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ARTICLE

Not realizing that you don't know: Fraction state anxiety is reduced by natural number bias

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Abstract

Background: Research has shown that mathematics anxiety negatively correlates with primary school mathematics performance, including fraction knowledge. However, recently no significant correlation was found between fraction arithmetic performance and state anxiety measured after the fraction task. One possible explanation is the natural number bias (NNB), a tendency to apply natural number reasoning in fraction tasks, even when this is inappropriate. Students with the NNB may not realize they are answering incorrectly.

Aims: The aim is to examine whether a misconception, namely the NNB, can influence students' fraction state anxiety.

Sample: The participants were 119 fifth- and sixth-grade students categorized as belonging to an NNB group ($n = 60$) or a No-NNB group ($n = 59$), according to their NNB-related answering profile on a fraction arithmetic task.

Methods: Group differences were examined for state anxiety and performance on a fraction and a whole number arithmetic task and self-reported trait mathematics anxiety.

Results: The NNB group reported lower fraction state anxiety than the No-NNB group, but there was no significant difference in trait mathematics anxiety. Furthermore, the NNB group reported lower fraction state anxiety than whole number state anxiety, while the opposite was true for the No-NNB group.

Conclusion: The present study suggests that students' perceptions of their own performance influence their state anxiety responses, and students with a NNB may not be aware of their misconception and poor performance. Not taking

into account qualitative differences in low performance, such as misconceptions, may lead to misinterpretations in state anxiety-performance relations.

KEYWORDS
arithmetic, fractions, mathematics anxiety, misconception, natural number bias, primary school, state anxiety

INTRODUCTION

Mathematics anxiety refers to a negative emotional state in relation to mathematical situations and tasks (Ashcraft & Moore, 2009; Richardson & Suinn, 1972). Recent studies have differentiated between trait and state mathematics anxiety, with the former referring to general anxiety towards mathematics and the latter to anxiety assessed during a mathematics-related situation (Conlon et al., 2021; Demedts et al., 2022; Orbach et al., 2020). In general, meta-analyses have shown that mathematics anxiety is negatively correlated with performance already in primary school (Barroso et al., 2021; Namkung et al., 2019). Yet, Halme et al. (2022) recently found no relation between performance and state anxiety on a fraction arithmetic task. Furthermore, the fraction arithmetic task did not result in significantly higher state anxiety than a whole number arithmetic task. This was surprising as increased task difficulty was shown to relate to higher state anxiety (Trezise & Reeve, 2018), and children were shown to have poorer attitudes towards fractions than whole numbers (Sidney et al., 2021).

One possible explanation for this surprising result by Halme et al. (2022) concerning the fraction state anxiety-performance relation is a common misconception about fractions, namely the natural number bias (NNB). The NNB is a tendency to apply the properties of natural numbers to rational number tasks, even when this is inappropriate (e.g., adding denominators and numerators together as in $1/2 + 1/2 = 2/4$). Consequently, the present study investigates whether having a strong NNB influences fraction state anxiety responses of late primary school children.

Mathematics trait and state anxiety

Mathematics anxiety is typically assessed with self-report questionnaires designed to assess trait mathematics anxiety. Individuals with higher trait mathematics anxiety have lower mathematics performance within a variety of mathematical tasks (Ashcraft & Kirk, 2001; Chan & Tang, 2020; Sorvo et al., 2017; Starling-Alves et al., 2022; Trezise & Reeve, 2018). Furthermore, a stronger relation has been found between trait mathematics anxiety and performance on tasks requiring more complex mathematical skills than foundational ones (Caviola et al., 2022; Namkung et al., 2019). As complex tasks require more cognitive resources, mathematics anxiety may result in lower performance by overburdening cognitive resources, such as working memory (Ashcraft & Kirk, 2001; Ramirez et al., 2013; Vukovic et al., 2013). This may lead students with mathematics anxiety to use less efficient strategies when solving tasks (Ramirez et al., 2016). Notably, the direction of the relation between mathematics anxiety and performance is highly debated, as some studies suggest that mathematics anxiety results in poor performance and others vice versa (Carey et al., 2016; Foley et al., 2017).

