DOI: 10.30570/2078-5089-2018-91-4-106-121

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VOTING THEORY: CUI BONO?[1]

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[1] The author gratefully acknowledges the constructive comments of the referee on an earlier version.

Abstract. The theory of voting has largely developed independently of the mechanism design research, but with the introduction of the concept of strategic voting the two traditions found a common ground. This happened some fifty years ago. Yet, despite the voluminous literature that has emerged since then, the impact of voting theory on the design of political institutions remains marginal. Often the assumptions are deemed too simplistic or too abstract or plainly "out of this world". It looks as if there is a demand for research that aims at building bridges over the wide gap that exists between the abstract social choice results and the behavioral-institutional realities characterizing political systems of today and tomorrow. We illustrate the applicability problems by discussing a relatively recent proposal for electoral reform of the single-member constituency system in electing the members for the House of Representatives in the United States. The proposed reform would seem to solve a major flaw in the existing system. As is often the case, this comes with a price, though: the proposal is plagued with problems of its own. However, the voting theory results have a wide area of applicability beyond voting. Yet the applicability of the voting theory results in these areas have remained largely unexplored. This article aims at suggesting some applications. Most straightforward ones pertain to multiple criteria decision making.

Keywords: social choice, incompatibility theorem, participation, Condorcet extension, electoral reform

Introduction

Voting theory is a somewhat ambiguous concept and it is, therefore, appropriate to start by asserting what is meant by it in the present consent. It is a theory that focuses on how the opinions of individuals are transformed into collective opinions through voting procedures. Obviously, collective opinions are often achieved through other mechanisms than voting (e.g. through negotiation and bargaining or simply through imposition), but our focus is on voting. The theory of voting sometimes refers to the study of the behavioral aspects of elections such as to the studies on the determinants of voting behavior (socio-economic background variables, attitudes towards minorities, etc.). In this article we adopt the first mentioned interpretation and look at voting from the institutional point of view. This means that we focus attention to the performance of voting procedures as opinion aggregations devices: what properties do they have as opinion-aggregating devices?

The main corpus of the theory that focuses on voting is the social choice theory. It studies the rules that connect individual opinions with social outcomes, be these either elected candidates, selected policy options or multi-member bodies. The starting point of the modern social choice theory is usually dated in the late 1940's and early 1950's when Duncan Black, Kenneth May and Kenneth Arrow presented their path-breaking works on the subject[2]. While this dating is correct insofar as the continuous social choice tradition goes, the precursors of the theory can be found in 13th and 15th century (Ramon Llull, Nicolaus Cusanus) and most certainly in the late 18th century just prior to the Great French Revolution (Jean-Charles de Borda, marquis de Condorcet)[3]. Several important contributions were also made in the latter half of the 19th century[4].

[2] Black 1948; Arrow 1951; May 1952, 1953.

[3] The history is presented in some detail in McLean and Urken 1995. See also Szpiro 2010; Tangian 2014.

[4] Edward John Nanson and Charles Lutwidge Dodgson; both included in McLean and Urken 1995.

The main difference between the earlier contributions and the post-1940's ones is that the former were primarily preoccupied with a problem related to a specific voting procedure and presented ways of rectifying the problem, while the latter open a more abstract perspective in defining various desirable properties (desiderata) of voting procedures and providing results on the compatibility or incompatibility of the desiderata. Alongside the work producing those results some research effort has also been invested in finding out the choice-theoretic properties on specific voting procedures.

Over the past seven decades a voluminous literature has emerged on the desirable and undesirable features related to voting rules. Some apparently negative findings have been dubbed paradoxes. Usually these pertain to situations — real or fictitious — where the choice resulting from the application of a given procedure is counterintuitive, surprising or bizarre, given the reported opinions of the individuals. A case in point is Borda's paradox which occurs when a group of voters using the plurality voting elects a candidate that would be defeated by a majority of votes in every possible pairwise election of the candidates. What makes this paradoxical is the intuitive idea that what is best in the view of a plurality of voters is *eo ipso* the best in all pairwise comparisons where it is present. The mere possibility of this kind of paradox led Borda to propose the abandonment of the plurality voting system and to suggest another procedure, today known as the Borda count, to take its place. As reported by Black[5] Borda had short-term success in the French Academy which adopted the Borda count for the election of its officers. However, this success was not to last long. In the beginning of the 19th century the Borda count was overturned.

