A Comparison of the Perceived Difficulties in Learning High School Mathematics from the Perspective of Teachers and Students

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The study is based on the theory of teachers' knowledge of students, specifically those who have difficulties learning mathematics, and how their knowledge is expressed concerning students' self-identification. The participants in this study were Namibian high school mathematics teachers (N=6) and their 12th-grade students (N=23), all of whom participated in semi-structured interviews. The students were interviewed about their views of difficulties experienced in learning mathematics, whereas the teachers were interviewed on how they described and identified these students. Participants described their views of school achievement, activeness in class, students' pace, level of asking questions, and math anxiety. The students evaluated themselves, and the teachers evaluated their students. When comparing the responses of teachers and students, the researcher found substantial agreement in how the two groups regarded low achievement in school assessments. In contrast, a low agreement was found regarding student anxiety in learning mathematics. The knowledge and relationships revealed in this study could play a critical role in providing opportunities for teachers to reflect on their knowledge of students, especially when addressing individual students' learning needs. However, this study was limited by a lack of evidence from classroom practice, which could constitute an essential factor in strengthening the findings.

According to Tamir (1991), teachers need professional knowledge and skills to function successfully. Teachers need to have knowledge about how students learn, including different teaching and learning strategies, and this knowledge should be reflected in their classroom practice (Ball et al., 2008; Shulman, 1986, 1987). Additionally, a teacher who knows students' misconceptions and what contributes to learning difficulties can help those students succeed (Ball et al., 2008). This knowledge enables teachers to accomplish their tasks (Ben-Peretz, 2011) and promotes practical teaching that supports students' learning (Ball et al., 2008).

This paper focuses on descriptions of Mathematical Learning Difficulties (MLD), how students express their learning difficulties, and how teachers perceive students' difficulties when learning mathematics based on teachers and their students' knowledge and experiences. Such knowledge relates to: (i) teachers' competence and knowledge about their students, and (ii) students' competence and understanding of mathematical concepts (Hamukwaya & Haser, 2021). The importance of studying this relationship was influenced by contradicting views that emerged from previous interviews conducted with Namibian 11th graders in 2018 (Hamukwaya & Ruttenberg-Rozen, in review). The results of the 2018 interviews revealed that while teachers identified 115 students as experiencing difficulties in learning math, 88 of those students self-identified as experiencing no difficulties in their learning. For a better understanding of the reasons for the differing descriptions and views about difficulties in learning mathematics that existed between the teachers and their students, the author of this study decided to investigate teachers' knowledge of students and their characteristics and build on previous literature in the field (e.g., Ball et al., 2008; Shulman, 1987, 1986).

The rationale for this study was that teachers and their students might have different descriptions of MLD for various reasons. Moreover, the descriptions of difficulties in learning mathematics and classroom work allow teachers to assess students' progress in

math. In the literature, scholars such as Jitendra et al. (2013) define MLD among students as they struggle to grasp the basic mathematical knowledge and skills required to comprehend the subject effectively. However, students with MLD can improve their understanding of mathematical concepts if given additional learning support (Hamukwaya & Haser, 2021). Therefore, the present study narrowed its attention to the knowledge of students with MLD. The notion of students' knowledge was utilized to study the teachers' perceptions (Hill et al., 2008) and the students' perceptions about experiencing MLD. This knowledge, which is the focus of this paper, might be crucial for teaching practices, especially for improving learning among students with MLD. Studies of this nature are needed in mathematics education to improve teachers' practice (Ben-Peretz, 2011).

Researchers (e.g., Ball & Bass, 2002) have drawn mainly on teachers' data to conceptualize the notion of knowledge regarding students. However, this paper utilizes both teachers' and students' points of view to describe MLD among students. Although some researchers have focused on teachers' knowledge of students and others on MLD, no research has found links between these two aspects. Furthermore, little is known about Namibian math teachers' knowledge of students and their characteristics. Thus, the author believes that analyzing this relationship may yield important findings that could assist in discerning any misalignment between teachers' and students' views, which could then improve teachers' knowledge of students with difficulties in learning Mathematics. The following research questions guided the study:

1. What are the descriptions employed by teachers when they sense that their students experience difficulties in learning mathematics?
2. What are the descriptions of high school students when they experience difficulties in learning mathematics?
3. What is the nature of the relationship between teachers' and students' descriptions employed to classify difficulties in learning mathematics?

Literature Review

Although this paper addresses only one aspect of teachers' pedagogical knowledge, namely knowledge about students and their characteristics, it is essential to understand the knowledge required for teaching from a broad perspective before presenting the literature on MLD. Therefore, literature about the knowledge required for teaching and the concept of difficulties in learning mathematics are discussed.

Knowledge Required for Teaching

The literature emphasizes seven types of knowledge necessary for teaching: general pedagogical knowledge; content knowledge; pedagogical content knowledge; knowledge of learners and their characteristics; knowledge of educational contexts; knowledge of educational ends, purposes, and values; and curriculum knowledge (Ball et al., 2008; Shulman, 1986). Pedagogical content knowledge contributes to teachers' awareness of how students learn and understand various teaching and learning strategies (Shulman, 1987). This knowledge is general and practical (Ball et al., 2001; Ben-Peretz, 2011; Tamir, 1991).

