Robust Stabilization for Non-linear Applications of Cyber physical Systems

A. Tahir and M. Pervaiz

Abstract—The stabilization of unstable non-linear compositions is an important and complex phenomenon across the worldwide. The purpose of this research is to model a robust control design for the stabilization of non-linear, unstable, and under actuated applications of Cyber Physical Systems (CPS) using the High Performance Sliding Mode Control (HPSMC) methodology, which has a negligible control errors (chattering) phenomenon. This study focuses on the designing of feedback controllers by means of HPSMC for two applications of CPS i.e., inverted pendulum system and mass spring damper system, both representing non-linear structures. The SMC procedure is based on saturated approximation of the control laws in order to obtain balanced and controlled behavior at, 1) minimum stabilizing time, and 2) unstable equilibrium points while producing negligible chattering occurrences. This technique provides a robust platform for the non-linear, unstable, and under actuated applications of CPS. The systems' narrations, control objectives, and implementation results of robust feedback controllers are drawn attention to evaluate the reduced control error performance.

Index Terms—Single Input Multiple Output Systems, Sliding Mode Control.

I. Introduction

WHEN the physical processes, system's parameters, and networking are integrated together, the system is known as Cyber Physical Systems (CPS) [1]. A CPS is usually composed of a central computational controller for controlling actuators; actuators for controlling operational physical devices; operational physical devices for affecting physical quantities and sensors for monitoring the physical quantities [2]. CPS has a huge number of applications (e.g. avionics, robotics, defense systems, communications systems, and medical devices etc.) for testing and manipulating the control algorithms.

In the field of engineering, a considerable amount of literature has been published on stabilization of inverted pendulum however; one major issue in early research conduct is robustness. Some of the work is described here for reference purposes.

In order to swing up the pendulum position, the author presented both a non-linear heuristic and an energy controller in [3]. To maintain the balance state at an upright location of the pendulum, its work introduces a linear quadratic regulator state feedback optimal controller. The outcome of heuristic controller is a repetitive signal at the appropriate moment. The optimal controller i.e. optimal state feedback controller is based on a model that is linearizable around the upright closed to the balanced state. Furthermore, [3] lacks the behavior of chattering reduction in the control signal.

Nasir et. al. [4] compared the time specification performance between the conventional controller PID, and the modern controller SMC for the balancing of an inverted pendulum. In addition, the pendulum's and cart's settling time is slightly high.

The responses of the control performances using three different controllers are proposed in [5]. The authors projected the scheming of PID, robust fuzzy logic, and the SMC controllers for controlling the speed of a nominal third order linear time-invariant model of a motor. The step response performance, applied to the nominal, and two perturbed motor plants of each controller are also discussed in this article. It is concluded from the output figures given in [5] that the SMC technique is more robust than either the fuzzy or the PID methodology.

Yadav et. al. [6] illustrated the controlling techniques for the balancing of an Inverted Pendulum as, the conventional PID controller and some different type of fuzzy logic controllers. The pendulum angle is controlled in the upright position by the fuzzy controller. Comparisons of fuzzy PD and conventional PID controllers are also described. It was noticed that a better performance is achieved using fuzzy PD+PID controller. Moreover, a slight perturbation can cause the poor system performance as, the fuzzy systems need prior knowledge and the controller would not work properly if a case was missed.

The authors in [7] suggested a plan that reduces the chattering phenomenon of SMC by establishing a low pass filter in the control signal. The proposed solution can maintain the control accuracy with the use of a huge disturbance estimator gain. This paper also proves the robustness of the SMC practice.

For the stabilization of the rotary inverted pendulum, Anvar et. al. [8] recommended a sliding mode feedback control scheme in which the genetic algorithm based state feedback control and SMC are combined. The pendulum can be stabilized using the proposed methodology. However, the chattering phenomenon is still visible in the control signals.

High Performance Sliding Mode Control (HPSMC) is a particular sort of changeable structure control that is highly robust in nature [9, 10]. This technique is one of the leading schemes for the applications of CPS, and is used to manage highly non-linear, unstable, and under actuated systems [11, 12].

