration skills, the present study only used items with very small numerosities. Children’s responses followed a bimodal distribution pattern revealing that variation in SFON on these items is not a result of variation in enumeration accuracy, but rather an indication of children’s SFON tendency. Second, we show that preschool SFON predicts arithmetic fluency and number line estimation measured seven years later, but not rational number knowledge or mathematical achievement, even after taking into account other early numerical skills, IQ and working memory. Finally, early SFON does not predict later reading skills or math motivation, supporting previous findings of the domain specificity of SFON (Edens & Potter, 2013; Hannula & Lehtinen, 2005; Hannula et al., 2010; Hannula-Sormunen et al., 2015).

Construct validity of SFON

To explore the construct validity of SFON, we examined the number of inserted items during the imitation task for each child’s responses on every trial (e.g. Hannula & Lehtinen, 2005). The distribution of the children´s responses on each trial showed that they were either providing an accurate number of items (i.e. the same number of items given by the adult modelling the task) or a wildly inaccurate number of items (e.g. all available items). Based on these patterns, it is clear that the obtained accuracy curves do not correspond to typical individual differences in enumeration accuracy measures (like those used in e.g., Sella et al., 2016). If the task measured enumeration accuracy, a fairly normal distribution of the children’s responses around the correct answer would be expected. Instead, the bimodal shape of the distribution and the very small number of accuracy errors revealed that variation on these SFON tasks is a result of individual differences in focusing on numerosity.

Furthermore, the high level of precision of the accurate answers indicates that providing such an accurate answer could not be a nonnumeric behavior. Since the items are removed from view, mere matching based on non-numerical aspects would not be as consistently accurate as is seen in these responses. Even tracking “one berry, then another berry” is numerical, as each instance is defined as containing only a single berry. Given there are eight possible responses for each trial (as there are e.g. 8 berries available for feeding), chance responses using such a non-numerical interpretation would lead to more variation around the accurate answer. In contrast, Baroody and Li (2016) used tasks in which their items remained visible, and thus allowed the use of non-numerical strategies such as the overall amount of items. As well, these tasks used numbers of items going clearly above children’s number recognition skill levels, making it impossible to differentiate between focusing on numerosity and enumeration accuracy.

Inter-individual differences in SFON tendency have been identified now in a number of samples of varying ages throughout childhood (e.g. Hannula & Lehtinen, 2005). As well, in general, SFON tendency has been found to have rank-order stability within individuals across task context and multiple measurement points, even though different task contexts and design have elicited variations within individuals in SFON responses (e.g. Rathe et al., 2017). The present study included only a small subset of possible SFON tasks, namely, imitation tasks with very small numerosities, which capture only a portion of the construct of SFON tendency. Thus, while SFON tendency is expected to vary in a more continuous manner on a larger-scale, the distribution of SFON responses in the present study was bimodal, indicating that a categorical classification was appropriate. We therefore acknowledge that in future studies, with more varied SFON tasks, a more scalar variable may be more appropriate.

Early SFON as a domain specific predictor of later mathematical skills and knowledge

Previous findings indicate that SFON measured across subitizing and counting ranges is a domain specific predictor of mathematical skills (Hannula & Lehtinen, 2005; Hannula-

Sormunen, Lehtinen et al., 2015). The present study extends these findings by determining that SFON measured solely with very small numbers predicts formal mathematical skills seven years later, even after controlling for age, early counting skills, IQ, and verbal working memory. These results underline the importance of children´s early mathematical experiences for the development of mathematical skills and support the conclusions of the domain specificity of the relation between SFON tendency and mathematical skills.

Several studies focusing on symbolic number knowledge recognize its importance as a mediator between informal and formal math knowledge (Martin et al., 2014; Purpura et al., 2013). Martin and colleagues (2014) showed that digit knowledge measured in preschool accounted for variance of grade one math outcomes over and above counting predictors. The fact that SFON tendency explains additional variance in mathematical skills, after digit naming is taken into account, underlines the relevance of this specific attentional component on later mathematical skills.

While SFON accounted for significant variance in later number line estimation and arithmetic fluency, no evidence of a specific effect could be identified for rational number knowledge and mathematical achievement which is contradictory with previous findings (Hannula-Sormunen et al., 2015; McMullen et al., 2015). However, the achievement test used in the present study is different from the standardized test used by Hannula-Sormunen and colleagues (2015), and we controlled for IQ, age and verbal working memory, in addition to digit naming and cardinality skills. While their mathematical achievement test covered a wide range of mathematical skills and knowledge, in the present study, the standardized achievement test was heavily focused on curriculum-based math achievement, especially on fraction and decimal number skills and knowledge. Therefore, both the rational number test and mathematical achievement test focused mostly on a relatively complex aspect of mathematical development, while arithmetic fluency and number line estimation captured more basic mathematical skills.