Knowledge for teaching includes aspects that make a subject difficult or easy to learn (Shulman, 1987). It also incorporates appropriate teaching strategies to address learners' difficulties and foster meaningful student understanding (Mishra & Koehler, 2006). This knowledge enables teachers to accomplish their tasks and ensure that their students master the subject, promoting student learning (Ben-Peretz, 2011). It is also vital for ensuring their

students' mastery of the subject matter, as teachers' knowledge influences their actions (Ben-Peretz, 2011) and enables them to execute their role. Furthermore, the knowledge teachers possess leads to different understandings of classroom practice, for example, providing adaptive teaching instruction (Oudman et al., 2018). When teachers understand how students learn, they can teach successfully (Lederman et al., 1994). This indicates that teachers' knowledge is an essential element of teaching and classroom practices.

The present study contributes to teachers' knowledge of students' and learners' characteristics. In the literature, knowledge about students is regarded as a key component of pedagogical content knowledge (Kleickmann et al., 2017). It is a significant domain of teachers' practical knowledge and includes knowledge about the specific students that teachers teach (Mayer & Marland, 1997). This knowledge enables teachers to focus on students' needs and recognize opportunities to foster understanding (Asquith et al., 2007). Understanding their students (Jang et al., 2009) plays a critical role in planning, teaching, and carrying out instruction (Ball et al., 2008). This indicates that teaching goes beyond what is being taught, as it also requires understanding the students.

Teachers employ knowledge of their students, such as prior academic success and personal characteristics, to provide adequate student activities that cater to and suit individual student needs in the classroom (e.g., Even & Tirosh, 2008). Such knowledge also allows teachers to adjust their expectations of students by considering different students' learning abilities and their academic problems, such as becoming frustrated or being likely to experience difficulties with specific academic tasks (Mayer & Marland, 1997). In turn, this knowledge contributes to identifying the best ways to interact with students and, thus, is regarded as critical. Mayer and Marland (1997, p. 18) state that "if you don't know your students, you can't teach them."

When teachers know their students, they can assess what students understand or do not understand, which is necessary to provide sufficient teaching instruction (Oudman et al., 2018). Hence, teachers have a special responsibility as they are the source of students' understanding (Shulman, 1987). It seems that teaching cannot be successful if teachers lack understanding of their students. Thus, teachers need to anticipate what students are likely to think, what they will find confusing, what difficulties they will experience and what they will find interesting and motivating (Mayer & Marland, 1997). Such anticipation may lead to effective teaching and learning. This indicates that teachers' knowledge of students is essential for teaching students with difficulties in learning mathematics, the present study's focus.

The Concept of Difficulties in Learning Mathematics

The phrase difficulties in learning mathematics have different meanings in the literature. Morgan et al. (2009) relate difficulties in learning mathematics to gaps in mathematics proficiency and low numerical processing skills. In comparison, Karagiannakis et al. (2014) describe difficulties in learning mathematics as various obstacles that lead to difficulties in processing numbers. In general, difficulties in learning mathematics relate to deficits in developing mathematical skills, dyscalculia, or difficulty in learning or comprehending arithmetic relationships (Baccaglini-Frank & Di Martino, 2020; Kaufmann & Von Aster, 2012). Hamukwaya and Haser (2021) consider the concept to refer to any limitations that hinder students' mathematical learning. From a psychological point of view, difficulties in learning mathematics hinder the cognitive learning processes necessary for understanding mathematics (Mazzocco, 2007). These deficits are believed to affect students' abilities in their academic performance in math.

Students with difficulties in learning mathematics struggle in the subject, complete their work at a slow pace, have difficulty representing mathematical concepts (Wang, Du, & Liu, 2009), and perform lower than their peers (Jitendra et al., 2013). Scholars have identified some reasons for students' difficulties in learning mathematics: a poor foundation, unwillingness to learn, teachers' incompetence (Montague, 1992: Voigts, 1998), and the language of instruction (Siyepu, 2013). Furthermore, some scholars argue that fear resulting from past experience, math avoidance, and the myth of math being a difficult subject might hinder one's success (e.g., Bekdemir, 2010). To address the needs of students who have difficulties in learning math, Torbeyns et al. (2004) suggest the need for effective teaching techniques at a slower pace to attain students' learning goals. According to Kember (1997), teachers should also design appropriate and inclusive demanding tasks that influence the quality of the learning outcomes. These tasks may positively impact students who have a poor background in mathematics and poor mathematical knowledge.

To target students with difficulties in learning mathematics more effectively, teachers should understand the characteristics of their students, as this appears to be one of the features that affect teaching (Shulman, 1987). In schools, how students view themselves as math learners influences their learning of the subject. In this sense, students develop their characters based on their classroom learning experience of math (Anderson, 2007). According to Wenger (1998), learning occurs through social participation, and such participation includes thoughts and actions and demonstrates that students are part of the mathematics community (Anderson, 2007). Further, Anderson (2007) states that participation develops students' mathematical concepts and skills and contributes to their personality as math learners.

