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HUMANIORA

Spontaneous Focusing on Numerosity in the Development of Early Mathematical Skills

by

Minna M. Hannula

ACADEMIC DISSERTATION

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TURUN YLIOPISTO
Turku 2005

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To Jouni and Kasperi

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ABSTRACT

The aim of the present set of longitudinal studies was to explore 3-7-year-old children's Spontaneous FOcusing on Numerosity (SFON) and its relation to early mathematical development. The specific goals were to capture in method and theory the distinct process by which children focus on numerosity as a part of their activities involving exact number recognition, and individual differences in this process that may be informative in the development of more complex number skills. Over the course of conducting the five studies, fifteen novel tasks were progressively developed for the SFON assessments. In the tasks, confounding effects of insufficient number recognition, verbal comprehension, other procedural skills as well as working memory capacity were aimed to be controlled. Furthermore, how children's individual differences in SFON are related to their development of number sequence, subitizing-based enumeration, object counting and basic arithmetic skills was explored. The effect of social interaction on SFON was tested.

Study I captured the first phase of the 3-year longitudinal study with 39 children. It was investigated whether there were differences in 3-year-old children's tendency to focus on numerosity, and whether these differences were related to the children's development of cardinality recognition skills from the age of 3 to 4 years. It was found that the two groups of children formed on the basis of their amount of SFON tendency at the age of 3 years differed in their development of recognising and producing small numbers. The children whose SFON tendency was very predominant developed faster in cardinality related skills from the age of 3 to 4 years than the children whose SFON tendency was not as predominant. Thus, children's development in cardinality recognition skills is related to their SFON tendency.

Studies II and *III* were conducted to investigate, firstly, children's individual differences in SFON, and, secondly, whether children's SFON is related to their counting development. Altogether nine tasks were designed for the assessments of spontaneous and guided focusing on numerosity. The longitudinal data of 39 children in *Study II* from the age of 3.5 to 6 years showed individual differences in SFON at the ages of 4, 5 and 6 years, as well as stability in children's SFON across tasks used at different ages. The counting skills were assessed at the ages of 3.5, 5 and 6 years. Path analyses indicated a reciprocal tendency in the relationship between SFON and counting development. In *Study III*, these results on the individual differences in SFON tendency, the stability of SFON across different tasks and the relationship of SFON and mathematical skills were confirmed by a larger-scale cross-sectional study of 183 on average 6.5-year-old children (range 6;0-7;0 years). The significant amount of unique variance that SFON accounted for number sequence elaboration, object counting and basic arithmetic skills stayed statistically significant (partial correlations varying from .27 to .37) when the effects of non-verbal IQ and verbal comprehension were controlled. In addition, to confirm that the SFON tasks assess SFON tendency independently from enumeration skills, guided focusing tasks were used

for children who had failed in SFON tasks. It was explored whether these children were able to proceed in similar tasks to SFON tasks once they were guided to focus on number. The results showed that these children's poor performance in the SFON tasks was not caused by their deficiency in executing the tasks but on lacking focusing on numerosity.

The longitudinal *Study IV* of 39 children aimed at increasing the knowledge of associations between children's long-term SFON tendency, subitizing-based enumeration and verbal counting skills. Children were tested twice at the age of 4-5 years on their SFON, and once at the age of 5 on their subitizing-based enumeration, number sequence production, as well as on their skills for counting of objects. Results showed considerable stability in SFON tendency measured at different ages, and that there is a positive direct association between SFON and number sequence production. The association between SFON and object counting skills was significantly mediated by subitizing-based enumeration. These results indicate that the associations between the child's SFON and sub-skills of verbal counting may differ on the basis of how significant a role understanding the cardinal meanings of number words plays in learning these skills.

The specific goal of *Study V* was to investigate whether it is possible to enhance 3-year-old children's SFON tendency, and thus start children's deliberate practice in early mathematical skills. Participants were 3-year-old children in Finnish day care. The SFON scores and cardinality-related skills of the experimental group of 17 children were compared to the corresponding results of the 17 children in the control group. The results show an experimental effect on SFON tendency and subsequent development in cardinality-related skills during the 6-month period from pretest to delayed posttest in the children with some initial SFON tendency in the experimental group. Social interaction has an effect on children's SFON tendency.

The results of the five studies assert that within a child's existing mathematical competence, it is possible to distinguish a separate process, which refers to the child's tendency to spontaneously focus on numerosity. Moreover, there are significant individual differences in children's SFON at the age of 3-7 years. Moderate stability was found in this tendency across different tasks assessed both at the same and at different ages. Furthermore, SFON tendency is related to the development of early mathematical skills. Educational implications of the findings emphasise, first, the importance of regarding focusing on numerosity as a separate, essential process in the assessments of young children's mathematical skills. Second, the substantial individual differences in SFON tendency during the childhood years suggest that uncovering and modeling this kind of mathematically meaningful perceiving of the surroundings and tasks could be an efficient tool for promoting young children's mathematical development, and thus prevent later failures in learning mathematical skills. It is proposed to consider focusing on numerosity as one potential sub-process of activities involving exact number recognition in future studies.

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Käpälämäki, April 26th, 2005

Minna M. Hannula

LIST OF EMPIRICAL STUDIES

This doctoral thesis is based on the following five studies reported in four original articles. The studies are referred to in the text by their Roman numerals:

I	Hannula, M.M., & Lehtinen, E. (2001). Spontaneous tendency to focus on numerosities in the development of cardinality. In M. Panhuizen-Van Heuvel (Ed.), *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education, 3,* 113-120. Drukkerij Wilco, The Netherlands: Amersfoort.
II and III	Hannula, M. M., & Lehtinen, E. (in press). Spontaneous focusing on numerosity and mathematical skills of young children. *Learning and Instruction.*
IV	Hannula, M. M., Räsänen, P., & Lehtinen, E. (2005). *Development of counting skills: Role of spontaneous focusing on numerosity and subitizing-based enumeration.* Manuscript submitted for publication.
V	Hannula, M. M., Mattinen, A., & Lehtinen, E. (in press). Does social interaction influence 3-year-old children's tendency to focus on numerosity? A quasi-experimental study in day care. In L. Verschaffel, E. De Corte, G. Kanselaar, & M. Valcke (Eds.), *Learning environments to promote deep conceptual and strategic learning.* Studia Paedagogica. Leuven: Leuven University Press.

1. INTRODUCTION

Although there has been a clearly growing interest in early mathematical development during very recent years, there are still considerable challenges both *in research,* to delineate different developmental trajectories and factors related to individual differences, and *in practice,* to establish developmentally appropriate assessments and effective, systematic ways of promoting this development (e.g., Ginsburg & Golbeck, 2004; Tudge & Doucet, 2004). In a recent review, Baroody, Lai and Mix (in press) state:

> Early childhood educators have long viewed young children and mathematics education like water and oil, as things that do not mix. As a result, they have tended to focus on literacy and to overlook numeracy. Balfanz (1999) observed that this has not always been the case. Indeed, "some of the founding figures of early childhood education, like Fredrick Froebel and Maria Montessori, …advanced the notion that young children are capable of complex mathematical thought and enjoy using mathematics to explore and understand the world around them" (Balfanz, 1999, p. 3). Based on their careful observations of children in natural settings, both Froebel and Montessori incorporated rich mathematical experiences into their early childhood programs.

Major differences in school-aged children's mathematical skills and emerging learning difficulties in mathematics already at the beginning of the school career suggest that there are preschoolers who lack the basic counting skills and arithmetic knowledge which are necessary for later development in mathematics (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Hanich, Jordan, Kaplan, & Dick, 2001; Porter, 1998). From both scientific and educational perspectives, there is an urgent need for studies charting, longitudinally, numerical development and delineating the early factors of later success. A growing body of research suggests that one of young children's most relevant learning environments is their everyday surroundings, potentially rich in embedded mathematical thinking and informal mathematical knowledge (e.g., Fuson, 1988; Ginsburg, Inoue, & Seo, 1999; Mix, 2002; Nunes & Bryant, 1996; Seo & Ginsburg, 2004; Tudge & Doucet, 2004). In particular, indicators for the amount of mathematically meaningful learning processes taking place in these situations need to be uncovered. The current work aims to add to our knowledge of these learning processes of young children.

Currently, nearly all our knowledge of young children's number recognition skills is based on studies that explicitly direct children's attention to the aspect of number. Such studies fail to capture in method or theory the process by which children focus on numerosity, and individual differences in this process that may be informative in the development of more complex number skills. Recognition of exact number when the number of items is utilized in action is not an automatic process, but an intentional act. This proposal is based on the notion that number is not a property of the physical world itself, but rather is determined as a result of how we choose to carve up the physical world into individual elements. There is no one number that describes a particular portion of matter (Wynn, 1998). Imagine, for example, a scene from a window: there may be one area of forest, but the same forest consists of 20 trees, 3 pines, 17 spruces, and yet a different number of twigs and needles. Depending on one's goals regarding the view from the window, one might also choose to focus on different aspects of the view: the colours of the autumnal forest, the shapes of branches moving in the wind, the sizes of leaves on the ground, or the numbers of trees waiting for felling.

The aspect of exact number is a particular aspect in natural surroundings. In a natural scene a person needs to focus attention on the aspect of number in the set of items a person is interested in, and the set of items has to be defined before one can determine the exact number of any items. Hence, exact enumeration requires specific attentional processes. It is not a totally automatic process. The current work on young children's spontaneous focusing on numerosity is based on this specific attention-demanding feature in the recognition and utilization of exact number of items in action. It is proposed that learning to focus attention on the aspect of number in one's surroundings is accordingly one of the significant elements of early mathematical development, providing the necessary practice in utilizing number recognition processes. Thus, differences in the extent to which children focus on numerosity and utilise their number recognition skills in their surroundings might explain the differences in the development of early mathematical[1] skills. To some children, the world may appear to be full of numerosities and opportunities for practising early mathematical skills, whilst others might focus on other features in the environment and involve themselves much less with pre-mathematical ideas. The aims of the present set of longitudinal studies was to explore 3-7-year-old children's Spontaneous FOcusing on Numerosity (SFON) and its relation to early mathematical development.

The present doctoral dissertation consists of a theoretical and methodological summary and the five original, empirical studies. In the summary section, first, a theoretical framework for exact number recognition, the phenomenon of SFON, and early mathematical development related to SFON tendency is presented. Second, the methodological solutions of this work are described. Third, the set of studies is overviewed with regard to the overall and specific aims of the doctoral dissertation. Fourth, a critical examination of the theoretical and methodological basis of this work is conducted, and fifth, the challenges for future studies on this phenomenon are presented.

1.1. Recognition of exact number of items

The current set of studies is closely related to earlier research on how exact numbers of items can be recognized. Two processes has been proposed to enable recognition of exact numbers of items (Dehaene & Laurent, 1994; Jensen, Reese, & Reese, 1950; Kaufman, Lord, Reese, & Volkman, 1949; Piazza, Giacomini, Le Bihan, & Dehaene, 2003; Sathian, Simon, Peterson, Patel, Hoffman, & Grafton, 1999) as distinguished from approximate number recognition (Feigenson, Dehaene, Spelke, in press; Lemer, Dehaene, Spelke, & Cohen, 2003). These are a fast, highly accurate parallel apprehending of items up to ca. three to four, often labeled as *subitizing*, and much slower, and more error-prone, serial attentional shifts demanding, language-dependent *verbal counting*, which functions also for enumeration of larger numbers (e.g., Jevons, 1871; Sathian et al., 1999; Trick, Enns, &

[1] It is acknowledged that the word "mathematical" is used in the current work in a considerably narrow sense, referring to early number concept and numerical skills, although mathematical thinking includes a broad array of other important concepts and skills, e.g., spatial, logical, problem solving skills.