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The HPSMC scheme is tested on two CPS applications; 1) the inverted pendulum system, and 2) the mass spring damper system. This HPSMC technique can be applied to the linear, non-linear, continuous, and discrete systems, which are explored in the next work of [11, 12]. The main goals of this work are; 1) system description, 2) analysis of open loop system behavior (control objectives), 3) designing and verification of robust feedback controller using the proposed control methodology.

The two electromechanical models i.e., inverted pendulum system, and mass spring damper systems are selected for testing due to the following main reasons.

1. They are highly non-linear, unstable, and under actuated systems in nature.
2. Easily available for laboratory usage (in most (academic institutions).
3. Even though it is a non-linear system, it can be a linear system for quite a broad range of variation without much error.
4. Provides good practice for forthcoming control engineers.

This paper is divided into the following parts. Control designing is presented in section II. The methodology is applied on two cases for testing, i.e., inverted pendulum system, and mass spring damper system. The comprehensive inverted pendulum section is described in III. In section IV, control technique for mass spring damper system is proposed. In last, conclusions and future work are discussed in V.

II. The Control Design

The HPSMC design approach consists of the following three modes:

1. Designing of a switching or sliding function so that the sliding motion specifies the design specifications.
2. Selection of an appropriate control law that will make the switching function attractive to the system state.
3. Inserting a variable boundary layer width, as the function of the angle between the sliding surface and state trajectory.

The investigated control law is given as [12],

$$[u] = [\omega] - [\Omega\, sat(s/\phi)] \tag{1}$$

Where, ω and Ω are control design parameters. ω is known as the equivalent control that dictates the motion of the state trajectory along the sliding surface. Ω is constant representing the maximum controller output. φ represents the boundary layer width. *sat(s/φ)* is a saturation function described as,

$$sat(s) = \begin{cases} s & if\ |s| \le 1 \\ \operatorname{sgn}(s) & if\ |s| > 1 \end{cases} \tag{2}$$

The design restrictions, reach-ability and sliding conditions are given in [12].

III. Case Study 1: Inverted Pendulum System

The Inverted Pendulum is a Single Input Multiple Output (SIMO) system. It is the best available electromechanical system for testing a broad category of control schemes. This type of SIMO has two degrees of freedom: (a) pendulum rotation, and (b) cart movement. Here the setup of one stage cart pole is considered [11].

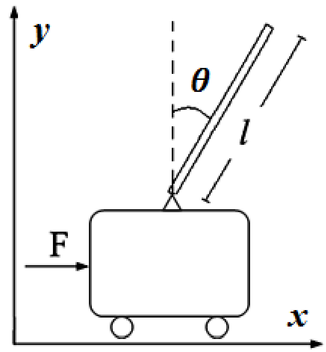

Figure 1: Representation of the Inverted Pendulum

The applied force on the cart is responsible for it's back and forth movement, and the pendulum catches its upright position due to the cart's shifting.

Keeping the view of pendulum's dynamics, the resulting non-linear sculpt, detailed mathematical modeling, and conventions are given in [11] for the inverted pendulum system.

$$\begin{aligned} &(\partial + m)\ddot{x} - ml\dot{\theta}^2 \sin\theta + ml\ddot{\theta}\cos\theta = F \\ &m\ddot{x}\cos\theta + ml\ddot{\theta} - ma\sin\theta = 0 \end{aligned} \tag{3}$$

The allotted values for the inverted pendulum system parameters are as follows,

TABLE I
System Parameters [11]

Symbol	Quantity	Assigned Values [a]
∂	cart mass	2.63 Kg
m	pendulum mass	0.162 Kg
l	pendulum rod length	0.255 m
a	gravitational force	9.8 m/s^2

[a] Units: Kg = Kilogram, m = meter, s = second.

A. Control Design Objectives

The Inverted Pendulum structure is a 4th order system and has four open loop poles. In order to check the whole behavior of the system, both linear as well as non-linear open loop analysis in this section were taken into account. For the linear system's analysis, we first linearized the system at equilibrium point $(\theta, \dot{\theta}) = (\pi, 0)$.

