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How Toxic is Public Debt?

Abstract

This paper deals with the debt-growth relationship using several time-series tools. The idea is to find out whether the inverse relationship between these variable can be detected without imposing any functional forms for the estimating relationship and whether the relationship does indeed reflect some nonlinear features. Thus recursive correlations with different orderings of the time-series are computed using the Reinhart & Rogoff panel data. After that, recursive correlations are re-estimated with data that are cyclically adjusted to reflect the structural features of these two variables. The nature of the relationship is also scrutinized by using various variable-parameter estimation techniques (Kalman Filter, Logistic functional form and recursive estimation). Finally, some analyses of causality are carried out using various transformations of the data up to the point of using various cross-sections of the data. The analysis shows that the inverse relationship between growth and debt is rather robust indeed and tends to support the "toxic debt" hypothesis rather than the cyclical debt accumulation hypothesis. Still the causality issue remains largely unresolved.

Key words: debt, government deficit, growth, causality

JEL code: E60, E62, E65

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1. Introduction

The burden of public debt has long been a controversial issue in economics, as is shown by e.g. the classical review article of Tobin (1965). More recently, the relevance of this issue has increased dramatically because of the rapid acceleration of debt growth in practically all countries. The topic has also become the subject of intensive research, reflected in the rapidly increasing number of papers that have been published. In particular, the paper by Reinhart and Rogoff (2010) has been frequently cited, probably because they found very large negative growth effects. The specific interesting feature in Reinhart and Rogoff (2010) was their finding of a 90% debt-to-GDP threshold. Reinhart and Rogoff argue that if public debt exceeds this level, the growth rate slows dramatically. This finding was later challenged by Herndon et al. (2013), who showed that the result was largely based on a computational error. A same results was obtained by Eberhardt and Presbitero (2013) who not only scrutinized the aggregate relationship but also cross-country differences. The 'computational error' debate has since then dominated public media almost to the extent that the main issue of whether public debt hinders economic growth has been forgotten[1].

Why, then, should public debt have important negative growth effects? Several explanations can be put forward (see e.g. Boskin (2012) and Feldstein (2012) partial surveys), ranging from crowding-out effects via higher interest rates to the income effects of higher debt service costs[2]. The list of explanations also include increased uncertainty due to unanticipated financial market reactions to higher debt (debt overhang) and loss of room for manoeuvre in monetary and fiscal policies due to the higher debt burden (see e.g. Mitra (2007) for an analysis of the debt constraint in economic policies). The income effects are particularly obvious in the case where public debt is owned by the foreigners. Servicing that debt in the future will require an increase in net exports, and that in turn requires a lower value of the terms of trade which obviously translates to lower real incomes in the home country. Increased uncertainty may in turn result from possible inability to manage with the debt serving costs. Higher debt serving costs imply need to increase taxes and thus higher debt means risk for higher taxes at some point of time in the future. This risk of higher taxes probably has some negative effect on investment, at least. When we speak about the loss of loss of room for

[1] The threshold idea has been rejected in practically all recent analyses, see. e.g. Pescatori et al (2014). They also found that the debt trajectory may well be equally important than the level of debt. The co-movements of the level and the change of debt are relatively poorly analyzed issues in the literature.

[2] The interest rate effects have mainly been analyzed in terms of government deficits, not so much for government debt. Perhaps, the most influential study is Laubach (2010) that considered the effects of (unanticipated) government deficits.

manoeuvre we have to keep in mind that this loss does not only show up in practical problems in current government financial operations (in changing the maturity, issuing new debt to roll over old debt, and so on) but it also affects the possibilities of increasing debt in the future simply because larger debt today makes it more difficult to increase debt tomorrow, possibly in a situation where the need for additional debt would be more acute than today.

If we consider these arguments, it is hard to conceive that there may indeed be a universal fixed threshold for negative growth-rate effects. Rather, we might expect that a negative growth-rate effect could be discerned for all values of the debt-to-GDP ratio. Moreover, if an effect exists, it is probably of long-term nature. In this respect it is different from the other causal explanation for debt – income growth relationship: the possibility that a permanently poor income growth causes deficit and debt problems for the government. By contrast, a more appealing hypothesis is that temporary (cyclical) changes in the income growth rate show up in deficits causing a changes in the debt/GDP ratio without affecting the level of debt.

The causality issue has inspired a several analyses where the relationship between the debt/GDP ratio and past and future income growth is analyzed. Dube (2013) using average 5-year past and future GDP growth rates instead of contemporaneous rates arrives at a result that there is basically no relationship between these variables. The reverse causality (from growth to debt) results in his mind from the fact that countries which grow fast are able to pay down their debt which may well be the case. Cechetti et al (2011) and Kumar and Woo (2010) arrive at somewhat different result and also suggest that the problem of debt is more severe with high levels of debt. Cechetti et al (2011) do not only consider government debt but also corporate and household debt and arrive at similar conclusions. Jorda et al (2014) further show that the impact of public debt very much depends on the developments of private debt prior to recessions. Thus, at least from the policy point of view, we should not only focus on government debt but also the overall development of indebtedness. The problem with the analysis of causality is at least partly related to time horizon. The “toxic debt” hypothesis makes more sense in the long run while the reverse causality interpretation most conveniently suits to cyclical movements of output and debt. So, what is the level playing field? Probably the only solution is look at things from both angles of view.