Brodeur, 1996; Trick & Pylyshyn, 1994). The current set of studies proposes that in addition to these widely acknowledged exact number recognition processes, also an attentional process of focusing on the aspect of number, is needed for the recognition of exact numbers and for utilizing the numerical knowledge in action.

Several mechanisms have been proposed to explain the process of *subitizing*: a limited capacity of short-term memory (Klahr, 1973), a general capacity limit for the focus of attention (Cowan, 2001), a pattern recognition process (Mandler & Shebo, 1982), or fast preverbal counting (Gelman & Gallistel, 1992). Accordingly subitizing has been described as a part of the mechanism for the visual tracking of separate (Scholl, 2001; Scholl & Pylyshyn, 1999; Trick & Pylyshin, 1994) or individual (Simon, 1997; Uller, Carey, Huntley-Fenner, & Klatt, 1999) objects from the visual field.

There are several reasons to propose that exact number recognition and utilization in subitizing range is not a totally automatic, pre-attentional process. These are the specific attention-demanding nature of exact number recognition taking place in natural surroundings (described above), the examinations of the methods used in earlier studies on subitizing, and the theoretical models on subitizing-based enumeration and object-file-theories on object individuation (e.g. Feigenson & Carey, 2003; Spelke, 2003; Trick and Pylyshyn, 1994). Subitizing-based enumeration has traditionally been measured with tasks in which the subject's attention has been guided towards the aspect of numerosity. This has been done by questions like "how many dots are there, or, is there the same number of dots now as earlier ". Thus, these studies have not been measuring purely the participant's pre-attentional subitizing processes, but also the participant's goal-driven attempts to utilize their pre-attentional subitizing process for enumeration.

In the studies with SFON, we have made a conceptual distinction between the pre-attentional, perceptual subitizing mechanism and subitizing-based enumeration, i.e. utilizing this innate mechanism for enumeration tasks. In the latter case, when we focus on enumeration based on the subitizing mechanism (in practice, when counting is prevented and the participant is asked to enumerate the set), numerical cognition plays a developmentally significant role. Knowledge of cardinal number words for subitizable numbers, as well as utilizing skills for numerical aspects in action develop during early childhood years.

It seems that non-automaticity of exact number recognition within subitizable range of numbers have directly been proposed by very few studies, although discussion about attentional processes in perception of objects and object features has been active (for a review Scholl, 2001). Trick and Pylyshyn's (1994) mention that enumeration by subitizing occurs when one *decides* to enumerate, and the number of items in the display is within the subitizable range. According to their proposal, in the first prenumeric stage of subitizing-based number recognition, one is only conscious of some items in the display, and pre-numeric individuation information is processed. In the second, goal-driven stage, number recognition is processed and a numeric response is chosen.

In accordance with this proposal, it is suggested that utilizing enumeration based on a pre-attentional subitizing mechanism consists of elements on different levels of functioning. The pre-attentional subitizing mechanism produces only pre-numeric individuation information about the objects in the set (see Trick & Pylyshyn, 1994; Spelke, 2003; Wiese, 2003). There is neurological evidence of single neurons alarming for "oneness", "twoness", and "threeness" in primates (Nieder, Freedman, & Miller, 2002), thus supporting the idea of real basic-level mechanisms enabling perceptual recognition of small numbers.

However, when a set of objects is enumerated, goal-directed processes are needed, in addition to preattentional processes, to enable more conceptual recognition of the number. Especially in natural social settings, all possible embedded numerosity can not be considered, nor can it be automatically recognized in a fully mathematically numerical sense. In order to consider significant numerosity in the action, and to recognize the cardinal values of the sets of objects, the person needs to intentionally focus his or her attention on the numerical aspect of the target objects, not only to carve up the set of target objects, but also to activate the numerical recognition level, at which the choice of numeric response is made.

There is some evidence suggesting that the range for numbers of items recognizable by subitizing-based enumeration grows in children and even in adults. Starkey and Cooper (1995) proposed that the range for subitizing-based enumeration increases with age from 1-3 in under two-year-old children, to 1-4 in three- to five-year-olds, and up to 1-5 in adulthood. Children's range for subitizing-based enumeration is smaller, and the process of subitizing is slower than that of adults, indicating either that children can not utilize the full range of their subitizing ability, or that the subitizing range grows during childhood (Chi & Klahr, 1975; Trick et al., 1996). Recent findings of Green and Bavalier (2003) show that even adults' subitizing-based enumeration skills can develop with intensive practice. The computational modeling approach has been successful in producing an analogical effect of practice in the growth of quantified numbers of items on the subitizing range (Peterson & Simon, 2000). The set-size limit for subitizing-based enumeration can be exceeded when the items are in the form of overlearned canonical patterns, such as dice patterns, or other familiar sub-patterns (Wolters, van Kempen, & Wijlhuizen, 1987; Wender & Rothlegel, 2000). Frequent focusing on the numerosity of visual displays develops semi-symbolic knowledge about certain patterns describing certain numerosity, for example, dice patterns describing the numbers up to six. Utilizing regular numerical patterns anchored with cardinal number words might be one of the developing skills closely related to subitizing. The current work deals with the earliest forms of subitizing-based enumeration without considering later developing skills of partitioning and adding up subitizable sub-sets from larger sets.

When a person enumerates and the set-size limit of preattentional subitizing-mechanism is exceeded, he or she needs to invoke controlled *counting processes* to be able to determine exact number of items (e.g., Dehaene & Laurent, 1994; Jensen et al., 1950; Piazza, et al., 2003; Sathian et al., 1999). Hence, counting has widely been regarded as an intentional process that has to be voluntarily triggered. There are five how-to-count principles, which have to be respected when objects are counted (Gelman & Gallistel, 1978). These are one-to-one correspondence, constant order, order irrelevance, abstraction, and the cardinality principle. According to the *one-to-one correspondence principle*, one must count all the objects and count each of them once, and only once. If one were to count one object twice, skip one object, or count in the spaces between the objects in the set, one would come up with the wrong total. *The principle of constant order* means that whenever one counts one must produce number words in the same order each time. *The order irrelevance* and *abstraction principles* refer to the fact that when you count, it does not matter in which order you count the items, or what kind of things you count. According to *the cardinality principle*, the last number tag used is the cardinal value of the set (Gallistel & Gelman, 1978).

Counting includes keeping track of counted items, planning the moves of attention, and inhibition of previously counted items (Trick & Pylyshyn, 1994). Moreover, reciting the list of number words, coordinating it synchronically with a series of individuating acts, and activating the cardinal value of the last recited number word as the result of the counting, are needed (e.g., Fuson, 1988). Counting (contrary to subitizing-based enumeration) has been shown to be a resource demanding activity affected by working memory capacity (Camos & Barrouillet, 2004; Tuholski, Engle, & Baylis, 2001). The role of language and culturally-shaped actions in verbal counting has recently been recognized by the findings of Gordon (2004) and Pica, Lemer, Izard, and Dehaene (2004). They found an Amazonian tribe which has number words and exact enumeration as well as arithmetic skills, only for very small numbers. For larger sets they seem to rely on analog magnitude representations producing only approximate numerical values. It is, however, not yet clear, what is the role of other cultural practices in the formation of exact number recognition skills.

1.2. Focusing attention on the aspect of number

In the current work, it is proposed that exact number recognition when number is utilized in action, is an intentional cognitive process. It requires specific focusing on the aspect of number and triggering the utilizing process for regarding the numerical knowledge in action.

Generally, in cognitive and neurocognitive psychology, visual information processing has been conceptualized as consisting of both goal-directed top-down and stimulus-driven bottom-up processes (for reviews see Enns, 1992; Plude, Enns, & Brodeur, 1994; Ullman, 1984). The effects of cognition seem to appear in a top-down manner (Pylyshyn, 1999). The studies on selectivity in perceiving are based on the notion that the world provides us with an infinitely large body of information. In order to act, humans need to select some of that information and ignore the rest. Gibson and Pick (2000) state:

> "An active perceiver has the tasks of extracting the information that specifies relevant events and especially, of detecting information that specifies an affordance of the environment relevant to the perceiver's species, needs and powers. Learning to detect the information that specifies such a relationship is perceptual learning. Attempts at acting on such information contribute further information, serving to increase the specificity of what is detected…Information about the world is obtained in a continuous flow by an active perceiver. Cycles of perceiving follow one another, often in an exploratory fashion. Invariance over transformations can only be detected over time" (p. 149).

In the present work, our interest is on a specific affordance of items, the numerosity of a set of objects or events, and how children learn to detect and start actively to explore its usefulness in their own action. Acquiring the knowledge of what specifies the affordance can be a long-term process in which the child's developing perceptual systems provide information that is increasingly accessible for new purposes (Gibson, 1988; Gibson & Spelke, 1983). According to Gibson (1988), the explorative behavior of a child (or an adult) is a spiraling process providing the optimal conditions for learning about more and more distinctive features and affordances of objects, surfaces, and events. Child's detecting new affordances provides the means of differentiating the qualities of things. This learning forms the basis for abstraction and classifying things (Gibson, 1988). These processes are significant parts of early mathematical development. In particular, they are meaningful for learning exact number recognition and utilization, which require determination of the set of objects on some basis.

The number of items or events in a collection can contribute to the affordances of that collection (Gibson & Spelke, 1983). This makes the aspect of number one of the significant features of the collections for many human actions. Number is abstracted over the color, size, and spatial configuration of the set of objects (Gibson & Spelke, 1983). A single object (or also a set of objects) can be analysed in such a way that its universal structure (e.g., its numerosity) becomes a figure, while its accidental detail fades into the background (Olsson & Lehtinen, 1997). In the case of a young child's enumeration this means that the child's abstraction of numerosity as a property of a set is essential for number recognition. Thus recognition of the cardinality of a set could be conceptualized as being an abstraction process.

Children's general attentional development forms the basis for their ability to focus on numerosity and utilize numerical knowledge in action. Children's ability to select and adjust strategies according to requirements of tasks increases with age so that they learn to adjust their action according to task-relevant features of the tasks (Pick & Frankel, 1974). During preschool years children's time allocated to focused attention on specific tasks increases, and they become increasingly capable of inhibiting and controlling their actions (Backen Jones, Rothbart, & Posner, 2003; Gibson & Spelke, 1983; Kochanska, Murray, & Coy, 1997; Ruff & Rothbar, 1996). Children's perception in tasks which are repeated several times becomes progressively more economical through detection of only those features of the available information that have the greatest utility for performing the behavior required (Gibson & Spelke, 1983).

