



An Attempt to Understand Relational Quantum Mechanics

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Abstract

We search for a possible mathematical formulation of some of the key ideas of the relational interpretation of quantum mechanics and study their consequences. We also briefly overview some proposals of relational quantum mechanics for an axiomatic reconstruction of the Hilbert space formulation of quantum mechanics.

1 Introduction

This is an attempt to understand Rovelli's relational interpretation of quantum mechanics, RQM, as outlined in [1–5], further advocated in [6] and sharpened in [7]¹. We try to identify and express some of the key ideas of RQM within a systematic Hilbert space formulation of quantum mechanics. RQM follows the old idea that properties of things are relational, and they get actualized in interactions. This poses the question how to justify this point of view within the Hilbert space structures of quantum mechanics without facing the well-known measurement problem.

Our study is in line with the critical analyses of [8–12] though our emphasis is more on the formal side of scrutinizing the assumptions constituting RQM, as we read them.² We also consider briefly the possible role of the basic ideas of RQM in an axiomatic reconstruction of (the Hilbert space formulation of) quantum mechanics.

Several authors have examined the relational interpretation of quantum mechanics. Especially, as Rovelli writes in his excellent little book *Helgoland: making sense of the quantum revolution* [6, p. 142], "the world of philosophy has reacted to this interpretation

¹ We are grateful to Carlo Rovelli for informing us on the recent paper [7] which contains a slightly modified list of the defining assumptions of RQM.

² Though the RQM literature is already quite abundant, it seems to us that no common understanding of this interpretation has yet been reached. This may be due to the lack of formally rigorous presentation of the basic assumptions of the interpretation.

To the memory of Maciej J. Mączyński (1937-2022), our friend, colleague and mentor.

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in various ways: different schools of thought have framed it in different philosophical terms." From [4, p. 8] we may also read that "[t]here are several objections that come naturally to mind when one first encounters relational QM, which seem to render it inconsistent. These have been long discussed and have all been convincingly answered; ...I will not rediscuss them here. Relational QM is a consistent interpretation of quantum theory."³ Still, we dare to take our go with it, with the hope that our analysis could contribute to a further clarification of the content of RQM.

We try to identify step by step, in the order of increasing requisite, the assumptions which seem to constitute the relational interpretation. The key notions, as we see them, are: property (a value of a physical quantity) (Sect. 2.1), interaction (Sect. 2.2), strong correlation (Sect. 2.3), perspective as a measurement scheme (Sect. 2.4), repeatability (Sect. 2.5), and local collapse (Sect. 2.6).

Physical quantities and their possible values, interactions, correlations, measurement schemes and repeatable measurement schemes are standard ingredients of quantum mechanics, whereas local collapse (our term) is a specialized form of the controversial projection postulate. In a nutshell, in our analysis RQM consists of the statement that physical quantities take values through their repeatable measurement schemes together with an *ad hoc* local collapse to circumvent the objectification problem.

In applying the above set of rules in the context of sequential and joint measurement schemes, Sect. 2.7, we come at odds with a claim of RQM that "we all 'see the same world'" [3, Sect. 2.6]. In [7], however, relying on the analysis of Di Biagio and Rovelli [13], Adlam and Rovelli supplement RQM with a new postulate called *cross-perspective links* to reach intersubjectivity between 'all parties'. Though remedying the possible lack of objectivity of the description, our reading of it in Sect. 2.8 rather suggests that this postulate is a disguised form of the projection postulate leading to a 'global collapse' of the state of the interacting pair.

In the proposed axiomatic reconstruction of RQM, two postulates have been formulated to outline the basic event structure of the theory. Though interesting, these assumptions constitute only a beginning of an axiomatic reconstruction of the Hilbert space formulation of quantum mechanics. It remains to be seen if such a reconstruction could be completed to yield a (version of the) relational interpretation of quantum mechanics.

2 Hilbert space theory and RQM

Relational quantum mechanics does not intend to change the mathematical apparatus of quantum mechanics, but proposes, in particular, a way of attaching values to the variables of a physical system relative to other systems interacting with it. RQM also treats all physical systems, including measuring apparatus and even observers, equally on the same basis as quantum systems. Therefore, it is safe to start with the Hilbert space formulation of quantum mechanics, together with a full use of its theory of compound systems within which mutually interacting systems are described.⁴

³ In a recent paper Di Biagio and Rovelli [13] also write: "In recent works, Āaslav Brukner and Jacques Pienaar have raised interesting objections to the relational interpretation of quantum mechanics. We answer these objections in detail and show that, far from questioning the viability of the interpretation, they sharpen and clarify it."

⁴ In [7] it is explicitly stated that "[RQM] does not require us to add anything to the existing mathematical framework of quantum mechanics." The second postulate of RQM even states that "unitary quantum mechanics is complete", *ibid.*

2.1 General background

To fix our notations, consider a physical system S with its Hilbert space \mathcal{H} , complex and separable.⁵ Its physical quantities (variables, observables, or whatever term one prefers to use) are given (represented) as the normalized positive operator measures (semispectral measures) E defined on σ -algebras \mathcal{A} of subsets of sets Ω and taking values in the set $\mathcal{E}(\mathcal{H}) \subset \mathcal{L}(\mathcal{H})$ of effects, i.e. positive operators bounded above by the identity I ; here $\mathcal{L}(\mathcal{H})$ is the set of the bounded operators on \mathcal{H} . Intuitively, such sets Ω consist of the possible values or measurement outcomes of the observables whereas the sets $X \in \mathcal{A}$ are the bin sets within which the values or outcomes are noticed, observed, or registered. Typically, (Ω, \mathcal{A}) is the real Borel space $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$, or an appropriate subspace of it.

Among the observables $E : \mathcal{A} \rightarrow \mathcal{L}(\mathcal{H})$ there are the (real) spectral measures $P : \mathcal{B}(\mathbb{R}) \rightarrow \mathcal{L}(\mathcal{H})$, taking projection operators as values, and which are uniquely associated with the selfadjoint operators $A = \int x dP(x)$. Occasionally, we may refer to such observables as the sharp observables. Apart from their central importance, there is, however, no point to restrict attention to such observables only, and still less so to the observables given as discrete selfadjoint operators (with the spectral structure $A = \sum a_i P_i$).⁶

The other fundamental notion of the theory is the notion of a state as a positive trace class operator of trace one, $\rho : \mathcal{H} \rightarrow \mathcal{H}$, $\text{tr}[\rho] = 1$. Any observable-state pair (E, ρ) defines a probability measure $p_\rho^E : \mathcal{A} \rightarrow [0, 1]$ through $p_\rho^E(X) = \text{tr}[\rho E(X)]$, with the obvious, but important properties that if $p_\rho^E = p_\rho^F$, for all ρ , then $E = F$, and if $p_\rho^E = p_\sigma^E$, for all E , then $\rho = \sigma$. In particular, states can thus be identified with the totality of the probabilities $p_\rho^E(X)$, for all E and X .

For the pure states, the extremal elements of the convex set of states, we also use the notation $\rho = P[\varphi] = |\varphi\rangle\langle\varphi|$, with $\varphi \in \mathcal{H}$ being a unit vector. Occasionally, we may refer to pure states also as vector states and use unit vectors as their representatives.

In the so-called minimal interpretation of quantum mechanics, the meaning of the number $p_\rho^E(X)$ is the probability that a measurement of E on S in state ρ leads to a result in X , with the idea that these probabilities approximate the actual measurement outcome statistics obtained by repeating the same measurement under the same conditions many times. Though accepting the Born rule, RQM aims to go beyond the minimal interpretation.⁷

Remark 1 In classical mechanics, one may consistently assume that all physical quantities (as functions on the phase space of the system) always have well-defined, though possibly unknown values (which may vary in time). Thus, if we know, in which phase space point the system is, that is, we know the values of the canonical variables, we may compute the value of any other variable. Therefore, the phase space points may be referred to as the pure (maximal information) states of the system, other (mixed) states being expressed as probability measures on phase space and are used to describe possible ignorance (of what ever source) on the actual phase space point. One may thus assume that a classical system is always in a pure state, that

⁵ We use *Quantum Measurement* [14] as a basic reference to the standard results used in this writing.