Another characteristic of good math students is engagement, which often comes from direct learning experiences (Wenger, 1998). Much of what students know about learning math results from their engagement with math in the classroom, indicating that students are capable mathematics learners (Anderson, 2007). The present study addresses the relationship of Namibian high school Mathematics teachers' knowledge of students and their students' self-assessment concerning difficulties in learning mathematics, intending to develop a theory that could improve teaching practice and students' learning.

Significant learning difficulties described in the literature include low numerical processing skills, difficulty comprehending arithmetic relationships, and difficulty representing mathematical concepts. Researchers believe that these difficulties hinder students' academic progress, including their pace in completing mathematical tasks, understanding mathematical concepts and participation in Mathematics lessons, and their desire for and interest in the subject. Although scholars have researched teachers' knowledge of teaching and Mathematical Learning Difficulties, there is a significant gap in research considering descriptions of MLD from the perspectives of both teachers and their students. Therefore, the researcher explored how students expressed their learning difficulties and how teachers perceived them. The goal was to understand the descriptions (based on knowledge and teaching and learning experiences) of MLD as employed by Namibian high school mathematics teachers and the students they observed experiencing MLD. Therefore, this paper addresses a significant research gap by considering the emergence of students' mathematical identification.

Methods

Research Design and Approach

The qualitative research was conducted by employing in-depth semi-structured interviews to accomplish three purposes: (1) to allow data to emerge from the participants as they explained their feelings, experiences, thoughts, understandings, and beliefs (Selvi, 2008), (2) to explore the data in-depth, and (3) to enhance the credibility of the findings. Therefore, this study explores qualitative descriptions to understand how MLD is perceived (how students express MLD and how teachers perceive it). The teachers and students in this study were of utmost importance to achieve diversity and variation in the research findings.

Context

This qualitative study was conducted in Namibia, a country in Southern Africa. The Namibian basic education system consists of four main phases, namely Junior Primary (ages 6 to 9 in pre-primary to grade 3), Senior Primary (ages 10 to 13 in grades 4 to 7), Junior Secondary (ages 14 to 16 in grades 8 to 10) and Senior or Higher Secondary (ages 17 to 18 in grades 11 to 12). In Namibia, math is compulsory in every educational phase, and high school math (Grades 11 and 12) is divided into ordinary (core and extended) and higher levels. Generally, teachers advise students in 11th grade to pursue one of these levels based on their mathematical performance in tenth grade; however, most students opt to take core-level mathematics.

The teachers who participated in this study taught both core and extended mathematics, except for Teacher 4, who taught core and higher math; they were all teaching 11th and 12th-grade math at the time of the interviews. The interviews focused on 12th-grade math students whom their teachers had selected because they were experiencing difficulties when learning 11th-grade math. These students participated in the 2018 interviews. The students' learning capabilities were based on their teachers' perceptions, not on a psychologist's screening.

Participants

As previously mentioned, two groups of students were represented: those who self-identified as experiencing difficulties in learning mathematics and those who did not self-identify. Some math teachers who selected the 11th graders in 2018 were also interviewed. The names utilized in this paper are pseudonyms.

Students. Twenty-four students (12 females and 12 males) participated in a follow-up interview conducted in July 2019, and they formed the sample in the present study. In the 2018 interviews, 7 of these students self-identified as experiencing difficulties, whereas 17 did not. The students studied ordinary mathematics, an extended level in 11th grade, before changing to the core level in 12th grade. Their ages ranged from 16 to 19, and their educational and socio-economic backgrounds varied.

Mathematics teachers. Six mathematics teachers (two females and four males) who selected the students in 2018 were also interviewed in July 2019. However, one teacher (Teacher 5, female) agreed to participate in the study as she was the only teacher responsible for teaching 12th graders. She did not participate in the selection process because she was not present at the school at the time. The teacher who selected the students moved to a different school. All teachers held a bachelor's degree, which qualified them to teach 11th and 12th-grade mathematics. Their ages ranged from 25 to 40, and they all had more than

two years of teaching experience, except for Teacher 5, who had half a year of teaching experience when the interviews were conducted.

Data Collection

Through the relevant authorities via emails and follow-up telephone calls, permission was granted to conduct a qualitative analysis using semi-structured interviews at 23 randomly selected high schools. In May 2018, the participating mathematics teachers selected five students from each school who were considered to experience difficulties learning mathematics. Of the 23 high schools that participated in the semi-structured interviews conducted in 2018, the researcher selected six as a sample for a follow-up interview. The selection of schools was based on the accessibility of the schools to the researcher. The data collection was performed in July 2019 at the end of the second semester. The researcher informed the participants in advance about the study procedures, their rights to withdraw from the study should they wish, and measures for maintaining confidentiality and anonymity. Thirty students, five from each school, and six teachers voluntarily agreed to be interviewed; however, six students were absent when the interviews were conducted.