Finally, concerning the proposal of the specific attentional process for focusing on number, there is some partly controversial neuropsychological evidence on adult participants' focusing on number compared to other qualities of displays. In Fink, Marshall, Gurd, Weiss, Zafiris, Shah, and Zilles (2000), directed attention tasks, assessing the numerosity of a display increased neural activation in different areas of the brain than assessing the shape of the same displays. In Piazza et al.'s (2003) study, number and color recognitions differed in reaction times within both countable and subitizable set sizes, while in brain activations, the differences in number and color naming were significant only within countable set sizes. Even supposing the subitizing-mechanism is capable of producing individuation information on items, similar activations on color and number recognition in the subitizable range do not, however, reveal what kinds of contributions the activations make to the cognitive performance of the participants. Could they, e.g. accurately tell the number of the items presented in the color-naming condition? The findings of Ganel and Goodale (2003) show that recognition of the shape of objects happens in a holistic way, whereas utilizing the same aspect in activities requires analytical, conscious processes. Thus, it may be that, in particular, utilizing the aspect of exact numerosity in action does not occur automatically, but requires intentional processes.

1.3. Spontaneous focusing on numerosity

An intentional process of focusing attention on the aspect of exact number in the set of items or incidents is called focusing on numerosity. This can trigger exact number recognition processes and utilizing the recognized exact number in action. Adding the term 'spontaneous' to focusing on numerosity refers only to the distinction between explicitly guided focusing on the aspect of number, and the child's own, non-guided, i.e. spontaneous, focusing on numerosity, not to the origins of SFON in children. The expression of SFON tendency is used to describe a child's general tendency to spontaneously focus on numerosity across different task contexts and social situations. In the current work, it is proposed that SFON tendency indicates the amount of a child's spontaneous practice in utilizing exact enumeration in natural surroundings.

Ericsson and Lehman (1996) show that experts seem to be capable of "seeing" multiple possibilities to practice their skills in everyday situations, and this has been an essential part of their development from a very early age. In the same way, it is assumed that children's spontaneous focusing on numerosity frequently leads them to perceive different numbers of objects or events in their surroundings, and thus they get practice in numerical skills. This, in turn, develops their quantifying skills in several ways: not only broadening their counting range, but also, along with more developed quantifying skills, a larger and larger quantity may appear as a possible subject for exact enumeration. Moreover, knowledge about the use of quantifying means in different tasks will increase with practice, so the child may tend to focus more on numerosity in new more demanding tasks. Better skills in quantifying can also enhance children's further interest in using their skills, especially if they are socially valued. Goldin-Meadow, Alibali, and Church (1993) have proposed that children's first counting attempts are one of the first significant, easily noticed signals for adults to start providing guidance in quantitative skills. In sum, the current set of studies suggests that SFON tendency accounts for some independent variance in the early mathematical development during the childhood years. Could this tendency be enhanced by the means of social interaction?

In general, social interaction and the aid of more experienced members of society are crucial for the development of higher order processes in self-regulation, especially the controlling of attention (Rothbart, Posner, & Boylan, 1990; Ruff & Rothbart, 1996; Gauvain, 2001). Gauvain (2001) describes how the social context affords children structured opportunities to practice, refine, and extend their cognitive skills. Participation in shared activities requires and also develops understanding of the intentions of others, cultural learning, motivation to share psychological states with others, and specific forms of cognitive representations for shared actions (Tomasello, Carpenter, Call, Behne, & Moll, in press). Gleissner, Meltzoff, and Bekkering (2000) have shown that 3-year-old children already code human behavior and imitate it on the basis of hierarchically organized goals, which give them the opportunity to focus on relevant and more abstract aspects of the actions observed. The joint processes of children and adults direct children's attention to relevant aspects of tasks, and help children to acquire the culturally-based numerical tools necessary for living in a society. These joint processes also help children to understand the purposes of the tasks and certain cognitive strategies embedded in a variety of everyday activities (Gibson & Spelke, 1983; Gauvain, 2001; Mix, 2002).

On the basis of an intensive case study, Mix (2002) presents evidence of how a wide range of socially and linguistically mediated numerically meaningful activities in everyday family life support conceptual growth in early mathematical development (see also Seo & Ginsburg, 2004; Tudge & Doucet, 2004). With studies of older students, Lobato and colleagues have shown that the social and physical aspects of a certain situation can support students' attention toward particular mathematical properties rather than others in mathematics lessons (Lobato, 2003; Lobato, Ellis & Munoz, 2003). Studies on the mathematical activities of parents and their children have shown that children's better mathematical skills are related to parents' skills in modeling mathematical thinking in everyday contexts, and in contexts involving the usual preschool material (Andersson, 2001; Blevins-Knabe & Musun-Miller, 1996; Huntsinger, Jose, Larson, Krieg, & Shaligram, 2000; Linnell, & Fluck, 2001; Saxe, Guberman, & Gearhart, 1986). Tudge and Doucet (2004) were able to show substantial individual differences in the amount of 3-year-olds' mathematical activities appearing in the form of academic lessons or mathematical games and materials during their daily life. In different cultures and countries, children's early mathematical thinking can vary (cf. Gordon 2004; Huntsinger et al., 2000; Pica et al., 2004; Torbeyns, van den Noortgate, Ghesquière, Verschaffel, Van de Rijt, & Van Luit, 2002). Although these studies have not regarded focusing on numerosity as a separate process from other numerical skills, they provide good reason to suggest that socio-cultural factors might also have a role in the formation of SFON tendency. In one of the studies of the doctoral dissertation, in *Study V*, the specific question of the role of social interaction in SFON tendency in natural day-care surroundings was explored.

In *Study V*, specific attention of the day-care personnel towards children's activities including numerical focusing and deliberate directing of children's attention towards variation in small numbers of objects were used to enhance children's SFON tendency. The idea of making children more aware of the aspect of number by using deliberate variation is based on the proposal of Marton and Booth (1997). They propose that the variation in how people experience a certain phenomenon, and the specific meaning it has for them, is the most fundamental aspect of learning. Thus, to learn something means to become capable of experiencing various aspects of the set of items in a certain, specific way. These different aspects (e.g. shapes, colors, or numerosity of sets of objects) may appear serially, and as separate aspects on which to focus awareness. An aspect which someone is focally aware of is interpreted against the background of potential variation in the aspect. Thus, "threeness" is experienced against the background of a potential variation in the aspect of number, against "twoness" and "fourness", for instance (Marton & Booth, 1997). In this way, children's implicit knowledge about small numbers may become more explicit targets of intentional focusing, and children could learn the affordances of numerical aspects in a variety of everyday activities. As Lindahl and Samuelsson (2002) describe, when children's intention is focused on a specific phenomenon, they seem to look for a variety of situations where they can practice and explore the phenomenon they want to master. Similarly, children's interest in numerosity could lead to continuous search for variation in it.

1.4. Early mathematical development and SFON tendency

For decades there has been agreement on the fact that early mathematical development forms the necessary basis for later mathematical skills learnt during the school years (Fuson, 1988; Geary, 2000; Gelman & Gallistel, 1978; Karmiloff-Smith, 1995; Mix, Huttenlocher, & Levine, 2002; Nunes & Bryant, 1996; Piaget, 1965; Seo & Ginsburg, 2004). The development of mathematical skills and concepts of children at the age of 2 to 8 years has been described as being highly individual, both in the rate at which children reach essential mathematical skills and in substance within mathematical concepts, as well as in relationships among contextualized knowledge concerning different aspects of numbers (Fuson, 1988; Van de Rijt, & Van Luit, 1999). Aunola et al. (2004) showed that children's mathematical skills develop in a cumulative manner from the preschool to the first years of school, even to the extent that the initial mathematical skills in beginning of preschool were positively associated with their later growth rate: the growth of mathematical skills was faster among those who entered preschool with already higher mathematical skills.

This work focuses on the development of skills closely related to cardinality determination at the age of 3 to 7 years, and especially on how SFON is involved in this development. Cardinality is the quality of a set describing how many items there are in the set. In this work, the specific focus is on utilizing the exact number of items in a set. Next, an integrative view on earlier findings on cardinality-related skills (e.g. Feigenson & Carey, 2003; Fuson, 1988; Gelman & Gallistel, 1978; Baroody, Lai, & Mix, in press; Mix et al., 2002; Spelke, 2003; Wynn, 1990) in relation to how these findings might be linked to current theoretical ideas and empirical findings of SFON is provided (see Figure 1).

1.4.1. Early roots of cardinality recognition

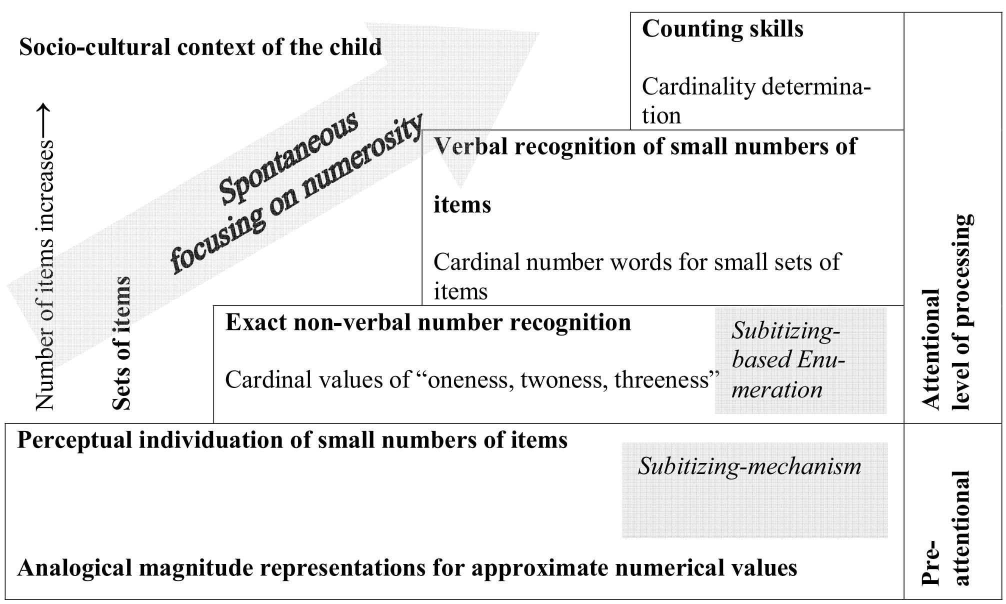


Figure 1. *The development of cardinality-recognition skills*

Cardinality-related skills start to develop very early. Two separate representational systems have been proposed to underlie number recognition skills in infancy: a fast but relative, imprecise discrimination of numerical magnitudes subject to set size ratio limit (analogical magnitude model), and an exact object tracking system operating (object file theory) in the small number range (Feigenson & Carey, 2003; Xu, Spelke, & Goddard, in press). These early systems for representing objects and approximate quantities are not unique to human beings, but can also be found in other animal species, like in birds, rats, salamanders, and different primates (for reviews see e.g. Dehaene, 1997; Hauser & Spelke, in press). Much controversy has surrounded the question of how numerical infants' early representations are (for reviews, see Baroody et al. in press; Feigenson & Carey; 2003; Uller et al., 1999; Wynn, 1998). Spelke (2003) has summarized the results on few-months-old infants' object individuation and numerical discrimination skills by proposing that infants represent objects in accordance with three spatiotemporal constraints on object motion. Infants represent objects as 1) cohesive bodies that maintain both their boundaries and their connectedness as they move, as 2) continuous bodies that move only on connected, unobstructed ways, and as 3) bodies that interact only if they come into contact (Spelke, 2003).