In the subsequent analysis we do not intend to estimate a threshold for the debt – income growth relationship. Nor do we intend to solve the issue of causality between debt and GDP growth. Rather we scrutinize the debt GDP-growth relationship with the intention of establishing whether this relationship is indeed negative and whether it displays any signs of discontinuities and, finally, whether the relationship comes closer to the long-run "debt depresses growth view" than to the conventional fiscal impact of growth slowdown (that can be put very simply: deficits and debt increase in depressions). Hence we carry out a recursive computation of GDP growth and debt-to-GDP average values (increasing the sample by one observation at time from smallest to largest) for the Reinhart and Rogoff data. In addition to this simple correlation/regression analysis we carry out two a bit more sophisticated analysis of relationships between growth and indebtedness. We estimate the relationship by using the Kalman Filter and thus allowing the regression coefficients to change over the level of debt. Alternatively, we estimate the relationship in a logistic form thus allowing for more flexible functional relationship for the relevant coefficients. We use recursive estimation so see whether relevant debt/GDP elasticity is indeed constant over time. Although we are well aware that we cannot solve the causality issue we try shed some light on the issue by considering some simple data tools like the (Granger) causality statistics to find out how ambiguous the results are with the data set we have. We also try to shed some light to the causality issue by scrutinizing the relationship between investment (both private and public) and the Debt/GDP ratio in section 3. As for the data, we use two samples: one for the 19 advanced economies and another for all 65 countries in our data for 1950–2008. In the first sample we have 1,102 data points and in the second we have 5,539 data points. In addition we use a more recent IMF data (Abbas et al (2010)) for the period 1948-2014 that include 156 countries and 5566 data points. Also some OECD data for control variables and data from the AMECO data base for an analysis of investment debt relationship are used.

2. Analysis of the bivariate relationship

We start with just by scrutinizing the data. The first step is to focus on the historical data on the debt/GDP ratio (compiled by the IMF) that are illustrated in Figure 1. The (unweighted) mean value of the debt/GDP ratio obviously suffers from sample selection bias (no data for low developing countries are available for the pre-WWII period). Contrary to common beliefs, indebtedness has not increased in all countries during the last decade. It has surely increased in high-income countries but the overall tendency has been quite different. Anyway, we can see that the period 1950-1980 was a time for exceptionally low debt/GDP ratio in the world – interestingly this was also the period rapid income growth for most countries. Otherwise the previous peaks in indebtedness coincide with the two Wold Wars and the Great Depression in the 1930s (the evolution of indebtedness in different country groups is reviewed in more detail in Abbas et al (2011)).

As for the GDP growth debt-relationship, we start the analysis by computing mean values of GDP growth and Debt/GDP rations in a recursive manner for the sample countries. The results of these analyses are reported in Figures 1-3. In Figure 1, we show the average values for 19 advanced economies, in the second those for all 64 countries in our data. In both cases we make recursive calculations starting from the smallest debt levels. Because that would give the high-debt countries relatively little weight, we also reverse the way of computing the average values, starting from the highest values of debt (Figure 3). The data are the same as Figure 1. The results of these calculations are strikingly robust for the different ways of computing the recursive values: there is always a negative slope in the debt-to-growth relationships. The growth rate loss may be as large as one percentage point, which is a shockingly high figure. Qualitatively, the shape of the relationship between the debt ratio and growth is similar to that obtained by Herndon et al. (2013) using locally smoothed regression, but we could argue that the recursive mean values are more sensitive to different segments of the data and less akin to the choice of the smoothness parameters.

Clearly, there is no single threshold that would change the pattern of slower growth with higher debt. Contrary to Reinhart and Rogoff (2010)'s proposition that growth problems arise only when very high levels of debt are reached, the graphs suggest that a moderate debt burden may already seriously hinder economic growth. This, of course, depends on institutions, policy credibility and so forth, but, even so, we argue that there is no harmless level of public debt. Some support to the nonlinear effect could, however, be obtained by using Kalman Filter technique in estimating the bivariate relationship between GDP growth and debt/GDP relationship:

$$\Delta y_t = g_0 + \rho_t * D_t + u_t \qquad (1)$$

$$\rho_t = \rho_{t-1} + e_t,$$

where Δy_t denotes the growth rate of GDP and D the (log of) Debt/GDP ratio. This simple model was estimated for the whole period 1820-2010 which includes 5539 observations that were ordered in an ascending order for all years and countries. The estimate of parameter g_0 was 5.787 (t-ratio 20.48) and the Log likelihood -16561. The final value of the state variable ρ turned out to be -.685 (21.69). The time path of (smoothed) ρ and fitted value of g are shown in Figure 5[3]. It appears that there is indeed some sort of kink in the relationship between these two variables but one has to be

[3] For the point of reference, see the estimates of a simple linear model in Table 1.

cautious in arguing on the behalf of a nonlinear impulse response. The scale differences are indeed rather small and they probably reflect the initial values of the coefficient ρ in the subsample where the debt/GDP ratio is practically zero.