[5] Black 1948.

Borda's paradox is but one early example of a result that was supposed to have implications for the choice of an election procedure. Yet, the evidence of those results having significant impact on electoral reforms or committee voting procedures is scant. Age-old systems are being used despite the results that show them to be vulnerable to various kinds of anomalies and paradoxes. In the author's experience the typical reaction of a politician or administrator to the results of social choice theory is initial interest followed by disbelief, amusement and eventually nonchalance. The correctness of the results themselves is seldom questioned. Those practitioners who maintain their interest in

the results are more likely to doubt the validity of the underlying assumptions about decision settings.

The main issues addressed in this article are:

1. Given that many procedures that fail on quite important social desiderata are widely used (e.g. plurality runoff voting), are there good reasons for ignoring the results?
2. Given that many social choice desiderata are incompatible with other such properties, are there fundamental properties that all reasonable voting systems should possess?
3. Is the community of scholars unanimous about these?

In what follows we shall first remind the reader of some of the best-known voting procedures for electing a single winner. We then briefly describe some well-known social choice results whereupon we revisit the plausibility of some reactions of the practitioners. We then move on to consider what we deem as the essential social choice desiderata. Thereafter we illustrate the theory-practice relation by discussing the relatively recent choice-theory motivated proposal of two Nobel Laureates for electoral reform in the United States. The final section provides some conclusions.

Some voting procedures

The following procedures have been introduced and discussed by many authors[6].

1. Plurality voting: each voter votes for one alternative and the winner is the one that gets more votes than any other.
2. Plurality runoff voting: voting as above. If one alternative gets more that 50% of the votes, it is elected. Otherwise a runoff contest takes place between the two largest vote-getters, whereupon the one with more votes is declared the winner.
3. Condorcet extension[7]: any system that results in the Condorcet winner when one exists. (Condorcet winner is an alternative that defeats all the others in pairwise comparisons by a majority of votes.)
4. The Borda count: each candidate is given a score on the basis of its position in individual rankings: the lowest rank gives *0* points, the penultimate rank *1* point, ... , the first rank $k-1$ points (if the no. of alternatives is k). The score of an alternative is the sum of the points it has been given by the voters. The alternative with the largest score is the winner.
5. The approval voting: each voter gives any alternative either *0* or *1* votes (approvals). The alternative with largest vote sum is the winner.
6. Copeland's method: for each alternative conduct all $k-1$ pairwise comparisons. The alternative's Copeland score is the number of alternatives it defeats by a majority of votes. The alternative with the largest score is the winner.
7. Dodgson's method: given a preference profile, determine for each alternative the minimum number of pairwise preference inversions needed to make that alternative the Condorcet winner. The alternative that needs a smaller number of preference switches than any other is the winner.

[6] E.g. Richelson 1979; Straffin 1980; Riker 1982; Nurmi 1987; Felsenthal 2012; Felsenthal and Nurmi 2018.

[7] The Condorcet extension is not a procedure but a set of procedures all characterized by a common property, viz. they elect the Condorcet winner when there is one in the profile of preferences under consideration.

8. Kemeny's median: given a preference profile generate all *k!* preference rankings and select the one that is closest to the given profile in the sense that the sum of pairwise agreements between the given preference rankings and the generated one is the largest.
9. The maximin method: determine for each alternative its minimum support in all pairwise comparisons. The alternative with the largest minimum support is the winner.
10. Black's method: if there is Condorcet winner in the given profile, it is the winner. Otherwise, the result is the same as in the Borda count.
11. Majority judgment: each alternative is assigned by voters a grade, i.e. a value from an ordinal scale (say, unacceptable, poor, fair, good, excellent) and for each alternative its median grade is determined. The alternative with the highest median grade is the winner.
12. Baldwin's rule: this is a Borda elimination system where an alternative with the lowest Borda score is eliminated at each round and the Borda scores re-computed for the remaining alternatives until the winner is found.
13. Nanson's method: otherwise the same as Baldwin, but at each stage the alternatives with the average or smaller Borda score are eliminated.
14. Schwartz' method: choose the smallest set of alternatives such that no alternative outside the set beats (by a majority) any alternative inside the set in pairwise comparison.
15. Young's method: choose that alternative that can be made the Condorcet winner by ignoring the preference rankings of the smallest number of voters.
16. The successive elimination: conduct in total $k-1$ pairwise comparisons according to a predetermined agenda so that at each comparison, the loser is eliminated and the winner faces the next alternative on the agenda. The winner of the $k-1^{th}$ comparison is the overall winner.
17. The alternative vote: a system where the winner is the alternative ranked first by the majority of voters. If no such alternative exists at the outset, one eliminates the alternative ranked first by the smallest number of voters and proceeds as if these were not present at all. The process continues until an alternative is found that occupies the first position in the rankings of a majority of voters.
18. Coombs' method: otherwise precisely like alternative vote, but the criterion for eliminations is the number of last ranks associated with each alternative: the one with the largest number of those is eliminated.
19. Range voting: each voter can assign any value from a range of positive real numbers to each alternative. The assigned numbers are summed for each alternative and the winner is the alternative associated with the largest sum.

