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Top Achievers in Mathematics in the End of Upper Secondary School

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Abstract: Important questions regarding mathematical giftedness are how and when it is possible to identify. To be identified as gifted, the student must have natural potential but also an appropriate mix of motivation, support, and challenges. This study is based on longitudinal data following students from 3rd grade in primary school to the end of upper secondary school between 2005 and 2015. We focus on top achievers (<2% of age cohort) of the national mathematics final exam at the end of upper secondary school. We investigate how accurately top achievers at the end of secondary school can be identified in 3rd, 6th, and 9th grades using national tests. We identify mathematical tasks that predict future top achievement and analyze how attitudes, gender, and parental background factors relate to high proficiency. Most top achievers had already been identified by 3rd grade and almost all of them by 9th grade. However, recognizing future top achievers was not very accurate, as they were indistinguishable from many students whose performance did not reach the same level over time. The best predictor for future top achievement was a student's ability to solve non-routine and atypical tasks in early school years.

Keywords: giftedness; longitudinal research; mathematics; top achievers

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1. Introduction

Giftedness is an elusive concept and in educational contexts it is often difficult to separate from high achievement [1–3]. Researchers debate the definition of giftedness and the contribution of innate abilities and the social environment in its formation [2,4]. To define and identify mathematical giftedness can be viewed as essentially the same problem, and there is a lack of systematic and consistent research about it [5]. In this study, we presume that success in mathematics is not based on specific innate abilities alone and that giftedness is not a static feature of a person. We see success in mathematics as *potential* in the same manner as Leikin [6]; this potential develops in the interaction of individual and social environmental factors in accordance with the socio-cognitive theory (see [7]).

Although the definition of giftedness is debated and the identification of giftedness is difficult, we examine top-achieving students with an assumption that most of them would be considered gifted in mathematics. Our aim is to investigate at which stage of basic education top achievers in mathematics can be identified and whether there are any predictive factors that could be identified and used to support the development of children's mathematical potential.

The study is based on the longitudinal data collected by the Finnish National Agency for Education (FNAE) and the Finnish Education Evaluation Centre (FINEEC). The data consist of students who were followed from 3rd grade of primary school to the end of upper secondary school between 2005 and 2015. The data were collected for the needs of national evaluations, and several reports have been made about mathematics national learning outcomes [8–11].

Top-achieving students were identified based on the matriculation examination of advanced mathematics. It is one of the final exams that are held twice a year simultaneously in all general upper secondary schools. In the Finnish context, there are no other official final exams. The Finnish education system consists of early childhood education, pre-primary education, basic education (grades 1–9), upper secondary education, and higher education (for more details, see [12]). After 9th grade, students apply for vocational upper secondary school or general upper secondary school. From general upper secondary school, they usually apply for higher education. In general, in upper secondary school, a student chooses whether to study mathematics via an intermediate or advanced syllabus. In addition, at the end of general upper secondary school, the student decides whether they will participate in the matriculation examination of mathematics. Our target group of this study is the most successful students in the matriculation examination of advanced mathematics.

In Finland, research of mathematically gifted students has been limited. Niemi and colleagues [13–15] examined high-achieving 9th graders from the same longitudinal data without the data of matriculation examination. In [13], they investigated how high achievers' mathematical competence developed during basic education and what kinds of factors predicted a student's development into a high-achieving student. In [14], they examined high achievers' choices when transitioning to upper secondary school and how their mathematical competence developed there. In [15], the focus was on high achievers' attitudes toward mathematics and how they developed from primary education to the end of upper secondary school.

In this article, we examine the top achievers in mathematics at the end of secondary education, with the aim of investigating when and how these students could be identified years earlier. More specifically, we examine what kinds of mathematical tasks are best for identifying future high performers and what other individual and social factors predict future achievement.

2. Mathematically Gifted Students

Mathematical giftedness is an ambiguous concept, and there is no consensus regarding its definition. A student who does well in mathematics is not necessarily gifted; on the other hand, a mathematically gifted student does not necessarily achieve high results in mathematics [16,17]. Mathematical giftedness generally refers to a high ability in mathematics, and the concepts of giftedness, high ability, and high achievement are often used interchangeably [3]. It is relevant whether high mathematical skills are seen as innate and immutable or as skills that can be developed. Students who believe that their mathematical skills are completely innate (fixed mindset) succeed less well than average students who are aware that they can develop their skills (growth mindset) [18].

Genetic qualities are seen as the basis for giftedness development. Yet, genetic readiness is not a sufficient premise for giftedness. In addition, persistent training is needed, among other things [19,20]. Krutetskii [21] sees mathematical giftedness as consisting of an individual collection of mathematical skills that enable success in mathematics, but at the same time, the student's internal motivation and the teacher's role in arousing interest are key factors in the development of mathematical competence. For example, Mönks and Katzko [22] agree that the social environment and motivation are central in the development of giftedness. Krutetskii [21] sees that mathematical skills can be developed but that developing into a top mathematician requires certain genetic characteristics that are related to the structure and function of the brain, among other things. According to a more recent view, mathematical giftedness is a combination of mathematical expertise and creativity [6]. Mathematically gifted individuals are characterized by the ability to do multifaceted problem solving, which is accompanied by flexible mathematical thinking. In addition, the cognitive factors of mathematically gifted individuals, such as the use of working memory or the orientation of attention, are exceptional [3,6].

Alternative concepts have been presented alongside mathematical giftedness. Mathematical *potential* [5] is a concept that reflects the dynamic perspective of mathematical skills. Leikin [5] sees that mathematical potential can develop into mathematical talent if an individual with mathematical potential is offered challenging learning opportunities that match their individual abilities, personality, and affective characteristics. The U.S. National Council of Teachers of Mathematics (NCTM) developed the concept of mathematical *promise*, which also emphasizes the influence of circumstances on the development of competence [23]. The NCTM defines mathematical promise as consisting of four interacting components: ability, motivation, beliefs, and experiences/opportunities. All of these should be developed for the student to achieve their highest possible mathematical performance [23].

It is problematic to distinguish high performance from giftedness. Students who clearly do better than average in their studies are usually defined as gifted. An above-average performance on a single school mathematics test is not sufficiently reliable for identifying mathematically gifted students. We can identify excellent performance in school mathematics and talk about *high-achieving* students [24], but at the same time, we cannot recognize all students who, for example, underachieve on a specific test. Conventional mathematics tests do not recognize the diversity of high-achieving students and giftedness, because they often focus on measuring basic skills. For example, in Australia and New Zealand, mathematically gifted students have been identified by selecting those ranked at or above the 90th percentile on a mathematics multiple-choice test, the Progressive Achievement Test (PAT). The PAT has been found to have an accuracy of 78% in identifying gifted students [25]. Another common method used in gifted program identification is that the teacher nominates potentially gifted students for further testing, referred to as the nomination stage. However, studies have shown, e.g., [26], that the nomination stage can result in a false negative rate that easily exceeds 60%.

We need tools to distinguish exceptionally talented students from others, cl. [27]. In addition to high mathematical competence, to distinguish the gifted students from other well-achieving students, we need to detect more specific characteristics of the students. These characteristics may include the ability to apply mathematical thinking in novel situations. A distinction can be made between creative mathematical competence and competence of school mathematics [28]. Students with high competence in school mathematics have the potential to produce new results and achievements that also have social value. Mathematically gifted individuals are seen as capable of high-level problem solving and inductive thinking. They have a high ability for logical reasoning, high confidence in their own abilities, and internal motivation for mathematics [29,30]. Mathematically gifted students are often identified by their ability to solve complex problems and their ability to think mathematically well beyond that of their age group [31].