According to the *"analogical magnitude model"* the infant's success on number discrimination tasks, as well as the human brains' determination of numerical order between numbers relies on imprecise mental magnitudes. These are hypothesized variables in the brain, varying systematically with number and other quantitative dimensions of experience. They form the basis for the subjective sense of magnitude. The cardinal value of a set of items is represented by a mental analog magnitude that is distinguished from the objective magnitudes that they represent. This magnitude mechanism behaves according to Weber's Law in discriminating between the quantities as a function of the ratio between them (Dehaene, 1997; Gallistel & Gelman, 1992; Gelman & Gallistel, 2004; Meck & Church, 1983; Wynn, 1998). Spelke (2003), and Feigenson and Carey (2003) propose that the analogical magnitude model represents large numbers of objects or events as sets with approximate cardinal values, and allows for numerical comparisons between sets. It fails to represent the members of sets as persisting, numerically distinct individuals, and therefore it also fails in adding or subtracting one. Mix et al. (2002) suggest that the approximate quantity representations form the basis for later numerical development.

Infants' implicit number representations are capable of representing small numbers of items exactly, and as persistent, numerically distinct individuals as described by an attention-based *"object file theory"* (Carey, 2002; Cohen & Marks, 2002; Feigenson & Carey, 2003; Spelke, 2000; Uller et al., 1999; Wynn & Chiang, 1998, but see also Xu, Spelke & Goddard, in press, for evidence on infants' failure in small number discrimination with continuous quantity controls). According to this proposal (Feigenson & Carey, 2003) infants assign a visual index of attention to each item in the display, which leads them to open an object file in short-term memory providing an experience of unique, bounded and cohesive objects. The object file is a mid-level representation in between earlier representations of unbounded features and later representations of object kinds. The object file is linked to the real-world object via the index capable of following the target object through time and space. The number of object files that can be opened is defined by the number of parallel indexes available for attending to. This leads us to the limits of subitizing-based enumeration explained by Trick and Pylyshyn (1994) (see Section 1.1.). Thus, the limits on set sizes of object tokens that may be simultaneously attended to and tracked seem to be within the same range. This supports the proposal that there is a shared system functioning both in object individuation in infancy and in the object-based basic mechanism of attention in adults (Carey & Xu, 2001).

Feigenson and Carey (2003) provide evidence of 12- to 14-month-old children who seemed to, first, maintain object-file-based representations on individual objects, and second, be able to compare object-file models via one-to-one correspondence to establish numerical equivalence. According to Spelke (2003), the concept "set of individuals" is central to counting, simple addition and subtraction, and all natural number concepts. The development of this concept is essential for a young child's piecemeal growth in understanding what oneness, twoness and threeness mean (Spelke, 2003; Wynn, 1992b). It is also essential for focusing on numerosity, because the numerosity is in particular the quality of a set requiring focusing on the set of individuals, not only the individuals.

The child needs to learn to match the pre-numeric, pre-attentional basic-level perceptual information about the exact values of small numbers with the socio-culturally supported non-verbal and verbal expressions of small cardinal values. The transitions from perception-based representations of infants via exact non-verbal representations to number-word representations have been described by Mix et al. (2002). According to their model, before number words can express exact cardinal and ordinal values, a child needs to develop the ability to mentally represent small numbers non-verbally and exactly. Exact non-verbal representations allow children to identify, represent, compare, and categorize sets of objects within a very small range of numbers. These skills are gradually integrated into activities with verbal number words (Mix et al., 2002; Wynn, 1990). Knowledge of exact cardinal values and number words related to these cardinal values, as well as utilizing skills for numerical aspects in action develop during early childhood years. In the development of cardinality, a child moves gradually from the innate, preverbal perceptual ability to individuate and discriminate small numbers of objects to the ability to represent exact cardinal values for oneness, twoness, and possibly threeness non-verbally in action (Mix et al., 2002).

According to Spelke's view (2003), language, especially verbal number words and language related to enumeration, enables the transition to more developed number representations. Children learn the meanings of words by relating these words to their preexisting concepts, which are made explicit by their core knowledge systems for representing approximate cardinal values of large sets and precise, distinctive representations of small numbers of individuals (Spelke, 2003). Children treat number words as referring to specific, unique numerosities before they know exactly which numerosity each number word refers to (Sarnecka & Gelman, 2004; Wynn, 1992b). Wynn's longitudinal study (1992b) shows how children first learn the cardinal meanings of "one, two and three" in a piecemeal fashion. After this, there seems to be a relatively sudden shift in cardinality recognition skills which results in cardinality meanings for all number words in the child's number word sequence.

Karmiloff-Smith (1995) theorized that the number-relevant information available to young children is implicit in procedures for processing environmental input. The components of this knowledge subsequently become explicitly defined by the process of redescription. Redescription is the process by which children's representations become more manipulable and flexible, allowing conscious access to knowledge. Focusing attention on the exact number of objects or incidents in one's surroundings, i.e. the attentional process enabling the utilizing of the preattentional subitizing mechanism for quantification, is an essential skill to be learnt before the child can utilize his or her innate mechanisms efficiently for utilizing quantification in his or her action (Hannula, Räsänen, & Lehtinen, 2005). From very early on, socio-culturally supported ways of utilizing numerosity in action play a role in learning to utilize the subitizing mechanism for exact enumeration. In utilizing numerosity in action, intentional focusing on numerosity is a necessary intermediate process. How easily the child considers numerosity in his or her action indicates the amount of practice the child is acquiring in his or her surroundings. The same items in the child's environment arouse different interpretations: props for naming, mouthing and banging, as well as for quantification (Gelman, 1990). Which action occurs in a given setting is determined mostly by the child's goals, but it is also affected by the child's skills and concepts. The child's focusing on numerosity produces practice in recognizing and utilizing numerosity in the meaningful everyday contexts of the child.

1.4.2. Object counting skills

Cardinal and counting situations are at first separate and different situations for children, but they are gradually connected when children understand that object counting procedures are not an isolated set of activities, but have a result, which describes how many items there are in the set (Bermejo, 1996; Fuson, 1988). Bermejo (1996) has highlighted the fact that cardinality should not be considered as a component of object counting, but rather as a means for determining cardinality, differentiating between procedure and result (cf. cardinality principle by Gelman & Gallistel, 1978). Counting can not be a prerequisite for cardinality recognition, because cardinality of a set can also be defined by estimation or subitizing. Therefore the relationship between counting and cardinality is more likely to be a contextual or instrumental than a causal one (Bermejo, 1996). Cardinality determination based on counting requires both explicit knowledge of the relation between numbers and quantity, and the attentional procedures for focusing on the units which are counted (Bialystok & Codd, 1997).

Sophian (1998) has suggested that children's conceptual knowledge about numbers is dynamically related to their goal-based numerical activities: conceptual advances facilitate new goals and corresponding activities, which in turn provide the input for further advances. According to the reciprocal views of Saxe et al. (1987) and Sophian (1998) on object counting development, socially structured goals of quantification change along with the development of skills and direct children's attention to different aspects of numbers and the ways in which others use them. The present study addresses the developmental role of children's spontaneous practice in utilizing exact enumeration in action. It is not only the socio-cultural mediation of numerical cognition which develops the child's skill to focus on the aspect of numerosity and to utilize innate and cultural tools for enumeration, but also focusing on the aspect of numerosity develops the child's enumeration skills by activating the enumeration process and thus producing practice in enumeration.

Thus, it has been suggested that the skills for object counting emerge as a result of the integration of a child's first cardinal numbers and the learning of culturally-based verbal counting procedures (Bermejo, Morales, & deOsuna, 2004; Fuson, 1988; Wynn, 1992b; 1998). Bermejo (1996) has described children's development in counting-based cardinality recognition by stating that children progress through the following levels in their performance of answering the question of "how many items there are": (1) not understanding the task of enumeration, (2) providing a number word sequence as an answer, (3) counting again, (4) repeating the last counting word (cardinality principle by Fuson, 1988; Fuson, Pergament, Lyons, & Hall, 1985), (5) giving the largest number as an answer, and (5) true cardinality understanding.

The remarkably slow development of counting-based cardinality recognition skills (e.g., Fuson, 1988; Wynn, 1990) could be explained by the major differences in nonverbal and verbal number recognition systems (Wynn, 1992ab), as well as by the demanding integration of different representations of individuals and sets, namely, object files and approximate numerical magnitudes (Spelke, 2003), and the need for lots of practice in acquiring accuracy in counting procedures. Learning to recite the list of number words not only accurately but also in one-to-one correspondence with pointing acts takes practice. This learning provides a child with the basis for constructing the hows and whys of counting, namely, Gelman and Gallistel's (1978) counting principles, as well as the essential and nonessential features of correct counting (Briars & Siegler, 1984; Cowan, Dowker, Christakis, & Bailey, 1996; Freeman, Antonucci, & Lewis, 2000), and the use of counting for solving tasks requiring numerical inferences and comparison of the cardinal values of sets (Sophian, 1987; 1995; Sophian, Wood, & Vong, 1995). When the basic skills of counting items in lines are stabilizing, children move on to learning the marking strategies necessary in counting random arrangements of objects (Fuson, 1988). However, their counting range, as well as the range of truly numerical number sequences, in which cardinal and ordinal aspects of numbers are integrated, remains limited to rather small numbers (Fuson, 1988), and they benefit from overt pointing gestures while counting objects (Alibali & DiRusso, 1999) during kindergarten years.

In our studies on SFON we have assumed that the better the subitizing-based enumeration skills a child has, the better basis she or he has for understanding the purposes of counting procedures in determining the cardinality of a set (Hannula, Räsänen, & Lehtinen, 2005). The practice in subitizing-based enumeration produced by SFON may also make a child more aware of numerical aspects of many everyday contexts and situations, in which verbal counting of objects can be utilized once these skills are developing.

1.4.3. Number sequence skills

A necessary sub-skill of object counting is reciting the list of number words. The number sequence production is a verbal culturally-based skill indicating the child's participation in socio-cultural activities, in which the number word string has been used. Learning the first words of this string has been described as a serial recall task, in which the cardinal and ordinal aspects, as well as the base-ten structure of number words are only later embedded in the sequence of number words (Fuson, Richards, & Briars, 1982; Fuson, 1988). The rules for constructing the number word sequence are first based on, and affected by, the linguistic structure of the number word sequence more than its mathematical nature, such as the cardinal values of the number words (e.g., Miller & Stigler, 1987; for a review, see Skwarchuk & Angling, 2002). Eventually, the sequence of number words becomes a mental construction of numberline including exact cardinal meanings for number words and exact ordinal relations between them. This can be used as a problem-solving tool in counting of items, and number words become countable objects themselves (Fuson et al., 1982).

Fuson (1988) has classified the development of number sequence elaboration skills into the following five levels through which number sequence skills are built:

(1) *string level*: the number words form a forward-directed, undifferentiated whole,
(2) *unbreakable list level*: the number words are separable, but the list of them can only be produced by starting at the beginning,
(3) *breakable chain level*: parts of the chain can be produced also from other starting points than one,
(4) *numerable chain level*: the number words have become units in a numerical sense; thus sets of number words can represent a numerical situation and can be counted and matched,
(5) *bidirectional level*: the number words can be easily produced and used in both directions.