⁶ Such a restriction would only hide some of the crucial assumptions concerning the possible measurements or value assignments of such observables.

⁷ In the above wording, minimal interpretation does not address the question of the meaning of probability, that is, it does not take a position on the question of the interpretation of the notion of probability. Though a position could be taken (modal frequency, propensity, ...), it is, in this context, unnecessary and would just hide our intentions.

is, all the variables have well-defined values, though in the case of incomplete information one needs to rely on a probability distribution of the values coded in a mixed state.⁸

In quantum mechanics one cannot consistently assume that all the quantities E of a system would always have well-defined values. Still, one may assume that the system is always in a well-defined state ρ , that is, the system is always characterized by the totality of the probability measures p_ρ^E . Even in case of pure states, some of those probabilities are always nontrivial ($\neq 0, 1$), and in the case of mixed states, a classical type of ignorance interpretation of the probability measures p_ρ^E is ruled out by the nonunique decomposability of mixed states into pure states.⁹ \square

RQM does not consider state as a fundamental notion but still uses it exclusively in two different roles. First, it appears as a bookkeeping means of the probabilities $p_\rho^E(X)$: "[state] is a theoretical device we use for bookkeeping information about the values of variables of [a physical system] S actualized in interactions with [another physical system] S' , values which can in principle be used for predicting other (for instance future, or past) values that variables may take in other interactions with S' ." [4, p. 6]. In [4, p. 4] it is further clarified that technically the notion of state ρ is computed from the expectation values $\text{tr}[\rho A]$, A being a (bounded) self-joint operator, equivalently, from the totality of the probabilities $p_\rho^E(X)$, for all E and X .¹⁰ The second use of this notion appears in Sect. 2.6, and it defines the state of a system with respect to another system, after the two systems have interacted with a consequence that a physical quantity of the first system has taken a definite value (indicated by a strong correlation).

2.2 RQM and interactions

In an attempt to go beyond the minimal interpretation, one typically faces the question under which conditions a physical quantity may be said to possess a definite value. For instance, if one knows that a hydrogen atom is in its energy ground state, one may wish to say that the energy has then the value -13.6 eV, the smallest energy eigenvalue of the atom in the appropriate units. More generally, in any state ρ , if $p_\rho^E(X) = 1$, then one may consistently assume (i.e. use the way of speaking without contradictions) that E has a value in X ,¹¹ or, alternatively, that the (sharp or unsharp) property $E(X)$ pertains to the system.¹² To cut short the used vocabulary, we may also refer to an effect $E \neq 0$ as a possible value of an

⁸ The structure of the set of probability measures on phase space is that of a (Bauer) simplex, which allows one to express any mixed state uniquely as a 'generalized convex combination' of the extreme points, the pure states. Clearly, they are the point measures identified as the phase space points.

⁹ Any mixed state has a continuum of different decompositions into pure states. A full characterization of the possible decompositions of a mixed state is given in [15], see also [14, Thm 9.2.].

¹⁰ Clearly, one does not need all the observables here, but one is not enough unless it is informationally complete (and thus necessarily given as a noncommutative semispectral measure), or some prior information is available. Especially, the statistics of a complete set of mutually commuting (sharp) observables does not, in general, suffice to determine the state of the system. However, RQM does not explain the meaning of these expectation values in that interpretation.

¹¹ In the case of continuous observables, like position and momentum, further care is needed in such a formulation.

¹² Consistently with that, one may also say that a property (sharp or unsharp) $E \in \mathcal{E}(\mathcal{H})$ is objective in a state ρ if either $\text{tr}[\rho E] = 1$ or $\text{tr}[\rho(I - E)] = 1$. We emphasize that this way of speaking does not mean a commitment to the so-called 'eigenvalue-eigenstate link', the assumption that if a result (eigenvalue) has been observed, then the state is determined to be the corresponding eigenstate.

observable E if $E = E(X)$ for some $X \in \mathcal{A}$, and we recall that any effect E defines a dichotomic observable with the values E and $I - E$, and with the outcome space $\{1, 0\}$.

In any state ρ there are observables E and (nontrivial) bin sets X such that $p_\rho^E(X) = 1$. By the same token, in any state ρ there also are plenty of observables F with bin sets Y such that $0 \neq p_\rho^F(Y) \neq 1$. Facing this basic situation, RQM poses the old question and proposes an answer, *interaction*:

- (I) When and how a probabilistic prediction about the value of a variable $[E]$ of a physical system S is resolved into an actual value $[X]$? The answer is this: when S interacts with another physical system S' . [4, p. 5].

Accordingly, assume that S interacts with another system S' , with a Hilbert space \mathcal{H}' , and assume that the interaction can be described by a unitary operator $U : \mathcal{H} \otimes \mathcal{H}' \rightarrow \mathcal{H} \otimes \mathcal{H}'$. Given that S and S' are initially, before the interaction, dynamically and probabilistically independent of each other, with the respective (pure) states $\rho = |\varphi\rangle\langle\varphi|$ and $\sigma = |\phi\rangle\langle\phi|$, the interaction then transforms the initial (unentangled) state $\rho \otimes \sigma = |\varphi \otimes \phi\rangle\langle\varphi \otimes \phi|$ to the final, typically entangled state $U(\rho \otimes \sigma)U^* = P[U(\varphi \otimes \phi)]$.¹³ Since the two systems S and S' are in a fully symmetric position,¹⁴ the question arises: which of the plurality of observables E and E' of S and S' , with nontrivial probabilities $0 \neq p_\rho^E(X) \neq 1$ and $0 \neq p_{\sigma'}^{E'}(Y) \neq 1$, should or could take values through U with respect to S' and S , respectively? For instance, S could be a proton and S' an electron and the interaction might be the Coulomb one (depending on the mutual distance), the pair (S, S') forming a hydrogen atom. Without further qualifications it is difficult to decide which proton variables take values, and which values, with respect to the electron, and vice versa. The proposed answer is:

- (SC) they are among those observables whose *values* get *strongly correlated* in the interaction.¹⁵

This leads us to consider such correlations.

2.3 RQM and correlations

A preliminary mathematical problem of RQM is thus to determine under which conditions any two effects $E \in \mathcal{E}(\mathcal{H})$ and $F \in \mathcal{E}(\mathcal{H}')$ are strongly correlated in the state $U(\rho \otimes \sigma)U^*$. To answer this question, we identify the effects E and F with the dichotomic observables $\{I - E, E\}$ and $\{I - F, F\}$, with the outcome spaces $\{0, 1\}$, and say that they are strongly correlated in the state $U(\rho \otimes \sigma)U^*$, if the corresponding dichotomic probability measures μ

¹³ RQM is somewhat ambiguous with respect to the notion of state, and, in particular, that of an isolated system. The assumption that S and S' are initially characterized by the states ρ and σ is in accordance with the examples discussed in [1–4] and writing $\rho \otimes \sigma$ just points to the fact that the two systems are independent of each other (and of the rest of the world). However, we are also told that "there is no meaning to 'the state of an isolated system'" [3, Sect. 3.1]. On the other hand, Dorato [16, p. 254] argues: "In RQM *relata* (isolated quantum systems, or parts) with state dependent dispositional properties ought to be regarded as existent, since there is no relation without *relata*".

¹⁴ It is a basic hypothesis of RQM that all systems (including observers) are treated on the same level, Hypothesis 1 of [1], a point underlined also by Dorato [16] and Pienaar [11]. On the other hand, van Fraassen [8], seems to deviate from this reading of RQM.

¹⁵ In the primary RQM literature the word *correlation* appears without the prefix *strong*, see, e.g. [1, p. 9]. However, in all the discussed examples the relevant correlations are strong.

and ν , with $\mu(\{1\}) = \text{tr}[\rho^f E]$, and $\nu(\{1\}) = \text{tr}[\sigma^f F]$, are strongly correlated with respect to their coupling γ , with $\gamma(\{1, 1\}) = \text{tr}[U(\rho \otimes \sigma)U^* E \otimes F]$, where $\rho^f = \text{tr}_{\mathcal{H}'}[U(\rho \otimes \sigma)U^*]$ and $\sigma^f = \text{tr}_{\mathcal{H}}[U(\rho \otimes \sigma)U^*]$ are the subsystem states, the partial traces of $U(\rho \otimes \sigma)U^*$ over \mathcal{H}' and \mathcal{H} , respectively.