Recent research has examined the relation between mathematics performance and state anxiety that is typically assessed during, or after completing a mathematics task. State anxiety measured after a mathematical task is shown to be positively correlated with trait mathematics anxiety (Conlon et al., 2021; Demedts et al., 2022; Halme et al., 2022; Orbach et al., 2019, 2020). However, some studies suggest that state and trait mathematics anxiety can have different relations with performance (Halme et al., 2022; Orbach et al., 2019, 2020). Furthermore, studies have shown that state anxiety responses can vary in

relation to task characteristics, such as mathematics problem difficulty and time pressure (Demedts et al., 2022; Punaro & Reeve, 2012; Trezise & Reeve, 2018). For instance, higher task difficulty is related to higher state anxiety or worry. This aligns with research in another motivational framework, situational interest, in which multiple factors affect situational interest towards mathematical tasks including, individual interest, task difficulty, and competence beliefs (Høgheim & Reber, 2019; Koskinen et al., 2023). Yet, more research is needed examining which task characteristics could affect the relation between state anxiety and mathematical performance.

Studies examining the relation between mathematics anxiety and performance in specific subdomains of primary school mathematics beyond whole number knowledge are limited (Barroso et al., 2021; Namkung et al., 2019). To address this gap, recent studies have shown that mathematics anxiety also negatively correlates with fraction magnitude knowledge (Starling-Alves et al., 2022). Likewise, Halme et al. (2022) found trait and state mathematics anxiety to correlate with performance in several subdomains relevant to primary school mathematics, including whole number and rational number knowledge. Contrary to what was expected, they found no significant relation between fraction arithmetic performance and state anxiety, while fraction arithmetic performance was negatively correlated with trait mathematics anxiety. The authors suggested that the surprising result regarding fraction state anxiety might be explained by the NNB. Students with an NNB may believe that they are answering the items correctly, even though they are not, and report less anxiety than would be expected based on their actual performance level. Thus, the present study investigates the influence of the NNB on state anxiety responses measured after a fraction arithmetic task.

Natural number bias

Despite the crucial role of fractions in mathematics development, students have great difficulties in understanding their unique properties (McMullen & Van Hoof, 2020; Reinhold et al., 2020; Siegler et al., 2011; Vamvakoussi et al., 2018; Vamvakoussi & Vosniadou, 2004), even at the end of secondary school (Van Hoof et al., 2018). This creates a bottleneck in students' mathematical attainment, as fractions are an important stepping stone in understanding more advanced mathematics, such as algebra (Booth & Newton, 2012; Siegler et al., 2012). One of the most investigated phenomena linked to difficulties with fraction learning is the natural number bias, which refers to the tendency to apply natural number properties in rational number tasks, even when this is inappropriate (Ni & Zhou, 2005). This leads to misconceptions such as assuming that the magnitude of a fraction increases when its denominator, numerator, or both increase (Stafylidou & Vosniadou, 2004). For example, 1/4 is believed to be larger than 1/3, since 4 is larger than 3. Another misconception is assuming that a fraction addition task is solved by adding the natural numbers in the numerators and denominators, leading to systematic mistakes such as claiming that 1/2 + 1/3 equals 2/5 (Jarrah et al., 2022; Siegler et al., 2011).

It is important to note that the NNB is not an all-or-nothing issue, as shown by the qualitatively different profiles of students (González-Forte et al., 2020; Van Hoof et al., 2018). Students with a very clear NNB profile show a naïve understanding of fractions as having the same properties as natural numbers in various fraction tasks. Students with a more advanced understanding of fractions may answer in a natural number-based way only on some tasks. Lastly, some students show a mathematically correct understanding of fractions already at the end of elementary school, with no signs of the NNB (González-Forte et al., 2020; Van Hoof et al., 2018). Consequently, natural number-based reasoning is not always a naïve misconception, but it could be a deliberate strategy some students choose to use when they are unsure how to solve a new fraction task, as suggested by Alibali and Sidney (2015).