The above systems constitute a vast majority of those used in choice situations where one and only one candidate or policy alternative is to be chosen. About half of them are Condorcet extensions (Copeland, Dodgson, Kemeny, maximin, Black, Baldwin, Nanson, Schwartz, Young, successive elimination).

Some classical results in brief

By far the best-known result in modern social choice theory is Arrow's impossibility theorem. Its first version was presented in 1951, but the following is its 1963 form[8].

Arrow's impossibility theorem (1963). No social welfare function satisfies the following conditions:

- (U) unrestricted domain, i.e. the function is to be defined for all possible n- tuples of individual preference relations;
- (I) independence of irrelevant alternatives, i.e. the collective preference between any two alternatives depends on the individual preferences between these two and on nothing else;
- (P) Pareto, i.e. if all individuals prefer one alternative to another, then so does the collectivity;
- (D) non-dictatorship, i.e. there is no individual whose preference coincides with the collective preference on all pairs of alternatives, regardless of the preferences of other individuals.

Social welfare functions assigns to each n-tuple of connected and transitive individual preference relations a (collective) connected and transitive preference relation. In other words, such a function provides not only the set of winning candidates (or policy alternatives, as the case may be), but a ranking over the set of candidates.

Since the conditions U, I, P and D are often regarded as plausible properties, the theorem is of negative nature. It should be observed, though, that no voting procedure actually being used satisfies the condition I[9]. Hence, this condition is not generally felt compelling.

Another well-known result is known as the Gibbard-Satterthwaite theorem since it was independently proven by Alan Gibbard and Mark Satterthwaite[10]. It is based on two definitions. Firstly, a social choice function is manipulable (by individuals) if and only if there is a situation (that is, a set of alternatives and an n-tuple of individual preferences) and an individual so that the latter can bring about a for him/her preferable outcome by preference misrepresentation rather than by truthful revelation of his/her preference ranking, ceteris paribus. Secondly, a social choice function is non-trivial (non-degenerate) if and only if for each alternative x, there is a preference profile so that x is chosen.

Gibbard-Satterthwaite theorem. Every universal and non-trivial resolute social choice function is either manipulable or dictatorial.

In other words, when any resolute social choice function is applied a situation may be encountered where at least one individual gets a better outcome for himself/herself by misrepresenting his/her preferences. Strictly speaking this theorem is not widely applicable since e.g. all the above 18 procedures can result in a tie between two or more alternatives. Hence, they are not resolute in the sense of the Gibbard-Satterthwaite theorem. By focusing on the procedures one by one, it can, however, be shown that the theorem applies to them as well. In fact, shortly after the publication of the theorem Peter Gärdenfors proved that all Condorcet extension social choice functions (i.e. those that are not necessarily single-valued) are manipulable[11].

[8] *Arrow 1963. After Arrow's theorem a plethora of other similar theorems have been presented (see e.g. Kelly 1978; Aleskerov 1999).*

[9] *This is not strictly accurate, viz. there are procedures aggregating (e.g. averaging or taking median values) cardinal scores that satisfy I. Thus, for example the majority judgment procedure satisfies condition I. I am grateful to the referee for pointing this out.*

[10] *Gibbard 1973; Satterthwaite 1975.*

[11] *Gärdenfors 1976.*

One of the basic rationales of democratic decision making is that the individuals benefit from taking part in collective decision making. In social choice theory this rationale is expressed as a principle known as participation. An important result by Herve Moulin[12] shows that participation is incompatible with the Condorcet winner criterion.