In the USA, the Scholastic Aptitude Test (SAT) has been used to identify participants aged 12–13 in the Study of Mathematically Precocious Youth (SMPY). The aim of the SMPY is to identify talented children and support the development of their exceptional skills [27,32,33]. One part of the SAT is the measurement of mathematical skills (SAT-M). Originally, the SAT was designed to measure students' readiness for university studies. For the purpose of the SMPY, the test measures skills such as algebra and geometry, which children have not yet been taught at this stage. Several children have been found to exceed the entrance requirements of many top universities [27]. Prior to the SAT, almost all students were required to earn scores within the top 3% on a conventional achievement test, and the final selection criteria has varied from a top-0.01% to a top-3% criterion [27].

Students who are successful in mathematics have been found to solve non-routine tasks better than others [34]. For routine tasks the student already has a familiar strategy to solve the task, but solving non-routine tasks requires a flexible use of strategies and demands creativity and originality to create new types of solution methods [34–36]. Abstract conceptualization has been found to be a significant predictor of success in mathematics, regardless of the type of task [37].

Among topics of mathematics content, knowledge of fractions and whole-number division in elementary school have been found to predict better algebra-related knowledge and mathematics achievements in high school [38]. In Finland, high achievement on the 9th-grade national testing of learning outcomes and high skills in geometry on the 6th-grade test predict high achievement in the upper secondary level [15]. The high-achieving 9th-grade students in mathematics have been found to perform clearly better than their age group in tasks related to plane geometry, the perimeter of a parallelogram, and the shape of a function, as well as tasks where an easy equation solution requires justification [39] (p. 49).

The reason to identify mathematically gifted students and support the development of their potential is that they have exceptional opportunities to contribute to society. Longitudinal studies conducted by the SMPY demonstrate that gifted individuals have reached leadership positions, and many of them are outstanding creators [27,40,41]. The SMPY is a significant longitudinal study, but it has been conducted within the unique context of the American education system. In other countries there has been much less longitudinal research regarding the identification and development of mathematically gifted students. In the SMPY, the identified individuals have been followed forward. However, there is a need for research that also looks backward, aiming to identify whether gifted students could have been predicted earlier. It is also important to investigate what kind of tasks would be suitable for identifying individuals in the context of a different educational system and to explore the effort of individual and environmental factors as well. Finland forms an interestingly different educational system for examining the development of giftedness, as there is hardly any streaming of students according to their achievement until grade 10 and variation of student achievement between schools is low.

3. Individual and Environmental Factors behind Mathematical Competence

According to Bandura's socio-cognitive model [7], mathematical skills develop in the interaction of individual and environmental factors. This study focuses on examining some central background factors and aims to find the factors that predict mathematical talent. Individual factors determine mathematical competence, but environmental factors are relevant for the manifestation and development of the competence.

3.1. Individual Factors

Individual factors include, for example, the individual's previous math skills, attitudes toward mathematics, and gender. A student's previous math skills in elementary school have been found to be a significant predictor for whether the student is among high-achieving students in 9th grade [13]. Strong mathematical skills in basic education predict that the student will do well in mathematics also at the upper secondary level [14,35].

Several studies have shown that there is a positive correlation between mathematical competence and math-related attitudes [7,42–45]. The relationship between attitudes and competence has been studied especially from the perspective of self-beliefs such as self-concept, self-efficacy, and self-confidence. Mathematical self-concept, that is, individuals' concept of their mathematical skills, has been found to explain proficiency better than other attitudes, as it controls individual behavior and choices [7,46]. Several studies, e.g., [47–49], have shown that those who trust their own abilities are the most successful in studies of mathematics. In the PISA 2012 study, mathematical self-concept and performance confidence have been found to be the strongest explanations for mathematical competence in Finland [50]. According to the international comparative analysis based on the PISA data [51], the effect of mathematical self-efficacy on competence is relatively small in Finland compared to other countries. On the other hand, the effect of mathematical competence on self-efficacy is one of the largest in Finland. According to the national longitudinal analysis, mathematical competence in elementary school affects mathematical attitudes in higher grade levels [52,53]. The better a student's competence is in 9th grade, the more positive their perception of mathematics as a school subject and the more likely they are to choose studies in general upper secondary school and advanced mathematics [10]. The connection between competence and attitudes becomes stronger with age when perceptions become more realistic [42].

Although the gender differences in mathematics have narrowed, girls generally apply for university places and jobs in the field of mathematics less often than boys, both internationally, e.g., [54,55], and in Finland [56]. In addition, boys are overrepresented among the best achievers in mathematics, e.g., [24,57,58]. In Finland, the proportion of boys among the best achievers in mathematics is slightly higher than the proportion of girls [59]. In general, there is more variation in skills among boys than girls [60,61]. According to a meta-analysis by O'Dea and co-researchers [61], the greater variation among boys has remained the same over the past 80 years, and it is especially visible in mathematics and science subjects. For such a variability hypothesis, it has been suggested that a partial explanation can be found in heredity [62]. An alternative explanation for the smaller proportion of girls among the best math performers is that girls are giving up mathematics [63] to focus on and invest in other subjects. This can be seen very early on [64].

Mathematics is associated with strong gendered stereotypes that determine girls' and boys' perceptions of themselves as math learners from an early age. Already at the beginning of school, girls estimate that they are weaker than boys in mathematics, even though there is no difference in the mathematical skills between girls and boys [65,66]. According to Oppermann and co-researchers [67], gender affects the desire to study mathematics already in 2nd and 3rd grade. Girls' perceptions of their own mathematical competence deteriorate more strongly than boys' as the school years progress and the gap between genders increases [10,52,66]. Gender differences in self-confidence were larger in Finland than in many other PISA countries [51]. Girls' weaker self-confidence can be seen, for example, in the choice between intermediate and advanced math in upper secondary school and STEM choices [68,69]. In addition, in the end of upper secondary school, female students experience more negative emotional states than male students at every proficiency level [11]. On the other hand, Niemi [24] showed that the high-achieving girls' attitudes developed in different ways than girls' attitudes in the average. In the study, high-achieving girls' attitudes developed in the upper level of comprehensive school and in upper secondary school to a higher level than high-achieving boys' attitudes. In elementary school, high-achieving girls might not have received as much support with their attitudes and motivation as boys, which could explain their less positive attitudes at an early stage [24].

3.2. Environmental Factors

Environmental factors include, among others, parents' socioeconomic status and learning environment. In this study, we focus on students' parental background, which includes information on whether the parents have completed a matriculation examination and students' perceptions of parental support for studying mathematics. Several studies have shown that the students' parental background and socioeconomic status strongly explain students' mathematical competence, e.g., [70,71]. According to the study based on PISA data, the connection between socioeconomic background and the learning outcomes of Finnish students has strengthened in recent years [60,72,73]. However, there is reason to approach the inequality of learning outcomes with caution. There is considerable measurement error in the PISA data regarding parents' education [39] (p. 29), [74]. This is related to the response bias of the parents' education level when the information is collected from the students and researchers ignoring the documented increase in the education level of Finns [74]. In addition, recent measurement of socioeconomic background has become more precise [39].

Variables measuring socioeconomic background are considered to be stronger explanations of variation in learning outcomes than the student's gender [75]. The latest TIMSS and PIRLS studies show that the starting level of 4th graders is related to socioeconomic background, and basic education can only partially equalize these differences [75]. However, the connection between socioeconomic background and learning outcomes is weaker when the student's cognitive skills or previous skills are considered [76,77].

Socioeconomic status is defined in different ways in different contexts, and there is no generally accepted way of measuring socioeconomic status [78,79]. The definition is mostly based on the parents' income, education, and information about their profession, i.e., social and economic status. In the PISA studies, socioeconomic status is defined with the ESCS index, which includes cultural status in addition to social and economic dimensions. Parents' education is considered one of the key components of socioeconomic status and has been used in national evaluations of learning outcomes as a simple indicator of socioeconomic status. In Finland, parents' completed matriculation examination has clearly explained differences in competence in different national studies, e.g., [8–11,80,81]. Parents' education seems to play a key role in the development of mathematical competence from 3rd grade up to the upper secondary level [9,10].