Remark 2 To recall the meaning of the above notion, consider any two probability measures μ and ν (on the real line) and let γ be any one of their couplings (joint measures), that is, $\mu(X) = \gamma(X \times \mathbb{R})$ and $\nu(Y) = \gamma(\mathbb{R} \times Y)$ for all X, Y . If one of them is a point measure (Dirac measure), then γ is necessarily the product measure, $\gamma = \mu \times \nu$, and their covariance is 0. Assume, thus that neither of them is a point measure. By definition, their normalized correlation coefficient with respect to γ is strong (or rather, perfect), that is, $\text{cor}(\mu, \nu | \gamma) = \pm 1$, exactly when $\int xy d\gamma(x, y) - \int x d\mu(x) \int y d\nu(y) = \pm \Delta(\mu)\Delta(\nu)$, where $\Delta(\mu), \Delta(\nu)$ are the standard deviations of μ, ν , and this is the case exactly when μ and ν are completely dependent with a linear function $h(y) = ay + b$, that is, $\gamma(X \times Y) = \nu(h^{-1}(X) \cap Y)$, for all X, Y , where $a = \pm \Delta(\mu)/\Delta(\nu)$ and $b = \epsilon_1 - a\epsilon_2, \epsilon_1 = \int x d\mu, \epsilon_2 = \int y d\nu$ ($a > 0$ for positive and $a < 0$ for negative correlation). □

Any two effects $E \in \mathcal{E}(\mathcal{H})$ and $F \in \mathcal{E}(\mathcal{H}')$ are thus strongly correlated in state $U(\rho \otimes \sigma)U^*$ exactly when all the probabilities $\text{tr}[\rho^f E]$, $\text{tr}[\sigma^f F]$, and $\text{tr}[U(\rho \otimes \sigma)U^* E \otimes F]$, are equal. Clearly, if the two measures are point measures then the (sharp or unsharp) properties are objective in the respective states of S, S' .

For two observables E and E' of S and S' , the strong correlation of any of their effects $E(X)$ and $E'(Y)$ after the interaction (in state $U(\rho \otimes \sigma)U^*$) is hereby characterized. To extend this notion to cover all possible values of the observables, it is natural, and, in fact, necessary, to discretize the observables. For that end, let (X_i) and (Y_j) be any partitions of the (real) value spaces of E and E' into disjoint bin sets, the so-called reading scales. The values of E and E' with respect to these reading scales are then strongly correlated if for each $E(X_i)$ there is an $E'(Y_j)$ such that the pair of effects is strongly correlated in state $U(\rho \otimes \sigma)U^*$. Finally, discretized or not, the observables E and E' are (by definition) strongly correlated in state $U(\rho \otimes \sigma)U^*$ if the (real) probability measures p_{ρ}^E and $p_{\sigma}^{E'}$ are strongly correlated with respect to their joint probability measure γ , with $\gamma(X \times Y) = \text{tr}[U(\rho \otimes \sigma)U^* E(X) \otimes E'(Y)]$.¹⁶

With the assumptions $\rho = |\varphi\rangle\langle\varphi|$ and $\sigma = |\phi\rangle\langle\phi|$, one may use a Schmidt decomposition of the state vector $U(\varphi \otimes \phi)$ to construct examples of pairs of effects and observables whose values are strongly correlated in this state. Indeed, let

$$U(\varphi \otimes \phi) = \sum_i \lambda_i \sum_{m=1}^{n_i} \xi_{im} \otimes \eta_{im}, \tag{1}$$

be such a decomposition, with $\lambda_i > 0, \lambda_i \neq \lambda_j, i \neq j, \sum n_i \lambda_i^2 = 1, n_i$ the degeneracy of λ_i , and $\langle \xi_{im} | \xi_{jn} \rangle = \delta_{ij} \delta_{mn} = \langle \eta_{im} | \eta_{jn} \rangle$. Let

$$P_i = \sum_m |\xi_{im}\rangle\langle\xi_{im}|, \quad R_i = \sum_m |\eta_{im}\rangle\langle\eta_{im}|, \tag{2}$$

so that

¹⁶ A detailed study of various equivalent formulations of the notion of quantum perfect (strong) correlations have been carried out by Ozawa [17].

$$\rho^f = \sum_i \lambda_i^2 P_i, \quad \sigma^f = \sum_i \lambda_i^2 R_i \tag{3}$$

are the spectral decompositions of ρ^f and σ^f . We note that this decomposition is (essentially) unique¹⁷ only when all $n_i = 1$. Also, the projections P_i , resp. R_i , need not sum up to I , that is, the vectors $(\xi_{ij})_{i,j}$, resp. $(\eta_{ij})_{i,j}$ need not form an orthonormal basis of \mathcal{H} , resp. \mathcal{H}' .

As an immediate observation, any (sharp) observables having the collections of projections $\{P_1, P_2, \dots\}$ and $\{R_1, R_2, \dots\}$ as part of their spectral structure have their corresponding ‘values’ P_i and R_i strongly correlated in the vector state $U(\varphi \otimes \phi)$. If, for some k , $n_k > 1$, that is, the projections P_k and R_k are n_k -dimensional, then also any finer observables with those $P[\xi_{km}]$ and $P[\eta_{km}]$ as spectral projections are strongly correlated, too. Moreover, for any unitary operators U_k, U'_k acting in the subspaces generated by the vectors $(\xi_{kj})_j$ and $(\eta_{kj})_j$ define other refined observables whose values are now strongly correlated. It is worth noting that the S -observables defined by different rotations U_k are mutually incompatible. A well-known example of such a case is given by a pair of qubits brought through an interaction to one of the Bell states.

Though incomplete, the above discussion already shows that strong value correlation is not enough to specify which of the possibly uncountably many observables of S take values with respect to S' , and vice versa. Further information is needed, and in search for that we follow an idea expressed in [3, Sect. 1.3]:

(P) In the relational interpretation, any interaction counts as a measurement, to the extent one system affects the other and this influence depends on a variable of the first system. Every physical object can be taken as defining a perspective, to which all values of physical quantities can be referred.

The primary RQM literature does not give an explicit definition of the notion of a perspective but in the examples discussed they appear as what is called measurement schemes. This invites us to take a step to formulate the idea of a *perspective* as a *measurement scheme*.

2.4 RQM - perspectives as measurement schemes

Let us assume that S' observes S , using the interaction U and fixing one of its quantities Z (with a value space (Ω, \mathcal{A})) as the means of observing or witnessing S , that is, as a pointer or read out observable. If S' is able to witness S in each of its initial states ρ ,¹⁸ then it is a simple mathematical fact that the probabilities $\text{tr}[U(\rho \otimes \sigma)U^* I \otimes Z(X)]$, $X \in \mathcal{A}$, σ a fixed state of S' , define a unique observable M , with the value space (Ω, \mathcal{A}) , of S such that these probabilities are those of M in state ρ .¹⁹

In the language of the quantum theory of measurement, system S' , with its Hilbert space \mathcal{H}' , the pointer observable Z , the initial (‘ready’) state σ , and the interaction $U : \mathcal{H} \otimes \mathcal{H}' \rightarrow \mathcal{H} \otimes \mathcal{H}'$, in short, $\mathcal{M} = (\mathcal{H}', U, \sigma, Z)$, is called a measurement scheme for

¹⁷ See, e.g. [14, Theorem 3.15].

¹⁸ Less is enough, e.g., pure states suffice.

¹⁹ We take this to mean that the “influence depends on a variable of the first system”.

measuring the observable M on S .²⁰ The above mentioned probability reproducibility condition can equally be written as $p_\rho^M(X) = p_{\sigma^f}^Z(X)$, for all X, ρ , where σ^f is the final state of S' .

Clearly, also S may witness S' , using the same interaction, but fixing one of its observables as a pointer. Again, if S can witness S' in each state of S' , then the above can be repeated, but with a changed perspective. The roles of S and S' are obviously asymmetric in these two processes (as will become even more evident below).