[12] *Moulin 1988.*

Moulin's theorem. If there are more than three candidates and at least 25 voters, no Condorcet extension satisfies the participation condition[13].

[13] *The minimum number of voters has subsequently been reduced to 12 by Brandt, Geist, and Peters 2017.*

For those who are convinced about the strength of the Condorcet winner criterion, this is certainly bad news. In fact, Joaquín Pérez showed that most Condorcet extensions suffer from the strong version of the no-show paradox which occurs when a group of identically-minded voters, by abstaining, brings about the best outcome from their point of view, whereas their voting according to their preferences, *ceteris paribus*, would result in a less preferred outcome[14].

[14] *Pérez 2001.*

These are but a small sample of the numerous incompatibility results achieved in social choice theory. The point here is to stress the nature of the results. Firstly, they state that it is possible that outcomes of voting are encountered such that, while they satisfy the requirements of one desideratum, they do not satisfy those of another desideratum. Secondly, the results point to a possibility, not a necessity of such outcomes. Thirdly, they are of general rather than context-specific nature. This means that under some constraints regarding the profiles that are admissible, it may well be that the incompatibility is transformed into a compatibility in the constrained domain. Fourthly, the results say nothing at all about the frequency of incompatible outcomes.

Sen and Maskin on electoral reform

On the basis of these remarks one might argue that the social choice theory should play only a subordinate role in the design of real world voting procedures. Yet, there are cases where one can argue on the basis of social choice criteria that some proposal for electoral reform is unlikely to bring about improvement in mapping the voters' preferences into collective choices. We illustrate this point by a relatively recent proposal by Eric Maskin and Amartya K. Sen for replacement of the U.S. electoral system which is largely based on the plurality voting system in single-member constituencies[15].

[15] *Maskin and Sen 2016. For a more extensive and scholarly account by the same authors, see Maskin and Sen 2014. For an earlier discussion focusing on the single transferable vote system, see Doron and Kronick 1977; Allard 1995; Nurmi 1997.*

In a nutshell, the proposal starts with the observation that the plurality rule (in primaries and in elections at large) may result in Borda paradoxes (see *Table 1*), i.e. in elected candidates that would possibly be defeated by all other contestants in pairwise majority contests. In *Table 1* and elsewhere in this article the columns represent voter preferences over the candidates so that the higher an alternative is located in the column, the more preferable it is from the point of view of the voters indicated by the column head. Thus for example, in *Table 1* four voters prefer candidate A to candidate C and the latter to candidate B.

In Maskin and Sen's proposal the voters are allowed to express their full preference rankings instead of indicating just their most preferred candidate. Given these rankings, it is easy to conduct pairwise majority comparisons of candidates. Now, sometimes the pairwise majority comparisons would lead to an indisputable winner, viz. the Condorcet winner. This would be declared the

Table 1 **Borda's paradox: the plurality winner (A) would be defeated by all other contestants (B and C) in pairwise comparisons by a majority**

4 voters	*3 voters*	*2 voters*
A	B	C
C	C	B
B	A	A

winner in Maskin and Sen's proposal. In those cases where a Condorcet winner does not exist in the reported profile, the proposal suggests that the plurality runoff system be used as a secondary (tie-breaking) device.

As it stands, the proposal is questionable for the following reasons: (i) the Condorcet winner is not always the intuitively most plausible choice; (ii) the Condorcet extensions are vulnerable to various paradoxes that do not afflict some other procedures; (iii) the absence of a Condorcet winner would leave us with a system (plurality runoff procedure) that is not a Condorcet extension, but nevertheless suffers from a similar paradox as the Condorcet extensions.

The following profile (*Table 2*) introduced by Peter C. Fishburn casts doubt on the universal plausibility of the Condorcet winner as the optimal choice[16].

[16] *Fishburn 1973: 147.*

Table 2 **Fishburn's example**

1 voter	*1 voter*	*1 voter*	*1 voter*	*1 voter*
D	E	C	D	E
E	A	D	E	B
A	C	E	B	A
B	B	A	C	D
C	D	B	A	C

Here the Borda winner E seems more plausible choice than the Condorcet winner D since the former has equally many first ranks as D, strictly more second and third ranks and no voter ranks it worse than third, whereas D is ranked next to last by one voter and last by one voter. Ergo: it is not always clear that the Condorcet winner is always the most plausible choice.