As a final check of the functional form, we estimated the bivariate relationship in the following logistic form

$$\Delta y_t = \alpha + \beta*(1/(1+\exp(\theta*D_t/Y_t)))+\tau*D_t/Y_t+ u_{t,} \qquad (2)$$

where Δy denotes GDP growth and D/Y the Debt/GDP ratio. Here parameter θ controls the curvature of the non-linear relationship. The estimates are reported in Table 2 and the fitted values in Figure 7. Quite clearly the relationship resembles the one obtained by the Kalman Filter model (1). Thus, with very small debt levels GDP growth is, *ceteris paribus*, relatively high but with high and very high level of debt the growth rate of GDP settled down to something like two per cent growth rate.[4]

Could the inverse relationship between growth and debt reflect some outlier country or period observations? No, that does not seem to be the case. In Table 4 we report LAD estimates for the simple equation and in Figure 8 we report the coefficient estimates of the debt/GDP ratio with the IMF data for 1948-2012 for each single country and for each calendar year. Quite clearly, the inverse relationship applies for the majority of periods and countries. In terms of time, it is only the 1990 when the relationship is very weak (or nonexistent) while for the set (156) countries, the positive outliers are mainly developing countries.

3. Interpreting causality between debt and growth

Of course, the plotted values (of correlation) can be interpreted in different ways from a causal point of view and our simple techniques cannot really solve the problem, although we may point out that the use of a lagged debt-to-GDP ratio instead of its contemporaneous value does not make any

[4] A similar result is obtained by using recursive estimates the log linear equation reported in Table 1. For space reasons, the graphs are not reported here.

difference to the shape of the curvatures. The problem is that debt-to-GDP ratios are highly persistent, so that different lead-lag relationships are not very informative in solving the problem of causality. The conventional causality tests (Table 3, two first sets of statistics) that we have carried out would suggest that causality is bi-directional, so that it would require an extreme view to argue that causality runs only from output growth to debt.

Moreover, it is a bit hard to imagine that a temporary slowdown in economic growth, or just cyclical variations of output growth could produce and almost monotonically increasing debt level that has been a symptom for many countries during the last decades. If we consider a typical public balance reaction to a macroeconomic (output) shock we may write the following equation for the public sector balance:

$$Def_t/Y_t = a + b\Delta y_t + u_t, \qquad (3)$$

where Def denotes the public sector balance (positive values indicating surplus and negative deficit). The coefficient of Δy reflects the impact of income changes on taxes and expenditures (due to both automatic stabilizers and eventual counter-cyclical policy rules). The value of coefficient b could be thought to be something like 0.5 but the main thing is that the kind of relationship would imply a linear relationship between output growth and the **change** of debt/income level, not a relationship between the output growth and the **level** of debt[5]. In fact, the fiscal rule relationship (3) would not say anything about the level of debt assuming both automatic stabilizers and policy rules are related in a symmetric way to cyclical movements of output. Only if we had a situation where the fiscal authority reacts to permanent slowdown of economic growth (which the authority falsely interprets a temporary demand shock) by progressively increasing public expenditures we would observe a negative relationship between economic growth and the level of debt[6]. This may become

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[5] That is because of the debt accumulation identity: $D_t/Y_t = (1+r_t-\Delta y_t)D_{t-1}/Y_{t-1} + Def_t/Y_t$, which comes close to the identity $\Delta(D_t/Y_t) = Def_t/Y_t$. Buti and Sapir (1998) considered the sensitivity of government deficit with respect to output gap $d(Def_t/Y_t)/d(y_t-\hat{y})$ denoted by ε, and ended up with estimate of ε being of the magnitude of 0.5. See Bénassy-Quéré et al (2010) for more extensive treatment of this issue. Thus basically we have two competing relationships: one between Δy and D/Y and another between $\Delta(D/Y)$ and Δy. Because the first could be expressed in first differences in the form $\Delta^2 y$ and $\Delta(D/Y)$, we may compare the explanatory power of these relationships vis a vis $\Delta(D/Y)$. In the data, both are negatively (and significantly) correlated with $r_1 = -0.063$ and $r_2 = -0.115$. So both stories get some support from the data. Given the non-nested nature of relationships (1) and (2), it is a bit hard to interpret VAR results of the type Lof and Malinen (2014) where a bivariate relationship between (per capita) GDP growth and (per capita) debt (not the debt/GDP ratio) is analyzed.

[6] This kind of policy was indeed pursued in some countries in the 1970s after the first oil shock (which was assumed to be just temporary) but it is difficult to find other instances of similar kind. An intuitively more realistic scenario would be such where the debt/GDP ratio goes down due to rapid growth of income (and prices). Developments right after the WWII would perhaps (also) reflect this kind of process.

evident if we eliminate entirely cyclical variations from the data. In drawing Figure 4, we have used 10-year moving averages of GDP growth and debt-to-GDP instead of annual rates in computing the recursive mean values. Clearly, the negative slope remains the same.

A similar result emerges if we estimate the simple bivariate relationship between Δy and Debt/y so that the dependent variable is not the actual growth rate of Δy but a cyclically adjusted growth rate, or alternatively even the sample mean of the growth rates (Table 4). The, we effectively have a cross-section regression between average growth rate and the average debt/GDP ratio. Quite clearly, the coefficient of Δy is negative in all cases suggesting that negative relationship between these variables does not only reflect cyclical variations of output growth and debt but also a long-run relationship between these variables. As for the magnitude of the results, they are roughly of the same size as in Kuman and Woo (2010) and Cecchetti et al (2011).
Notice also that if run the Granger causality test statistics between Δy and Δ(D/Y) (Table 3) the results differ from those between Δy and (D/Y) even though we still end up with a bidirectional relationship. It would make more sense to think that output shocks affect government deficit than government debt. On the other hand, government debt, not so much government deficit, may have a causal effect on output growth. The basic nature of results seems to be the same even though we extend the lag structure up to 10 yours (see the last set of statistics in Table 3).