Remark 3 It is a fundamental result of quantum mechanics, an application of the dilation theory, that for each observable E of S , there is a system \tilde{S} , with a Hilbert space \mathcal{K} , an interaction $U : \mathcal{H} \otimes \mathcal{K} \rightarrow \mathcal{H} \otimes \mathcal{K}$, a read out observable Z (which can be chosen to be sharp) and a ready state σ (which can be chosen to be pure) such that the defined observable is the observable E , that is, $p_\rho^E = p_{\sigma^Z}^Z$ for all ρ ; for a detailed exposition, see, e.g. [14, Chpt 7]. Clearly, if we first fix S' , with its Hilbert space \mathcal{H}' , a ready state σ , and an interaction U , then only those observables E of S come into play which, for all ρ and X , are of the form $\text{tr}[\rho E(X)] = \text{tr}[U(\rho \otimes \sigma)U^* I \otimes Z(X)]$ for some S' -observable Z . □

Let us go on with the assumption that in the interacting pair (S, S') , S' is the observing system in a pure state σ , and that the interaction U defines the observable E through a sharp S' -observable Z . That is, the perspective is defined by the 4-tuple $\mathcal{M} = (\mathcal{H}', U, \sigma, Z)$, which defines E in the sense that for any ρ , $p_\rho^E(X) = p_{\sigma^Z}^Z(X)$ for all X . In general, E is not sharp. A necessary and sufficient condition for the values $E(X)$ and $Z(X)$ to get strongly correlated through \mathcal{M} is that all the probabilities $\text{tr}[U(\rho \otimes \sigma)U^* E(X) \otimes Z(X)]$, $p_{\sigma^Z}^Z(X)$, and $p_{\rho^E}^E(X)$, are the same. Since $p_\rho^E(X) = p_{\sigma^Z}^Z(X)$, the requirement $p_{\sigma^Z}^Z(X) = p_{\rho^E}^E(X)$ gives $p_{\rho^E}^E(X) = p_\rho^E(X)$, which is the first-kind property of \mathcal{M} . Clearly, this is not enough to provide strong value correlations.²¹ The additional requirement $\text{tr}[U(\rho \otimes \sigma)U^* E(X) \otimes Z(X)] = p_{\sigma^Z}^Z(X)$ is, by $p_{\sigma^Z}^Z(X) = p_\rho^E(X)$, the repeatability property of \mathcal{M} , which is stronger than the first kind property. To take up this notion, we find it useful to recall briefly some relevant aspects of conditional states in measurement schemes.

Remark 4 Consider the state $U(\rho \otimes \sigma)U^*$ and define, for any X , the (unnormalized) state

$$I \otimes Z(X)U(\rho \otimes \sigma)U^* I \otimes Z(X), \tag{4}$$

which can be normalized if $\text{tr}[I \otimes Z(X)U(\rho \otimes \sigma)U^* I \otimes Z(X)] = p_\rho^E(X) \neq 0$.

Using a characterization of classical conditional probability together with an application of Gleason’s theorem, the state (4) can be interpreted as a conditional state, giving rise to conditional probabilities with the condition $I \otimes Z(X)$. (For details, see [21].) With partial tracing, one gets the corresponding (unnormalized) subsystem states:

$$\text{tr}_{\mathcal{H}'}[I \otimes Z(X)U(\rho \otimes \sigma)U^* I \otimes Z(X)] = \text{tr}_{\mathcal{H}'}[U(\rho \otimes \sigma)U^* I \otimes Z(X)] = \mathcal{I}^M(X)(\rho), \tag{5}$$

$$\text{tr}_{\mathcal{H}}[I \otimes Z(X)U(\rho \otimes \sigma)U^* I \otimes Z(X)] = Z(X)\sigma^f Z(X). \tag{6}$$

When $p_\rho^E(X) \neq 0$, we let $\rho^f(X)$ and $\sigma^f(X)$ denote the normalized versions of (5) and (6).

²⁰ The term measurement scheme, also called premeasurement [18–20], is used to emphasize that a full measuring process may require further properties, like a capacity of recording a result. We return to that question in Sect. 2.6. We also stress that the notion of a measurement scheme does not imply any restrictions to the nature of the system S' .

²¹ The so-called standard measurement schemes give a plenty of such examples, see, for instance, [14, Sect. 10.4.].

The conditional interpretation of the states $\rho^f(X)$ and $\sigma^f(X)$ is slightly different. Whereas $\sigma^f(X)$ is, in the above Kolmogorov-Gleason sense, the state of S' after the interaction with the condition $Z(X)$, and, clearly, in that state the property $Z(X)$ pertains to S' , the interpretation of the state $\rho^f(X)$ is more involved. First of all, the probability $\text{tr}[\rho^f(X)E(X)]$ is not equal to one, in general, so that its interpretation as a conditional state of S after the interaction with the condition $E(X)$ is not warranted without further specifications. We return to that later. However, for any S -observable F , with a value space (Ξ, \mathcal{B}) , the (Kolmogorovian joint) probability measure $\gamma(Y \times X) = \text{tr}[U(\rho \otimes \sigma)U^* F(Y) \otimes Z(X)]$, $Y \in \mathcal{B}$, $X \in \mathcal{A}$, is well defined and one has

$$p_{\rho^f(X)}^F(Y) = \text{tr}[\rho^f(X)F(Y)] = \frac{\text{tr}[U(\rho \otimes \sigma)U^* F(Y) \otimes Z(X)]}{\text{tr}[U(\rho \otimes \sigma)U^* I \otimes Z(X)]} = \frac{\gamma(Y \times X)}{\gamma(\Xi \times X)}, \tag{7}$$

which shows that $p_{\rho^f(X)}^F(Y)$ is, in the classical sense, the conditional probability $\gamma(Y \times \Omega | \Xi \times X)$ of the event $Y \times \Omega$ given the event $\Xi \times X$ with respect to the probability measure γ . In quantum mechanics with its minimal interpretation, the product $p_\rho^E(X)p_{\rho^f(X)}^F(Y) = \text{tr}[\mathcal{I}^M(X)(\rho)F(Y)]$ is often read as a sequential probability: the probability that a measurement of F leads to a result in Y given that a previous measurement of E with \mathcal{M} in state ρ led to a result in X .²²

Classical conditional probability is additive with respect to the disjoint partitions of the conditioning event, but this is not the case for the Kolmogorov-Gleason notion as applied in equations (4) and (6). Therefore, if, for instance, (X_i) is a partition of Ω into disjoint bin sets X_i , one has

$$\rho^f = \mathcal{I}^M(\Omega)(\rho) = \sum_i \mathcal{I}^M(X_i)(\rho) = \sum_i p_\rho^E(X_i)\rho^f(X_i), \tag{8}$$

given that $p_\rho^E(X_i) \neq 0$. However, one may also define the Kolmogorov-Gleason conditional state $\sum_i p_\rho^E(X_i)\sigma^f(X_i)$ with respect to the partition (X_i) of Ω (for details, see, [22]), but, in general, this conditional state is not σ^f , that is, the equality

$$\sigma^f = \sum_i p_\rho^E(X_i)\sigma^f(X_i) \tag{9}$$

may fail.

If Z is sharp (projection valued) and $\sigma = P[\phi]$ a pure state (as we have assumed with the choice of the perspective \mathcal{M}), then for any $\rho = P[\varphi]$, and for any reading scale (X_i) , the (validity of the) equality (9) is equivalent to the mutual orthogonality of the component states $\rho^f(X_i)$, that is, $\rho^f(X_i)\rho^f(X_j) = 0$ for all $i \neq j$. In this case there is a strong correlation between the component states $\rho^f(X_i)$ and $\sigma^f(X_i)$. For details, see [23, Thms 3.11 and 7.2] or [14, Thm 22.1]. Still, the values of E and Z with respect to a reading scale (X_i) need not be strongly correlated. \square

2.5 RQM – correlations via repeatability

As already noticed above, a necessary and sufficient condition for \mathcal{M} to produce strong value correlations between the observables E and Z is the equality of the probabilities

²² Here \mathcal{I}^M denotes the (completely positive) instrument, operation valued map, uniquely defined by \mathcal{M} . We wish to underline that the sequential probability is a bi-probability (on pairs of bin sets) and its interpretation does not presuppose any form of a collapse, that is, it does not presuppose that the state of S would have collapsed, in between, to the (unnormalized) state $\mathcal{I}^M(X)(\rho)$.