Nonetheless, studies have shown that students with a strong NNB are unaware that they are solving fraction tasks in an incorrect way. Merenluoto and Lehtinen (2004) stated that even students in upper secondary school can be unaware of their misconception and show overconfidence in their natural number-based answers. Similarly, González-Forte et al. (2023) showed that seventh-grade students with a clear NNB profile had high confidence levels, when answering a fraction task incorrectly with a

natural number-based answer. Moreover, these students were reluctant to adapt their reasoning, when confronted with a student's answer who had reasoned correctly. Likewise, the present study focuses on students with a clear NNB profile, as finding low-state anxiety amongst these students would support the notion that the NNB is an actual naïve misconception meaning students are unaware they are solving fraction tasks incorrectly.

Present study

The present study is part of a larger project investigating the relations between mathematics anxiety and performance in late primary school children. It aims to extend the findings by Halme et al. (2022) by assessing whether fraction state anxiety responses differ between low-performing students with qualitatively different fraction understanding. Previous studies have shown that students with a strong NNB are unaware of their incorrect reasoning and show overconfidence in their incorrect answers (González-Forte et al., 2023; Merenluoto & Lehtinen, 2004). Therefore, students with a clear NNB may also have low fraction state anxiety despite their poor fraction understanding. Halme et al. (2022) suggested that the lack of a significant correlation between fraction performance and state anxiety could be due to some students having an NNB.

Consequently, our first research question is whether fraction state anxiety responses differ between students with low fraction performance and a clear NNB (i.e., answering all fraction items consistent with an NNB) compared to students with low fraction performance and no signs of an NNB (Research question 1). Our hypothesis is that students with an NNB have lower fraction state anxiety compared to low performers without the NNB (Hypothesis 1). If our hypothesis is supported, we will further examine (1) whether fraction state anxiety correlates with the strength of the NNB, (2) whether the presence of the NNB group explains the lack of a significant correlation between fraction performance and state anxiety in the whole sample analysis by Halme et al. (2022).

Fractions are shown to be more difficult to learn than whole numbers (Siegler et al., 2011), and both children and adults have more negative attitudes towards fractions than whole numbers (Sidney et al., 2021). As students with a clear NNB have high confidence in their rational number understanding (González-Forte et al., 2023; Merenluoto & Lehtinen, 2004), they may not find fractions to be more difficult or anxiety-inducing than whole numbers. Therefore, we also examine whether the relation between state anxieties differs between a fraction arithmetic task and a whole number arithmetic task in the two groups (Research question 2). We expect that only low performers without NNB have higher state anxiety on the fraction task than the whole number task (Hypothesis 2). We also verify that there were no significant group differences in whole number performance that may explain differences in state anxiety responses.

METHODS

Participants

The whole sample consisted of 412 fifth- and sixth-grade students from 27 classrooms within 10 Finnish schools located in two similar-sized urban municipalities in Southern Finland. The main analysis of the present study relates to a subsample of 119 fifth- and sixth-grade students, who had low performance on the fraction arithmetic task due to either (a) a clear NNB or (b) no NNB. Students who answered all the fraction arithmetic task items in a natural number-based way (e.g., $1/3 + 1/3 = 2/6$) were categorized into the NNB group ($n = 60$). Students who had all answers incorrect on the fraction arithmetic task items due to some other reasoning than the NNB (e.g., $1/2 + 1/4 = 2/4$) were categorized as the No-NNB group ($n = 59$). There were very few students who had all answers incorrect due to some other reasoning than the NNB ($n = 13$); therefore, the criteria were expanded to include students with the two

same denominator items as correct (i.e., 1/3 + 1/3 & 3/4–1/4). This means that students in the No-NNB group had a maximum of two out of eight correct, and all incorrect answers had a different source than the NNB.

The descriptive information for both groups is shown in Table 1. The groups only differed in the amount of fraction instruction completed by Time point 1 with majority of the NNB group (96.7%) not having completed the ongoing academic year's fraction instruction. Yet, fraction instruction typically begins in the third grade and continues throughout primary school. Subtraction and addition of fractions with the same denominator are in the fourth-grade curriculum, while fractions with different denominators are covered latest in the fifth grade.

Permissions for conducting the present study were received from the municipalities, principals, and classroom teachers and written consent was obtained from the students' guardians. Students were informed that taking part is voluntary and they can withdraw from the study at any point. Ethical permission was granted from the University of Helsinki and the study followed the ethical guidelines of the Finnish National Board on Research Integrity (TENK).

