The Principle of the Indiscernibility of Identicals Requires No Restrictions

Abstract

There is a certain argument against the principle of the indiscernibility of identicals (PInI), or the thesis that whatever is true of a thing is true of anything identical with that thing. In this argument, PInI is used together with the self-evident principle of the necessity of self-identity (”necessarily, a thing is identical with itself”) to reach the conclusion $a=b \rightarrow \Box a=b$, which is held to be paradoxical and, thus, fatal to PInI (in its universal, unrestricted form). My purpose is to show that the argument in question does not have this consequence. Further, I argue that PInI is a universally valid principle which can be used to prove the necessity of identity (which in fact is how the argument in question is usually employed).

Keywords: indiscernibility; Leibniz’s law; substitutivity; intensionality; necessity of identity

1. The Barcanian argument

There is an argument, deriving from Ruth Barcan (1947), that some, e.g. Dale Jacquette (2011), take as a refutation of the (unrestricted) principle of the indiscernibility of identicals (PInI), or the thesis ”whatever is true of a thing is true of anything identical with that thing”. The argument in question – let us call it the Barcanian argument (BA) – is as follows (Jacquette 2011, p. 107):

1. $a=b \rightarrow \forall X(Xa \leftrightarrow Xb)$	PInI
2. $a=b$	assumption

3. $\Box a=a \rightarrow \Box a=b$	1 & 2 (for $\lambda x(Xx) = \lambda x(\Box a=x)$)
4. $\Box a=a$	necessity of self-identity
5. $\Box a=b$	3 & 4
6. $a=b \rightarrow \Box a=b$	2 & 5, conclusion

According to Jacquette, the conclusion 6 refutes PInI (in its general, unrestricted form) because it may be ”only logically contingently true that $a=b$” (Jacquette 2011, p. 108). The purpose of this paper is to show that this argument does not have the mentioned consequence: it does not refute PInI (in its universal, unrestricted form). Further, I intend to argue, in contradistinction to Jacquette and some others, that PInI is a universally valid principle and an argument very much like BA may rather be used as a demonstration of the necessity of identity.[1]

[1] This is how the argument is standardly taken (especially in its Kripkean form), i.e., not as a refutation (by reductio) of PInI but as a validation of the necessity of identity. See, for instance, Maunu (forthcoming).

Much of the material in this paper is familiar from the literature. My motivation for writing this paper is that even some renown philosophers seem to misconceive PInI (thus misleading students by their writings). Furthermore, I have noticed (via personal communication) that some philosophers find it hard to understand that the necessity of identity thesis has nothing to do with rigid designation for it can be expressed by means of definite descriptions (see the end of Section 5 below).

2. PInI and substitutivity

According to *the principle of the indiscernibility of identicals*, or PInI for short,

> (PInI) If the object *a* is the same as the object *b*, then whatever is true of *a* is true of *b* (and whatever is a property of *a* is a property of *b*).

This is not to be confused with *the principle of substitution*, PS:

> (PS) If in a true statement (or sentence) some expression *e* is replaced by (i.e., substituted with) an expression co-referential with it (i.e., with an expression sharing the referent with *e*), a true statement (sentence) results.

PS says something about the substitutivity of *signs* whereas PInI concerns *objects* (referents). Nevertheless, several influential philosophers seem to have made this confusion (and many of them have thereby expressed doubts about PInI, or, at least about the universal applicability of it). For example, in Quine 1961 [1953] we find (p. 139, emphasis removed):

> One of the fundamental principles governing identity is that of substitutivity – or, as it might well be called, that of indiscernibility of identicals. It provides that, given a true statement of identity, one of its two terms may be substituted for the other in any true statement and the result will be true.

Writings by prominent philosophers in which the indicated confusion is made include the following: Barcan 1947, Marcus 1961, Marcus 1986, Quine 1961 [1953] (just quoted), Quine 1976

[1953], Quine 1960 (p. 167), Hintikka 1957, Geach 1963, Wiggins 1965, Dummett 1973 (p. 270), Haack 1978 (pp. 183–84), Simons 1998, Taylor 1998 (pp. 45–46, 189f.), Morris 2006 (pp. 118, 129).[2]

[2] For example, Morris 2006, p. 118: ”It is a basic law of identity that if *a* is the same thing as *b*, whatever is true of *a* is true of *b*. That means that if we begin with a truth about an object, in which the object is referred to by one name, we should still have a truth if we refer to the same object by a different name.”