In addition to socioeconomic background, studies have shown that the importance of parents' support for learning is central, e.g., [10,82,83]. The support, attitudes, and influences given by parents are reflected in the child. If the parents give the child support and are interested in the child's schooling, the child will do better in school, e.g., [82]. The connection of support to competence has been studied among the students of the upper secondary level on the national evaluation of mathematical competence [10]. According to the results, support significantly explains competence in both general upper secondary school and vocational upper secondary school. In general upper secondary school, the connection can be seen more strongly: The more the student felt they had support for their studies, the higher their competence was. The difference in competence between the extreme groups who received parental support corresponded to two years of studies. Educated and high-income parents invest especially in boys' education and guide them more strongly to go to general upper secondary school [83].

4. Research Questions

The aim of the study is to find out at which stage mathematical talent can be identified and what kinds of factors predict students' development into top achievers in mathematics. We explore how well the different methods identify top achievers. Such identification has two types of potential errors: false negative and false positive. A false negative is an outcome where the model does not identify all the top achievers. A false positive is an outcome where the model incorrectly predicts someone be a top achiever.

1. At which stage of basic education can the top achievers in mathematics be identified?

Hypothesis 1. *The majority of top achievers in mathematics can be distinguished from other students of advanced mathematics during elementary school. In 9th grade, the group is formed most clearly when the students have made the decision about whether they will apply to general upper secondary school or to a vocational education track and whether they intend to study intermediate or advanced mathematics if they go to general upper secondary school* [10,13].

2. What kinds of factors predict students' development into a top achiever in mathematics?

    2.1 What kinds of mathematical tasks predict development into a top achiever in mathematics?

    Hypothesis 2.1. *Mathematically talented students stand out in their ability to solve atypical and non-routine tasks for their age* [34,37]. *Mathematically talented students are characterized by the ability to solve multiple problems and have flexible mathematical thinking* [40].

    2.2 What background factors predict development into a top achiever in mathematics?

Hypothesis 2.2. *The majority of the top achievers are boys* [24,57,58,63]*. The best predictors are probably the parents' education, e.g.,* [70,71,75]*; parental support, e.g.,* [10,82]*; and the student's positive self-concept, e.g.,* [47–50].

5. Methodological Solutions

The data consist of two different types of data: the data of the national longitudinal evaluation of mathematical learning outcomes and the data of matriculation examination results of advanced mathematics at the end of upper secondary general education (grade 12). The national longitudinal data were collected by FNAE and FINEEC. The different datasets are sample-based and nationally representative. The data consist of the same students' mathematics test results, attitudes toward mathematics, and background factors that were collected in grade 3 (2005), grade 6 (2008), grade 9 (2012), and at the end of upper secondary school (grade 12) (2015). The results are mainly based on these longitudinal data.

Longitudinal data were supplemented by the students' grades on the matriculation examination of advanced mathematics in 2015. This study examines the students who were the most successful on the national matriculation examination of advanced mathematics.

5.1. The Data Set and Participants

The total sample of the national longitudinal data consists of 3896 students. From this total sample, we examined the students who attended general upper secondary school and passed the matriculation examination of advanced mathematics in the spring of 2015 (n = 490). The students who passed the matriculation examination of advanced mathematics were a selected group of students. Of those who completed their matriculation examination in the spring of 2015, 39% completed the exam of advanced mathematics.

The target group in this study was selected of the most successful students in the matriculation examination of advanced mathematics in the spring of 2015. They received the best grade ("laudatur") on the matriculation examination (n = 37). We call them top achievers in mathematics. The best grade was given to 7.4% of those who took the exam of advanced mathematics. That represents 2.9% of all matriculation graduates and 1.4% of the entire age cohort. Researchers [84–86] have defined that students who score in the top five percent of standardized academic tests are high achievers. Heller [87] has suggested that the best 6–10% of an age cohort are referred to as academically gifted, the best 3–5% as highly gifted, and the top 1–2% as extremely gifted. Applying similar criteria to our sample, students who received the highest grade on the matriculation examination of advanced mathematics were considered mathematically extremely gifted. The top achievers were a limited and selected group of mathematics-oriented and motivated students. Getting the highest grade requires problem-solving skills and skills in applying mathematical knowledge, which are seen as one part of mathematical giftedness. Additionally, it requires motivation to study mathematics and receiving appropriate support to develop mathematical skills.

The comparison groups in this study were the students who achieved the second-best grade ("eximia cum laude approbatur") on the matriculation examination of advanced mathematics (n = 109) (high achievers in mathematics) and other students who completed the matriculation examination of advanced mathematics (n = 344). The comparison is primarily focused on the top achievers and the others, but in order to gain a deeper understanding of the phenomenon, we also seek to uncover differences between top achievers and the high-achieving students.

5.2. Measurements

The national evaluation of learning outcomes in mathematics was based on the targeted learning outcomes, content areas, and criteria set in the curricula of basic education and upper secondary school in this study [88–90]. The national achievement tests of mathematics measured competence in three content areas in every grade: (1) numbers, calculations and algebra, (2) geometry, and (3) data processing, statistics, and probability. In addition, in the upper secondary level, algebra and functions were their own content areas. The tasks, assessment criteria, and scoring instructions for the tests have been drawn up by a group of experts. In addition, experts and pre-testing have been used to evaluate the quality of the task sets. The tests include three task sections: mental math (A), multiple-choice tasks and short answers (B), and tasks that require justification (C). In addition, the tasks are classified into different categories in terms of content area, difficulty level, and depth of knowledge required [91,92]. Because of the linking requirements in the national tests, the test items are not released. Hence, they cannot be published as such in this study. Table 1 shows the number of sections corresponding to the areas of mathematics, the maximum scores, and reliabilities in the tests given to different grades.

Table 1. The contents of mathematics in the tests given to different grades.

	Grade	Number of Items	Maximum Raw Score	Reliability (α)
Overall mathematical competence	3rd	38	44	0.86
	6th	39	52	0.85
	9th	68 [1]	84 [1]	0.94
	General upper secondary school	29	52	0.87
	Vocational upper secondary school	33	46	0.84
Numbers, calculations, and algebra	3rd	22	24	0.81
	6th	21	28	0.78
	9th	36	40	0.88
	General upper secondary school	3	3	0.27
	Vocational upper secondary school	3	3	0.26
Geometry	3rd	10	14	0.67
	6th	10	14	0.66
	9th	16	22	0.83
	General upper secondary school	7	14	0.73
	Vocational upper secondary school	7	14	0.65
Data processing, statistics, and probability	3rd	6	6	0.55
	6th	8	10	0.47
	9th	7	9	0.61
	General upper secondary school	2	2	0.34
	Vocational upper secondary school	5	5	0.56
Algebra	General upper secondary school	6	8	0.71
	Vocational upper secondary school	6	8	0.71
Functions	General upper secondary school	11	31	0.82
	Vocational upper secondary school	12	22	0.66

[1] Contains five tasks of functions.

In order to be able to compare the results of tests from different grades and different versions, the scores of the tests were compared, i.e., brought to a common standard by FINEEC. IRT modeling, based on Item Response Theory [93,94], was used in the comparison. In the data of this study, the test in 9th grade was chosen as the base level for comparison because it serves as the last common measurement point before the transition phase to secondary education (for details see [10] (pp. 213–214)).