Procedures

This study is part of a larger longitudinal data collection, and the present study reports on the data from the first measurement point gathered in early November (Halme et al., 2022). Due to the COVID-19 situation, the measurements were conducted at school on a digital platform and guided by the participants' classroom teacher. Students completed the digital test individually on their own computer. Before each task, there were instructions on how to complete the task including information on the duration. The duration of each task was limited to ensure that each participant had the same amount of time per task, with a total duration of 34 min for the test battery. The test battery was presented in the following order: spontaneous focusing on relations, trait mathematics anxiety, fraction self-efficacy, guided multiplicative relations, adaptive rational number knowledge, fraction magnitude knowledge, whole number arithmetic knowledge, and fraction arithmetic knowledge. After each task, except fraction magnitude knowledge, students rated their level of anxiety during the task, as a measure of state anxiety. The present study only includes trait mathematics anxiety, and measures of performance and state anxiety for fraction arithmetic knowledge and whole number arithmetic knowledge.

Measures

Mathematics anxiety

Trait mathematics anxiety was measured with nine statements of which six statements were about general mathematics adapted from Sorvo et al. (2017) (i.e., I feel anxious about mathematics; I get anxious

TABLE 1 Descriptive characteristics of the low-performing students with or without NNB.

	NNB group ($n = 60$)	No-NNB group ($n = 59$)	Whole sample ($N = 412$)	Group difference NNB vs. No-NNB
Age, mean (*SD*)	11.86 (.58)	11.95 (.66)	11.89 (1.03)	$t(117) = .79$, $p = .43$, $d = .15$
Female, *n* (%)	33 (55.00)	25 (42.37)	208 (50.49)	$\chi^2(1) = 1.90$, $p = .17$
Fifth grade, *n* (%)	33 (55.00)	27 (45.76)	190 (46.12)	$\chi^2(1) = 1.02$, $p = .31$
Completed fraction instruction, *n* (%)	2 (3.33)	26 (37.29)	127 (30.83)	$\chi^2(1) = 21.30$, $p < .001$

Note: Completed fraction instruction means that the students had completed the ongoing academic year's fraction instruction. All students have had fraction instruction since the third grade.

when I have to do mathematics exercises; I get anxious when I have to do mental arithmetic; I get anxious when I have to start mathematics homework; I feel tension, when I have to do mathematics tasks; I am worried that I will not learn mathematics) and three equivalent statements modified to be about fractions (i.e., I am anxious about fractions; I get anxious when I have to do fraction tasks; I am worried that I will not learn fractions). These statements were specifically chosen to not include characteristics related to test or social anxiety. The scale was continuous from 1 to 5 including descriptions "not at all" and "very much" respectively, and faces that changed from less anxious to more anxious (see Figure 1). As all the nine items loaded onto one component within the whole sample and in the two groups, the average score for the nine items was used as the measure for trait mathematics anxiety. Cronbach's alpha was .94 for the whole sample.

State anxiety was assessed after each task with a one question prompt to evaluate the level of anxiety a student had experienced during the task, "How anxious were you while doing the previous task?" This was measured on the faces scale (Figure 1).

Fraction arithmetic knowledge

Students were asked to solve eight items of fraction addition and subtraction: (1) 1/3 + 1/3; (2) ¼ + ½; (3) 3/8 + ¼; (4) 2/5 + ½; (5) 3/7 + 1/3; (6) ¾ − ¼; (7) 4/6 − ½; (8) 4/5 − ¼. The students had 150 s to complete the task. Students received one point for each correct answer with a maximum of eight points. Cronbach's alpha was .84 for the whole sample.

Whole number arithmetic knowledge

Students were asked to solve a fill-in-the-blank task by filling in the missing number that completed the equation. There were six items: (1) 12 + __ = 11 + 15; (2) 6 * __ = 2 * 15; (3) 2 * 16 = __ * 8; (4) 54–48 = 18/__; (5) __/7 = 6/3; (6) 50/2 = 27 − __. The students had 150 s to complete the task. Students received one point for each correct answer with a maximum of six points. Cronbach's alpha was .86 for the whole sample.