The verbal number sequence skills of the majority of Finnish school-beginners have developed to a level where children can not only count accurately up to over 50, but can also start counting from a given number forward and backward under 20, as well as count a prescribed number of steps forward and backward from a given number (Keranto, 1981; Lehtinen & Kinnunen, 1993; Kinnunen, Lehtinen, & Vauras, 1994). The skills of number sequence elaboration before school age predict children's later success in arithmetic skills at school (Kinnunen et al., 1994). Poor proficiency in counting backwards, which depends on learning both forward sequence and number-before relations can interfere with children's skills to make numerical comparisons or solve subtraction tasks (Baroody, 1984).

1.4.4. Basic arithmetic skills

The last early mathematical skills of interest in the present set of studies are basic arithmetic skills. They enable adding and subtracting with verbal number words, and they develop together with cardinality recognition and number sequence skills but also with separate schemes of quantitative increasing and decreasing. Similarly to the early basis for exact number recognition, procedures for numerical computations are also built up in infancy on an analogical magnitude-based estimation for representing numerosities and object files individuating small exact numbers of items (e.g., Wynn, 1992a; McCrink & Wynn, 2004) and in toddlerhood, on experiences combining and separating sets of objects with these nonverbal skills forming the basis for the development of conventional verbal methods of arithmetic (Levine, Jordan, & Huttenlocher, 1992). Numbers become more meaningful as they are incorporated into different cognitive activity patterns. Resnick and Greeno (1990) propose that children can perceive and reason about aggregations of substances and objects before they represent them in systematic ways. Furthermore, important distinctions between different kinds of aggregations develop as part of children's conceptual growth regarding numbers and quantities (see also Mix et al., 2002). The physical and social world of young children provides extensive opportunities for them to develop concepts about amounts of material, their comparisons, and the effects of various kinds of actions on these amounts.

According to Resnick and Greeno's hypothetical model (1990), these concepts form a set of protoquantitative schemata, which refer to amounts of material, to quantities. They do not yet contain elements that refer to precise measures of the quantities involved. Children's protoquantitative schemata organize their reasoning about quantities in ways that capture many of the major relationships among quantities that are later to be given precise measurement and expression via numbers and arithmetic (Resnick & Greeno, 1990). The study of Zur and Gelman (2004) demonstrates the close link between verbal counting and arithmetical skills already in 3- and 4-year-olds. According to Resnick and Greeno's proposal (1990), children's knowledge of number facts and their methods of counting to find answers to addition and subtraction tasks are gradually integrated into a unified set of relations. Thus the number facts are based on counting methods for addition and subtraction questions, and the numbers represent members of sets with true cardinal values (see also Fuson, 1988).

In the current work, it is proposed that due to the significant role of understanding cardinality values for number words, children's SFON can be related to their basic arithmetic skills. Focusing on exact numbers of objects in addition to other aspects of sets of objects changing while sets of objects are manipulated, could be one of the essential processes giving meaning for numerical increasing and decreasing. Thus, focusing on numerosity could have two kinds of role in arithmetical development, first, as providing exercise in cardinality recognition and formation of cardinal and ordinal relations to number words, but also supporting the very operation of numerical addition and subtraction by providing the basic condition for understanding numerical operations. On the other hand, variation in the aspect of number is provided by addition and subtraction operations, which could also aid a child to focus on the aspect of number (cf. Marton and Booth, 1997).

2. AIMS

The present work provides an examination of 3–7 –year-old children's spontaneous focusing on numerosity and mathematical skills in four studies conducted in three longitudinal research projects. The overall aims shared by all the five studies were to,

1) study whether there is a separate process of focusing on numerosity, that is theoretically and methodologically distinct from children's activities involving exact number recognition,
2) develop novel, age-appropriate tasks and methods to quantify individual differences in children's spontaneous focusing on numerosity,
3) study the stability of the SFON tendency across different task contexts (in all studies) and ages within longitudinal studies (*in Studies II and III*), and
4) examine the relationship between SFON tendency and the development of early mathematical skills closely related to exact number recognition skills including subitizing- and counting-based enumeration, number sequence production and elaboration, and basic arithmetic skills.

In addition to these aims shared by all the studies, there were also the following specific aims:

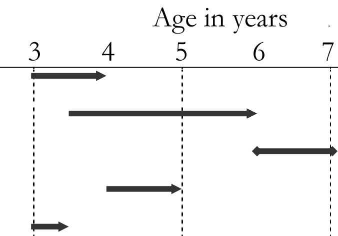
1) to study domain specificity of SFON tendency by controlling the effects of non-verbal and verbal IQ on the association between SFON and mathematical skills (*Study III*),
2) to study whether failure in SFON tasks is explained by children's inability to deal with SFON task requirements other than focusing on numerosity by testing whether those children who failed in SFON tasks were able to execute the tasks when their attention was guided towards the aspect of number in the tasks (*Study III*), and
3) to explore the potential influence of social interaction on children's SFON tendencies (*Study V*).

3. METHODS

For the present set of studies, mainly longitudinal methods were selected. This enabled the studying of young children's mathematical development, and especially the role of long-term general SFON tendency in this development. The five studies comprising the doctoral dissertation are based on data (see Table 1) gathered in three research projects on children under school age. In Finland, children start school in August of the year they turn seven. Before this, more than 95 percent of children go to government-funded preschool aimed at promoting their school-readiness skills during the year before the school start. The participating children were either in day-care or preschool. Some children were also being cared for at home by their parents for short periods during the longitudinal studies. All the data gatherings were conducted at day-care centres familiar to the children.

Table 1. *Data gathering in different studies. Note, Studies II and III are reported in one article, and Study III is a cross-sectional study, while the other studies are longitudinal.*

Study / Research project
Study I / Pythagoras (N = 39)
Study II / Pythagoras (N = 39)
Study III / Origins of Exclclusion (N = 183)
Study IV / Pythagoras (N = 39)
Study V / SFON Enhancement (N = 34)



3.1. Participants

Studies I, II, and *IV* were conducted within the framework of Pythagoras project, which was a broader follow-up study of children's mathematical development from the age of 3 to 6 years. Pythagoras project was funded by the Academy of Finland, and it was led by professor Erno Lehtinen, (Department of Teacher Education and Centre for Learning Research, University of Turku, Finland). The sample of the research project was gathered from children in seven day-care centres located in socio-economically varying districts of a Finnish town with 170 000 inhabitants. Written consent forms were sent to the parents of Finnish-speaking children without any developmental delays, and having their 3rd birthday within two months from the beginning of the follow-up. The parents of all but one child gave their permission. The data collection was complete providing data on 39 children (18 girls and 21 boys) in their SFON and mathematical skills. The children's ages varied from 2 years 10 months to 3 years 2 months when the follow-up began. At the end of the follow-up, the children were in 19 different day-care centres, or home-care settings. The educational background of children's parents was slightly lower than for Finnish adults in the age group 25-49 (see Table 2).

Study III was a part of a larger longitudinal study called Origins of Exclusion in Early Childhood, in which children's cognitive, motivational and social development was followed from a year before preschool (age 5.5 years) to the end of the second school year (age 8.5 years). The project was funded by the Academy of Finland, and it was led by professor Kaarina Laine (Department of Education, University of Turku, Finland). At the beginning of the longitudinal project, all parents of children reaching the age of six years during the year 2001 in ten kindergartens were asked for their informed consent. Nine per cent of the parents refused to give their written consent. The kindergartens were selected from socio-economically different districts of a Finnish town with 170 000 inhabitants. The sample represented an urban Finnish population in day care except for a slight over-representation of boys and a weaker educational background in parents of the children in the sample (see Table 2). The participants of *Study III* were 183 preschoolers (102 boys, and 81 girls) aged from 6 years 0 months to 7 years 0 months (M = 6.51 years, SD = 4 months). The sample included 18 children from immigrant families with at least moderate skills in the Finnish language, and 17 children with special educational needs.

Study V was a part of a research project on SFON enhancement. The project was led by professor Erno Lehtinen (Department of Teacher Education and Centre for Learning Research, University of Turku, Finland) and the work for the current dissertation was funded by the Finnish Graduate School of Educational Sciences and Finnish Cultural Foundation. The participants in *Study V* were 34 three-year-old (range from 2 years 10 months to 3 years 2 months) children, who were in day care in seven Finnish urban kindergartens in middle-class districts of a town with 170 000 inhabitants. The experimental group consisted of 17 children from three groups of children in three kindergartens. The control group of 17 children was selected from 24 children at four kindergartens by matching children's age, SFON scores, cardinality-related skills (the number of recognised and produced numbers of objects and the level of counting attempts) and gender to the corresponding characteristics of the experimental group. The children were from native Finnish speaking families, and they had no developmental delays. All the parents with children meeting these criteria in the participating kindergartens gave written consent. The educational background of the parents was representative of that of average Finns aged 25-49 (see Table 2).

Table 2. *The educational level of the mothers and fathers of the participating children and a representative sample of Finnish 25-49 –year old men and women. Due to missing data on parent's educational level, the numbers of participating children and parents differ in* Studies III *and* V.

The educational level	Studies I, II,VI, %		Study III %		Study V %		Finnish 25-49-year-olds[1] %	
Mothers and fathers n	M: 39	F: 39	M:151	F: 139	M: 31	F: 31	M:	F:
Below upper secondary education	46.2	61.6	43.0	47.5	16.1	35.5	20.6	25.3
Upper secondary qualification	38.4	17.9	28.5	25.2	35.5	25.8	42.0	46.7
Tertiary degree	15.4	20.5	28.5	27.3	48.4	38.7	37.4	28.0

[1] source Statistics Finland, 1997

There are some indications about Finnish children's early numeracy measured by modified versions of Early Numeracy Test (Van Luit, Van de Rijt, & Pennings, 1999) compared to children in other countries. Cross-national comparison of 4-7 –year-old children in Finland, Hong Kong and Singapore showed that Finnish children's number sense was weaker than in the other two countries (Aunio, Ee, Lim, Hautamäki, & Van Luit, 2004), while the other comparison of 6-year-old's number sense indicated that Finnish children's skills were at the level of English, and slightly above Swedish children (Hiltunen, 2003). Accordingly, the study of Van de Rijt et al. (2003) showed only negligible differences in 5- to 7-year-old children's skills in a variety of European countries.

3.2. Assessments of SFON and mathematical skills

All the measures of the studies are presented in Table 3. Detailed descriptions of the measures can be found in *Studies I-V*. One purpose of the current work was to design suitable diagnostic tasks for young children's mathematical skills in familiar everyday task contexts following the guidelines of Donaldson (1978). She demonstrated that young children's mathematical skills are best revealed by tasks in everyday contexts, in which they can show their procedural skills, and conceptual understanding in action or play. The focus of the measures was mainly on the child's understanding of the mathematical goals of the tasks instead of immediate comprehension of certain verbal expressions. Hence, the role of comprehension of specific verbal instructions in the task contexts was minimised by presenting all the tasks with necessary clarifications. This was done due to several difficulties in testing skills of young children arise from their limited language skills, which may constrain both their responses to the task and also their initial awareness of the task demands (Porter, 1998). Children's performance level throughout the age between 4 and 6.5 years is higher on nonverbal problems than on story problems and number-fact problems (Levine, Jordan, & Huttenlocher, 1992). Children with difficulties only in mathematics are superior to children with reading and mathematics difficulties in areas that may be mediated by language but not in ones that rely on numerical magnitudes, visuospatial processing, and automaticity (Hanich et al., 2001). For these reasons, a special effort was made to present all the SFON, guided focusing and mathematical tasks of the studies in ways which would support children's linguistic comprehension. Furthermore, when selecting the materials for the tasks, efforts were made to ensure that basic visuo-motorical skills, such as pointing at items and picking up items would not substantially limit children's success in the tasks. A familiarization session before the assessments was used to decrease the effect of a strange person conducting the task situations with the participating 3-year-old children.