One possible way of finding out the nature of the causality effect would be an analysis where output growth is explained by the some initial level of debt. That comes close to using long lags in past and future growth rates and debt (see e.g. Dube 2013). We carried the experiment in such a way that all initial levels of debt (more precisely, the debt/GDP –ratio) for 1948-2012 were used in the analysis where the dependent variable was the average growth rate between the initial period and 2014. The results of this analysis are reported in Figure 8. The interpretation of the graph is rather straightforward: most of the time the relationship is negative but for (the initial debt levels of) the 1980s and 1990s no such relationship can be detected.[7]

Obviously, we cannot make a final verdict on causality on the basis of these two variables only. It may well be that in a more general setting, the outcome is different. Take for instance that the possibility that the dominating exogenous variable is demographics. Deteriorating demographics

[7] We carried out a similar exercise in another round by ”explaining” the current level of debt by the previous mean growth rates (starting from 1948). Also in this can the coefficient was mainly negative but the negative values concentrated to the early part of the sample period.

could well cause a slowdown in economic growth and, at the same time, an increase in government expenditures. In this case, bidirectional causality between economic growth and debt would only reflect the impact of this “third” variable. A quick look at the data would not, however, support this view.[8]

4. Debt and investment

Comparing GDP growth and government indebtedness is not necessarily very informative because the growth of GDP components may show completely different character. Take a simple example. GDP growth and indebtedness may be positively correlated because public consumption is financed by accumulating debt. Expansive (or contractionary) fiscal policy would at least in the short run produce a positive association between income growth and debt.

A useful check would be to compare the investment share of GDP (or the share of public investment out of total public expenditures). If debt depresses investment, which is probably a typical interpretation of the “toxic debt” hypothesis, we would expect to find a negative relationship between the investment share and the government indebtedness. If causality runs from investment to debt we should interpret the evidence as suggesting that negative investment shocks translate to income growth problems which in turn cause financing problems within the government. The bigger is the investment shock, the more debt increases. Alternatively we could interpret the evidence from the point of structural problems: investment slowdown (that is caused by some third variables) causes a slowdown of GDP growth. If government tries to maintain the previous expenditure (growth) level it is only possible by continuously accumulating debt.

As for empirical evidence, the recent paper by Salotti and Trecroci (2012) ends up with results that favor the “debts causes the slowdown of investment (and productivity)” interpretation that obviously makes better sense for the trend-like movements of these two time series.

It has sometimes been argued that debt accumulation is caused by government’s additional expenditure to public investment – possibly as a reaction to a reduction of private investment. It

[8] To see that, we carried out an experiment where the ratio of working-age population to total population (Dem) was introduced into the simple bivariate regression. It did not make a noticeable difference in results irrespectively of the way the equation was estimation (Table 5). This does not, of course, mean that the case for causality is settled so easily. Panizza and Presbitero (2012) do in fact show (with OECD data) that the relationship does no more exist when proper instruments are used. Clearly, the case requires more analysis.

would be relatively easy to examine whether this conjecture is right by comparing these to time series.

That is done in Figure 9, were we first compare the share of (private) investment of GDP and the Debt/GDP ratio (the recursive mean values is computed in the same way as in Figure 1-4). In the right-hand-side panel, we compare the share of public investment of total government expenditures. The outcome is quite clear: investment shares decrease almost monotonically along with the debt/GDP ratio. It is easy to reconcile this outcome from the point of view of the toxic debt hypothesis but if we consider the reverse causality alternative we have rather adopt the structural view which says that governments react to investment slowdown by continuously accumulating debt. As for the public investment share, the interpretation is less ambiguous: increase in debt is clearly not caused by new public investment projects. Rather the borrowed money is used to transfers and public consumption. When there is no debt, the share of public investment of all expenditures is above 13 per cent but when debt goes to extreme levels (say, over 100 per cent of GDP), the share drops roughly to 7 per cent. A couple of decades ago, David Aschauer (1989) showed that a slowdown of public investment could be one of the major reasons why economic growth had decreased dramatically since the 1970s. If his estimates are indeed applicable we could argue that the adverse development of public investment would indeed have contributed to slowdown of economic growth especially because it seems that no offsetting change has occurred with private investment.

5. Discussion

There is no doubt that there is an inverse relationship between debt and growth but the economic implications of this finding are not equally clear because the causality issue cannot be easily solved. As pointed out earlier, we tend to favor the interpretation the excessive debt causes slowdown of growth rather than the opposite. Partly this is because the relationship not only shows up in actual data but also in cyclically adjusted data and even more in the cross-section data. Moreover, if (cyclical) income shocks caused the debt problems we would perhaps see the effect with the low levels of debt but not so much with the very high values of debt. In other words we would expect the debt/GDP -ratio elasticity to fall. But is seems that the relationship is rather linear than U-shaped. The behavior of investment, particularly public investment, is also something that rather poorly corresponds to the explanation that the increase of debt is just a result of fiscal policies. Of course, these findings do not exclude the possibility that both causal explanations are present at the same time. Rather, the question is of the relative (quantitative) importance of these explanations. To study that, we need more data, and in particular we need data for control variables.

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Table 1 **Estimation results of a simple linear bivariate model**

constant	4.249	(44.18)
Debt/GDP	-.014	(10.06)
R2 = 0.017, SEE 4.807, Log likelihood = -16555		
constant	5.898	24.97
log(Debt/GDP)	-.703	10.64
R2 = 0.020, SEE = 4.801, Log likelihood = -16549		

The dependent variable is the growth rate of GDP. The sample period is 1820-2010.