The tasks of the matriculation exam of advanced mathematics are based on the curriculum of advanced mathematics of general upper secondary education [88,91] and their targeted learning outcomes and content areas. The curriculum of advanced mathematics includes 10 compulsory courses, but in practice, students taking advanced mathematics complete at least 12 courses. The minimum number of courses for the curriculum of intermediate mathematics is six courses. It has been observed that students who have completed more than 13 courses have a clear increase in their level of mathematical competence during their studies in general upper secondary school [11]. In addition, a varying number of elective courses is available depending on the school. The minimum number of all courses required to complete general upper secondary school is 75. The matriculation examination of advanced mathematics is one of the final exams that are held twice a year simultaneously in all general upper secondary schools. The exams are drawn by the Matriculation Examination Board (MEB), and the MEB assesses the tests of all students after a preliminary assessment by teachers. Because we had access only to the final grades, we were not able to calculate reliabilities on the task sections as we did with the national tests.

A shortened version of Fennema's and Sherman's [95] attitude scale adapted to national needs has been used in investigating students' attitudes toward mathematics [96]. The shortened version has three dimensions: liking mathematics, self-concept of mathematics, and experience of the usefulness of mathematics; each of them is measured by five statements. For 3rd grade, a shortened version of the standard scale was used, and the wording was modified to be more concrete. The aspect of finding the subject usefulness was not included for grade 3 because the questions were largely related to postgraduate studies and working life. Attitudes were investigated according to the dimensions shown in Table 2. The table also shows the number of sections corresponding to the dimensions, maximum scores, and reliabilities according to the different grades.

Table 2. The dimensions of the attitude measurements in different grades.

	Grade	Number of Items	Maximum Score	Reliability (α)
Overall attitude toward mathematics [1]	3rd	8	32	0.86
	6th	15	60	0.88
	9th	15	60	0.91
	General upper secondary school	15	60	0.92
	Vocational upper secondary school	15	60	0.91
Self-concept	3rd	4	16	0.79
	6th	5	20	0.82
	9th	5	20	0.88
	General upper secondary school	5	20	0.86
	Vocational upper secondary school	5	20	0.87
Liking mathematics	3rd	4	16	0.88
	6th	5	20	0.89
	9th	5	20	0.90
	General upper secondary school	5	20	0.92
	Vocational upper secondary school	5	20	0.91
Usefulness of mathematics	6th	5	20	0.81
	9th	5	20	0.53
	General upper secondary school	5	20	0.83
	Vocational upper secondary school	5	20	0.83

[1] Contains self-concept and liking mathematics in 3rd grade.

Information about students' background factors was collected by questionnaires in connection with the achievement tests. In this study, explanatory factors were examined from the perspective of individual and environmental factors. Individual factors included the data describing the student's test success (competence by tasks and overall), attitudes toward mathematics, and gender. Environmental factors were related to parental background, which included parental support for studying mathematics and information on whether the parents had completed a matriculation examination.

5.3. Statistical Analysis

We used logistic regression analysis with stepwise selection to find out which factors separated top achievers from other students of advanced mathematics [97–99]. Some of the explanatory variables were dichotomized (e.g., matriculation examination completed by parents, parental support). The significance of the factors in the model was tested with Wald's X^2 test. However, using the Wald test size involves the risk of rejecting variables because the Wald test size remains small as the standard error increases [98] (pp. 746–747), [100]. In this study, the risk of variable rejection remained small.

The effectiveness of the models produced by the regression analyses was described using Nagelkerke's measure [101]. It should be noted that Nagelkerke's R^2 value does not give an exact degree of explanation of the model like the square of the co-correlation coefficient in a linear regression model, but it gives a sufficiently reliable estimate of the proportion of the observation that the model is able to explain. The effect size was measured using the odds ratio. The value of the odds ratio $Exp(B)$ is a coefficient that indicates the risk level of belonging to the studied group when the explanatory variable increases by one unit. The odds ratio was used to describe how far two probabilities or relative proportions are, but the odds ratio does not directly describe the relationship between the probabilities [102].

When possible, we computed effect sizes using Cohen's *d* and Cohen's *h* values [103]. Cohen's *d* can be used as a measure of the effect size between two independent sample means, and it describes the standardized difference. Cohen's *h* can be used as a measure of the effect size between two proportions. It describes the arcsine-transformed difference. For a large effect, Cohen's values should exceed 0.80.

6. Results

First, we investigated the top achievers' mathematical competence in 3rd, 6th, and 9th grade. The aim was to find out at what stage the top achievers can be identified and how well we can identify them at an early stage. Second, we investigated the kinds of mathematical tasks in 3rd, 6th, and 9th grade that predict development into a top achiever in mathematics and how well it can be predicted. Third, we explored the connection between attitudes and high mathematical proficiency and the connection between gender and high mathematical proficiency. Finally, from the environmental factors, we investigated which kinds of parental backgrounds explain high mathematical skills.

6.1. Top Achievers' Mathematical Competence during Basic Education

The top achievers were partly identified from others in 3rd grade (Figure 1). The figure shows the percentages of each student group. We found that 54% of top achievers ranked in the decile describing the highest competence in 3rd grade. The rest of the top achievers ranked in deciles 6–9.

When we examined the distribution of the highest decile of 3rd graders more closely, it consisted of 14.8% top achievers (n = 20), 33.3% high achievers (n = 45), and 51.9% other students (n = 70). Hence, most of the students in the highest decile in 3rd grade were students who later completed the exam of advanced math but did not achieve the best grade.

Further analysis with logistic regression showed that the mathematical competence in 3rd grade explained 6.1% of the development into a top achiever on the exam of advanced math (Nagelkerke's R^2 = 0.061). The odds ratio of the effect was 3.46 (B = 1.24; $S.E.$ = 0.35; $p < 0.001$).

Sixth grade seemed to be a meaningful stage to identify top achievers; 73% of the top achievers were in the highest decile based on the 6th-grade test (Figure 2). The other top achievers were placed in deciles 6, 8, and 9. The high achievers and the other students succeeded quite well in 6th grade.

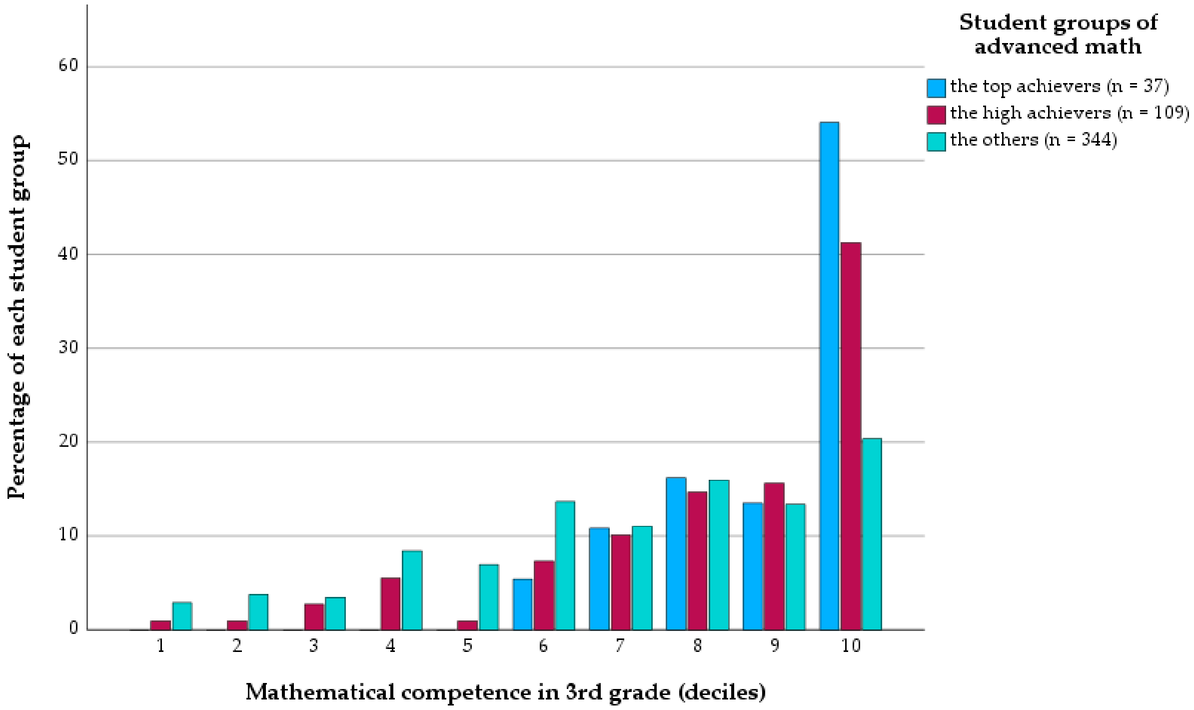

Figure 1. Mathematical competence in 3rd grade.