It is often stated that the Borda winner is highly unstable under modifications (additions or removals) of the alternative set. In other words, the winning alternatives will often differ when some alternatives are removed from the profile, ceteris paribus. The Condorcet winner in contrast does not change if alternatives are removed from a given profile. However, the Condorcet winner is unstable under another type of profile modification that intuitively speaking should not make any difference in the voting outcomes, viz. adding or removing a group of voters whose preferences constitute a perfectly symmetrical Condorcet cycle[17]. An example of such cycle is shown in *Table 3*.

[17] *See Saari 1988, 1990; Nurmi 1999.*

Table 3 A perfectly symmetrical Condorcet cycle with 9 voters

3 voters	*3 voters*	*3 voters*
A	B	C
C	A	B
B	C	A

Each alternative in *Table 3* is ranked first, second and third by equally many voters. So, the profile suggests that the outcome should be a tie consisting of all three alternatives. So then, removing this kind of "component" from any larger profile or adding it to some profile should not change the winners. Yet, adding such a profile to another one with a Condorcet winner may well dispose the winner. Consider the profile of *Table 4*.

Table 4 A profile with a strong Condorcet winner

5 voters	*3 voters*
A	B
B	C
C	A

Here A is not just a Condorcet winner, but a strong one at that since it occupies the first rank of more than 50% of the voters. So, one would expect that adding the profile of *Table 3* to create a 17-voter profile would result in A remaining the Condorcet winner. This is, however, not the case. Instead of A, B is the Condorcet winner in the augmented profile. We notice that B remains the stable Borda winner under the addition of the Table 4 profile to the Table 3 profile. Hence, it seems that the Borda count may produce more stable outcomes than the pairwise majority voting systems. In sum, while Condorcet winners tend to be stable outcomes with respect to adding or removing candidates, they are not stable under adding or removing voters.

In the absence of a Condorcet winner Maskin and Sen propose the use of the plurality runoff system. This has its advantages, but due to its serious flaws cannot be considered a plausible method even as a tie-breaker. Firstly, it is not monotonic (see *Table 5*).

Table 5 Plurality runoff is not monotonic

6 voters	*5 voters*	*4 voters*	*2 voters*
A	C	B	B
B	A	C	A
C	B	A	C

After the runoff between A and B, the outcome in *Table 5* election is A, but if A gets some additional support, ceteris paribus, so that the 2 BAC voters become ABC voters, A is no longer the winner (C is). Hence additional support, other things being equal, may turn winners into non-winners in the plurality runoff procedure.

Unlike many other positional procedures, the plurality runoff suffers from a particularly serious version of the no-show paradox known as the P-BOT paradox[18]. An instance of the paradox occurs in profiles where a candidate, say x, wins, but when a group of identically-minded voters all ranking another candidate, say y, last joins the electorate, ceteris paribus, then y wins. In other words, the abstaining voters in the original profile are clearly better off abstaining since by voting according to their preferences they bring about the victory of the (in their opinion) worst candidate who would not be elected if they would abstain. The following 14-voter profile gives an instance of the P-BOT paradox (see *Table 6*).

[18] Felsenthal and Tideman 2013, 2014. For a comprehensive discussion on monotonicity-related properties, see Woodall 1997.

Table 6 An instance of the P-BOT paradox in plurality runoff voting

6 voters	*3 voters*	*5 voters*
A	B	C
B	C	A
C	A	B

Here C wins after the runoff between A and C. Suppose now that a group of 3 voters with an identical preference ranking as that of the original 3-voter group in the middle, i.e. BCA, joins the electorate which now consists of 17 voters. In this expanded electorate the runoff is between A and B, whereupon A wins. Yet, A is the last-ranked candidate of the 3 newcomers. Possibilities like this do not encourage voters to the polls, rather to the contrary.