The researcher conducted face-to-face, semi-structured interviews after working hours at the participants' schools. A common interview framework guided the discussion. Teachers and their students were interviewed individually about their perceptions of difficulties in learning mathematics. The interviews were audio-recorded and transcribed. The researcher was interested in exploring the relationship between the teachers' descriptions of the assessment criteria for MLD and their students' criteria regarding MLD. The driving question was whether the teachers agreed with their students and, conversely, whether what the teachers indicated about their students agreed with what the students indicated about themselves.

In the interview, the students responded in the language they felt comfortable with, either English or Oshiwambo, students' native language. The researcher is a native speaker of Oshiwambo and was thus able to ask further questions for clarification in both languages and transcribe and analyze the data. During the interviews, open questions that sought clarification were asked. The teachers' interviews lasted longer than those of the students; most of the students' interviews lasted 40 minutes, while the teachers' interviews lasted about one hour. In the interview protocol, the discussion was guided by six questions for the teachers and nine for the students (see Appendix A). The questions specifically addressed how the students expressed learning difficulties and how the teachers perceived the students' learning difficulties. Below are two lists of the most important questions for this study that guided the interviews:

Mathematics teachers' interview guide: (1) How would you characterize a student with difficulties learning mathematics? What are their characteristics, the things they do, and things they cannot do in mathematics lessons? (2) What are the characteristics of the following students [names of the students] in terms of what they do and what they cannot do in mathematics class? How is their performance? What learning support do you give them? (3) What do you do once you have identified that learner A or B is experiencing difficulties or struggling to learn mathematics?

Twelfth-grade students' interview guide: (1) How has your performance been in 12th-grade mathematics lessons? (2) Do you think you experience difficulties when learning mathematics? (3) Based on your observations and experiences, why do you think some high school students experience difficulties when learning mathematics? Can you elaborate? Are there school-related factors? What are they?

Data Analysis

The researcher transcribed the interviews and replayed the recordings several times to ensure accuracy. Then, a theory-driven content analysis that enabled qualitative data reduction through codification and theme development was conducted (Creswell, 2009). Open coding was employed to understand participants' expressions, how they perceived MLD and the possible relationship between teachers and their students' views of difficulties in learning mathematics. Following this, the transcriptions were read thoroughly, and significant words and common phrases based on the three research questions that guided the study were highlighted. The researcher conducted this process to investigate the relationship between teachers and their students' perceptions when identifying MLD among students. Both the teachers and students provided some of their ideas and specific observations. Afterward, the interview transcripts were read several times, and thematic codes were developed.

Common expressions and understandings were noted, and keywords were identified, including "participation," "pace," "level of asking questions," "fear," "panic," "confused," "not vocal," "fail," and "achievement." Based on the research questions, the researcher selected data about each theme and gathered quotations to ensure validity and accuracy of the coding process in the qualitative research, which also helped to preserve the participants' voices. The analysis produced five criteria: anxiety about mathematics, no class participation, low achievement in school assessments, slow working pace, and the asking of low-level questions.

Furthermore, the data were classified and highlighted the effect size of the responses. The features were listed, counted, and added together, and more frequent aspects that had the most agreement were noted. Experts in the field reviewed the analysis process and gave feedback that ensured and supported the validity of the study findings. Moreover, all participants were treated as individuals, and privacy was ensured using pseudonyms.

Findings

The analysis presented in this section includes data from only 23 students. One of the students (Loide) identified herself as not experiencing difficulties in learning mathematics. This was confirmed by Teacher 2, as Loide had been performing well in mathematics since the 11th grade. The teacher might not have followed the selection instruction that only students considered to be experiencing difficulties in learning mathematics should be identified. Consequently, this might explain the different views about learning difficulties between students and their teachers, as were reflected in the earlier 2018 interviews. Thus, the relationship between Loide and her teachers was not considered in this study.

The following subsections present findings of how the students expressed their learning difficulties and how teachers perceived the students' learning difficulties.

Mathematics Teachers' Perceptions of their Students' Learning Difficulties

In the interviews, teachers were asked to provide descriptions of how they identified students they observed to experienced MLD. Table 1 contains some excerpts of the teachers' responses. The first column contains the teachers' statements and descriptions of students, and the second column contains the themes that emerged related to the criteria employed to identify each student. As reflected in Table 1, five themes emerged from the data: low achievement in school assessments, not being active in class, slow work pace, asking low-level questions, and Mathematics anxiety.