Taylor 1998, pp. 45–46: ”It is surely a reasonable principle that if a = b, then whatever property a has, b must have too. This is Leibniz's principle of the indiscernibility of identicals. But, as soon as one states this seemingly unproblematic principle, there arise apparent counterexamples. ... [Russell's example 'George IV wished to know whether Scott was the author of Waverley' ....] Frege's approach attempts to preserve the logical sanctity of Leibniz' law .... Russell too seeks to preserve the logical sanctity of Leibniz' law ....” (Those who think, as Taylor apparently does, that PInI is in need of some kind of ”saving”, must confuse PInI with PS.)

Simons 1998, pp. 678–79: ”[PInI] is uncontroversial, but needs careful formulation to exclude non-extensional contexts. For example, in 'John believes that *x* defeated Mark Antony', substituting the names 'Octavian' and 'Augustus' for *x* may yield different truth-values ...”.

On the other hand, the mentioned confusion has often (and a long time ago) duly been noticed and corrected in the literature, for example, in Smullyan 1948, Thomason & Stalnaker 1968, Stalnaker 1977, Cartwright 1971, Kripke 1971, Kripke 1980 (p. 3), Plantinga 1974 (p. 15), Maunu 2002.[3] It is pointed out in many of these writings that while it is obvious that PS does not hold for intensional statements (which may be shown by arguments like BA), this is not true for PInI, or at least cannot be taken as true without a separate argument. In view of this fact, it is surprising that Jacquette (2011) takes it for granted that BA refutes PInI (in its general form), and seeks to find out which restrictions are needed to save PInI. (And he seems to be talking about PInI and not PS, because he states that if $a=b$ then ”all of the properties of *a* are properties of *b*, and conversely” (Jacquette 2011, p. 112).)[4]

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[3] For example, Plantinga 1974, p. 15: ”if *x* has *P* essentially, then the same claim must be made for anything identical with *x*. If 9 is essentially composite, so is Paul’s favourite number, that number being 9. This follows from the principle sometimes called ‘Leibniz’s Law’ or ‘The Indiscernibility of Identicals’:

> (3) For any property *P* and any objects *x* and *y*, if *x* is identical with *y*, then *x* has *P* if and only if *y* has *P*.

Like Caesar’s wife Calpurnia, this principle is entirely above reproach. [Footnote:] Apparently Leibniz himself did not clearly distinguish (3) from:

> (3') Singular terms denoting the same object can replace each other in any context *salva veritate*

a ‘principle’ that does not hold for such excellent examples of language as English.”

[4] However, possibly Jacquette, despite his just-quoted statement, is confused, and is conflating PInI and PS; and he may be talking about logical necessity, understood in such a manner that only PS is relevant. But then his conclusion does not concern PInI but is only that co-referential terms cannot

It seems clear that PInI is not in need of saving or restricting at all. In my view already the following is a convincing proof of the general validity of PInI: If something is true (or is a property) of an object, *a*, and we ask whether it is true (or a property) of something identical with *a*, we must of course answer in the positive because anything identical with *a* is *a* itself – this does not depend on whether this ”something identical with *a*” is called ‘*b*’ or ‘*c*’ or whatever. Whether we use rigid designators (e.g. proper names) or nonrigid designators (e.g. definite descriptions) is inconsequential: if the capital of Russia (theC, for short) is the same as the most populous city in Europe (theP), then whatever is true of the city that is in fact theC (= Moscow) is true of the city that is in fact theP (= Moscow).

3. A refutation of PInI by extensional properties?

As expected, one set of truths or properties Jacquette regards as refuting PInI are those involving intensionality; for example, modality as in the very formulation of BA given above. I shall deal with such truths shortly. Meanwhile, let us consider Jacquette’s claim that there are also entirely extensional properties that disprove PInI by an argument parallel to BA. Well, *that* would be hugely astonishing, if there were such properties (but, in reality, there are not and could not be: with respect to extensional logic PInI is, in effect, tantamount to PS). As an alleged example of such a property, Jacquette (2011, pp. 114–15) gives $\lambda x(a{=}x \rightarrow b{\neq}x)$.[5] Jacquette is here seriously mistaken. The (allegedly) PInI-refuting argument parallel with BA – call it BA' – with this property (instead of

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always be interchanged salva veritate, which is hardly news.

[5] This is Jacquette’s ”F_5”.