But are there then systems that would guarantee the choice of a Condorcet winner, on the one hand, and satisfy participation condition, on the other? In view of Moulin's theorem described above, clearly no. An examination of various types of no show paradoxes reveals, however, that there are significant differences between Condorcet extensions in terms of their vulnerability to the no show paradoxes[19]. If the profiles are divided into two classes with the first including those where a Condorcet winner exists and the latter being characterized by the absence of the Condorcet winner, it turns out that all Condorcet extensions are invulnerable to the paradox whereby an additional support to the winner turns it into a non-winner. However, all Condorcet extensions may suffer from this paradox if the initial profile does not include a Condorcet winner. When it comes to the opposite kind of paradox, viz. one where by voting a group of voters may bring about the victory of their last ranked candidate, a majority of Condorcet extensions are vulnerable to it under both Condorcet and non-Condorcet domains. As an exception one can mention Young's rule. Thus, from the point of view of selecting a voting

[19] Felsenthal and Nurmi 2017: 83.

procedure that satisfies the Condorcet winner criterion and does quite well in terms of participation condition, this rule would seem a plausible candidate for electoral reform.

Opinions, however, differ with respect to the plausibility of the Condorcet winner criterion. Some of the reasons for doubting it have been presented above. Without debating this issue further, it is worthwhile to note that the suggested reform resorts to two separate logics for election: one for profiles with a Condorcet winner and the other for those profiles where no such candidate exists. Leaving aside the former logic, one may wonder the rationale for choosing the plurality runoff method for profiles without a Condorcet winner. There are quite a few others available. To take the simplest, the plurality rule would be an obvious alternative in that it is monotonic both in fixed and variable electorates. However, Maskin and Sen want to avoid the Borda paradoxes. Hence, this alternative is to be rejected. Yet, there is another one, viz. the Borda count. This would, thus, amount to using any Condorcet extension when a Condorcet winner exists and the Borda count when no such winner is to be found in the profile. A look at the list procedures we started with reveals that this procedure has in fact been proposed before by Black and is thus known as Black's method.

Would Black's method then be a more appropriate alternative for the current U.S. election system? From a theoretical point of view it would. To wit, the Borda count is not vulnerable to the no show paradoxes either in fixed or variable electorates. However, when combined with Condorcet extensions as in Black's method, the no show paradoxes re-appear as shown in *Table 7*.

Table 7 Black's method is vulnerable to the strong no-show paradox

voter 1	*voter 2*	*voter 3*	*voter 4*	*voter 5*
D	E	C	D	E
E	A	D	E	B
A	C	E	B	A
B	B	A	C	D
C	D	B	A	C

Here D is the Condorcet winner and, hence, is elected by Black's method. Now, suppose that voter 5 decides to abstain, ceteris paribus. Then the Condorcet winner disappears and E emerges as the Borda winner. It is thus elected by Black's method. Yet, E is the first-ranked alternative of the abstaining voter. Clearly this voter is better off abstaining.

Even so, Black's method also contains a mixture of two different winning intuitions: positional (stemming from Borda count) and binary (being a Condorcet extension). A worthwhile alternative to both Black's method and the proposal of Maskin and Sen is Nanson's method[20]. As explained above, it is a Borda elimination procedure that is based of Borda scores of candidates.

[20] *Nanson 1883.*

At first the Borda scores are computed for all candidates and then those with the average or smaller Borda score are eliminated. Maintaining the relative positions of the remaining candidates, the new scores are computed as if the eliminated candidates had not been present and again those candidates with the average or smaller Borda score are eliminated. The process continues until the winner is found.

The main motivation of the procedure is to guarantee the election of an eventual Condorcet winner. This follows from Nanson's insight according to which — although the Condorcet winner is not necessarily the Borda winner in a profile — there is a weak connection between Condorcet winners and their Borda scores: the Condorcet winner's Borda score is always strictly larger than the average Borda score. This can be proven in a pretty straightforward manner[21]. Thus, the elimination process guarantees that the eventual Condorcet winner is the winner of Nanson's procedure.

[21] *Nurmi 1987: 46.*

This comes, however, with a drawback: Nanson's method not only may lead to a no show paradox (as all Condorcet extensions, by Moulin's theorem), but is also nonmonotonic in fixed electorates[22]. This puts it basically in the same class as Maskin and Sen's proposal. An example of its vulnerability to the no show paradox is presented in *Table 8*[23].