Table 3. *The ages at which children's SFON, guided focusing on numerosity, mathematical skills, and controlling variables were measured in different studies. Note! X = the skill was measured repeatedly, at the ages of 3 years, 3 years 1 months, and 3 years 6 months.*

			Study		
Measure	I	II	III	IV	V
SFON					
Imitation					
Visible objects	3				X
Disappearing objects		4 and 5		4 and 5	X
Two kinds of disappearing objects		6	6.5		
Cardinality context	3				
Model		6	6.5		
Finding		6	6.5		
Selection					X
Observations of SFON at day-care					3
Guided focusing tasks			6.5		
Mathematical skills					
Cardinality recognition	3, 3.5, 4	3.5			X
Counting attempts					X
Subitizing-based enumeration				5	
Number sequence production		3.5, 5, 6		5	3.5
Number sequence elaboration			6.5		
Object-counting		5 and 6	6.5	5	
Basic arithmetic skills			6.5		
Controlling variables					
Raven's non-verbal IQ			6.5		
Comprehension of verbal instructions			6.5		

The SFON, guided focusing, and mathematical tasks were designed in the studies. When the studies were conducted there were no standardised tests for under school-aged children's mathematical skills in Finland. Earlier tasks of Salonen et al. (1994) and Fuson (1988) formed the basis for the number sequence and object-counting tasks used for children at the ages of 5, 6, and 6.5 years. The tasks used for children at the ages from 3 to 4 years were specifically aimed to capture the cardinality recognition and production skills with a task which did not require knowledge of verbal number words, but was based on children's skills in recognising exact numbers of items.

3.3. SFON and guided focusing assessments

The aim of the SFON assessments was to obtain a reliable indicator for a child's general SFON tendency across different task contexts. This kind of indicator captures the extent of a child's focusing on numerosity, and thus the amount of practice acquired in utilizing enumeration skills in his or her surroundings. In order to enable the measuring of children's spontaneous behavior, only novel tasks or, at least, tasks with novel materials and/or contexts can be used each time. Furthermore, when presenting the SFON tasks, no use can be made of any phrases which could suggest that the tasks were somehow mathematical or quantitative. In order to hinder the confounding effect of number recognition skills on the measures of SFON, the SFON tasks have to include only such small numbers of items or incidents, which all children should be able to recognize. Moreover, the SFON tasks should not exceed children's memory capacity, visuo-motorical, or verbal comprehension skills, so that if the child focuses on number in the task, he or she is capable of proceeding in the task in accordance with his or her numerical focusing target.

The scoring of the child's SFON was based on analyses of video-recorded task situations in all other studies except for *Study III*, in which structured observations following the same criteria as the analyses of video-recordings were used to evaluate whether the child focuses on numerosity in the task. All the child's (a) utterances including number words (e.g. "I'll give him two berries"), (b) use of fingers to express numbers, (c) counting acts, like a whispered number word sequence and indicating acts by fingers and/or head, (d) other comments referring either to quantities or counting (e.g. "Oh, I miscounted them"), or, (e) interpretation of the goal of the task as quantitative (e.g. "I gave an exactly accurate number of them"), were identified. The child was scored as focusing on numbers, if she or he produced the correct numerosity, and/or, if she or he was observed presenting any of the aforementioned (a - e) quantifying acts. Furthermore, in one SFON task, the Finding task used for 6 and 6.5 years old children, a structured interview immediately after the task was used to evaluate, whether the child had considered the number in the task.

Fifteen novel tasks were progressively developed for the SFON assessments over the course of the five studies (see Table 3) with increasing theoretical understanding resembling the cycles of a design experiment (The Design-based Research Collective, 2003). During this process, more and more critical conditions for the methods to meet the demands of assessing children's spontaneous focusing on numerosity were recognized, and taken into account in the subsequent SFON assessments. The aim was to distinguish between a child's spontaneous focusing on numerosity (or lack thereof), and children's basic number recognition, counting and other procedural skills. Basic method for this was using only such small numbers of items in the tasks that every child is able to recognize them. In order to capture better the child's general SFON tendency, the number of different SFON tasks used at one measurement point was increased. Also tasks developed later with disappearing items were better controlled than the first SFON tasks with visible objects. When the items disappear from sight, the child cannot use any geometrical hints for producing similar sets of items. Because of this, the probability of producing the accurate number of items in the task by other means than focusing on numerosity is very low, and high certainty about the child's actual focusing on numerosity is reached.

Furthermore, guided focusing versions of the SFON tasks were developed to specifically test that SFON tasks do not exceed children's cognitive skills. With the guided focusing tasks used in *Study III*, it was possible to specifically test whether those children who did not spontaneously focus on numerosity could execute the SFON tasks, when explicitly guided to look at, and to count relevant numbers in the tasks. The aim was to confirm that it is the lack of spontaneous focusing on numerosity, not the lack of adequate counting and other procedural skills, which lead to failure on SFON tasks.

In order to explore whether SFON assessments are a valid indicator of a child's more general SFON tendency, observational data on children's SFON in day-care settings, in addition to the SFON assessments, were gathered in *Study V*. The analyses showed that children's SFON in the assessments and in day-care settings were related.

The experimenter's role is very critical in the assessment of SFON tendency. It is well-known how expectations of the experimenter can influence the behavior of the participant (Clever Hans effect). In order to overcome the effects of the experimenter's expectations, in the longitudinal Pythagoras project when the children were six years old all the SFON tasks were presented by a novel experimenter who knew nothing about the children's former skills. Similarly in the Origins project, the experimenters did not have any knowledge about the children's mathematical skills or their previous SFON tendency. There was no experimenter effect on SFON in the quality or number of SFON observations.

One of the methodological advances obtained during the series of studies was reached in *Study III*, in which the broader assessment of children's cognitive and social skills enabled the presentation of the SFON tasks among non-mathematical tasks. This allowed us to completely hide the quantitative purpose of the SFON tasks. In the same study, structured observations and interviews were used to record children's focusing on numerosity in the SFON tasks. The aim of this development was to make the SFON assessment less time-consuming and more suitable for diagnostic purposes in young children's education.

3.4. Statistical methods

Six different kind of statistical analyses were used in the present set of studies (Table 4).

Table 4. *Statistical analyses conducted in different studies.*

Statistical analyses	Study	Purpose of the analyses
Repeated measures ANOVA	I, V	Group comparisons
Intraclass correlation	II, III	Stability of SFON
Structural equation modelling	II	Longitudinal association of SFON and mathematical skills
Partial correlation	III	Controlling the effects of other variables on the covariance of SFON and mathematical skills
Mediation analyses	IV	Association between SFON, subitizing and verbal counting skills
Spearman's rank-order correlation	V	Covariance of SFON observations and assessments

All the statistical analyses, except for the path analyses were done with SPSS (versions 8-12) following the guidelines of Field (2000), Sobel (1982), and Baron and Kenny (1986).

For studying developmental differences between groups *repeated measures ANOVA's* were run together with post-hoc comparisons (in *Study 1*) and planned contrasts (*Study V*). For examining stability in SFON across two measurement points in *Studies II and III*, *average-measure intra-class correlation* was used. This method is both theoretically appropriate and statistically sensitive approach to detect the existence of systematic errors, and thus evaluate the test and re-test reliability (Yen, Lo, 2002). Furthermore, it allows generating correlation coefficient for more than two variables simultaneously. By *partial correlation* in *Study III* it was possible to control the effects of third variables on the covariance of SFON and mathematical skills. Due to violation of normality in the tested variables and a small sample size, Spearman's rank-order correlation was used in *Study V*.

Structural equation modelling, specifically path analyses of observed variables was used in studying the stability and the direct and indirect associations of SFON and mathematical skills longitudinally in *Study II*. The small sample size and the very few contemporary indicators of specific skills did not allow the formation of measurement models including latent variables.

First theory-based conceptual models on associations between variables were developed. Next the path analyses were conducted with LISREL 8.51 (Jöreskog & Sörbom, 2001), using the maximum likehood method in fitting the models to the covariance matrix. The maximum likehood method is recommended by West, Finch and Curran (1995) for small sample sizes when the distributions are not substantially non-normal (skewness below 2 and kurtosis below 7). These criteria were met in our study, in which skewness was 1.88 and kurtosis 4.16 in the most non-normal variables. Some regression associations had statistically non-significant regression co-efficients, and therefore, non-significant associations were removed step-wise, resulting in the final models. The non-significant associations were removed one by one, always removing the associations with the lowest t-value. The procedure was halted when all remaining *t*-values reached a minimum of 1.645, which is associated with a one-sided significance level of 5%. One-sided statistical testing for hypotheses was used throughout the analysis because the direction of all associations was hypothesized in advance.

In evaluating the goodness of fit of the models to the sample data, not only the traditional χ^2 (Chi Square) was used, but also two indicators recommended for small samples by Hu and Bentler (1999): the Standardized Root Mean Square Residual (SRMR) and Comparative Fit Index (CFI). The absolute fit index called SRMR indicates whether, on average, the correlations between observed variables, as predicted by the model, differ by no more than 0.10 from the actual correlations. CFI is an incremental, noncentrality-based fit index. A statistically significant ($p < .05$) χ^2 value relative to degrees of freedom indicates that the observed and estimated covariance matrices differ (Hu & Bentler, 1999; Schumacker & Lomax, 1996). Hu and Bentler (1999) recommended the following approximate cut-off values for rejecting the model: for SRMR greater than 0.09, and for CFI smaller than 0.95. Due to slight violation of normal distribution in observed variables, the results of path analyses were confirmed by running the analyses also with normal score-transformed variables. The results of the path analyses based on such a small sample size have to be considered with caution, and as indicative only.

Mediation analyses (Baron & Kenny, 1996; Sobel, 1982) was used in *Study IV* for studying the direct and indirect associations between SFON, subitizing-based enumeration and verbal counting skills. To test for mediation, one should estimate three regression equations (Baron & Kenny, 1996). These are, first, regressing the mediator on the independent variable, second, regressing the dependent variable on the independent variable, and third, regressing the dependent variable on both the mediator and the independent variable. A variable functions as a mediator, when it meets the following conditions: (1) variations in levels of the independent variable significantly account for variations in the mediator, (i.e. association a) (2) variations in the mediator account for variations in the dependent variable (i.e. association b), and (3) when associations a and b are controlled, a previously significant relation between independent and dependent variables is no longer significant. Sobel's test (1982) was used to test for the significance of indirect associations from independent to dependent variable via the mediator.

4. AN OVERVIEW OF THE EMPIRICAL STUDIES

In the following overview, the set of five studies reported in the following four articles is briefly described in respect to the main research questions of the doctoral dissertation.