Table 1
The way high school Mathematics Teachers Perceive Learning Difficulties

Teachers' identification of students	Criteria for difficulties in learning mathematics
Ombili has learning difficulties and struggles, but he is a hard worker. […] He always asks if there is something that he does not understand, but his questions are below his level of learning. Mostly, he asks, "How do you get that?" He fails tests and examinations. In Grade 11, he was a new student, and he did not interact with peers. [Teacher 1 excerpt]	Asking low-level questions and low achievement in school assessments
Lempy is a struggler, and she is scared of examination. Most of the time, she panics, is confused, and gets lost. Sometimes she panics about simple tasks. She is not a vocal person; thus, she never asks if she does not understand. [Teacher 1 excerpt]	Mathematics anxiety (panic, confusion, being lost when doing simple mathematics tasks) and Not participating in class (non-vocal and never asking questions when not understanding)
Tamba is not serious with his schoolwork. He is struggling and having learning difficulties in mathematics. In Grade 11, his absenteeism from school was high. However, in Grade 12, the performance is still not good. He is not vocal, and he never participates in class. [Teacher 1 excerpt]	Low achievement in assessment
I observed Kletus sleep most of the time in the mathematics lessons, and her performance is very low, and I think this is what makes her have learning difficulties. [Teacher 3 excerpt]	Not participating in class Low achievement in assessment
Ekandjo forgets fast and is confused. [Teacher 4 excerpt]	Low achievement in assessment
Most of the time, Stella doesn't have an idea of the way in which to solve mathematics problems. She is slow and struggles. [Teacher 5 excerpt]	Mathematics anxiety Slow pace
Elias is not really a struggler as he achieves better in mathematics, although there are times he doesn't understand, and that makes him perform low. [Teacher 6 excerpt]	Low achievement in assessment

Low achievement in school assessments (14 responses). The teachers voiced the opinion that students experiencing difficulties in learning mathematics came from poor mathematics learning backgrounds, i.e., students who had not mastered basic mathematics skills in earlier grades were "unable to" solve simple mathematics problems. As a result, they "perform poorly in various activities." For example, Teacher 1 stated that Ombili's classroom activities were usually poorly done. Furthermore, the teachers argued that students' low achievement in mathematics could also result from difficulties when using a calculator. Some of the teachers mentioned students with MLD missing lessons and assessments and being "absent a lot" (especially in the cases of Taamba, Tauno, and Nandy). The teachers indicated that missing lessons caused students to achieve poorly in school assessments. Based on the teachers' views, low student performance indicated learning difficulties. As a result, students could fail 11th grade and, consequently, affect their 12th-grade performance; this may be another criterion that teachers employed in identifying difficulties in learning mathematics.

Not active in class (seven responses). The teachers shared that some students who were observed to have MLD "sleep in classes" and "do not participate." For instance, Kletus and Katyi were reported by Teacher 3 and Teacher 5, respectively, to sleep during class most of the time. Meanwhile, Teacher 3 observed that Tauno only rarely asked questions when he did not understand. The teachers' responses indicated that these students were not vocal or shy; they "hardly approach the teachers" and demonstrated a lack of interest in learning mathematics. This indicated that teachers assessed this criterion according to students' behavior in the classroom.

Slow work pace (three responses). Some teachers stated that students with MLD were slow at attempting the given questions. They took a long time to answer a single question or "else you find a student leaves questions unanswered and some are just underlining words." For example, Teacher 5 indicated that Stella worked slowly to complete her tasks though she tried her best. This implies that Stela was a positive struggler in learning mathematics. The transcripts further revealed that the students' slow work pace might result from poor mathematical learning backgrounds if students had not acquired the necessary basic math skills in previous grades.

Asking low-level questions. This theme was only mentioned by Teacher 1. When interviewed, Teacher 1 stated that students with MLD asked questions below their level that were sometimes off-topic. According to Teacher 1, these students often did not know where to start when trying to ask questions. When discussing Ombili, Teacher 1 indicated that "his questions are below his level of learning." The teacher noted that Ombili's low-level questions resulted from a poor background in mathematics in earlier grades. However, Teacher 1 indicated that Ombili was trying his best and willing to learn mathematics, as his performance had improved compared to his achievements in 11th-grade mathematics.

Math anxiety. Only Teacher 1 expressed that student experiencing difficulties in learning mathematics appeared uncomfortable, lost, and confused while teaching. For example, Teacher 1 stated that Lempy "panics, is confused and gets lost." Sometimes, Lempy might even panic over a simple task, according to Teacher 1. This illustrates that math anxiety is one criterion for identifying MLD based on data.

How do Students Express MLD

Four indicators of how the students expressed their learning difficulties emerged from the data and are presented below in order of decreasing importance.

Low achievement in school assessments (17 responses). Most of the students lamented that they performed poorly in their school activities and failed to provide correct answers. Thus, all student participants changed from the extended level to the core level for 12th grade. For example, Ombili stated that "I fail to get correct answers". Even though he was willing to learn, his low achievement resulted from previous school assessments. Some students revealed that they had experienced low achievement in mathematics in previous grades. Other students expressed that the situation had worsened due to the new learning environment. Having new teachers and classmates hindered them from approaching teachers when they were experiencing difficulties in learning mathematics, for example, in the cases of Joan and Lempy, which negatively influenced their performance.

On the other hand, Agnes attributed the low achievement to poor calculator skills; not knowing how to operate a scientific calculator resulted in incorrect answers correctly. Findings also revealed that low achievement in mathematics resulted from math anxiety (easily forgetting preparations for a test or examination and becoming confused), a poor foundation of learning content, and working at a slow pace. In contrast, some students believed their poor performance was due to a "lack of concentration in classes." Though students voiced their thoughts about failing, most saw an improvement in their 12th-grade math performance compared to the 11th grade.