The Routh's Hurwitz criteria was then applied to check the system's characteristics. In the meanwhile, the phase portrait analysis was used to describe the nature of non-linear system. Using Routh's Hurwitz criteria, the tabulation has the form,

$$\begin{array}{llll} s^4 & 1 & -8.807 & 0 \\ s^3 & (4.441e-16) & 0 & 0 \\ s^2 & -8.807 & 0 & 0 \\ s^1 & -17.614 & 0 & 0 \\ s^0 & -17.614 & 0 & 0 \end{array} \quad (4)$$

From above, it is clear that there is one sign change in the first column of Routh's tabulation; one root of this system is present in the right-half of the s-plane that causes the system to be unstable.

For a clear view point of the qualitative features of the pendulum's angle trajectories, we used the technique of phase plane analysis. The phase portrait is shown in Figure 2.

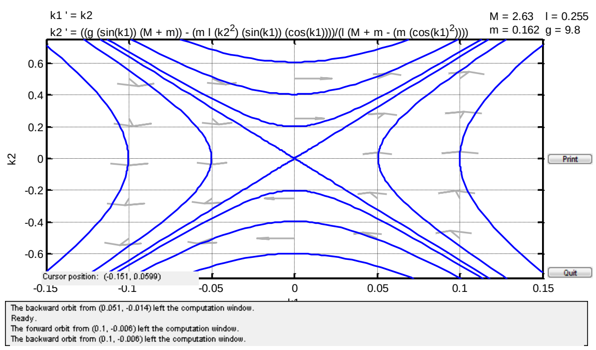

Figure 2: Phase Portrait of the Pendulum's Angle

The plot in Figure 2 illustrates that there exists a saddle point where one of the poles is < 0 and the second pole presents on > 0. Because of the second unstable pole, almost all of the system trajectories diverge to infinity.

We checked the controllability of the system using controllability test to ensure whether the inverted pendulum system is controllable or not. The controllability matrix of the inverted pendulum is given in (5),

$$S = \begin{bmatrix} 0 & 1.4911 & 0 & 13.1327 \\ 1.4911 & 0 & 13.1327 & 0 \\ 0 & 0.3802 & 0 & 6.0760 \\ 0.3802 & 0 & 6.0760 & 0 \end{bmatrix} \quad (5)$$

The number of uncontrollable states are = 0. S is a non singular matrix, as its determinant is 16.5360, and its rank is equal to the system order. Therefore, the system is controllable.

B. Control Design and Stability Analysis

By taking the reference of the sliding surfaces and the control design parameters from [12] and [11] respectively,

$$s_\theta = c_1\beta_1 + c_2\beta_2 \quad , \quad s_x = c_1\beta_3 + c_2\beta_4$$

$$\omega_\theta = \frac{1}{\cos\beta_1}\left[c_1\beta_2 lq + (\partial + m)n - po\right]$$

$$\omega_x = -c_1\beta_4 q - rp + oa$$

We kept the state space representation of equation (3) in equation (1), we derived the following chattering free HPSMC control laws for balancing the pendulum in an upright position and cart position at a specified location;

$$[u_\theta] = [\omega_\theta] - [\Omega_\theta \, sat(s_\theta / \phi)] \qquad (6)$$

$$[u_x] = [\omega_x] - [\Omega_x \, sat(s_x / \phi)] \qquad (7)$$

Equations (6) and (7) represent the control laws for the pendulum angle and cart position respectively. Figure 3 shows a block diagram of the control illustration for stabilization of the inverted pendulum system.

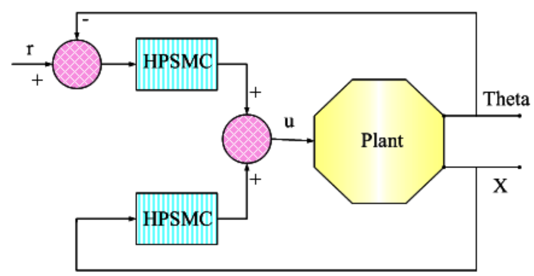

Figure 3: HPSMC Controller for the Inverted Pendulum System

In order to analyze the stability performance, the technique of Lyapunov's Stability Analysis was used. Keeping the sliding surfaces in positive definite scalar functions, the resultant equations lead to the given form,

$$\dot{V}_\theta = \frac{s_\theta}{lq}\left(c_1\beta_2 lq + (\partial + m)n - po - u_\theta \cos\beta_1\right) \qquad (8)$$

$$\dot{V}_x = \frac{s_x}{q}\left(c_1\beta_4 q + rp - ao + u_x\right) \qquad (9)$$