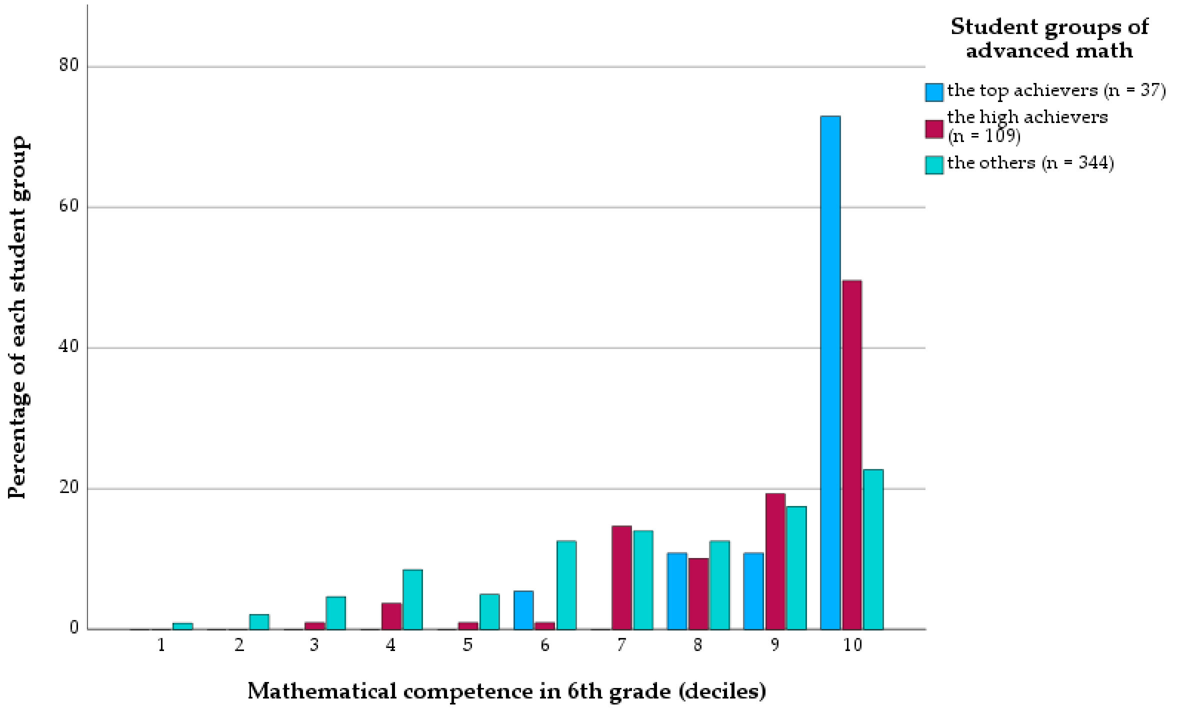

Figure 2. Mathematical competence in 6th grade.

When we examined the distribution of the highest decile more closely, it consisted of 17.0% top achievers (n = 27), 34.0% high achievers (n = 54), and 49.0% other students (n = 78). Hence, the highest decile was a mix of all students completing the advanced math exam, with only some achieving the best grade on the final exam.

Further analysis with logistic regression showed that the mathematical competence in 6th grade explained 13.3% of the better performance on the exam of advanced math (Nagelkerke's R^2 = 0.133). The odds ratio of the effect was 6.57 (B = 1.88; $S.E.$ = 0.38; $p < 0.001$). The odds ratio almost doubled from 3rd grade to 6th grade.

The top achievers were identified even more clearly in 9th grade (Figure 3). Of the top achievers, 86% ranked in the highest decile in 9th grade.

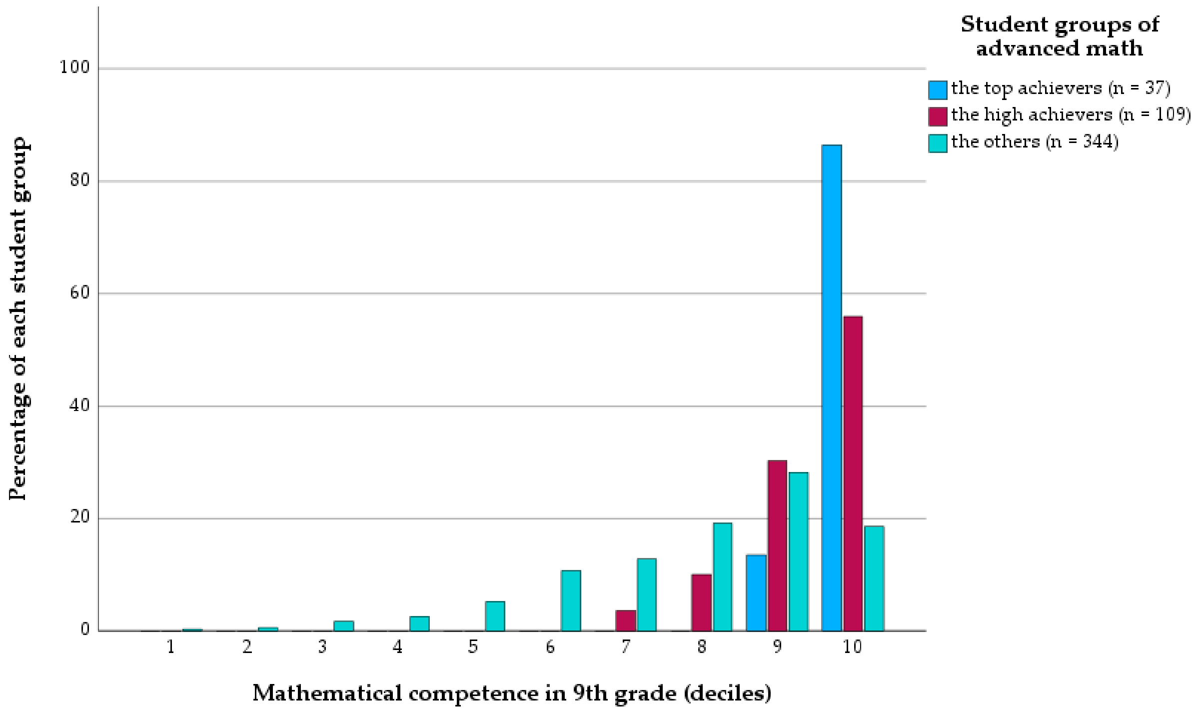

Figure 3. Mathematical competence in 9th grade.

When we examined the distribution of the highest decile of 9th grade more closely, it consisted of 20.4% top achievers (n = 32), 38.9% high achievers (n = 61), and 40.8% other students (n = 64). In the highest decile of the 9th-grade test, one-fifth were future top achievers, but this group included so many students that top achievers could not be identified as better than others based on the 9th-grade test. There were only four future top achievers in the highest decile, whose grade was higher than the highest of non-top achievers.

Further analysis with logistic regression showed that the mathematical competence in 9th grade explained 24.1% of the better performance on the exam of advanced math (Nagelkerke's $R^2 = 0.241$). The odds ratio of the effect was 16.79 ($B = 2.82$; $S.E. = 0.49$; $p < 0.001$). The odds ratio almost tripled from 6th grade to 9th grade and was five times larger in 9th grade than in 3rd grade.