$\text{tr}[U(\rho \otimes \sigma)U^* E(X) \otimes Z(X)]$, $p_{\sigma'}^Z(X)$, and $p_{\rho'}^E(X)$, for all X and for any ρ . It turns out that this is exactly the repeatability property of \mathcal{M} .²³

A measurement scheme \mathcal{M} is said to be repeatable if its immediate repetition does not lead to a new result, meaning that the sequential probabilities $p_{\rho}^E(X)p_{\rho'(X)}^E(X)$ should be equal to $p_{\rho'}^E(X)$. There are many equivalent formulations of this notion (see, for instance, [14, Def. 10.3., Ex. 10.9.10]) and obviously one is: for any ρ, X , if $\text{tr}[\rho E(X)] \neq 0$, then $p_{\rho'(X)}^E(X) = 1$, that is, $\text{tr}[U(\rho \otimes \sigma)U^* E(X) \otimes Z(X)] = \text{tr}[\mathcal{I}^{\mathcal{M}}(X)(\rho)E(X)] = p_{\rho}^E(X) = p_{\sigma'}^Z(X)$. Clearly, this means that the property $E(X)$ pertains to the system in state $\rho'(X)$, though it does not mean that the conditional state $\rho'(X)$ would be a conditional state in the sense of (6). For that still further specifications would be needed, see the below Remark 5.

The repeatability condition is a strong condition implying, in particular, that the observable E is discrete [24, 25] (see, also, [14, Thm 10.4.]), that is, there is a countable set $\Omega_0 = \{\omega_i \mid i \in \mathbb{I} \subset \mathbb{N}\} \subset \Omega$ such that each $\{\omega_i\} \in \mathcal{A}$ and $E(\Omega_0) = I$. In that case there is a natural reading scale with $X_i = \{\omega_i\}$. Also, the repeatability implies that both the observables E and Z as well as their ‘values’ $E(X_i)$ and $Z(X_i)$ are strongly correlated in $U(\rho \otimes \sigma)U^*$ for all ρ [23, Theorems 5.5. and 6.3.]. If, in addition, the states $\rho^f(X_i)$ are mutually orthogonal, then one also has that $\sigma^f = \sum_i p_{\rho}^E(X_i)\sigma^f(X_i)$, meaning that in this case both of the final states ρ^f and σ^f have, in view of the measurement scheme \mathcal{M} , natural decompositions (8) and (9) in terms of the final component states $\rho^f(X_i)$ and $\sigma^f(X_i)$ with the relevant weights $p_{\rho}^E(X_i)$. This happens, in particular, if E is sharp, that is, projection valued.

Remark 5 If in addition to the repeatability, the measurement scheme \mathcal{M} is also nondegenerate, that is, the possible final states $\{\rho^f \mid \rho \text{ a state}\}$ separate the set of effects, then E is sharp (projection valued). Further, if the measurement scheme is also d -ideal, d for discrete, that is, for any ρ and ω_i , if $p_{\rho}^E(\{\omega_i\}) = 1$ implies $\rho^f(\{\omega_i\}) = \rho$, then \mathcal{M} is equivalent to a Lüders measurement of E , that is,

$$\mathcal{I}^{\mathcal{M}}(X)(\rho) = \sum_{\omega_i \in X} E(\{\omega_i\}) \rho E(\{\omega_i\}),$$

for all X and ρ .²⁴ Clearly, if E is a sharp discrete observable, then there is a perspective, a measurement scheme \mathcal{M} , with the given properties. In this case, the (unnormalized) state $\mathcal{I}^{\mathcal{M}}(X)(\rho)$ is (also) a Kolmogorov-Gleason conditional state with respect to the partition $(E(\{\omega_i\}))$ of $E(X)$. Though extensively used in original sources of RQM, this seems not to be an essential requirement of the interpretation. \square

According to RQM, the *strong value correlation*, and thus the *repeatability* assumption plays a crucial role in a process where a possible value of an observable gets resolved into an actual value. These assumptions lead to the natural decompositions (8) and (9). But, as is well known, the decompositions (being nonunique) do not carry a description of ignorance (Remark 1) on the actual values of the two observables and hence do not easily justify value assignments, a justification cap known as the objectification or the measurement problem. (For an extensive discussion of the subject matter,

²³ PUZZLE 2 of [8] expresses some reservations for the use of repeatable measurement schemes in Rovelli’s approach. Due to the equivalence of this notion with the strong value correlations, the same doubt might equally well concern perspectives (measurement schemes) producing such correlations.

²⁴ See [26, Sect. 4.3], or [14, Thm 10.6.].

we refer to [19, 20], or [14, Chpt 22]). A final step is still to be taken. Before that two further comments are due.

Remark 6 Fixing a perspective \mathcal{M} to resolve a possible value of a discretized observable E , for which $p_\rho^E(X_i) = p_{\sigma'}^Z(X_i)$ for all i , does not avoid the fact that the interaction U , even if repeatable, nondegenerate and d-ideal for E in the sense of Remark 5, may create strong correlations (in some states) also between the values of some other observables of S and S' , observables which may be incompatible with E and Z . Indeed, if the Schmidt decomposition of $U(\varphi \otimes \phi)$ contains degenerate eigenvalues λ_i , then any pointers Z associated with mutually incompatible rank-1 refinements of the corresponding projections R_i would give such examples. \square

Though important, discrete observables do not exhaust all the observables and many realistic measurement schemes, with discrete pointers, define (discrete) unsharp observables. Neither do repeatable measurements exhaust the measurements of discrete observables.

Remark 7 In [4, Sect. 2(a)] we find an argument for the discreteness of quantum mechanics:

[T]he major physical characterization of quantum theory is that the volume of the region R where the system happens to be cannot be smaller than $2\pi\hbar$:

$$\text{Vol}(R) \geq 2\pi\hbar,$$

per each degree of freedom. This is the most general and most important physical fact at the core of quantum theory. This implies that the number of possible values that any variable distinguishing points within the region R of phase space, and which can be determined without altering the fact that the system is in the region R itself, is at most

$$N \leq \frac{\text{Vol}(R)}{2\pi\hbar}$$

which is a *finite* number. That is, this variable can take discrete values only.

There is no compact region R of the phase space where the ‘system is’ or ‘happens to be’ (in the sense that the ‘values’ of the canonical variables would be localized in such an R , see the below Example 1); thus the first inequality is trivial $\infty \geq 2\pi\hbar$ and hence the case for the finiteness of N does not arise. Neither does the so-called unsharp (phase space) localization help in this respect, see, for instance, [27, 28]. \square

2.6 RQM and local collapse

With the tools collected above we now try to express the full RQM answer to the question “When and how a probabilistic prediction about the value of a variable of a physical system is resolved into an actual value?”, the proposed answer being “when the system interacts with another system”.

Let the totality of the probabilistic predictions concerning the system S be expressed by a state ρ . Let S' be any other system able to define a perspective through a measurement scheme $\mathcal{M} = (\mathcal{H}', U, Z, \sigma)$ with respect to which values of the quantity E of S can be

referred to in the sense that the predictions $p_\rho^E(X)$ concerning E are now reproduced as the probabilistics $p_{\sigma'}^Z(X)$ concerning Z .

In order to approach the ‘RQM value assignment’ through strong value correlations, one needs to assume that the perspective \mathcal{M} is repeatable, that is, with \mathcal{M} we also fix a reading scale (X_i) , with respect to which the scheme is repeatable (w.r.t. (X_i)).²⁵ In addition, one may assume that the states $\rho^f(X_i)$ are pairwise orthogonal²⁶ so that both $\rho^f = \sum_i p_\rho^E(X_i)\rho^f(X_i)$ and $\sigma^f = \sum_i p_\rho^E(X_i)\sigma^f(X_i)$ as well as $\text{tr}[\rho^f(X_i)E(X_i)] = 1 = \text{tr}[\sigma^f(X_i)Z(X_i)]$ for all i . In this case, all the relevant correlations are strong. Nevertheless there is no way to justify that the (discretized) observables $X_i \mapsto E(X_i)$ and $X_i \mapsto Z(X_i)$ would have one of the possible values $E(X_i)$ and $Z(X_i)$, unless one of the weights $p_\rho^E(X_k) = 1$; all that can be said on these values is their strong probabilistic coupling in the state $U(\rho \otimes \sigma)U^*$,

$$\text{tr}[U(\rho \otimes \sigma)U^* E(X_i) \otimes Z(X_j)] = \delta_{ij}p_{\sigma'}^Z(X_j) = \delta_{ij}p_\rho^E(X_i).$$

To close up this justification gap RQM poses the following postulate (see, e.g. [4, p.7]):

(LC) In the $S - S'$ -interaction one of the possible ($p_\rho^E(X_i) \neq 0$) values $E(X_i)$ is resolved into an actual value $E(X_k)$, say, relative to S' , and this accompanied with the assumption that the state of S with respect to S' is then $\rho^f(X_k)$.