After placing the control laws from equation (6), and (7) into (8), and (9) respectively, the following equation was obtained in both cases;

$$\dot{V} = -\eta |s| < 0 \qquad (10)$$

Where, η = 1. The system trajectories are guaranteed due to this positive constant η. These trajectories strike the sliding surface in finite time. Further, the sliding condition is verified by s. Thus, the behavior of the Inverted Pendulum system becomes stable.

C. Results

The results are accumulated on the initial condition of the $\pi + 0.01$ radians to the pendulum. The model is implemented in Matlab® – Simulink® environment with the singular point $(\theta, \dot{\theta}) = (\pi, 0)$.

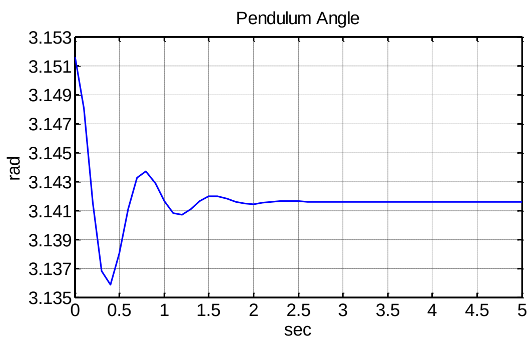

Figure 4: Regulation Result of Pendulum Angle

Figure 4 depicts the settled behavior of the Inverted Pendulum at an upright position for approx. after 2 seconds.

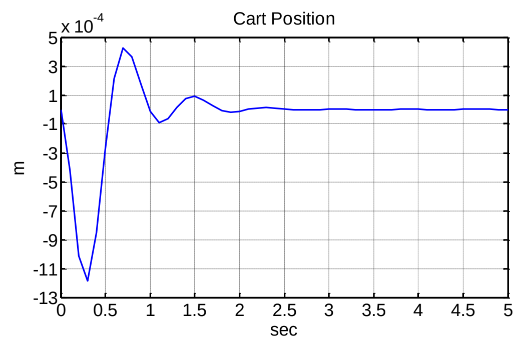

Figure 5: Regulation Result of the Cart Position

The cart position stabilizes at a preferred location in Figure 5 in about 2 seconds.

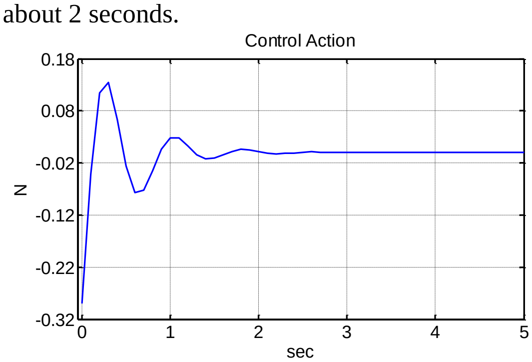

Figure 6: Inverted Pendulum Control Signal

Figure 6 illustrates that for the balancing of the inverted pendulum system, approximately 0.1347 Newton force needs to be applied to the plant.

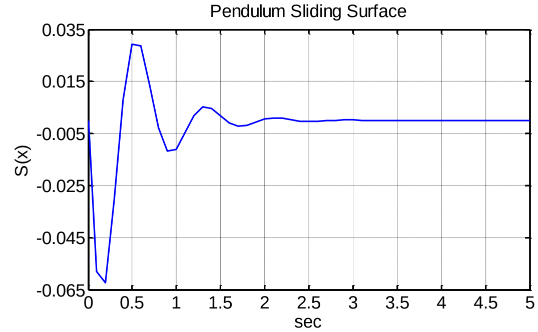

Figure 7: Pendulum Model Sliding Surface

Figure 7 describes the performance of switching surfaces for the pendulum control.