Table 2 **Estimates of a logistic regression model**

$\Delta y_t = 1.899 - 5.415*(1/(1+\exp(-.020*D_t/Y_t)))$
$t_1 = 4.96$, $t_2 = 7.62$, $t_3 = 3.89$, $R^2 = 0.020$, SEE = 4.802, Log likelihood = -16549

$\Delta y_t = 6.400 - 2.645*(1/(1+\exp(-.009*D_t/Y_t))) - .011*D_t/Y_t$
$t_1 = 9.90$, $t_2 = 3.78$, $t_3 = 2.31$, $t_4 = 4.32$, $R^2 = 0.021$, SEE = 4.801, Log likelihood = -16547

Δy = GDP growth, D/Y = Debt/GDP

Table 3 **Pairwise Granger Causality Tests**

Null hypothesis:	**Obs**	**F Statistic**	**Prob.**
Δ(D/Y) does not Granger Cause Δ(Δy)	5196	16.82	0.000
Δ(Δy) does not Granger Cause Δ(D/Y)		12.44	0.000
Δ(D/Y) does not Granger Cause Δ(Δy)	2969	16.19	0.000
Δ(Δy) does not Granger Cause Δ(D/Y)		4.16	0.016
Δ(D/Y) does not Granger Cause Δy	5238	6.85	0.001
Δy does not Granger Cause Δ(D/Y)		14.65	0.000
Δ(D/Y) does not Granger Cause Δy	2984	4.33	0.013
Δy does not Granger Cause Δ(D/Y)		10.04	0.000
(D/Y) does not Granger Cause Δy	3278	3.94	0.000
Δy does not Granger Cause (D/Y)		3.55	0.001

The larger (smaller) sample corresponds to the sample period 1820-2010 (1950-2010). The last set of results makes use of the IMF data for 156 countries. In this case, the lag length is 10.

Table 4 **Estimates with various transformations of the GDP growth variable**

Dep.var	***constant***	***Debt/Y***	***R^2***	***SEE***	***Estimation setting***
mean(Δy)	4.127 (237.11)	-.007 (25.81)	0.051	1.051	cross-section all data
mean(Δy)	4.361 (128.49)	-.008 (19.67)	0.064	1.196	cross-section 1960-2010
mean(Δy)	3.723 (65.83)	-.009 (14.41)	0.085	1.430	cross-section 1980-2010
Δy	4.59 (41.80)	-.010 (6.31)	0.011	4.973	panel
Δy	3.966 (47.58)	-.011 (7.18)	0.011	4.970	panel
Δy	3.869 (3.90)	-.006 (3.90)	0.178	4.639	panel CF + TF
MA(Δy,5)	4.168 (67.08)	-.011 (10.62)	0.064	2.385	panel
MA(Δy,5)	4.003 (73.08)	-.011 (11.68)	0.064	2.378	panel, GLS
MA(Δy,10)	4.103 (63.74)	-.011 (9.56)	0.086	1.951	panel
MA(Δy,20)	3.986 (67.81)	-.009 (8.51)	0.079	1.665	panel
MA(Δy,5)	3.960 (77.50)	-.007 (8.23)	0.411	1.947	panel CF + TF
AR2(Δy)	3.872 (66.97)	-.008 (8.40)	0.029	2.502	panel
AR2(Δy)	3.633 (100.05)	-.003 (5.37)	0.612	1.635	panel CF + TF
Δy	4.952 (27.31)	-.012 (4.36)	0.003	11.13	IMF panel
Δy	4.484 (54.68)	-.013 (10.88)	0.012	11.14	IMF panel LAD
Δy	5.267 (22.13)	-.017 (4.69)	0.061	10.95	IMF panel +CF

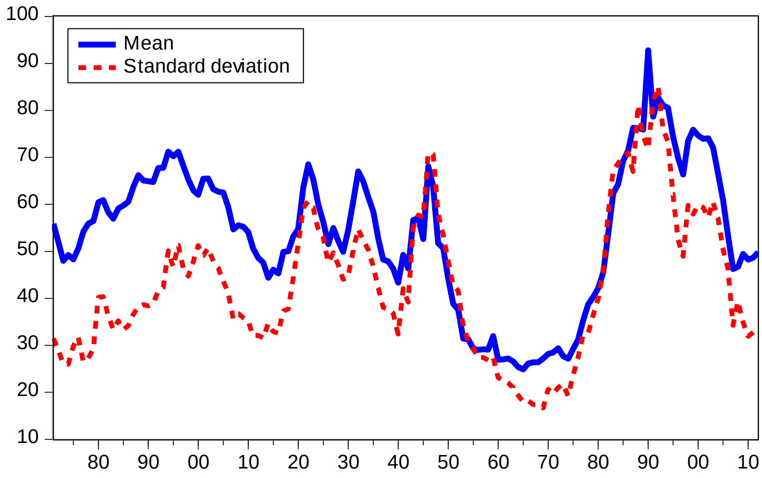
Mean(Δy) denotes the sample mean of Δy. MA(Δy,5) denotes the moving average of Δy with 5 lagged terms. AR2(Δy) denotes the predicted values of Δy with an AR(2) model with fixed cross-section (CF) and fixed period effect: $\Delta y = 2.98 + .20\Delta u_{-1} + .05\Delta y_{-2} +$ CF + TF. All estimates, unless otherwise indicated, are OLS estimates. In the panel data, the number of data points is 5539. In the cross-section data, we have 65 countries. In the IMF panel, the number of countries is 156 and data points 5566.