$\lambda x(\Box a{=}x)$ of BA), is as follows:

1. $a{=}b \rightarrow \forall X(Xa \leftrightarrow Xb)$	PInI
2. $a{=}b$	assumption
3'. $(a{=}a \rightarrow b{\neq}a) \rightarrow (a{=}b \rightarrow b{\neq}b)$	1 & 2 (for $\lambda x(Xx) = \lambda x(a{=}x \rightarrow b{\neq}x)$)
4'. $a{=}a \rightarrow b{\neq}a$	(?)
5'. $a{=}b \rightarrow b{\neq}b$	3' & 4'
6'. $a{=}b \rightarrow (a{=}b \rightarrow b{\neq}b)$	2 & 5', conclusion

However, there is no reason at all to accept 4', in contradistinction to the uncontroversial 4 in BA. Further, as Jacquette (2011, p. 107) sees it, in BA ”the game is over” for PInI already at the third step, by which he means, I gather, that what we have in 3, viz. $\Box a{=}a \rightarrow \Box a{=}b$, is inconsistent or paradoxical (and, hence, fatal to PInI). Now, in BA' the third step is $(a{=}a \rightarrow b{\neq}a) \rightarrow (a{=}b \rightarrow b{\neq}b)$. There is nothing suspicious about this – indeed, it is a logical truth! (If $a{=}b$, then the antecedent (of the main implication) is false; if $a{\neq}b$, then the consequent is true.) Accordingly, 3' is independent from any application of PInI: were anything bad to follow from 3' (though, of course, it will not), PInI would not be responsible for it.

In short, Jacquette’s following claim is entirely groundless (Jacquette 2011, p. 114):

> Interestingly, it turns out that we do not actually need logical necessity in the formulation of a similarly embarrassing application of [PInI]. We can do so also by defining the property [...] $\lambda x(a{=}x \rightarrow b{\neq}x)$. Now, if we assume that $a{=}b$, and bring back $\neq$ for further collaboration, then under [PInI], we derive by parity of inference with [BA] something that is fatal once again to the logical consistency of [... PInI ...]: $a{=}a \rightarrow a{\neq}b$.

4. PInI and intensional properties

Let us turn then to properties that involve intensionality – properties that at least may *seem* to threaten PInI. These include modal properties (as in BA) and epistemically intensional properties, which Jacquette (2011, pp. 110–11), following Roderick Chisholm, calls ”converse intentional properties”, e.g. ”having the property of being believed by *d* to have the property *F*”, or $\lambda x(Bd(Fx))$, as Jacquette (2011, p. 110) formalizes this. Concentrating on modal properties (other intensional properties may be dealt with analogously), the mistake in arguments like BA (as alleged refutations of PInI) has in fact many times pointed out and solved in the literature. (See the references in section 2 above.) Because this solution seem to have gone unnoticed by Jacquette and others who think that BA rebuts PInI, I think I should repeat it here.

The original general form of PInI is, ”for every x and y, if $x=y$, then whatever is true of x is true of y, and conversely”, or ”all of the properties of x are properties of y, and conversely”, or, preliminarily, $\forall xy(x=y \rightarrow \forall X(Xx \leftrightarrow Xy))$. Specifying this to singular terms, we see that, rather than being what is given in step 1 in BA (i.e., $a=b \rightarrow \forall X(Xa \leftrightarrow Xb)$), the true singular term form of PInI is,

1*. $a=b \rightarrow \forall X(\lambda x(Xx)a \leftrightarrow \lambda x(Xx)b)$.

(This is not denied by Jacquette: he holds throughout Jacquette 2011 that the paradox producing property (or one of them) is $\lambda x(\Box a=x)$.)

Then, instead of 3 we have at most

3*. $\lambda x(\Box a=x)a \rightarrow \lambda x(\Box a=x)b$.

(At this point we may notice that the step 3 of BA, i.e., $\Box a=a \rightarrow \Box a=b$, is obtainable from 1* and $a=b$ only if '*a*' and '*b*' are rigid, i.e., refer to the same object with respect to every possible world. In that case the conclusion 6 of BA is unproblematic.) With

4*. $\lambda x(\Box a=x)a$

(which is indisputable for a rigid '*a*') we obtain

5*. $\lambda x(\Box a=x)b$

and, finally,

6*. $a=b \rightarrow \lambda x(\Box a=x)b$.

Concisely:

1*. $a=b \rightarrow \forall X(\lambda x(Xx)a \leftrightarrow \lambda x(Xx)b)$

2. $a=b$

3*. $\lambda x(\Box a=x)a \rightarrow \lambda x(\Box a=x)b$

4*. $\lambda x(\Box a=x)a$

5*. $\lambda x(\Box a=x)b$

6*. $a=b \rightarrow \lambda x(\Box a=x)b$

Here, '*b*' occurs outside the scope of the necessity operator (i.e., in an extensional or transparent or de re position), and so 6* says only that if $a=b$, then it is true of b that it is necessarily identical with a, which is entirely unproblematic even for a nonrigid 'b' (assuming that 'a' is rigid). For example, if 12 = the number of apostles, then it is true of the number that is in fact the number of apostles (= 12) that it is necessarily identical with 12, i.e., $\lambda x(\Box t=x)n$ holds, where 't' stands for 12 and 'n' is a shorthand for 'the number of apostles'.