We call this hypothesis a *local collapse*, a thing which happens to S only with respect to S' .²⁷

To underline the *ad hoc* nature of this postulate, let us recall that S and S' are just any two quantum systems brought into an interaction which correlates strongly the values of the chosen observables. In particular, there is no detector indicating the occurrence of a special result after the interaction.²⁸ The postulate (LC) simply stipulates that some E -value occurs in the interaction and due to the strong correlation the state of S w.r.t. S' is then an eigenstate of the corresponding E -value.²⁹ Though not explicitly mentioned in the original RQM literature, one might expect, in accordance with the below assumption (CPL), Sect. 2.8, that a similar thing happens to S' : the pointer Z takes the corresponding value, and the state of S' w.r.t. S is a corresponding eigenstate.

2.7 RQM – sequential perspectives

From the point of view of the world around S , the probabilities for the possible values of the observables of S (after the interaction) now vary depending on the perspective: in view of S' they are those coded in the state $\rho^f(X_k)$ whereas for any other system S'' they are those coded in ρ^f , that is, those arising from $U(\rho \otimes \sigma)U^*$, which is the state of $S - S'$ w.r.t. S'' .

²⁵ Equivalently, one may choose the pointer observable Z to be discrete. Including reading a scale gives more flexibility in considering the possible value assignments of discretized versions of continuous observables like position or momentum.

²⁶ If E is sharp, the orthogonality requirement is redundant.

²⁷ Dorato [16] calls this process a "primitive, mutual manifestation of dispositional properties". Dorato also writes [*ibid.* p. 260] that "the manifestation in question ought to be regarded as *de facto* irreversible; otherwise no stable measurements would be available".

²⁸ See also Brukner's paper "Qubits are not observers" [12].

²⁹ This assumption resembles another *ad hoc* assumption, called the eigenvalue-eigenstate link: if one knows the value of a quantity, then the state has to be a corresponding eigenstate.

As emphasized also in [1, 4, 7], this means that the observable E of S could take different values with respect to other perspectives.

Consider, in sequence, the interactions $S - S_1$ and $S - S_2$, modelled by the schemes \mathcal{M}_1 , and \mathcal{M}_2 , respectively, with the initial probabilistic information on S being coded in state ρ . Since the unitary operator U_i acts nontrivially only in the Hilbert space $\mathcal{H} \otimes \mathcal{H}_i$, $i = 1, 2$, the possibilities for the values of the observables E_i of S defined by the perspectives \mathcal{M}_i are, in succession, those coded in the states ρ , and $\mathcal{I}^{\mathcal{M}_1}(\Omega)(\rho) \equiv \rho_1^f$ ³⁰

Let us assume that the measurement schemes have been tuned to be repeatable with respect to the corresponding reading scales (X_i) , and (Y_j) . According to local collapse, in each case some of the possibilities $p_\rho^{E_1}(X_i) \neq 0$, and $p_{\rho_1^f}^{E_2}(Y_j) \neq 0$, will resolve into actual values, say $E_1(X_l)$, and $E_2(Y_m)$, with the state of S with respect to S_1 , and S_2 , being the normalized versions of $\mathcal{I}^{\mathcal{M}_1}(X_l)(\rho)$, and $\mathcal{I}^{\mathcal{M}_2}(Y_m)(\mathcal{I}^{\mathcal{M}_1}(\Omega)(\rho))$, respectively. We may have chosen all the perspectives such that they not only define the same observable, that is, $E = E_1 = E_2$, but even so that their instruments are the same, $\mathcal{I} = \mathcal{I}^{\mathcal{M}_1} = \mathcal{I}^{\mathcal{M}_2}$. Due to the repeatability property, the respective local (unnormalized) states of S are then $\mathcal{I}^{\mathcal{M}}(X_l)(\rho)$, and $\mathcal{I}^{\mathcal{M}}(Y_m)(\rho)$, with the corresponding probabilities $p_\rho^E(X_i)$ for the bin sets X_i , Y_m . Clearly, the values taken by the observable in question in this sequence of interventions, need not be the same.

This is to be compared with the fact that one may also consider the above sequence as a sequential (joint) intervention, with the following sequential probabilities:

$$\text{tr}[\mathcal{I}^{\mathcal{M}_2}(Y)(\mathcal{I}^{\mathcal{M}_1}(X)(\rho))],$$

with X , Y , being the bin sets in the respective value spaces. These probabilities define a bioobservable E_{12} of S whose possibilities in state ρ are just these probabilities. Using the dual instruments, this observable is

$$\begin{aligned} E_{12}(X, Y) &= \mathcal{I}^{\mathcal{M}_1}(X)^*(\mathcal{I}^{\mathcal{M}_2}(Y)^*(I)) \\ &= \mathcal{I}^{\mathcal{M}_1}(X)^*(E_2(Y)), \end{aligned}$$

with the marginal observables

$$\begin{aligned} E_{12}(X, \Omega) &= E_1(X) \\ E_{12}(\Omega, Y) &= \mathcal{I}^{\mathcal{M}_1}(\Omega)^*(E_2(Y)). \end{aligned}$$

If, again, the measurement schemes as well as the reading scales are chosen as above, then these probabilities are simply

$$\text{tr}[\mathcal{I}^{\mathcal{M}_2}(Y_j)(\mathcal{I}^{\mathcal{M}_1}(X_i)(\rho))] = \text{tr}[\mathcal{I}^{\mathcal{M}}(Y_j \cap X_i)(\rho)] = \delta_{ij} \text{tr}[\mathcal{I}^{\mathcal{M}}(X_i)(\rho)],$$

with $\mathcal{I}^{\mathcal{M}} = \mathcal{I}^{\mathcal{M}_2} = \mathcal{I}^{\mathcal{M}_1}$, and $E_{12}(X_i, Y_j) = E(X_i \cap Y_j) = \delta_{ij}E(X_i)$. Again, by local collapse, one of these possibilities gets resolved in the process, the bioobservable E_{12} takes a value $E_{12}(X_k, X_k)$, say, and the state of S with respect to the system $S_1 - S_2$ is (the normalized form of) $\mathcal{I}^{\mathcal{M}}(X_k)(\rho)$.

There is, however, no conflict between the two views, since their perspectives are different. In the sequential case, we have a (joint) perspective monitoring the possibilities of the

³⁰ For simplicity, we omit the possible time evolution between the interventions and assume that the observables E_i have the same outcome space Ω (usually $\Omega = \mathbb{R}$).

biobservable E_{12} of S in the state ρ , whereas in the first case we have two different perspectives monitoring (separately in succession) the S -observables E_1, E_2 , in the respective states ρ, ρ_1^f .

Finally, we return briefly to the question of correlations coded in the state $U(\rho \otimes \sigma)U^*$ arising from a repeatable measurement scheme $(\mathcal{H}', U, \sigma, Z)$ of E (w.r.t. a fixed reading scale). Since now $p_{U(\rho \otimes \sigma)U^*}^{E \otimes Z}(X_i, X_j) = 0$, for all $i \neq j$, any suitable perspective would see (according to RQM) one of the possible pairs $(E(X_i), Z(X_i))$ as the 'value' of the $S - S'$ observable $E \otimes Z$. Clearly, there is no reason to expect that the pair of values taken by $E \otimes Z$ with respect to the chosen perspective is the same as the value $E(X_k)$, say, of E taken by the perspective $(\mathcal{H}', U, \sigma, Z)$, a value strongly correlating with the value $Z(X_k)$ of Z . However, in [3, Sect. 2.6] one reads:

(S) Prima facie, RQM may seem to imply a form of perspective solipsism, as the values of variables realized in the perspective of some system S' are not necessarily the same as those realized with respect to another system S'' . This is however not the case, as follows directly from quantum theory itself. The key is to observe that any physical comparison is itself a quantum interaction. Suppose the variable E of S is measured by S' and stored into the variable Z of S' . This means that the interaction has created a correlation between E and Z . In turn, this means that a third system measuring E and Z will certainly find consistent values. That is: the perspectives of S' and S'' agree on this regard, and this can be checked in a physical interaction.