In summary, we can state that the top achievers were identified partly as a different group already in 3rd grade but recognizability improved by the end of 9th grade. Only five top achievers remained unidentified in 9th grade. However, it should be noted that the tests also identified students who did not achieve the best grade on the matriculation exam.

6.2. Tasks That Predict Future Achievement in Mathematics

Among the tasks of the national test of 3rd grade, the tasks presented in Table 3 best predicted development into a top achiever. The table shows the basic statistics for these tasks, including how large a proportion of top achievers, high achievers, and other students was able to solve the tasks. According to the model, the variables that best predicted higher future achievement were the tasks involving addition with crossing hundreds (Exp(B) = 4.98) and conceptual and abstract understanding of geometric concepts (Exp(B) = 4.73). These variables demonstrated quite strong statistical significances.

Table 3. Math tasks in 3rd grade that predicted high proficiency of advanced math.

Variable	*B*	*S.E.*	*Exp(B)*	*p*	*Percentage of Correct Answers (Top Achievers/High Achiever/Others)*
Addition task with crossing hundreds	1.61	0.76	4.98	0.034	93.5%/76.5%/79.0%
Conceptual and abstract understanding of geometric concepts: line segment and infinite	1.55	0.51	4.73	0.002	83.9%/55.1%/50.9%
Pre-algebra, missing number task (a · b = c +__)	1.32	0.51	3.72	0.010	83.9%/68.4%/49.5%
Conceptual and visual understanding of geometry	1.07	0.40	2.92	0.007	61.3%/45.9%/27.5%
Constant	−6.44	0.98	0.002	<0.001	

Nagelkerke's R^2 = 0.205.

The tasks can be seen as measuring algebra skills, mastery of mathematical concepts (e.g., line segment, edge, infinite), and visual understanding. The tasks required non-routine mathematical skills. Further analysis showed that the odds ratio between the *sum* variable consisting of these non- routine tasks in 3rd grade and the top achievement was 3.70 (B = 1.31; $S.E.$ = 0.25; $p < 0.001$).

Of the top achievers, 35.5% completed each of the above non-routine tasks correctly and the rest of them completed at least half correctly. Of the students who completed all of the non-routine tasks correctly, 25.0% were top achievers, 22.7% were high achievers, and 52.3% were other students.

Table 4 shows the kinds of tasks in 6th grade that best predicted development into a top achiever. According to the model, the variables that most accurately predicted higher future achievement were the tasks involving understanding of means (*Exp(B)* = 7.46) and understanding of complementary events in probability (*Exp(B)* = 7.46). The odds ratios for these variables were stronger compared to the tasks in 3rd grade. However, there was also greater uncertainty ($S.E.$ = 1.03), which indicates a higher potential for false positives and false negatives.

In 3rd grade and in 6th grade, tasks were characterized by non-routineness and understanding of mathematical concepts. Further analysis showed that the odds ratio between the sum variable consisting of these non-routine tasks in 6th grade and the top achievement was 3.11 (B = 1.13; $S.E.$ = 0.22; $p < 0.001$). The explanatory power remained almost the same as in 3rd grade.

Almost half of the top achievers (48.6%) completed each of the above non-routine tasks correctly. Of all the students who completed all of the non-routine tasks correctly, 20.9% were top achievers, 36.0% were high achievers, and 43.0% were other students. The proportion of other students decreased by about 9 percentage points, and the proportion of high achievers increased by about 13 percentage points. The proportion of top achievers remained almost the same.

Table 5 shows the kinds of mathematical tasks in 9th grade that best predicted high proficiency in mathematics. The tasks in 9th grade exhibited the highest odds ratios, indicating a strong association with future higher achievement. At the same time, the statistical significances were relatively low. According to the model, these tasks demonstrated lower levels of uncertainty. Notably, the ability to solve the task involving the area of a trapezium presented almost a 12-fold increased likelihood of being among the future top achievers.

Table 4. Math tasks in 6th grade that predicted high proficiency of advanced math.

Variable	*B*	*S.E.*	*Exp*(*B*)	*p*	*Percentage of Correct Answers (Top Achievers/High Achiever/Others)*
Understanding of mean, multiple choice	2.01	1.03	7.46	0.05	97.3%/84.4%/70.0%
Understanding of complementary event in probability, multiple choice	2.01	1.03	7.46	0.05	97.3%/80.7%/70.8%
Choosing the correct unit of measure	1.11	0.50	3.05	0.03	86.5%/71.6%/53.9%
Conceptual and visual understanding of geometry (same task as in the 3rd grade test)	1.09	0.55	2.97	0.05	89.2%/78.0%/62.5%
Quotative division task, mental math	0.76	0.38	2.13	0.05	67.6%/54.1%/32.7%
Constant	−8.21	1.51	0.00	<0.001	

Nagelkerke's R^2 = 0.208.

Table 5. Math tasks in 9th grade that predicted high proficiency of advanced math.

Variable	*B*	*S.E.*	*Exp*(*B*)	*p*	*Percentage of Correct Answers (Top Achievers/High Achiever/Others)*
Area of trapezium	2.45	1.04	11.53	0.02	97.1%/76.4%/51.4%
Estimating and computing powers	2.12	0.75	8.32	0.01	94.1%/59.4%/53.8%
Problem solving with equations and multiple steps	2.03	0.48	7.63	<0.001	83.8%/41.3%/24.1%
Geometric task with volume (problem solving)	1.56	0.77	4.77	0.04	94.6%/78.0%/59.3%
Constant	−8.75	1.41	0.00	<0.001	

Nagelkerke's R^2 = 0.386.

The 9th-grade tasks represented the importance of problem solving and geometric skills. The tasks resembled non-routine tasks, as in 3rd and 6th grade. Further analysis showed that the odds ratio between the sum variable consisting of these 9th-grade non-routine tasks and the top achievement was 7.62 (B = 2.03; $S.E.$ = 0.32; $p < 0.001$). The explanatory power was 2.5 times higher than in 3rd and 6th grade.

Of the top achievers, 73.5% completed each of the above non-routine tasks correctly. Of all the students who completed all of the non-routine tasks correctly, 36.2% were top achievers, 29.0% were high achievers, and 34.8% were other students. The proportion of top achievers and other students was almost the same.

In summary, it can be stated that non-routine tasks predicted future top achievement. The explanatory power was the highest in 9th grade, but not all of the top achievers could be primarily identified based on their ability to solve these tasks.

6.3. Attitudes That Predict Future Achievement in Mathematics

We did not find a statistical model of the attitudes of 3rd graders that could explain development into a top achiever in mathematics.

Among the attitudes of 6th graders, positive self-concept best predicted becoming a top achiever in mathematics (Table 6). The table shows top achievers', high achievers', and other students' mean levels of self-concept. According to the model, self-concept showed statistical significance; however, it did not effectively differentiate between achievers, as the odds ratios were low. The top achievers' perception of themselves was in line with almost the highest rating on the scale. The high achievers and the other students rated their self-concept at a high level.

Table 6. Attitudes toward mathematics in 6th grade (sum variables) that predicted high proficiency of advanced math.

Variable	*B*	*S.E.*	*Exp*(*B*)	*p*	*Mean of Attitude (1–5) (Top Achievers/High Achievers/Others)*
Self-concept	0.07	0.02	1.07	<0.001	4.6/4.3/4.1
Constant	−8.30	1.50	0.00	<0.001	

Nagelkerke's R^2 = 0.114.

Of all the attitude statements of the 6th graders, the statements presented in Table 7 best explained the development into a top achiever in mathematics. These individual statements showed moderate discriminability, particularly with the statement involving the perception of solving difficult math tasks (*Exp(B)* = 2.32). Although self-concept as a sum variable best predicted high proficiency in mathematics, among the attitude statements, the statement related to liking mathematics became one of the explanatory factors ("*Mathematics is one of my favorite subjects*."). The top achievers were more confident than others in their ability to solve difficult tasks and considered mathematics one of their favorite subjects more than others.