Table 5 **An experiment with a demographics control variable**

Dep. var	***constant***	***Debt/Y***	***Δy***	***Dem***	***R^2***	***SEE***	***Estimation setting***
Δy	1.880 (0.57)	-.026 (6.14)		.040 (0.80)	0.405	2.333	panel CF + TF
Δ(Debt/Y)	8.366 (1.28)		-.498 (4.44)	-.100 (1.02)	0.141	7.736	Panel CF+TF

The sample consists of 27 OECD countries for 1960-2010.

Figure 1 **Debt-to-GDP ratio in the IMF data for 1871-2012**



Data source: *www.imf.org/external/datamapper/index.php?db=DEBT*

Figure 2 **Debt-to-GDP ratio and GDP growth in advanced economies**

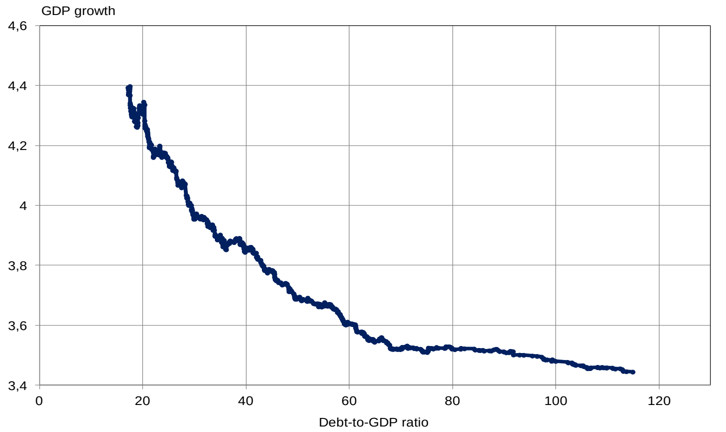


Figure 3 **Debt-to-GDP ratio and GDP growth for all countries**

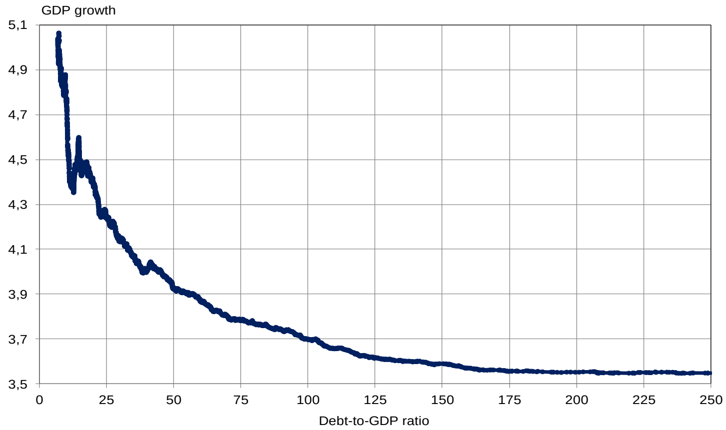


Figure 4 **Debt-to-GDP ratio and GDP growth from largest to smallest observation**

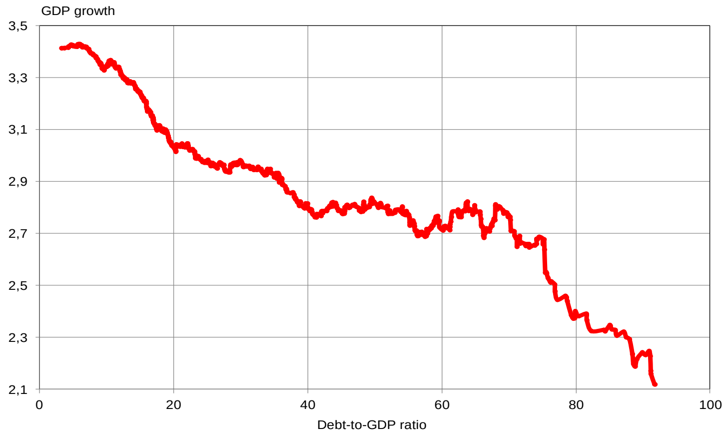


Figure 5 **10-year moving average of GDP growth and Debt-to-GDP ratio**

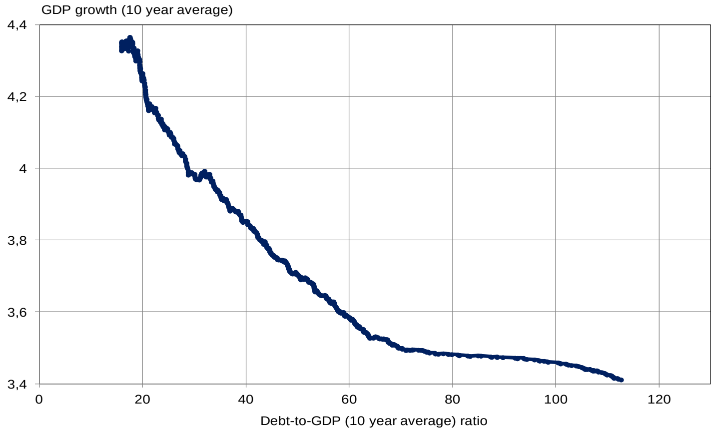


Figure 6 **Kalman filter estimates of the GDP coefficit and the fitted value GDP growth**