Math anxiety (14 responses). Many students expressed their views regarding anxiety in learning mathematics. For example, Lempy confirmed that in 11th grade, she was confused and scared due to the new learning environment. In addition, Ageno said she "gets confused and scared" regarding examinations, while Nandy indicated that she "gets upset and stressed when failing tests or examinations." This illustrates that different circumstances caused students to experience anxiety, which might have contributed to experiencing MLD.

A slow pace (five responses). Some students, who were slow to catch up or slow in completing tasks, stated that they could not finish within a given timeframe; thus, when answering questions, they started with those they understood and skipped others. During the interview, Stella lamented that she had no idea what to do when solving some mathematics tasks; she felt stuck and spent too much time solving tasks. This confused her, and, as a result, she provided wrong answers and, thus, failed. Based on the students' observations, one can argue that working at a slow pace could indicate MLD among students.

Not active in class (three responses). Only three students mentioned that they did not participate in class. Jason stated that "I do not participate in class," though he asked questions when he did not understand. Agnes said she "does not like to participate" in class because she did not feel free to ask questions in the teacher's presence but would rather wait for the teacher to leave the class and then ask her peers. Helvi voiced the view that she did not participate in class because she preferred to listen, and when she did not understand, she would ask her fellow students in the class. Some students expressed that anxiety made them reluctant to approach their teachers for help. These results indicate that, even though students did not participate in class, they sought help from other students when they found themselves experiencing difficulties in learning mathematics.

The Relationship Between Teachers' and Students' Views of Difficulties in Learning Mathematics

Based on the overall patterns that emerged from the responses (see Table 2), most students and teachers expressed views regarding low achievement in school assessments among students experiencing difficulties in learning mathematics. Furthermore, more than half the students' responses indicated that they experienced feelings of anxiety when

learning math at school. This aspect was only mentioned in a small number of the teachers' responses. Moreover, the data indicated that both students and teachers commented on the students' slow work pace. Additionally, more teachers' responses mentioned students not participating actively in math classroom activities than students' responses. In their responses, the teachers remarked on students asking low-level questions, but none of the students' responses revealed this aspect. Furthermore, 17.4% of the teachers' responses implied that some students were not experiencing difficulties in learning. In contrast, only a small number of the students' responses (4.3%) indicated that they were not experiencing difficulties.

Table 2
Overall Patterns (in percentage) of the Responses of Teachers and Students

Patterns of the responses	Teachers	Students
Low achievement in school assessment	64.0%	74.0%
Feelings of anxiety	4.3%	56.5%
Slow work pace	17.4%	13.0%
Students not participating actively	30.4%	13.0%
Students asking low-level questions	43.5%	0.0%
Students not experiencing difficulties in learning	17.4%	4.3%

After exploring the relationship between teachers' and students' descriptions and views, alignment was found only in some categories. Table 3 displays how some experiences matched and others did not match. Specifically, the table presents the similarities and differences between the teachers and their students, whether they agreed, and the difficulties the students experienced. For example, regarding anxiety, a relationship only existed between the views of Teacher 1 and Lempy. In contrast, no relationships were found between other students who shared their math anxiety and the views of their teachers in this regard. Ageno, for example, expressed feeling anxiety, while in contrast, Teacher 5 perceived that Ageno had no difficulties in learning. This might reveal the difficulty teachers have in identifying anxiety among students.

The findings also revealed a relationship between most students and their teachers regarding low achievement in school assessments (as shown in Tables 2 and 3). For example, Visto stated that "I never pass math"; in support of this, Teacher 6 stated that most of the time, Visto's activities were poorly completed, and her mathematical background was weak. On the other hand, some students indicated low performance, though their teachers did not directly express this. For example, Stella stated that her performance was not good, while Teacher 5 observed that Stela worked slowly and did not participate in-class activities. Similarly, Jason expressed that he was slow to catch up and complete his tasks, which caused him to struggle in learning math, whereas Teacher 6 reported that Jason did not perform well in Mathematics.

Table 3
Similarities and Differences found in Perceived Learning Difficulties

Categories identified	Descriptions matched	Differences in descriptions
Math Anxiety	One similarity i.e., Teacher 1 and Lempy	Thirteen differences e.g., no difficulties in learning (Teacher 6); felt anxiety (Ageno)
Low achievement in school assessments	Thirteen similarities	Five differences e.g., low achievement (Teacher 1); felt math anxiety (Nandy)
Did not participate in class activities	No similarities	Six differences e.g., did not participate in class activities (Teacher 3); low mathematics achievement (Kletus)
Slow pace	One similarity i.e., Teacher 5 and Stella	Two differences e.g., low achievement in mathematics assessment (Teacher 6); slow pace (Jermy
Low-level questions	No similarities	One difference i.e., questions below his level of learning (Teacher 1); he enjoys asking questions and seeks assistance from his teachers and peers (Ombili)