If, on the other hand, 'a' is assumed to be nonrigid, then there is no reason to accept 4*: It is not true, we are entitled to presume, that the number of apostles was necessarily the number that was in fact the number of apostles (= 12), i.e., it is not the case that $\lambda x(\Box n=x)n$. The simple mistake in BA consists in the illegitimate importation of singular terms into modal context. In short, for modal or in any other way intensional F, we cannot equate Fa and $\lambda x(Fx)a$ – for example, $\Box a=b$ is not generally equivalent with $\lambda x(\Box a=x)b$.

The solution to the (alleged) problem posed by BA may be summed up as follows. First, if 'a' and 'b' are rigid designators, the conclusion 6 of BA is unproblematic. Secondly, the exact form of PInI is 1*, not 1 of BA. Thirdly, for a modal $\lambda x(Gx)$ one cannot without further ado get from $\lambda x(Gx)c$ to Gc, if 'c' is not rigid. Accordingly, with $Xx = (\Box a=x)$, what we get from 1* and $a=b$ is 3* – and this together with 4* yields 6*, which holds even for a nonrigid 'b', if 'a' is rigid. Fourthly, if 'a' is not rigid, 4* does not hold, which blocks the route to 6*.

All in all, BA does not disprove PInI. Nor is any other refutation possible: because *only one object* (called 'a' or 'b' or whatever) is in play in PInI, we cannot have any difference in what is true of or what is a property of this one and only one object: what PInI really states is just that whatever is true (or, a property) of an object, is true (or, respectively, a property) of *it*, which everyone should accept as a self-evidently true principle.

5. The necessity of identity

We have seen that BA cannot be used to discredit PInI. On the contrary, because PInI is universally valid, the following non-question-begging[6] BA-style argument can be devised for demonstrating the necessity of identity, in a manner that is not dependent on the nature of the singular terms used:

1*. $a=b \rightarrow \forall X(\lambda x(Xx)a \leftrightarrow \lambda x(Xx)b)$	PInI
2. $a=b$	assumption
3''. $\lambda xy(\Box x=y)aa \rightarrow \lambda xy(\Box x=y)ab$	1* & 2 (for $\lambda x(Xx) = \lambda x(\lambda y(\Box x=y)a)$)
4''. $\lambda xy(\Box x=y)aa$	necessity of self-identity
5''. $\lambda xy(\Box x=y)ab$	3'' & 4''
6''. $a=b \rightarrow \lambda xy(\Box x=y)ab$	2 & 5'', conclusion

In contrast to BA, this is a valid argument. The entirely unproblematic conclusion is that if *a=b* then it is necessary that the former is the latter. It does not matter whether '*a*' and '*b*', as used in this argument, are rigid designators (e.g. proper names) or nonrigid designators (e.g. definite descriptions): if the capital of Russia ('theC') is the same as the most populous city in Europe ('theP'), then it is true of the city that is in fact theC and the city that is in fact theP that necessarily, the former (= Moscow) is the latter (= Moscow), i.e., $theC=theP \rightarrow \lambda xy(\Box x=y)(theC)(theP)$. The identity involved in the true "TheC is theP" is, like any identity, a necessary identity, even though "TheC is theP" does not express a necessary truth. The necessity of identity is independent from

[6] Jacquette may be seen as begging the question against the necessity of identity in his statements such as "the property is intuitively that of being logically necessarily identical to entity *a*, a property which certainly *a* has but *b* does not have, when it is only logically contingently true that *a* = *b*" (Jacquette 2011, p. 108).

rigid designation, the necessary truth of some identity statements is not.[7]

6. Conclusion

BA cannot be used to refute PInI in its universal form – it does not force us to place any restrictions on PInI in order to save it. Extensional properties most certainly cannot be utilized in any argument such as BA. Intensional (including modal) properties may seem to hold more promise, but, we have seen, they fail to show the untenability of PInI as well. PInI is a universally valid principle: If $a=b$, then only one object is in play, and there cannot be any variation in the truths and properties of this one and only object. Further, this principle can be used to demonstrate the necessity of identity.[8]

[7] Cf. Kripke (1980, 3): ”If ‘*a*’ and ‘*b*’ are rigid designators, it follows that ‘$a = b$’, if true, is a necessary truth. If ‘*a*’ and ‘*b*’ are *not* rigid designators, no such conclusion follows about the *statement* ‘$a = b$’ (though the *objects* designated by ‘*a*’ and ‘*b*’ will be necessarily identical).” See also Maunu (forthcoming).

[8] I thank the anonymous referees of *Synthese* for useful suggestions that improved this paper.

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