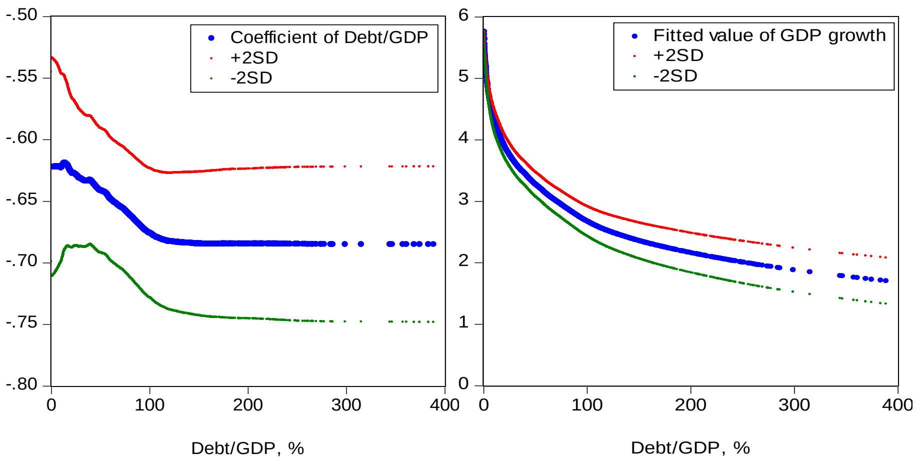
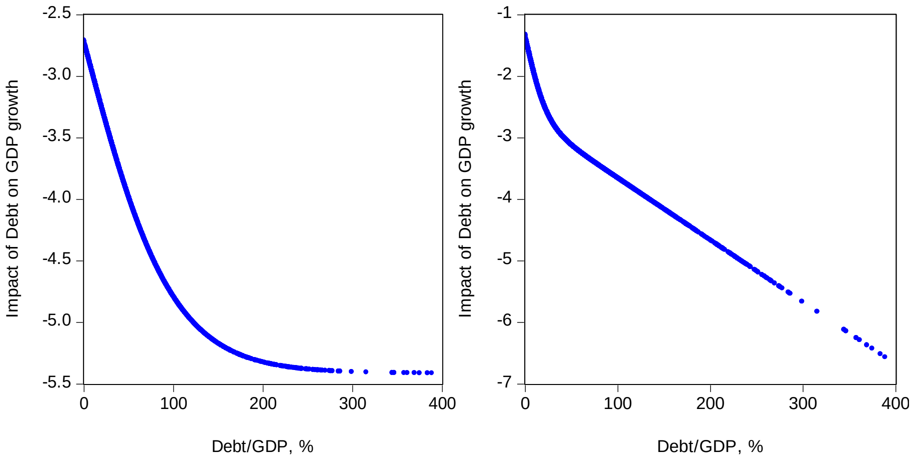
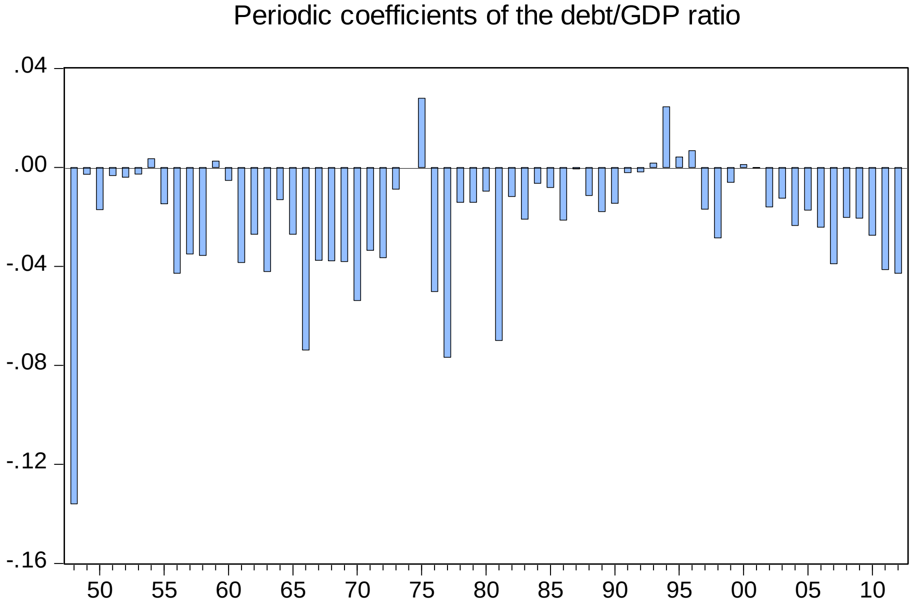