Table 7. Attitude statements in 6th grade that predicted high proficiency of advanced math.

Variable	*B*	*S.E.*	*Exp*(*B*)	*p*	*Mean of Attitude (1–5) (Top Achievers/High Achievers/Others)*
"*I can solve even difficult math tasks.*"	0.84	0.29	2.32	0.004	4.5/4.0/3.9
"*Math is one of my favorite subjects.*"	0.38	0.16	1.39	0.037	4.1/3.5/3.2
Constant	−7.29	1.31	0.001	<0.001	

Nagelkerke's R^2 = 0.112.

In 9th grade, a positive self-concept best explained a high proficiency in mathematics (Table 8). As in 6th grade, the self-concept showed statistical significance but did not effectively differentiate between achievers (*Exp(B)* = 1.03). The self-concept of all students who completed the exam of advanced math was at a high level. The gap between different groups decreased. High achievers' and other students' self-concept remained at the same level, but top achievers' self-concept decreased a bit.

Table 8. Attitudes towards mathematics in 9th grade (sum variables) that predicted high proficiency of advanced math.

Variable	*B*	*S.E.*	*Exp*(*B*)	*p*	*Mean of Attitude (1–5) (Top Achievers/High Achievers/Others)*
Self-concept	0.03	0.01	1.03	0.030	4.4/4.3/4.0
Constant	−4.88	1.12	0.01	<0.001	

Nagelkerke's R^2 = 0.029.

In the attitude statement of 9th grade, one statistically significant variable was found to distinguish the top achievers from others (Table 9). The top achievers identified themselves as good at mathematics. The odds ratio of that variable was the highest of all attitude statements in 6th and 9th grade. The difference was clearest between the top achievers and the other students.

Table 9. Attitude statements in 9th grade that predicted high proficiency of advanced math.

Variable	*B*	*S.E.*	*Exp*(*B*)	*p*	*Mean of Attitude (1–5) (Top Achievers/High Achievers/Others)*
"I think I am good at math."	0.98	0.33	2.66	0.003	4.7/4.5/4.1
Constant	−6.87	1.55	0.001	<0.001	

Nagelkerke's R^2 = 0.060.

In summary, we can state that the explanatory power of self-concept remained almost at the same level in 6th and 9th grade, and the effect on high proficiency was not very large. It is difficult to predict high proficiency in mathematics based on attitude levels because all the students who graduated with the exam of advanced math had a positive attitude toward mathematics.

6.4. Gender Differences among Top Achievers

A clear majority of top achievers of advanced math (73 percent) was boys (Figure 4). High-achieving students and other students had a more even gender distribution.

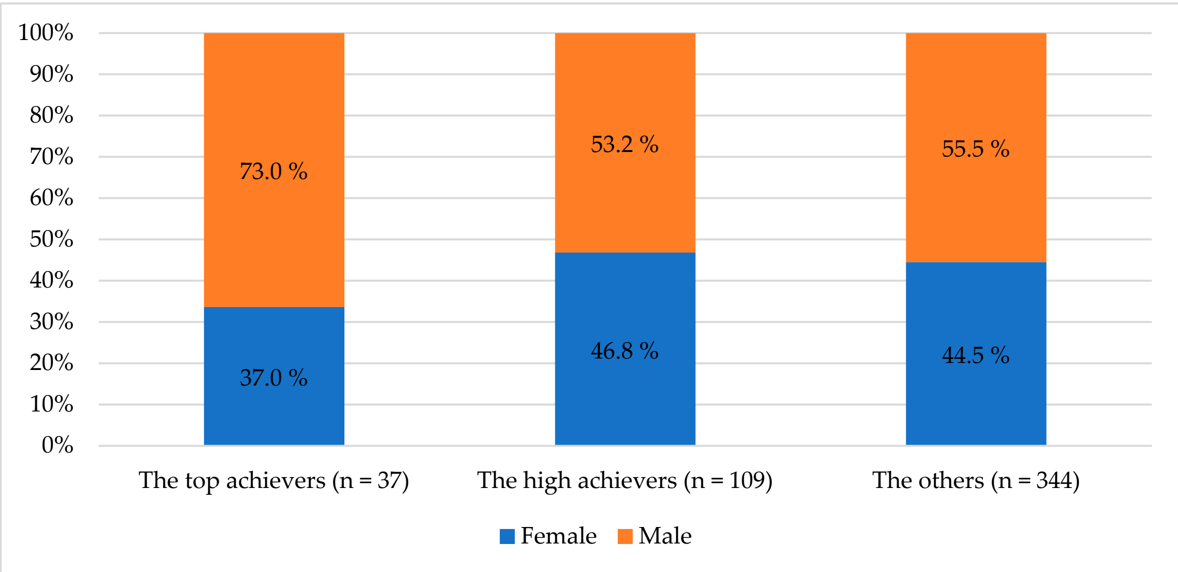

Figure 4. Gender distribution among students of advanced mathematics.

According to the binomial probability, the difference between top-achieving girls and top-achieving boys was statistically significant, and the effect size of the difference was large (*BIN* = 0.03; *Cohen's h* = 1.1).

Table 10 shows the proportion of boys and girls in the highest deciles in 3rd, 6th, and 9th grade.

Of top-achieving boys, 14.6% could be identified in 9th grade, and their proportion in the top decile increased by about 4 percentage points from 3rd grade. Of top-achieving girls, 5.7% could be identified in 9th grade, and their proportion in the top decile increased by 1.3 percentage points. Gender distribution was almost at the same level in all achiever groups.

Table 10. The proportion of top-achieving boys/girls, high-achieving boys/girls, and other boys/girls in the highest decile of 3rd, 6th, and 9th grade.

	Top-Achieving Boys/Girls	High-Achieving Boys/Girls	Other Boys/Girls
The highest decile in 3rd grade (n = 135)	10.4%/4.4%	21.5%/11.9%	31.1%/20.7%
The highest decile in 6th grade (n = 159)	12.6%/4.4%	21.4%/12.6%	34.0%/15.1%
The highest decile in 9th grade (n = 157)	14.6%/5.7%	24.2%/14.6%	27.4%/13.4%

6.5. Differences in Parents' Education and Support among Top Achievers

Both parents of about half of the top achievers and high achievers graduated with the matriculation examination (Figure 5).

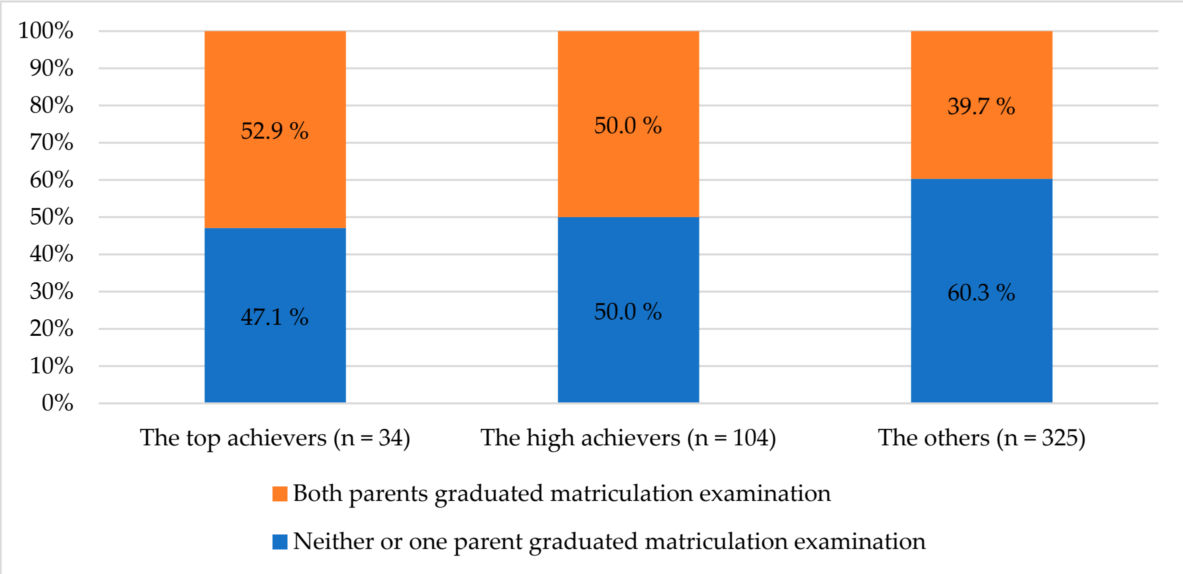

Figure 5. Distribution of parents' education among students of advanced mathematics.