4.1. Study I

Hannula, M.M. & Lehtinen, E. (2001). Spontaneous tendency to focus on numerosities in the development of cardinality. In M. Panhuizen-Van Heuvel (Ed.), Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education, 3, 113-120. Drukkerij Wilco, The Netherlands: Amersfoort.

In this article, we report on the first phase of the 3-year longitudinal "Pythagoras-project" with 39 children. The aim of the study was to explore whether there were individual differences in 3-year-old children's spontaneous focusing on numerosity, and whether these differences were related to the children's development in cardinality related skills from 3 to 4 years of age. The first two tasks for assessing SFON within children's number recognition range were developed in this study.

The participants were 39 Finnish-speaking children in day care. They had no developmental delays. Spontaneous focusing on numerosity was assessed individually by two video-recorded tasks, in which the child's attention was not specifically directed to the numerical aspects of the task. In the Imitation task the child was asked to imitate the experimenter when she gave different numbers of carrots to a toy hare. In the other task, comprising two different sections, the child was asked to help the experimenter in putting socks on one- and multilegged caterpillars by bringing the caterpillar as many socks as it needed. Spontaneous focusing on numerosity was assessed with a SFON section of the caterpillar task, in practice, from the first item of the task, in which a 2-legged caterpillar was introduced. Although this form of request includes a quantitative hint for adults, surprisingly all children did not focus on the exact number of legs on caterpillars, but brought either a handful of socks or only one sock to the caterpillar. Specific guidance for focusing on number of legs and socks was provided for every child immediately after the child's first trial to bring the caterpillar two socks so that all the children came to understand the quantitative goal of the task. Cardinality-related skills, particularly children's exact number recognition skills, were assessed by the following items with increasing number of legs on caterpillars.

The results showed that 14 children did not focus on numerosity in either both tasks (3 children), or in one task (11 children), whereas the remaining 25 children interpreted the numerosity immediately as a relevant factor in both tasks. The children were assigned to two groups according to their initial amount of SFON tendency. Separate 2 x 3 (group x age) ANOVAs with repeated measures were performed for their performance, together with Tukey's HSD tests, at the ages of 3, 3.5 and 4 years. The results on children either focusing on numerosity in both tasks (the spontaneous group), or not focusing on numerosity in both tasks (the non-spontaneous group) in their development of cardinality-related skills showed that the spontaneous group developed faster than the non-spontaneous group from the age of 3 to 4 years. The spontaneous group outperformed the non-spontaneous group at the age of four years, unlike at the age of 3 and 3.5 years, in their skills to recognise and produce small numbers of objects. Thus, it was concluded that children's development in cardinality recognition skills is related to their SFON tendency.

This study suggested some improvements for the SFON assessments. First, the SFON tasks should not include any verbal hints for focusing on numbers, and second, producing exactly accurate numbers of items in the SFON tasks should be possible only by focusing on numerosity, not on producing a similar-looking figure or area with the objects. The latter may be possible in the Imitation task.

4.2. Studies II and III

Hannula, M. M., & Lehtinen, E. (in press). Spontaneous focusing on numerosity and mathematical skills of young children. Learning and Instruction.

The aims of this article reporting on two studies (*II* and *III*) were to give information about young children's individual differences in SFON both longitudinally and with a variety of different tasks taking into account the methodological shortcomings of the SFON assessments in *Study I*. We assumed that within the frames of a child's existing mathematical competence, it is possible to distinguish a separate mental process, one which refers to the child's tendency to spontaneously focus on the aspect of numerosity and utilize his or her enumeration skills in various activity situations. We hypothesized that there should be some stability in a child's spontaneous focusing on numerosity across different tasks (*both Studies*), and across time (*Study II*). Furthermore, SFON tendency should be related to the development of mathematical skills during the phase when children normally establish their counting skills (*both studies*). The association between SFON and mathematical skills should not be explained by children's differences in non-verbal IQ, comprehension of verbal instructions, or by their lack of enumeration or other cognitive skills required for executing the SFON tasks (*Study III*).

Study II. Participants in the longitudinal sub-study were the same 39 children as in *Studies I* and *IV*. The part of the follow-up included in *Study II* began when children were 3.5 years old, and ended when the children were 6 years old.

The aims of *Study II* were to examine children's SFON longitudinally and to study how it was related to the formation of early mathematical skills. Our hypothesis was that young children vary in their SFON and that there is some stability in this tendency. In addition, we assumed that SFON is involved in the development of mathematical skills. Practice produced by a strong SFON tendency might result in an improvement in mathematical skills, and well developed mathematical skills could in turn increase children's subsequent spontaneous tendency to focus on numerosity.

The children's SFON was tested with an exact interval of one year, at the ages of 4, 5 and 6. For the SFON assessments five different novel tasks were used. One SFON task at 4, one SFON task at 5, and a set of three tasks at 6 years. The increase in the number of SFON tasks was the result of a deeper theoretical understanding of the phenomenon of SFON tendency, and especially the notion that by using several different SFON task contexts, we can increase the reliability and validity of the SFON indicator as long as there is stability across the SFON tasks used. The assessments of their mathematical skills included tasks of number sequence production at 3.5, 5 and 6, early cardinality recognition at 3.5, and object counting at the ages of 5 and 6. The results showed that children varied in their SFON and that the average measure of intraclass correlations showed stability in their SFON tendency across the SFON measures of the three measurement points. There were also individual differences in children's SFON related to their mathematical skills. Path models indicated a reciprocal development between SFON and mathematical skills.

In *Study III*, we studied individual differences in SFON and mathematical skills, and furthermore, specifically tested whether those children who did not spontaneously focus on numerosity, could execute the SFON tasks, when explicitly guided to focus on relevant numbers in the tasks. The aim was to confirm that it is the lack of spontaneous focusing on numerosity, not the lack of adequate counting and other procedural skills, which leads to failure on SFON tasks. Moreover, we wanted to control the effect of non-verbal intelligence or comprehension of instructions on the association between SFON and mathematical skills. The gender differences in the variables measured were also explored.

The participants of the cross-sectional study were 183, on average, 6.5-year-old preschoolers. Their age varied from 6.0 to 7.0 years. A set of three SFON tasks was designed for the study, one of which included subitizable numbers of items, and two required counting. The same tasks were also used in *Study II*.

The results show that SFON is involved in mathematical development even after children have achieved basic skills of cardinality recognition. The partial correlations, varying from .27 to .37 showed that the shared variance in SFON and mathematical skills was domain-specific, and not explained by children's differences in non-verbal intelligence and comprehension of verbal instructions. Moreover, the guided focusing tasks showed that the children's failure in SFON tasks was not caused by their inability to deal with the cognitive requirements of these tasks. In this study, we used structured observations instead of reviews of video recordings of SFON assessments as in other studies. The analyses of the experimenter effects showed that we were successful in creating an instrument with which all the experimenters were able to gather data reliably. In this large-sample study, gender effects were also studied. The results showed no gender effects in SFON or mathematical skills, except for one SFON task, in which girls outperformed boys.

In sum, the results of the studies give support to our main hypotheses concerning the existence and stability of SFON, as well as its positive association with mathematical skills. Within a child's existing mathematical competence, it is possible to distinguish a separate process, which refers to the child's tendency to spontaneously focus on numerosity. We also found considerable stability in this tendency across different tasks. Furthermore, SFON tendency seemed to be clearly positively related to the development of verbal counting skills.

4.3. Study IV

Hannula, M. M., Räsänen, P., & Lehtinen, E. (2005). Development of counting skills: Role of spontaneous focusing on numerosity and subitizing-based enumeration. Manuscript submitted for publication.

In *Study IV*, we investigated individual differences and stability in SFON tendency at the age of 4 to 5 years, in addition to associations between children's long-term SFON tendency, subitizing-based enumeration and verbal counting skills. In the literature, the role of subitizing-based enumeration has been suggested to be significant in counting development, but surprisingly, there seemed to be no empirical studies on the association of these two in young children. As a part of this study, a theoretical framework of subitizing-based enumeration different from the subitizing mechanism was created. We proposed that SFON is involved in the development of mathematical skills by producing practice in intentional recognition and utilizing numerosity in action. The role of SFON and subitizing-based enumeration in learning different sub-skills of counting was hypothesized to differ according to how significant a role integrating cardinal values for number words has for learning the sub-skill.

Participants were the same 39 children as in studies *I* and *II*. The children were tested for their SFON tendency at two measurement points with an exact interval of a year, at the ages of 4 and 5 years, and for their subitizing-based enumeration and counting skills in separate testing sessions at the age of 5 years. In order to obtain a more valid indicator for long-term SFON tendency, we used the average of SFON measures at two measurement points. This decision was supported by a significant intraclass correlation between the two SFON measures, which also showed that there was considerable stability during the follow-up. For the study, we designed a new computer-aided game with which we assessed children's subitizing-based enumeration skills.

Results of this explorative study showed that the children with a strong general long-term tendency to focus on numerosity tended to enumerate by subitizing larger numbers of items, and had better verbal counting skills at the age of 5 years. Furthermore, there was a positive direct association between spontaneous focusing on numerosity and number sequence production. The association between spontaneous focusing on numerosity and object-counting skills was significantly mediated by subitizing-based enumeration. These results indicate that the associations between the child's SFON tendency and sub-skills of verbal counting may differ on the basis of how significant a role understanding the cardinal meanings of number words plays in learning these skills. It was suggested that experimental studies should be carried out to clarify the direction of the causality between the variables explored.

4.4. Study V

Hannula, M. M., Mattinen, A., & Lehtinen, E. (in press). Does social interaction influence 3-year-old children's tendency to focus on numerosity? A quasi-experimental study in day care. In L. Verschaffel, E. De Corte, G. Kanselaar & M. Valcke (eds.), Learning environments to promote deep conceptual and strategic learning. Studia Paedagogica. Leuven: Leuven University Press.

The goal of this quasi-experimental study conducted in a highly naturalistic setting in day-care centers was to investigate whether it is possible to influence 3-year-old children's tendency to spontaneously focus on numerosity, and thus promote children's deliberate practice in recognizing and producing small numbers of objects or incidents.

Participants were 34 normally developed 3-year-old children representing Finnish urban children in day care. The experimental group consisted of 17 children from three groups of children in three kindergartens and a control group of 17 children in four kindergartens. The set of SFON tasks developed in our earlier studies was used to assess children's SFON tendency. Furthermore, the day-care personnel gathered observational data on children's SFON manifestation during day-care hours. A specific value of this study was in the information that it gave us abut how adults can notice and promote

SFON, and especially what kinds of difficulties there may be for adults in doing this. The children in the experimental group participated in activities aimed to increase their SFON tendency during their normal day-care hours for four weeks. The post-test was performed four weeks after the pre-test, and the delayed post-test five months after the post-test. In order to measure children's overall SFON tendency, three different tasks (and parallel versions of them) were presented at the pre-, post-, and delayed post-tests. Accordingly, cardinality related skills were assessed at every measurement point by a task in which the children's confident level of recognizing and producing numerosity as well as their separate cardinality-related skills were evaluated. Furthermore, the children's skill to count from one onward was tested at the delayed post-test.