Data analysis

Data were analysed using IBM SPSS Statistics 28. Independent samples *t*-tests were conducted to examine differences between groups (NNB vs. No-NNB). Pearson's correlations were conducted to examine the relation between the number of natural number-based answers and fraction state anxiety within the whole sample and within the low-performing participants (i.e., performance score ≤ 2 regardless of reasoning). In addition, Pearson's correlations were conducted to examine, whether excluding the NNB

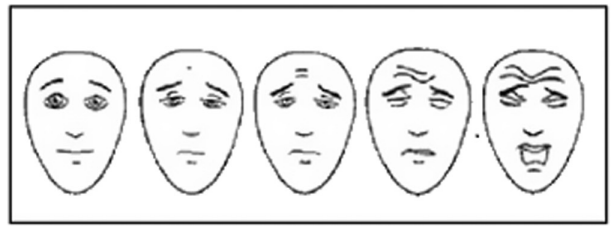

FIGURE 1 The faces scale used for trait mathematics anxiety and state anxiety. *Note*: This figure is adapted from Punaro and Reeve (2012).

group from the whole sample analysis would result in a significant negative correlation between fraction arithmetic performance and state anxiety. A two-way repeated measures ANOVA was conducted to examine group (NNB vs. No-NNB) by task type (fraction vs. whole number) interactions for state anxiety. Cohen's *d* is used to report effect sizes.

As an additional remark, there were six participants categorized as No-NNB who gave no answer on any of the items on the measures for whole number and fraction arithmetic knowledge. Two of them reported a 5 and four of them reported a 1 for state anxiety on both tasks. We did not exclude any of these participants, because we cannot be certain that the scores are due to a lack of knowledge or motivation. Excluding all six participants would only change the group difference in whole number performance from not significant to significant.

RESULTS

Fraction state anxiety

As illustrated in Table 2, independent samples *t*-tests comparing the NNB and No-NNB groups showed no significant difference in trait mathematics anxiety, $t(118) = -.63$, $p = .53$, $d = .12$. This is the case even when separately examining the items related to general mathematics (six items), $t(117) = .43$, $p = .67$, $d = .08$, and fractions (three items), $t(117) = .97$, $p = .33$, $d = .18$. Moreover, no significant group difference was found for whole number anxiety, $t(117) = -.14$, $p = .89$, $d = .03$. A significant group difference was found in fraction performance, $t(118) = 10.42$, $p < .001$, $d = 1.93$, and fraction state anxiety, $t(118) = 2.82$, $p = .006$, $d = .52$. They were both significantly lower in the NNB group compared to the No-NNB group confirming the first hypothesis that students with the NNB have lower fraction state anxiety.

As there was a significant group difference in fraction state anxiety, we further examined the relation between the strength of students' NNB (i.e., number of incorrect natural number based answers) and their fraction state anxiety. First, we examined the whole sample and found a small negative correlation, $r(411) = -.12$, $p = .02$, meaning a higher number of incorrect, natural number based answers related to a lower fraction state anxiety response. Next, we examined all students with low fraction performance (i.e., score ≤ 2), regardless of the reasoning (i.e., no NNB, some NNB, all NNB). We found a small negative correlation between the number of NNB answers and state anxiety amongst low performers, $r(270) = -.25$, $p < .001$. In addition, we examined whether excluding the NNB group from the whole sample analysis by Halme et al. (2022) would result in a significant negative correlation between state anxiety and performance on the fraction arithmetic task. We found a small, yet significant negative correlation, $r(352) = -.15$, $p = .01$, after excluding the NNB group. When excluding all students with any level of NNB (i.e., inclusion criteria NNB score = 0), the negative correlation between fraction performance and state anxiety became stronger, $r(193) = -.24$, $p < .001$.

TABLE 2 Descriptive statistics for mathematics anxiety and performance measures.