Furthermore, when discussing Ombili, Teacher 1 indicated that "his questions are below his level of learning." Interestingly, Ombili reported that he enjoyed asking questions and seeking assistance from his teachers and peers, as he was eager to learn and understand mathematics despite his poor performance. Moreover, a relationship between Stella and Teacher 5 was revealed regarding her slow work pace. Teacher 5 indicated that, although Stela worked slowly, she tried her best. However, no relationship was found between the views of teachers and four students who mentioned working at a slow pace. For example, Jermy said that he doubted his answers, which caused him to spend more time on a task, while Teacher 4 only observed Jermy's low achievement. A similar finding was revealed between Teacher 6 and Alfeus; in this case, Alfeus stated, "it takes me time" to solve mathematics problems, but Teacher 6 mentioned that Alfeus was not achieving well in math.

The findings in this study revealed no relationship between the teachers' views and their students regarding participation. However, a relationship between participation and low achievement in school assessment was indicated in comparing the teachers' responses on this theme. For instance, Teacher 3 observed that Tauno rarely asked questions when he did not understand; however, Tauno only felt he failed because he was not serious about his schoolwork. Furthermore, Kletus and Katyi were reported by Teacher 3 and Teacher 5, respectively, to sleep during class most of the time. However, Kletus and Katyi mentioned only that their achievement in mathematics was low.

Problems Encountered and Limitations

Firstly, some students seemed uncomfortable answering some of the researcher's questions, especially when they were asked to provide reasoning for their answers. Secondly, six of the students from different schools were absent on the day of the interviews, which affected the number of targeted participants. Thirdly, not all the identified students felt they were experiencing difficulties in learning mathematics; thus, data from one student were excluded from this study, affecting the number of participants. Fourthly, the mathematics teachers were interviewed only once; so, the findings might not reflect their actual perceptions and knowledge about the students. Fifthly, one of the teachers who identified students for the study moved to a different school. Her replacement may not have known the criteria for identifying the students for this study. Finally, this study was limited to the Namibian context, and it did not collect data from practices in the classroom. Classroom practices could prove essential in supporting the findings reported in this paper.

Discussion

The study illustrates that a lack of teacher involvement in selecting students with MLD might explain inconsistencies in the relationship between the views of the teachers and those of the students. One example is the case of Teacher 5 and three of her students. This teacher viewed these students as experiencing no difficulties in learning math; however, this contradicted her students' views, as they self-identified as experiencing difficulties. The difference suggests that the teacher did not yet know her students, as she was new at the school. This might also reveal that different teachers view students' learning differently. As mentioned in the Participants section, seven students who participated in this study self-identified as experiencing difficulties in learning mathematics during the interview conducted in 2018. However, all 23 students indicated experiencing hindrances and barriers in learning mathematics in 2019, though they expressed these difficulties differently. This demonstrates that seventeen students who, in 2018, indicated that they did not experience difficulty in learning math changed their perceptions.

The study shows that students with MLD were slow to solve given questions (Wang et al., 2009) because they could not remember formulas, became confused, or doubted their answers. Pau was one of the students who could not finish his activities within the given timeframe. Moreover, a primary factor affecting students' work pace was their difficulty with time management (Haarala-Muhonen et al., 2011). Thus, these students would not achieve well; they would likely fail tests and examinations. This can be a sign of learning difficulties, according to the responses from the teachers.

The study reveals that students' learning barriers, such as a new learning environment, affected math anxiety. The 11th graders felt uncomfortable and scared to approach teachers or peers when they did not understand. Additionally, the fear associated with students facing examinations or tests, that is, fear of failing, was another expression of anxiety (Ardi et al., 2019; Zhang, Zhao, & Kong, 2019). The test situation upset students and caused them to feel stressed (they felt panicky, confused, and lost), or they forgot formulas and, consequently, performed poorly (Hembree, 1990). Thus, this study revealed that anxiety might influence the speed at which students work on math problems, which could contribute to MLD. Participants suggested that teachers understood how to support students who experienced math anxiety. However, it seemed difficult for teachers to identify anxiety among students as only one teacher expressed concern regarding this aspect.

The literature emphasizes that students are expected to participate in verbal and written mathematics activities (e.g., Bennett, 2014; Moschkovich, 1999 & 2015). According to Jansen (2006), students' participation demonstrates their competence in the subject. Furthermore, participation strengthens students' identities as capable mathematicians (Anderson, 2007). This implies that when students do not participate, it is difficult for teachers to determine whether they understand the subject. In another study supporting this finding, participation caused teachers to understand the needs of Latino students in a reform-oriented mathematics classroom (Moschkovich, 1999). This suggests that teachers should support students who experience MLD in their class participation.