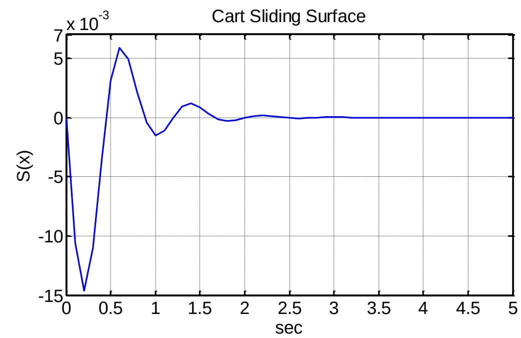

Figure 8: Cart Model Sliding Surface

The performance of switching surfaces for the cart control is shown in Figure 8.

IV. Case Study 2: Mass Spring Damper System

A 2nd order non-linear mass spring damping system is considered and illustrated in Figure 9 [13, 14],

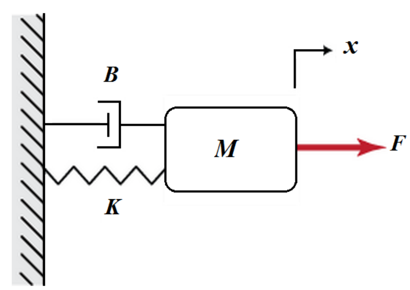

Figure 9: Mass Spring Damper System

The forces of a spring and viscous damper are directly proportional to the displacement and velocity of the mass respectively. Both forces are in the negative *x* direction due to the opposing effect of the motion of the mass. When the spring is unscratched then the position of the mass is zero. F represents the input force to the system. This system is also known as the Vander Pol Oscillator, and the governing equation of the mass spring damping system is given as [13].

$$M\ddot{x} = -B\dot{x} - Kx + F \qquad (11)$$

The allotted values for the mass spring damping system parameters are given in Table II.

TABLE II
SYSTEM PARAMETERS [9]

Symbol	Quantity	Assigned Values [a]
M	mass	1 Kg
B	damper	$x^2 - 1$
K	spring constant	1

[a] Units: Kg = Kilogram.

A. *Control Design Objectives*

In this section, linear and non-linear system's analysis techniques are presented. The mass spring damping system has two open loop unstable poles as,

$$0.5 \pm 0.866\, i \qquad (12)$$

According to Hurwitz criteria, the tabulation has the form,

$$\begin{array}{lcc} s^2 & 1 & 1 \\ s^1 & -1 & 0 \\ s^0 & 1 & 0 \end{array} \qquad (13)$$

There are two sign changes in the first column of the Routh's table, so there are two roots present in the right half of the s plane. Hence, the system is unstable. The phase portrait of the mass spring damping system has the following graphical representation,

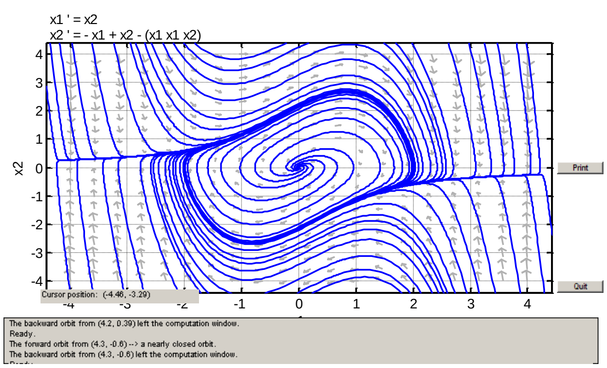

Figure 10: Phase Portrait of the Mass Spring Damper System

The phase portrait shows that the trajectories diverge to infinity except the origin point at (0, 0).

The controllability test is applied to this system and, the controllability matrix is given as,

$$S = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \qquad (14)$$

The number of uncontrollable states are = 0. *S* matrix is a non singular matrix, as its determinant is -1, and its order is equals to the system order. Therefore, the system is controllable.