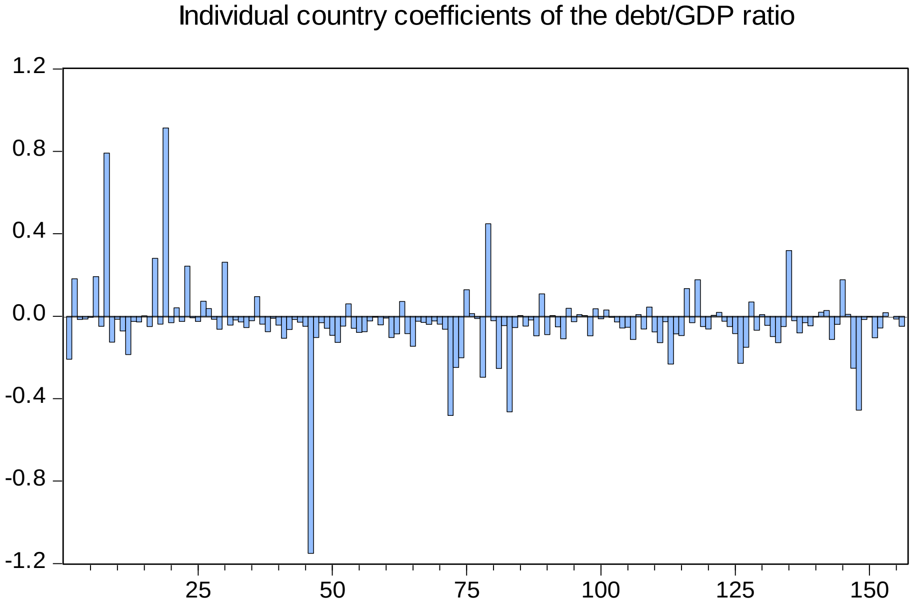
Figure 7 **Impact of Debt on GDP growth according to the logistic regression**

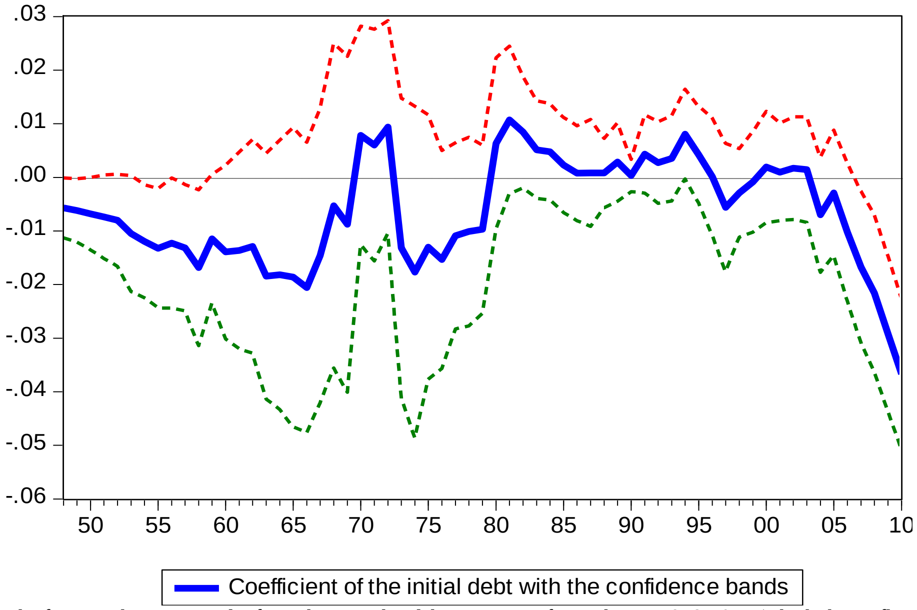


In the right-hand-side panel, coefficient of linear term τ is set to zero.

Figure 9 **Time-series and cross-section results with the IMF data**

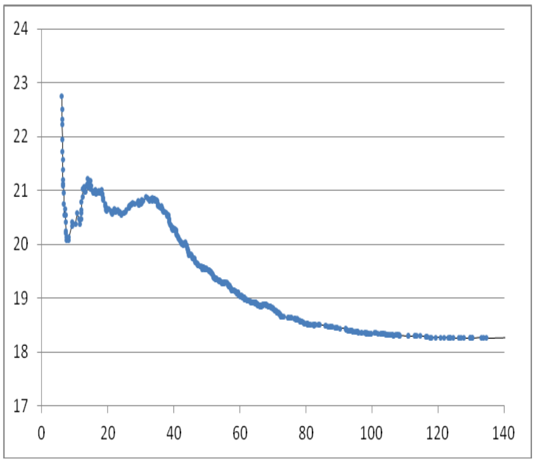


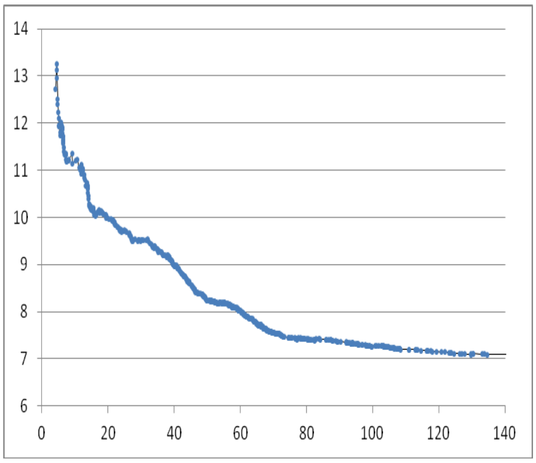




The first graph report results from the growth - debt regression for each rear 1948-2012 (which thus reflects cross-section evidence over time), the second results for each 156 sample country (reflecting time series evidence for individual countries) and the third results from a regression $gt_0 = \beta_0 + \beta_1 debt_{t0}$, where gt_0 denotes the mean value of GDP growth rate for t_0 – 2014 while $debt_{t0}$ is the initial debt level. Here t_0 goes from 1948 to 2012.

Figure 9 **Debt and investment**





In the left-hand-side panel, there is the relationship between private investment/GDP and the Debt/GDP ratio, while in the right-hand-side panel the relationship between the share of public investment of total public expenditure and the Debt/GDP ratio is displayed.