	NNB group ($n = 60$), mean (*SD*)	No-NNB group ($n = 59$), mean (*SD*)	Whole sample ($N = 412$), mean (*SD*), range	Mean difference (95% confidence interval) between NNB and No-NNB group
Fraction state anxiety	1.80 (1.17)	2.51 (1.55)	2.14 (1.28), 1–5	.71 (.21 to 1.21)
Fraction performance	0	1.20 (.89)	2.26 (2.21), 1–8	1.20 (.97 to 1.43)
Whole number state anxiety[a]	2.26 (1.32)	2.22 (1.45)	2.02 (1.26), 1–5	−.03 (−.54 to .47)
Trait mathematics anxiety	1.70 (.96)	1.59 (.91)	1.51 (.70), 1–5	−.11 (−.45 to .23)

[a]Missing data from the NNB group ($n = 1$) and the whole sample ($n = 3$).

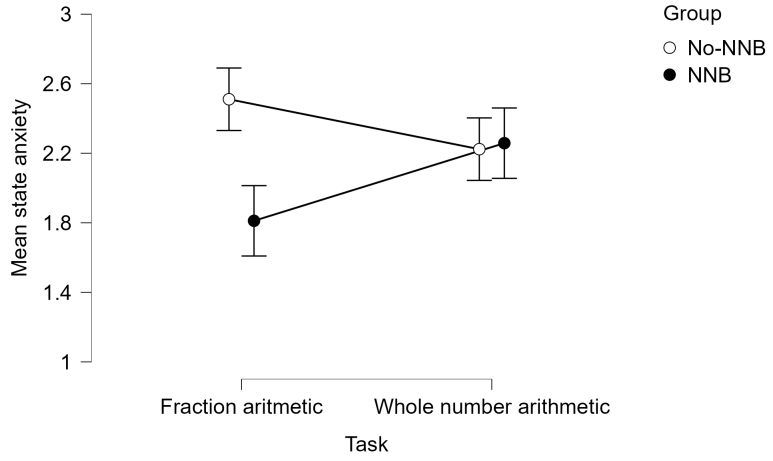

FIGURE 2 The group by task type interaction for state anxiety. *Note*: The error bars are 95% confidence intervals and the state anxiety scale is from 1 to 5.

Differences in state anxiety between fraction and whole number arithmetic

To address our second research question on the group differences for state anxiety between the whole number and fraction task, a two-way repeated measures ANOVA was conducted to examine group (NNB vs. No-NNB) by task type (fraction vs. whole number) interactions for state anxiety. There was no main effect of task type, $F(1, 116) = .70$, $p = .41$, $\eta_p^2 < .01$, but a significant interaction effect was found between group and task type, $F(1, 116) = 14.72$, $p < .001$, $\eta_p^2 = .11$. Simple effects showed that both groups had a significant difference between fraction and whole number anxiety, $p_{\text{NNB}} = .001$ and $p_{\text{No-NNB}} = .04$. Notably, the direction of the relation was opposite for the two groups, as shown in Figure 2 (see Figure A1 in the Appendix for state anxiety distributions). Confirming our second hypothesis, only the No-NNB group showed higher state anxiety on the fraction task compared to the whole number task. In addition, we examined whether a group difference in whole number performance explains the difference in state anxiety. The NNB group mean ($M = 2.63$ $SD = 2.36$) for whole number performance did not significantly differ from the No-NNB group mean ($M = 3.31$ $SD = 2.26$), $t(116) = 1.59$, $p = .11$, $d = .29$.

DISCUSSION

The present study examined whether fraction state anxiety responses differ between low-performing students with qualitatively different fraction understanding. This study highlights the novel finding that a misconception, such as the NNB, can influence the relation between state anxiety and mathematics performance. Results show that students with qualitatively different reasons for low fraction performance can have similar trait mathematics anxiety, while differing in their fraction state anxiety. More specifically, the NNB group had significantly lower state anxiety compared to the No-NNB group. We propose that students with an NNB may be unaware of their incorrect answers resulting in low performance and low anxiety. In addition, the NNB group reported significantly lower state anxiety on the fraction task than the whole number task, while the opposite was true for the No-NNB group. These results have implications for the research fields of mathematics anxiety and the natural number bias.