[22] *Smith 1973: 1036—1037.*

[23] *Felsenthal and Nurmi 2017: 59—60.*

Table 8 Nanson's method and the no show paradox

5 voters	*5 voters*	*6 voters*	*1 voter*	*2 voters*
A	B	C	C	C
B	C	A	B	B
D	D	D	A	D
C	A	B	D	A

Here Nanson's method results in B. If one of the right-most voters abstains, C — the alternative ranked first by this abstaining voter — wins. Thus, the strong version of no-show paradox appears.

Are the properties important?

The discussion of Maskin and Sen's proposal illustrates the typical dilemmas facing the institution designer: to secure the satisfaction of one's favorite desiderata, one may have to sacrifice the fulfillment of other important criteria. The concentration on Condorcet extensions and their incompatibility to monotonicity-related properties can be defended on the grounds that the Condorcet winner seems a quite popular solution concept among the social choice theorists. Yet, participation and its cognates seem equally pertinent desiderata. After all, a failure on participation undermines the very rationale of voting. It is, however, one thing to establish an incompatibility between criteria and quite another to say that these conflicts in choices are necessary in many, most or even all profiles. Incompatibility is shown by constructing or finding

one instance (profile) where the choice resulting from the procedure under investigation satisfies one criterion, but fails on another. The incompatibility result as such says nothing at all about the frequency with which one can expect the incompatible choices to be encountered in actual choice making bodies. The reason for this is that we in general do not know what kinds of profiles will emerge. Very often we do not even know which profiles have emerged in the past for the simple reason that voters have not been asked to provide this sort of information. Typically, just one name or number is written in the ballot slip and this is all that we know about the voter's preferences.

Hence, it could be argued that designing institutions merely on the basis of the theoretical desiderata misses a crucial point: the problems that theoretically plague a procedure may never surface in practice because of the peculiarities of the processes forming the preference profiles. Indeed, not much is known of the ways in which profiles or opinions are formed in various electorates. Thus far most work has focused on various kinds of domain restrictions of which the best-known is single-peakedness[24]. In single-peaked profiles there is a degree of consensus that basically guarantees the existence of a Condorcet winning candidate or policy alternative. Yet, in the advanced democratic systems all over the world the traditional left-right dimension of politics has been complemented with quite a few salient dimensions, such as environmental issues, immigration, military spending etc. With these new dimensions the idea of equilibrium political positions (like the Condorcet winner) becomes increasingly implausible. Rather, it is "chaos" that seems to prevail in multi-dimensional politics.

[24] *Black 1948.*

An important issue pertaining to application of theoretical criteria in proposing electoral system reforms is the hypothetical reasoning underlying some criteria. A case in point is monotonicity. It boils down to the requirement that additional support to the winning candidate, ceteris paribus, should never render it a non-winner. To establish that a monotonicity violation has occurred in a given election, one has to find out the entire preference profile and, moreover, to look at all possible improvements in the winner's position and determine the associated election outcomes. In addition, the monotonicity-related criteria are typically based on various ceteris paribus provisos, i.e. the changes in profiles that are examined are restricted to a considerable extent. These kinds of considerations undermine the direct applicability of these criteria in policy recommendations. Yet, the properties should call into question the plausibility of the outcomes resulting from non-monotonic procedures.

Voting theory without voters

Many properties dealt with in the theory of voting are directly transferable to settings involving just a single decision maker. In those settings the voters are replaced by criteria of performance as in the multiple criteria decision making (MCDM). Then the task at hand amounts to aggregating performance orderings of decision alternatives into an overall ranking or selecting the best alternatives on the basis of their performance in criterion-wise tests. To illustrate, *Table 1* above can be deemed as a result of assessing three alternatives A, B and C, in terms of three criteria of performance so that the first criterion has

the weight 4, the second has the weight 3 and the third weight 2. The problem is to combine the data of *Table 1* into an overall ranking of the three alternatives or into a choice of one of them using a reasonable system of aggregation. Similar "translation" can be done to all examples discussed above. In this new interpretation the voting theory criteria pertain to the rationality (in various senses) of outcomes reached by aggregation performance indicator values with each indicator having a pre-determined weight.

In this interpretation the aggregators that are vulnerable to the no show paradoxes have counterintuitive properties such as those exhibited in *Tables 5—8*. Adding or subtracting the weights of criteria may have bizarre consequences in decision outcomes under various procedures. Using non-monotonic aggregators may lead to absurd situations where an improvement of an alternative's performance in some criterion may actually render a winning alternative non-winning.