Some students in this study confessed that they do not participate or do not like to participate in class. Similar results were obtained by Zhao and McDougall (2008), as they found that some Chinese students hesitated to participate in class. However, in Zhao's and McDougall's study, this was due to a language barrier. Dkeidek et al. (2011) note that in some settings, participation, e.g., asking a question, can be seen as a sign of slow learning; they found that most Asian students attempted to avoid participation. The current study found that participation was more important for teachers in determining learning difficulties than for the students, who did not consider participation important as they could receive help from their peers. The students did not mention participation but rather how active they were, while teachers did not consider that. However, according to the literature, when considering the benefits of students' participation and interaction, it becomes clear why the criterion of participation was important to the teachers. The teachers seemed to link non-participation to difficulties in learning mathematics.

According to the literature, a direct link exists between participation and learning achievement, as they may predict each other (Howe & Tolmie, 2003; Webb et al., 2014). The present study provides insight into students who did not participate in mathematics lessons, which is observable in teachers' descriptions of students (e.g., "not vocal" and "never asks"). However, students themselves mentioned other categories, such as low achievement in school assessments. This reveals that students might experience difficulties in learning, thus causing them to perform poorly, which could, in turn, affect their level of classroom participation. On the other hand, some students indicated that their performance was low even when their teachers did not express the sentiment (e.g., Stella's views mentioned above).

It is claimed that teachers' knowledge of students predicts their achievement (Ball et al., 2008), the overall judgment of their performance, and an actual understanding of the student (Oudman et al., 2018). Nevertheless, Acharya (2017) found that students' achievement depends on their need for the subject matter and their interest and seriousness (p.2). The present study's findings reveal a high number of teachers and students who voiced low achievement as a reason for students experiencing difficulties in learning mathematics. Teachers argued that students with poor backgrounds in math were unable to solve basic mathematical questions, which could lead to low achievement. The findings demonstrate that when a student is not performing well, teachers might interpret this as the student experiencing difficulties in learning.

According to Woodward (1992), students' questions expose their thoughts and convey messages to the teacher regarding their conceptual understanding. Dkeidek et al. (2011) found that a poor learning foundation played a significant role in the development of the question-asking ability of students, and they consider asking questions to be a beneficial skill, as it activates students' prior knowledge (Schmidt, 1983). The findings of this study indicate that some students with MLD did ask questions. However, these questions were

often below the 12th-grade level, and teachers did not expect this. Interestingly, some of these students were curious and eager to learn and understand mathematics. This suggests that asking questions reveals that the student is interested in learning math. However, if students' questions benefit their academic understanding, one question requiring further explanation is why do students ask low-level questions?

Conclusions

The paper reports findings from a study that examined the relationship between Namibian mathematics teachers' assessments of their students' difficulty in learning mathematics and students' assessment of the same. The similarities and differences between the teachers' and students' perceptions were compared.

Five criteria in identifying learning difficulties among students were indicated. The fact that students were aware of their learning abilities was encouraging. Interestingly, the teachers and their students expressed MLD in different ways. Evidence confirming the relationship between the teachers' knowledge of students with MLD and the students' self-identification of MLD is important when developing theory in the subject since no teaching practice was observed.

This knowledge could be essential to educational policymakers and educators when addressing learning difficulties in mathematics classrooms. The findings presented in this paper contribute to an understanding of teachers' knowledge of students and students' knowledge of their learning. In addition, the knowledge shared in this study can be employed by teachers and utilized to prepare pre-service teachers. However, there appears to be a need for further work, such as discovering ways to assist teachers in identifying difficulties in learning mathematics more accurately. Some criteria, for instance, math anxiety, were observed by only a few of the teachers, which was significantly different from the views expressed by the students. Perhaps all teachers did not employ the same criteria, or their students' understanding was not comprehensive. The findings reported in this paper may not be similar in different contexts, for example, because of cultural differences.

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Appendix A

Twelfth-grade mathematics teachers' interview guide – 2019

1. How would you characterize a student with difficulties in learning mathematics? What are their characteristics, things they do and cannot do in the mathematics lesson?
2. From your observation, what is the average of students with difficulties in learning in your classes?
3. What are the characteristics of the following students in terms of things they do and what they cannot do in math class? How is their performance? What support do you give them?
4. What do you do once you identify that learner A or B is experiencing difficulties or struggling in learning mathematics?
5. Do you think it is important for teachers to support students with difficulties in learning mathematics?
6. Do you think there are influences on the national curriculum in relation to teaching students with difficulties in learning?

Twelfth-grade students' follow-up interview guide – 2019

1. How has your performance been in grade 12 mathematics lessons?
2. How do you realize that you know a specific mathematics concept?
3. Why do you want to learn mathematics?
4. What does mathematics include? How do you study these in order to understand them?
5. Do you think you have difficulties in learning mathematics?
6. What are your weaknesses and strengths in learning grade 12 Mathematics? Which topics were you good at in grade 12 math? Which topics were you not as good as you wanted to be? What are the reasons for being good at ... and not good at …?
7. Based on your observations and experiences, why do you think some high school students have difficulties in learning mathematics? Can you elaborate? Are there school-related factors? What are they?
8. What factors influence your learning of mathematics in grade 12?
9. Comparing last year to this year, what kind of changes do you identify in your mathematics achievement?

NB: Probing or follow-up questions will be asked to explore the issues raised during the interview discussion.