While meta-analyses show negative correlations between mathematics anxiety and performance (Namkung et al., 2019), Halme et al. (2022) showed that children's fraction arithmetic performance was negatively correlated with their trait mathematics anxiety, but not with their state anxiety. The present results confirm that this discrepancy is at least partly due to a specific subgroup of students, who show

an answering profile indicative of the NNB. Based on previous research in mathematics anxiety, the NNB group would be expected to have high state anxiety due to their poor performance (score of zero) on the fraction arithmetic task. In contrast, the NNB group had significantly lower state anxiety than the No-NNB group, who notably even had higher performance. Moreover, the group difference in fraction state anxiety was unlikely to be caused by trait mathematics anxiety, which was equivalent between the groups as supported by the effect sizes. Thus, the present study is the first to show that a misconception, such as the NNB, can result in low-state anxiety co-occurring with low performance. This emphasizes the importance of examining subgroups of students, as overlooking a misconception can result in inaccurate interpretations of state anxiety and performance relations.

Our second hypothesis was that fractions would be more anxiety-inducing than whole numbers to students without an NNB, but not necessarily more anxiety-inducing to students with an NNB. The current results confirm this hypothesis, as the students without any signs of an NNB had higher fraction state anxiety than whole number state anxiety. This result is in line with a previous study (Sidney et al., 2021) that found more negative attitudes towards fractions than whole numbers. In contrast, and unexpectedly, the students with an NNB found the whole number arithmetic task to be more anxiety-inducing than the fraction arithmetic task. This shows that the NNB group did not have lower state anxiety towards mathematical tasks in general, especially as the NNB group's state anxiety and performance relation on the whole number task was similar to the No-NNB group. There was only a performance-anxiety discrepancy on the fraction task. In addition, this result suggests that the NNB group may have perceived the whole number task to be more difficult than the fraction task. This could be due to differences in working memory demands of each task, as solving fractions with natural number-based strategies requires only addition or subtraction of two whole numbers, while the whole number task required solving multi-step missing value equations. Consequently, students with an NNB appear to have inaccurate appraisals of their own fraction understanding and task-related performance.

The current results also have theoretical implications for understanding the relation state and trait mathematics anxiety have with performance. First, in tasks with misconceptions, trait and state mathematics anxiety can have different relation with performance, especially amongst low-performing students. For instance, students with more NNB answers on a fraction task had lower state anxiety, while poor performance was related to higher state anxiety only in students without any NNB answers. Second, the current results suggest that state anxiety responses include situation-specific competence appraisals that can be influenced by various task characteristics, such as task difficulty (Demedts et al., 2022; Trezise & Reeve, 2018) and misconceptions. Thus, state anxiety can be a useful measure of subjective performance-related beliefs, especially within a specific task. Third, the present study supports suggestions that state anxiety reflects actual experiences of anxiety during mathematical situations (Goetz et al., 2013), while trait mathematics anxiety appears to not capture task-specific nuances. Consequently, measuring both state and trait mathematics anxiety is important to capture qualitatively different aspects of mathematics anxiety-performance relations.

As to the implications for the NNB research, the current results extend prior research on the NNB with a novel viewpoint of measuring state anxiety. Studies have previously shown that students with an NNB have high confidence levels on a fraction task (González-Forte et al., 2023). Consequently, students with a stronger NNB may experience more subjective certainty of the correctness of their natural number-based answers leading to lower anxiety. We propose that the co-occurrence of either high confidence or low anxiety with poor performance indicates that students with an NNB perceive their level of performance incorrectly due to their unawareness of their incorrect answers. This is further supported by the results of the whole number task, in which the NNB group had better performance, but also higher state anxiety. Consequently, the current findings support prior findings that students with a strong NNB profile might not be aware of their misconception (González-Forte et al., 2023; Merenluoto & Lehtinen, 2004), indicating that the NNB can be a naïve misconception.

It is important to keep in mind that almost all students in the NNB group had not yet had the ongoing academic year's fraction instruction. As fraction arithmetic is first covered in the fourth-grade curriculum, fifth graders (55% of the NNB group) have not yet accumulated many experiences with

fraction arithmetic, especially different denominator items. Thus, their certainty in the correctness of their NNB answers may come from a lack of knowledge or poor remembering of how to solve fraction arithmetic. In contrast, the high anxiety in the No-NNB group could be due to anxiety related to the additional working memory resources needed for remembering fraction arithmetic procedures and trying to apply them to calculate the fraction tasks correctly. Notably, the NNB group gave NNB answers even on the same denominator items, suggesting these students did not deal with these challenges. This strengthens the argument that the NNB group has lower fraction state anxiety due to their naïve fraction understanding. Fraction instruction could result in students realizing that fraction arithmetic is harder than they expected, and an accumulation of negative experiences is suggested to increase mathematics anxiety (Ashcraft & Moore, 2009). Consequently, fraction performance appraisals and state anxiety levels may change after instruction.