These findings raise interesting questions about the potential developmental mechanisms that might link SFON with very small numbers with later formal mathematical skills. There are two relevant aspects that distinguish between outcome measures. First, the nature of the numbers is different; the number line estimation and math fluency tasks involved operating with natural numbers, while the rational number test and a large part of the mathematical achievement test focused on rational numbers. Preschoolers who focus on numbers might be children who initiate very early deliberate practice in the area of natural number system (Lehtinen et al., 2017), but this practice may not always lead to support for rational number knowledge. Second, the nature of knowledge and skills needed for solving the tasks involving natural or rational numbers is slightly different. The arithmetic fluency task was time limited and focused on single digit arithmetic problems, which are typically already consolidated skills at the fifth grade. This kind of task requires fast fact-retrieval and procedural knowledge. Previous studies showed that fluency is obtained and predicted by efficiency to process numerical symbols and by reading skills (Bartelet et al., 2014; Koponen et al., 2013). The ability to quickly and effortlessly access magnitude information of numbers is a relevant predictor of both number line estimation and math fluency (Bartelet et al., 2013; Laski & Siegler, 2007; Moore & Geary, 2016; Vanbinst, Ghesquière & DeSmedt, 2015). Extensive practice - as indicated by SFON measures - in seeing and using numbers might also explain the predictive value of SFON on number line estimation since improvement in numerical skills was previously documented as a relevant predictor of knowledge of numerical magnitudes (LeFevre et al., 2013).

For rational number conceptual knowledge, the practice provided by SFON with small numbers might not be sufficient, since children have to go through substantial conceptual change in order to understand the nature of rational numbers in comparison with natural numbers (Kainulainen, McMullen, & Lehtinen, 2017; Stafylidou & Vosniadou, 2004). The lack of relation between SFON tendency and later rational number knowledge found in the present study is not in line with previous findings (McMullen et al., 2015). The discrepancy could be explained by the different number range and types of SFON used in the two studies and the different rational number conceptual knowledge tasks used by McMullen and colleagues. In this previous study, McMullen and colleagues used three SFON tasks with numbers failing into both subitizing and counting ranges, while we used only very small numbers, thus their measures may have conflated SFON tendency and enumeration skills to some extent. As well, they used a rational number test that included the more advanced concept of the density of rational numbers, something not included in the present study. Finally, McMullen and colleagues did not control for age, IQ, or verbal working memory in their prediction, which could partly explain why their predictive association remained significant.

SFON was not related to later reading skills and math motivation, which is in line with previous results (Hannula et al., 2010; Edens & Potter, 2013). Liking math in fifth grade was positively associated with digit naming and cardinality skills measured seven years earlier, which is in line with the skills oriented model of the development of liking math (Arens et al., 2016; Garon-Carrier et al., 2016; Marsh, Trautwein, Lüdtke, Köller, & Baumert, 2005). Different aspects of situational motivation (Lehtinen, Vauras, Salonen, Olkinuora, & Kinnunen, 1995) in relation to SFON should be considered for further investigation.

Limitations

There are some limitations to be taken into account when making generalizations from these findings. The associations we found between predictors and outcome variables should not be interpreted as causal ones. It is possible that other cognitive skills such as visual-spatial processing (Cirino, Tolar, Fuchs, & Huston-Warren, 2016; LeFevre et al., 2013) might affect relation between predictors and outcome variables. Even though the overall amount of explained variance was moderate, due to the rather small sample size, we limited the number of variables to those that were assumed to be theoretically most relevant.

Educational implications and conclusions

The present study indicates that spontaneous focusing on even very small numbers has a significant contribution to formal mathematical skills substantially later. Moreover, variation on these SFON tasks is not explained by enumeration accuracy.. Considering the long term effect of early SFON, these findings emphasize the importance of taking into account SFON tendency as a separate essential process in the assessment and education of preschool children's mathematical skills. Future studies should investigate whether the enhancement of SFON tendency together with other early numerical skills could prevent later learning difficulties in mathematics. Both explicit guidance and informal everyday activities can be used in early education to support development of SFON and counting skills (Hannula, Mattinen & Lehtinen, 2005). It seems children's early self-initiated practice as indicated by SFON tendency has a long-lasting, and specific effect on basic numeracy.

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