In our reading, this is a new independent assumption and it appears to be incompatible with the preceding ideas trying to exhibit the assumption (LC) of local collapse. Pienaar [11] calls this the assumption of *shared facts* and he also argues that it is inconsistent with the rest of the RQM assumptions, as understood by him. The claim that according to RQM "we all 'see the same world'" [3, Sect. 2.6] is not supported by our reading of the underlying assumptions constituting RQM. This conclusion is also in line with van Fraassen [8, Sect. 5.4] who, in his attempt to understand RQM, finds it necessary to propose an additional postulate in RQM to reach concordance between different observers.

2.8 RQM – cross-perspective links

To remedy the above defect Adlam and Rovelli have recently completed RQM with the postulate of *cross-perspective links* [7, Definition 4.1], which reads:

(CPL) In a scenario where some observer Alice [S'] measures a variable V [E] of a system S , then provided that Alice does not undergo any interactions which destroy the information about V [E] stored in Alice's physical variables [presumably Z], if Bob subsequently measures the physical variable [Z] representing Alice's information about the variable V [E], then Bob's [S''] measurement [presumably, with pointer Z'] result will match Alice's measurement result.³¹

To spell out this assumption, let $(\mathcal{H}', U, \sigma, Z)$ describe the measurement scheme, called Alice, of the variable E of S with the probabilities for the possible values of E being coded

³¹ The wording of this postulate is somewhat unfortunate in view of the basic ideology of RQM since it may mislead one to think that the quantum systems S' and S'' , called Alice and Bob, would possess some extraordinary properties beyond their pure quantum nature. See also [12].

in state ρ so that $p_\rho^E(X_i) = p_{\sigma_i'}^Z(X_i)$ for any $X_i \in \mathcal{R}$ (a fixed reading scale), and assume that the value X_k , say, has occurred. By the local collapse, the state of S with respect to S' is then $\rho^f(X_k)$. On the other hand, the states of S and S' with respect to any other system S'' should be ρ^f and σ^f , that is, those arising from $U(\rho \otimes \sigma)U^*$ as partial traces. Let us assume that Bob, given as the scheme $(\mathcal{H}'', U', \pi, Z')$, now measures the S' -variable Z using an interaction U' that couples only the systems S' and S'' . Thus the probabilities for the possible values of Z are coded in the state σ^f so that $p_{\sigma_i'}^Z(X_i) = p_{\pi_i'}^{Z'}(X_i)$ for all $X_i \in \mathcal{R}$ (Bob using the 'same' reading scale). Note that this is not a sequential measurement of S but a measurement on S (by $(\mathcal{H}', U, \sigma, Z)$) followed by a measurement on S' (by $(\mathcal{H}'', U', \pi, Z')$). Since we now have

$$p_\rho^E(X_i) = p_{\sigma_i'}^Z(X_i) = p_{\pi_i'}^{Z'}(X_i),$$

for any X_i , the conclusion that "if Alice recorded a result X_k , then the result of Bob has to match with that" seems to presuppose that in addition to the local collapse $\rho \mapsto \rho^f(X_k)$, the state of S' with respect to S'' has to be collapsed to $\sigma^f(X_k)$.³² It is a basic feature of quantum mechanics, there is no way for Bob to get with certainty the result Alice recorded unless the state of $S + S'$ w.r.t. S'' is an eigenstate of $I \otimes Z(X_k)$, that is, the state of S' w.r.t. S'' is an eigenstate of $Z(X_k)$. The meaning of the provision in (CPL) remains unclear to us.

This is to be compared with the three different cases discussed in Sect. 2.7, with assuming that Bob instead performs one of these measurements. Without repeating the whole discussion, we just note that if Bob realises, say, a joint measurement of $E \otimes Z$ on $S - S'$ in state $U(\rho \otimes \sigma)U^*$, any of the possible results (X_i, X_i) could be obtained, anyone with probability $p_\rho^E(X_i)$. Without assuming that the postulate of cross-perspective links holds also in this case, the conclusion that "we all 'see the same world'" is still unjustified. For the result of Bob to get matched with the result of Alice it is necessary the state of $S - S'$, with respect to Bob, S'' , needs to be such that the result $(E(X_k), Z(X_k))$ is certain, that is, the state of $S - S'$ with respect to S'' is such that its partial states are $\rho^f(X_k)$ and $\sigma^f(X_k)$. If one of them is pure, then that state is just the product $\rho^f(X_k) \otimes \sigma^f(X_k)$. Since S'' is arbitrary, this makes the local collapse to be global.

3 Axiomatic reconstruction and RQM: some remarks

In his initial 1996 RQM paper, Rovelli bases his approach on two main ideas:

1. That the unease [with the text book quantum mechanics] may derive from the use of a concept which is inappropriate to describe the physical world at the quantum level. I shall argue that this concept is the concept of observer-independent state of a system, or, equivalently, the concept of observer-independent values of physical quantities. [1, p. 1639]
2. That quantum mechanics will cease to look puzzling only when we will be able to derive the formalism of the theory from a set of simple physical assertions ("postulates", "principles") about the world. Therefore, we should not try to append a reasonable interpretation to the quantum mechanics formalism, but rather to derive the formalism from a set of experimentally motivated postulates. [1, p. 1639]

³² Clearly, it would be enough that the state of S' w.r.t. S'' would be an eigenstate of $Z(X_k)$.

Since the pioneering works of von Neumann [30] and Birkhoff and von Neumann [31] a lot of effort has gone to searching for a systematic, conceptually clear and mathematically rigorous reconstruction of quantum mechanics from physically motivated axioms. This has led, in particular, to the extensive theories of quantum logics and convex operational or generalized probabilistic theories. From the massive literature we mention only the classic monographs of Mackey [32], Varadarajan [33], Piron [34], Mittelstaedt [35], Beltrametti and Cassinelli [36], and Ludwig [37, 38]. Each of these approaches, when appropriately supplemented, leads to a conceptually clear, puzzle-free interpretation of quantum mechanics in accordance with the second of the above two ideas.³³ The plurality of the existing conceptual frameworks may, however, suggest that the epic question "how could the world possibly be the way [quantum] theory says it is?" [40, p. 4] will not receive a concerted answer. The powerful framework of generalized probabilistic theories has shown the usefulness of the dual concepts of states (as equivalent classes of preparations) and observables (as equivalent classes of measurements), which gives weight to the boldness of the first of the above ideas.³⁴

In recent years, many of the building blocks of these approaches have been slightly modified, reformulated, and improved to fit better for the needs of quantum information theory. Several reconstructions of the Hilbert space theory of finite level systems have thus been achieved, see, for instance, [41–45]. Finite level systems, typically qubits, suffice for most of the needs of quantum information technologies but leave out most of quantum physics. It remains to be seen if such reconstructions could be extended to reconstruct full quantum mechanics, and thus support a systematic interpretation of the whole quantum mechanics.

Here we shall briefly comment on some of such reconstructions which derive, at least partly, their motivations from the ideas of RQM.

Instead of developing systematic RQM, Rovelli outlines the starting point of such a reconstruction along the quantum logic frame indicating two of its basic postulates. In such a frame, a reconstruction is based on a set L of events (called also questions, or propositions, or experimental functions, or decision effects, or properties, depending on the point of view) concerning a physical system S , and one aims to provide a physically motivated mathematical structure for L such that, in the end, L could be identified with the projection lattice of an appropriate Hilbert space.

Without entering into full details, it suffices to recall that the two postulates of [1] concerning the "event structure" are:

Postulate 1 (Limited information). There is a maximum amount of relevant information that can be extracted from a system;

Postulate 2 (Unlimited information). It is always possible to acquire new information about a system.