The point is that aggregation is quite common outside strictly voting contexts. Thus, the results of voting theory are relevant in many other contexts than the ones where they were originally discovered. An excellent case in point is the recent study of Andrey Subochev, Fuad Aleskerov and Vladimir Pislyakov on rules for aggregating bibliometric indicators using axiomatic characterization of the rules as the point of departure[25].

[25] *Subochev, Aleskerov, and Pislyakov 2018.*

Concluding remarks

The role of voting theory in the design of institutions has thus far been marginal. Typically the theoretical properties of voting rules are not explicitly mentioned in the debates preceding electoral reforms. A notable exception to this is the discussion in the U.S. Congress on the specifics of the one-person-one-vote principle during the enlargement phases of the Union[26]. Also in other countries the reform debate has typically centered around the principles of proportionality, not so much on the committee voting rules. Intuition seems to play a significant role in reforms. Often procedures are proposed as solutions to one or more problems encountered in applying some systems. E.g. it is easy to envision that the plurality runoff system was proposed as a solution to problem affecting the plurality voting, viz. that the winner may be a candidate regarded as worst by the majority of voters. Similarly, Nanson's rule was explicitly designed to handle the problem associated by the Borda count, viz. that of not electing a Condorcet winner. Nonetheless, a systematic comparative approach to voting system choice is typically omitted. There may be good reasons for this. E.g. some theoretical properties are deemed irrelevant, the designers may have different views on the appropriate criteria etc. Moreover, some important criteria are based on hypothetical reasoning so that it is not at all easy to find out whether real world voting outcomes satisfy them in concrete profiles.

[26] *The developments are comprehensively discussed in Balinski and Young 1982.*

Above we have discussed briefly one recent reform proposal which seems to be based on a comparative analysis of pros and cons of various voting rules. The present writer is not convinced about the proposal but is pleased to observe that scholars of the very highest level of competence have presented and argued for a clear alternative to the present system in the U.S. elections. Decades

ago a proposal to abandon the first-past-the-post in the U.K. was made by the Electoral Reform Society and its campaign for the single transferable vote still continues. So, it is likely that no quick and decisive victories are at hand in the electoral reform campaigns. It still important that such proposals are made and thoroughly debated for it is only in this way that we learn the details of the mechanisms resulting in the collective choices.

The importance of voting theory is, however, not restricted to voting. The results are mutatis mutandis applicable to individual decisions making problems and many other areas calling for aggregating results obtained by measuring performances in different tests or contests. Thus, for example, in sports competitions such as decathlon, heptathlon, artistic skating, as well as tournaments of various kinds are potential areas of applicability of the voting theory.

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Х.Нурми

ТЕОРИЯ ГОЛОСОВАНИЯ: CUI BONO?

Ханну Нурми — профессор департамента философии, политической науки и современной истории факультета общественных наук Университета Турку (Финляндия). Для связи с автором: hnurmi@utu.fi.

Аннотация. На протяжении долгого времени теория голосования развивалась независимо от исследований дизайна механизмов, но около полувека назад, с введением концепта стратегического голосования, эти две традиции обрели точки соприкосновения. Впрочем, хотя за это время появился огромный массив работ по теории голосования, ее влияние на дизайн политических институтов остается минимальным. Часто ее положения воспринимаются как чересчур примитивные, или слишком абстрактные, или просто иллюзорные. Похоже, что назрела потребность в исследованиях, нацеленных на преодоление глубочайшего разрыва между абстрактными результатами теорий общественного выбора и поведенческими и институциональными реалиями политических систем сегодняшнего и завтрашнего дня.

Проблемы применимости положений теории голосования иллюстрируются автором на примере недавних проектов реформирования системы одномандатных округов на выборах в палату представителей Конгресса США. При том что предложенные реформы, по-видимому, исправят важнейшие изъяны нынешней модели, они, как это часто случается, будут иметь свои издержки, поскольку предусмотренные ими нововведения чреваты собственными проблемами. Тем не менее выводы теории голосования применимы в широком круге областей за пределами самого голосования. Однако возможности их использования во многом остаются нереализованными. В статье описываются и обосновываются некоторые варианты такого использования. Большинство прямых приложений касаются многокритериального принятия решений.

Ключевые слова: общественный выбор, теорема несоответствия, участие, расширение Кондорсе, электоральная реформа