Both parents of about 40% of other students graduated with the matriculation examination. Although the difference between the groups among other students seemed large, it was not statistically significant according to the binomial probability (*BIN* = 0.127; *Cohen's h* = 0.48). It should be considered that the expected value affected the result. According to that, 43% of other students' parents would have belonged to the group in which both parents graduated with the matriculation examination.

In relation to parental support, we did not find any statistical model to explain high proficiency in mathematics. In other words, the parental background of all students who completed the exam of advanced math seemed to be similar, and neither the parents' education nor parental support differentiated the competence among the students.

7. Discussion and Limitations

In this study, we investigated the development of top achievers' (< 2% of the age cohort) mathematical competence and at what stage in basic education they could be identified. In addition, we studied the kinds of mathematical tasks and individual and environmental factors that predicted development into a top achiever in mathematics. Next, we provide some implications and limitations of this study.

7.1. Implications of the Results

The results of this study have significant implications for understanding the development of top achievers in mathematics and their identification at different stages of basic education.

The top achievers were partly identifiable in 3rd grade from other students who graduated with the matriculation examination of advanced math. The ability to identify top achievers strengthened in 6th grade and was most clear in 9th grade, when students decided whether to apply to general upper secondary school and whether to choose an intermediate or advanced syllabus of mathematics. This confirmed previous research results, e.g., [10,13]. The mathematical content areas deepen and become more difficult when moving to a higher grade. It is easier to identify mathematical talent when comparing to one's own age cohort.

The results show that the ability to solve certain types of math tasks was a better predictor for identifying future top achievers. According to the results, top achievers did better in non-routine tasks than their age cohort on the national tests of 3rd, 6th, and 9th grade. These non-routine tasks required algebra skills, understanding of mathematical concepts, geometry skills, and problem-solving skills. This confirmed previous results that mathematically gifted students are characterized by the ability to solve an exceptionally diverse range of problems and have flexible mathematical thinking compared to their age cohort [29,31,34,37,40]. The predictive power of these non-routine tasks was highest in 9th grade. The ability to solve certain types of tasks yielded a good assessment of top achievers, but only 36% of those who solved all of the non-routine tasks in 9th grade were top achievers.

According to previous studies, e.g., [47–50], mathematical self-concept has a strong connection with mathematical skills. In this study, students' self-concept in 6th and 9th grade was related to high proficiency of mathematics, but the differences in self-concept between top achievers and others were not large. All students who graduated with the exam of advanced math had basically positive attitudes toward mathematics. The top achievers' perception of their ability to solve difficult tasks and their perception of themselves as being good at mathematics were stronger than others.

The proportion of boys among top achievers was higher than that of girls, and boys had more variation in their skills. The gender distribution was almost at the same level from 3rd to 9th grade, but boys were more likely to be identified than girls. We think that the higher proportion of boys is partially explained by some kind of selection. Girls give up studying mathematics and focus on and invest in other subjects instead of mathematics, cf. [63].

Referring to previous studies [10,70,71,75,82], we hypothesized that having parents who graduated with the matriculation examination and receiving greater parental support for studying predicted higher proficiency in mathematics. According to this study, whether the parents graduated with the matriculation examination or whether students had some parental support for studying did not have an effect on development into a top achiever in mathematics. However, we can consider that parents' education and parental support had effects on a student's decision to apply to general upper secondary school and to choose the syllabus of advanced mathematics.

Mathematical giftedness requires, among other things, the ability for flexible and abstract mathematical thinking and the ability to solve different kinds of mathematical problems. Mathematically gifted students can be identified, to a notable extent, as having higher mathematical skills and a higher ability to solve atypical and non-routine mathematical tasks compared to their age cohort. From this point of view, the matriculation examination of advanced mathematics is a good measure to identify mathematically gifted students with reliable accuracy. The proportion of those with the best grade from the exam of advanced mathematics was only 1–2% of the entire age cohort. Other researchers [84,85] have assumed that students who scored in the top 5% on academic tests were high achievers, and according to Heller [87], those scoring in the top 1–2% were extremely gifted.

7.2. Limitations

The study highlights that early identification of top achievers based on single tests is not highly accurate, and some top achievers may remain unrecognized. More than half of top achievers were in the highest decile in 3rd grade. The proportion of top achievers was 73% in 6th grade, and in 9th grade almost all top achievers were in the highest decile. The limitation suggests the need for additional assessment methods and comprehensive approaches to identify potential mathematical achievers at an early stage.

The highest decile in 9th grade included about 80% of high achievers and other students of advanced math, and the top achievers could not be identified to be better than others. We can believe that all students in the highest decile have potential, but not all of them are able to develop into top achievers. Only six percent of these students who graduated with the matriculation exam of advanced math achieved the best grade.

Identifying mathematical giftedness at an early stage is problematic, and a single test may not find all potential mathematical achievers, comp. [25]. For example, underachievement is possible for many different reasons. In addition, such a retrospective study is easily misleading. Mathematical potential can appear at an early stage, but some environmental factors are relevant for the development of the potential. Systematic studies of how we can actually predict mathematical skills would be beneficial. Important would be to control the interaction of many different individual and environmental factors. This complicates the designs notably.

Regarding the sample and methods, some limitations can be mentioned. It should be noted that the definition of top achievers is best suited to serve the Finnish education system with the matriculation examination system. It is a selected group that applies to general upper secondary school and chooses to study advanced mathematics, and the gifted individuals are those who achieve the best grade on the matriculation examination. The categorization data may result in some information loss, but on the other hand, the categorization data have enabled a different perspective and the use of analysis methods such as logistic regression analysis and the associated odds ratio. The study examined the predictability of different factors on future higher achievement. It should be noted that the phenomenon can also be explained by alternative models. It would be justified in future research to use several models, each including different or additive predictors compared these side by side, and to test whether adding individual factors increases the predictive power.

8. Conclusions

The results show that the ability to solve non-routine mathematical tasks in early years best predicted future high proficiency in mathematics. Then, a relevant question is whether it would even be possible to develop a diagnostic test that could be used accurately to identify mathematically gifted students in early grades. However, if we could identify potential students, we could offer them additional challenges, support, and encouragement to develop their potential. The test should consist of non-routine tasks that are atypical for the age level and measure things that have not yet been taught in school at that stage. Those tasks require conceptual understanding and problem-solving skills.

However, our results provide no justification for trying to identify top achievers from others in 3rd grade and separate them into special programs. The results show that it was also possible to become a top achiever from an average or lower level. Many other students had potential, scoring in the highest decile in 3rd, 6th, or 9th grade, yet did not achieve the best grade on the matriculation exam of advanced math. If students are separated into different groups by their skills, the mathematical potential of many students would remain easily unrecognized, and they would not have support to develop their potential. In addition, heterogeneous groups have been found to strengthen the self-concept of high-achieving students [24].

According to the results, the factors related to the student's parental background were not significant variables predicting high proficiency in mathematics. It was possible to develop into a top achiever regardless of only the parental factors. It is important to provide relevant challenges, support, and encouragement to develop mathematical potential.

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