B. *Control Design and Stability Analysis*

The equation (1) of chattering free HPSMC control law is applied to the mass spring damping system, in order to balance the system behavior. The following equation was obtained,

$$\omega = -c_1 x_2 + x_1 - x_2 + x_1^2 x_2 \qquad (15)$$

The block diagram of the controlling strategy is given as,

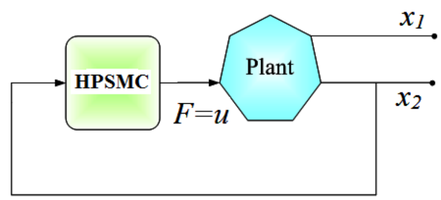

Figure 11: HPSMC Controller for the Mass Spring Damper System

For stability analysis, the resultant Lyapunov's Stability Analysis equation leads to the given form,

$$\dot{V} = s(c_1 x_2 - x_1 + x_2 - x_1^2 x_2 + u) \qquad (16)$$

After putting together the control laws from equation (1), and in (16), the same equation as in (10) was obtained. Hence, the system behavior is stable.

C. *Results*

The model is implemented in the Matlab® – Simulink® environment with an initial condition of the 1 meter.

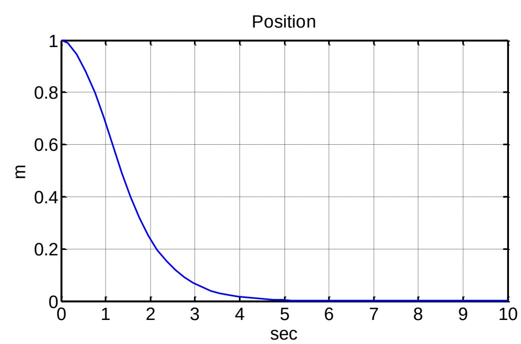

Figure 12: Position Regulation Result

Figure 12 depicts that the position is regularized in about 3.5 seconds.

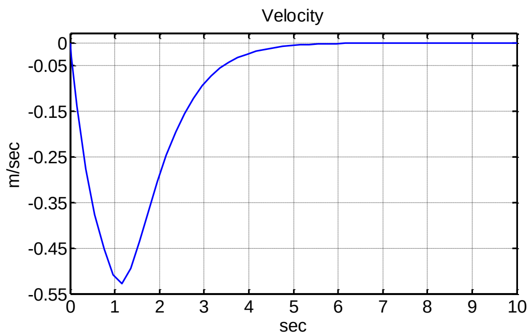

Figure 13: Velocity Regulation Result

The velocity of the system tends to zero in 4.5 seconds in Figure 13.

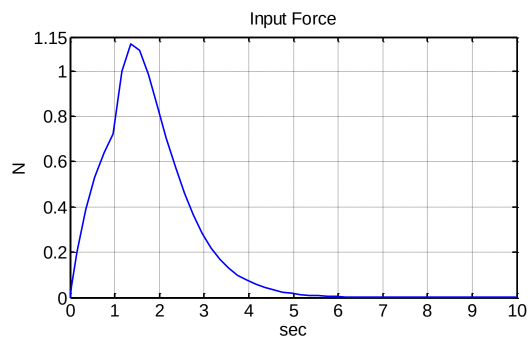

Figure 14: Control Signal

A 1.1199 N force is applied at the maximum to the mass spring damping system for regulated behavior as shown in Figure 14.

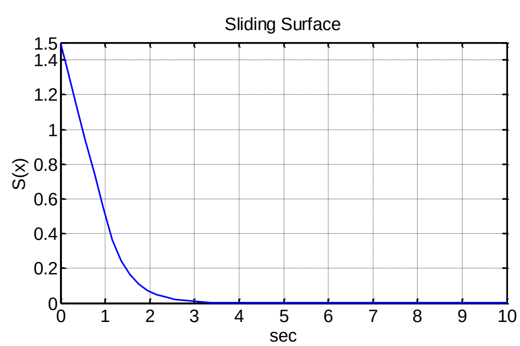

Figure 15: Sliding Surface

Figure 15 shows that in a controlled phenomenon, the sliding surface becomes zero in approx. 2.5 seconds.

V. CONCLUSION

The detailed description, analysis, and results shown in this paper define that HPSMC is a good robust technique for stabilization of non-linear CPS models. This technique is successfully implemented and tested on two mechanical models, i.e., (1) the inverted pendulum system, and (2) the mass spring damper system. Chattering free and stable performances, as well as minimum settling time behaviors are achieved using HPSMC control laws. Stability analysis of the robust feedback controller is also discussed. Lastly, the implementation results using Matlab® – Simulink® are highlighted.

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