Limitations and further research

This study had limitations that need to be considered when generalizing the results. First, the fraction task used in this study measures only basic addition and subtraction knowledge, while fraction knowledge consists of much more, such as magnitude and density understanding. Thus, research is needed to know whether these results are generalizable to all fraction knowledge tasks. Second, the variance within the anxiety responses is limited due to the low levels of anxiety within the Finnish population in general (Lee, 2009), making it important to replicate these findings with populations from other countries. Third, the study included only fifth and sixth graders, and these results may not be the same in other age groups. Thus, it is worth investigating this phenomenon in older students and even in adults. Lastly, further research should examine this phenomenon in other mathematics topics and subjects that are influenced by common misconceptions, such as statistics (Lem et al., 2013).

Another important aspect is that mathematics anxiety is suggested to overburden working memory resources (Ramirez et al., 2013), potentially making it harder to inhibit the incorrect, yet more intuitive natural number-based strategies. Solving fraction arithmetic with a natural number-based strategy should have a lower burden on working memory, as the correct fraction strategies require remembering and using more complex procedural rules. However, according to our results, the NNB answers were not a consequence of higher trait or state mathematics anxiety in the NNB group compared to the No-NNB group. Nevertheless, longitudinal research should examine whether students with high mathematics anxiety and low working memory capacity could be at risk for persistent NNB that continues even after fraction instruction. In addition, improved fraction understanding in students with an NNB may lead to higher state anxiety due to the higher working memory demands of correct answering strategies. As we only have correlational data to support that fewer NNB answers relate to higher state anxiety, we cannot confirm that an increase in fraction knowledge (i.e., reduced NNB) results in higher state anxiety. In our future research, we will examine the longitudinal development of fraction state anxiety and the NNB.

CONCLUSION

Overall, the current findings indicate that special attention should be given to the qualitative differences in profiles of students, such as whether students have a misconception or not, when examining relations between state anxiety and mathematics performance. Not taking these differences into account can have influences on the interpretation of results, such as understanding whether fractions are more anxiety-inducing than whole numbers. Thus, measuring state anxiety is valuable for understanding task-specific perceptions and examining individual differences related to task-specific factors, such as misconceptions. These appear to not be captured by trait mathematics anxiety.

AUTHOR CONTRIBUTIONS

Hilma Halme: Conceptualization; methodology; formal analysis; investigation; writing – original draft; writing – review and editing; project administration. **Jo Van Hoof:** Conceptualization; formal analysis; writing – original draft; writing – review and editing. **Minna Hannula-Sormunen:** Writing – review and editing; funding acquisition; supervision. **Jake McMullen:** Methodology; writing – review and editing; funding acquisition; supervision.

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CONFLICT OF INTEREST STATEMENT

All authors declare no conflict of interest.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

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APPENDIX A

PLOTS FOR STATE ANXIETY

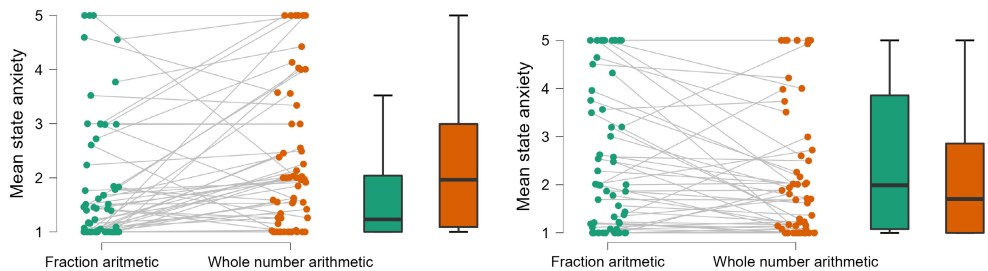

FIGURE A1 Distribution of state anxiety for the NNB group (left) and the no-NNB group (right).