Assuming that L has the structure of an orthocomplemented lattice, Grinbaum [46] formalizes the notion of "relevant information" as follows:

³³ As an example of some such complements, see, for instance [39].

³⁴ Note, however, that some of the axiomatic approaches, notable those of [34] and [35], do not take the notion of a state as a primitive concept.

an event $b \in L$ is *irrelevant* with respect to an event $a \in L$ if $b \wedge a^\perp \neq 0$; otherwise, b is relevant with respect to a , that is, if $b \wedge a^\perp = 0$.

For any $a, b \in L$, if $b \leq a$, then b is relevant to a . We recall that an orthocomplemented lattice is Boolean if all its disjoint elements, $a \wedge b = 0$, are also orthogonal, $a \leq b^\perp$, in which case a is relevant to b^\perp and b is relevant to a^\perp . In a non-Boolean lattice disjointness does not imply orthogonality and one may find mutually incompatible $a, b \in L$ such that both b and b^\perp are relevant to a .

Using a characterization of the orthomodularity given in [47], it is then shown [46] that Postulate 1, when interpreted strictly (see below), implies that L is orthomodular, that is, for any $a, b \in L$, if $a \leq b$ then $b = a \vee (b \wedge a^\perp)$. The proof depends on interpreting ‘‘maximal amount of relevant information’’ to mean that L is of finite length, that is any maximal chain $0 < a < b < \dots < 1$ has finitely many elements. The next example shows that this is an essential part of the proof.

Example 1 [39, Example 3] Consider the infinite dimensional vector space $V = \ell^2(\mathbb{Q})$ of the square summable sequences of rational numbers $q = (q_1, q_2, q_3, \dots)$ with the Hermitian form $f(q, p) = \sum_{i=1}^\infty q_i p_i$. The lattice $L_f(V)$ of f -closed subspaces is a complete, orthocomplemented, irreducible AC lattice of infinite length. Since the space V is not complete, it follows from a theorem of Solér [48] together with [49, Theorem 2.8.], that neither the lattice $L_f(V)$ nor the space V is orthomodular.³⁵ The vectors $e_1 = (1, 0, \dots, 0, \dots), \dots, e_n = (0, \dots, 0, 1, 0, \dots), \dots$ form an orthonormal basis in V and one may construct an increasing sequence of f -closed subspaces $V_1 = [e_1], V_2 = [e_1, e_2], \dots, V_n = [e_1, e_2, \dots, e_n], \dots$ such that the preceding is relevant with respect to the following one. Here V_n is the (n -dimensional, and thus f -closed) subspace generated by the vectors e_1, \dots, e_n . Clearly, this sequence is unlimited, showing that the above proof of orthomodularity is valid only under the assumption L has a finite length.

Along with $L_f(\ell^2(\mathbb{Q}))$, the projection lattice $\mathcal{P}(\mathcal{H})$ of a complex separable infinite dimensional Hilbert space \mathcal{H} contains plenty of sequences of mutually relevant events. In addition to such ordered sequences, consider, as an example, the spectral projections of the (canonical) position and momentum Q and P . They have the following properties: for any bounded sets $X, Y \in \mathcal{B}(\mathbb{R})$,

$$Q(X) \wedge P(Y) = Q(X) \wedge P(Y)^\perp = Q(X)^\perp \wedge P(Y) = 0,$$

showing that all such $P(Y)$ and $P(Y)^\perp$ are relevant to any $Q(X)$, and the other way round. □

This example suggests that the idea of ‘‘maximal amount of relevant information’’ of Postulate 1 should, perhaps, be interpreted as ‘‘maximal amount of mutually compatible relevant information’’ and ‘‘maximality’’ not to be restricted to the finite case.

As concerns Postulate 2, it is seen to imply, together with Postulate 1, that L is non-Boolean, perhaps, even irreducibly so, see also [50].

Despite the attractive nature of the two postulates, the (possible) implication that L is of finite length is ruling out most of quantum physics. Also, though the orthomodularity and the non-Boolean character of L are necessary, there are many alternative ways to get these properties, and, more importantly, there is still a long way to reach the Hilbert space

³⁵ A Hermitian space V is orthomodular if for each $M \in L_f(V)$, $M + M^\perp = V$.

realization for the event structure, see, for instance [39]. Finally, there is no hint how such a reconstruction might help to justify a relational interpretation of quantum mechanics, especially its core idea of ‘local collapse’, let alone the assumption of ‘cross-perspective links’.

A systematic information theoretic reconstruction of the N -qubit quantum theory, which derives some of its motivation from relational interpretation, is worked out in great detail in [45, 51]. Instead of following the GPT-approach, the authors construct a slightly different general frame of ‘landscape of qbits theories’ $(\mathcal{Q}, \Sigma, \mathcal{T})$, where $\mathcal{Q}, \Sigma, \mathcal{T}$ are, respectively, the sets of all possible (relevant) questions, states, and time evolutions describing the physical system S under consideration. The authors pose 4 [51] plus 1 [45] principles of information acquisition which determine the structure of the triple $(\mathcal{Q}, \Sigma, \mathcal{T})$ to be the familiar Hilbert space theory based on the N -fold tensor product of the qubit Hilbert space \mathbb{C}^2 . The first two of these principles are conceptually motivated by the above two postulates of Rovelli, whereas the other three formulate the requirements of information preservation (in-between interrogations), time evolution (of the catalog of knowledge, states), and question unrestrictedness (the physical realizability of the (relevant) questions). Even though this approach shares many features of RQM, there is, however, a fundamental difference: in RQM all the interacting systems S and S' are treated on the same basis as quantum systems, whereas here the observing system S' is a classical system (not described), able to detect the answers to the posed questions. A possible value of a physical quantity of the system S is here resolved into an actual value through a classical observer’s detection, an idea contradicting the very starting point of RQM. The so-called measurement problem is hereby ‘resolved’ by simply assuming that the state of the system S after interrogating a question ‘collapses’ according to the yes/no –answer to the relevant ‘eigenstate’.

Finally, an axiomatic reconstruction of quantum mechanics gives us mathematically coherent and conceptually clear, puzzle-free interpretations of quantum mechanics. Still, there remains the question of how the abstract Hilbert space theory should be applied in concrete cases, like atoms and molecules, or, say, photons. To the best of our knowledge, the most successful approach starts with first characterising the Poincaré and the Galilei invariant isolated and elementary quantum objects within the abstract Hilbert space theory and then extends this analysis to mutually interacting objects, a method initiated by the seminal papers of Wigner [52], Bargmann [53], and Mackey [54], and further developed, for instance in the monographs of Mackey [55], Varadarajan [33], and Ludwig [37], as well as in Cassinelli *et al.* [56]. Such an approach does not easily fit with the basic ideology of RQM.

In *Helgoland: making sense of the quantum revolution* we read that “[r]elational QM is a consistent interpretation of quantum theory”. In our reading of the basic assumptions of the relational interpretation of quantum mechanics, which goes along the studies of van Fraassen [8], Laudisa [10], and Pienaar [11], such a conclusion appears premature. With adding the postulate of cross-perspective links, Adlam and Rovelli [7] conclude that the thus updated RQM “guarantees intersubjective agreement between observers when they perform measurements on one another”. This may well be the case. However, in our reading, the price is a somewhat implicit return to the controversial projection postulate which is a crude way to avoid the measurement or objectification problem of quantum mechanics. We may thus close with the words of Laudisa [10] that “we have reasons to be much less optimistic toward the prospects of RQM: a lot of work needs to be done before RQM may aspire to become a satisfactory interpretational framework for the main foundational issues in QM”.

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Noted added in proof During the submission procedure of this paper further critical studies of RQM have been presented. While Lawrence, Markiewicz and Żukowski [57] argue that RQM is not an interpretation of quantum mechanics but a different theory, Huber [58] deduces on the given premises that either RQM is inconsistent or a disguised form of the many worlds interpretation. Stacey [59], on the other hand, casts serious doubts on the ability of the “cross-perspective link” to serve its purpose. This all adds weight on a call for a coherent reformulation of the relational interpretation of quantum mechanics.