Children's attention was directed to numerosity (how many objects or incidents there were) in organized, structured games organized by adults, as well as in everyday situations (dressing, cleaning up, and free play). The aim was to provide plenty of different tasks and situations, where a 3-year-old child could focus on the numbers of objects/incidents within a range of such small numbers that they can deal with. In everyday situations, the adults' aim was to notice the moments when a child paid attention to the number of objects or incidents. Focusing on the small numbers of everyday items embedded in everyday activities was given special emphasis. The question "How many of something?" in directing children's attention to the numerical aspects of sets and manipulation of numerosity in everyday material by games of giving, taking, making etc. was used to arouse children's interest in variation in numerosity. Counting of objects was in no way guided by the adults during the training. Instead, cardinal number words for 1-3 were regularly used, integrated in action.

The results show that it is possible to enhance children's SFON tendency by means of social interaction. There was an experimental effect on SFON tendency and subsequent development in cardinality-related skills from pre-test to delayed post-test in the children with some initial SFON tendency in the experimental group. The study also provided knowledge on how adults can create and promote moments of joint attention to the aspect of number, especially with those children who already have some SFON tendency. Further studies are needed to expand the findings to develop effective methods for arousing non-numerically focusing children's interest in number.

5. MAIN FINDINGS AND DISCUSSION

The aim of the present set of longitudinal studies was to explore 3-7-year-old children's Spontaneous FOcusing on Numerosity (SFON) and its relation to early mathematical development. The theoretical framework was based on, first, the specific attention-demanding nature of exact number recognition and utilization in natural surroundings, second, the theoretical models of subitizing- and counting-based exact number recognition processes, and, third, the models of development of number recognition and early mathematical skills during childhood years. Specifically, the role of child-initiated deliberate practice produced by a strong SFON tendency was theoretically linked to the current models of early mathematical development. Over the course of conducting the five studies, fifteen novel tasks were progressively developed for the SFON assessments. In these tasks, the aim was to control confounding effects of insufficient number recognition, verbal comprehension and other procedural skills as well as working memory capacity. Furthermore, it was empirically explored how children's individual differences in SFON are related to their development of exact number recognition, number sequence, subitizing-based enumeration, object counting, and basic arithmetic skills. The effect of social interaction on SFON tendency was tested.

Overall results across the five studies give support to the main hypotheses concerning the existence and stability of SFON, as well as its positive association with mathematical skills. Hence, it is concluded that within a child's existing mathematical competence, it is possible to distinguish a separate process which refers to the child's tendency to spontaneously focus on numerosity. There are significant individual differences in children's SFON at all measured ages. Moreover, stability was found in this tendency across different tasks assessed both at the same and at different ages. Furthermore, SFON tendency is positively related to the development of early mathematical skills with the correlations between SFON and different mathematical skills varying between .35 and .57 during the period from 3 to 7 years. As these results are based on both longitudinal and cross-sectional multi-method data of three different samples, and the results showed similar patterns, there is reasonable support for the validity of these main findings.

In addition to these general results, some specific questions on SFON were studied in individual studies. Alternative explanations for individual differences in SFON measures and covariance of SFON and mathematical skills were specifically explored in *Study III*. Partial correlations between SFON and object counting, SFON and number sequence elaboration, as well as SFON and basic arithmetic skills varied between .26 and .37 when the effect of comprehension of verbal instructions and nonverbal IQ was controlled. This showed that SFON accounted for significant amount of individual variance on mathematical skills. In all the studies comprising the present work, the specific methodological aim was to develop SFON tasks which would not exceed any participating child's procedural and numerical skills. Success in this was specifically tested in *Study III*. We explored whether all the children can actually carry out the SFON tasks by using guided focusing tasks. The results showed that children had particularly lacked focusing on the numerical aspect of the SFON tasks, not the quantifying means or other cognitive skills needed in the task. However, the results of the guided focusing tasks indicated the existence of a few children with special educational needs, children who might have difficulties in counting and retaining numbers in the SFON tasks. Thus, the hypothesis on SFON as a separate process of activities involving exact number recognition was supported by these findings. The quasi-experimental study indicated that SFON tendency is affected by social interaction (*Study V*). *Study III* showed that there were no gender effects in general SFON tendency, although in one SFON task the girls outperformed the boys. The studies on SFON suggest that individual differences in the amount and the quality of spontaneous focusing on numerosity might be of great importance for the children's early development of number concepts and skills.

5.1. Theoretical implications

The present set of studies has implications for our current theories on exact number recognition and children's early mathematical development. This work suggests that young children's early development, and especially their different developmental trajectories in exact number recognition and utilization cannot be sufficiently conceptualized in terms of earlier theories describing only the processes and skills which are used *after* a child has already focused attention on the numerical aspect of the task. The empirical part of this work shows convincingly that young 3–7 –year-old children have substantial individual differences in their self-initiated, spontaneous focusing numerosity in tasks in which their possible failure to consider exact numbers is not exhaustively explained by their inability to deal with the cognitive requirements of the tasks. It seems that this focusing process is a factor explaining some of the individual differences in children's mathematical development. It is proposed that number recognition and utilization processes need to be triggered in natural surroundings, and that when a child's tendency to spontaneously focus his or her attention on the aspect of number is very strong, this produces lots of practice in number recognition and utilization, and thus enhances the child's understanding of numerical aspects as affordances of sets. Accordingly, gradual proficiency in number recognition skills based on utilizing both subitizing- and counting-based procedures can enhance the child's tendency to focus on numerosity. In accordance with Sophian's (1998) views, during the childhood years, conceptual and procedural advances in enumeration enable and facilitate the child's SFON, which in turn leads to the child's own practice in utilizing numerosity in a variety of situations, which subsequently further develops enumeration skills.

Surprisingly, it seems that the earlier studies of exact number recognition in adults or in children have not theoretically or empirically regarded focusing on numerosity as an attentional sub-process of exact number recognition and utilization. The present study suggest that focusing on the aspect of exact number should be added to the necessary principles of exact quantifying (Gelman & Gallistel, 1978) describing how and why sets of items are enumerated. Focusing on the aspect of number requires determination of the set being perceived on some basis, and this is needed in exact cardinality determination based on both subitizing and counting in a natural environment. Not all possible subitizable or countable numbers of items in a natural setting can be brought to the conceptual, conscious levels of processing. Mechanisms of object individuation are mid-level processes, producing only prenumeric individuation information on the objects. Thus, it is hence proposed that an intentional process of focusing attention on the aspect of exact number in the set of items or incidents is needed for recognition of number on a conceptual level.

It triggers exact number recognition processes and utilization of the recognized exact number in action. The present set of studies shows that the suggestion of non-automaticity of exact number recognition when the number of items is utilized in action, is of relevance in the early mathematical development. In all but two of the 15 developed SFON tasks, only numbers of items in clearly subitizable set sizes were used. This may imply that at least in young children exact number recognition in subitizable set sizes is not an automatic process, but requires specific focusing on the aspect of number. An interesting question to be answered in future studies on both children and adults is how different the whole process of exact number recognition is in natural surroundings compared to the processes needed in traditionally used enumeration tasks involving readily predefined sets of very simple figures like dots. These differences might be significant for understanding the individual differences in early mathematical development taking place in natural surroundings.

A theoretically important issue of concern in the present study is a very essential one, but, at the same time, it is described by many researchers as a relatively neglected area in the research of early mathematical development (e.g. Ansari & Karmiloff-Smith, 2002; Baroody et al., in press; Ginsburg & Goldbeck, 2004). The factors producing individual differences both in typical and atypical mathematical development before school age have received markedly little attention in earlier studies. The focus on individual differences in children's spontaneous number recognition and utilization activities directs attention away from universal developmental patterns toward factors that explain variations in the degree to which children involve themselves more or less in number-related activities. The notion of SFON tendency emphasizes the significance of children's personal generalizing processes, or abstraction as Olsson and Lehtinen (1997) state. It especially takes seriously children's continuous generalization processes, which are guided by aspects of situations that the children find personally salient (Hannula & Lehtinen, 2004).

The present set of studies with child-specific measures of educationally relevant environments is a step toward answering the question of why children differ so much in mathematics development already at an early age. This question was recently raised by a twin study which controlled for genetic factors of children's mathematics performance (Oliver et al., 2004). The results of the study suggested that children, both twins and others, experience mathematics-relevant environments differently not only, as is broadly accepted, when they are reared in the same family and when they attend the same schools but even to some extent when they are taught in the same classroom by the same teacher (Oliver et al., 2004). In accordance with this conclusion, the results on substantial individual difference in SFON tendency suggest that it is by no means self-evident that, in natural social situations, all children would equally easily notice the number of objects, and utilize this information in action. This leads to considerable differences in children's practice with quantitative skills.

5.2. Practical implications

The present study has practical implications for young children's early education in mathematical skills and concepts. The notion of number as an affordance (Gibson & Spelke, 1983) brings the focusing on numerosity into the everyday context of children and adults. The aspect of number is one of the significant, useful features and affordances of the sets of items for many human actions. Mathematical thinking, including also utilizing exact number recognition is a significant part of perceiving and structuring the everyday surroundings, beginning already at a very early age. These skills form the basis for later number concepts. The substantial individual differences in SFON tendency during the childhood years suggest, firstly, that uncovering and modeling this kind of mathematical thinking embedded in everyday life could be an efficient tool for promoting young non-numerically focusing children's mathematical development and thus prevent later failures in school mathematics. Secondly, the role of sufficiently early diagnostics and interventions is critical. Special care should be taken of those children whose SFON tendency is very low. It would be essential for young children's educators to understand the relevance of children's learning to utilize even the earliest mathematical skills, and their learning to focus attention on mathematically meaningful aspects in a wide range of everyday activities, in addition to mathematics-specific preschool or kindergarten tasks. This is an important goal for in-service training of educators as there is evidence of how few mathematical experiences young children can be deliberately offered in day care. The observations of Tudge and Doucet (2004) showed that the amount of specific mathematical activities enhanced by parents can be larger at home than that of childcare professionals in day-care surroundings. Similarly, Mattinen's qualitative study (2005) on SFON enhancement includes detailed descriptions on adults' difficulties in noticing children's focusing on number, and especially promoting it. The parents and educators could acknowledge young children's mathematical skills more and systemically expand on children's mathematical thinking occurring as a part of children's activities. Some aspects of this suggestion have been already promisingly elaborated in several recent mathematical programs for young children (e.g., Baroody, 2004ab; Fuson, 2004; Greenes, Ginsburg, Balfanz, 2004; Griffin, 2004; Klein & Starkey, 2004; Seo & Ginsburg, 2004).

Mathematically meaningful focusing seems to be one of the specific forms of mathematical behavior in children whose mathematical skills develop optimally during childhood years. This aspect of everyday mathematical thinking should be included in children's learning materials at kindergarten and preschool levels. Students' difficulties in considering real world situations in school mathematics indicate that this aspect needs special attention also at a considerably later age (Verschaffel, Greer, & De Corte, 2000). Learning to focus on numerosity may be one of the significant steps on young children's way to learning to adopt mathematically meaningful perspectives on perceiving the world around them. Accordingly, a strong SFON tendency may be a significant factor in the creation of a rich, versatile, abstraction of variety of exact numerical meanings for number words. This kind of basis may form a numerical understanding which is well anchored in versatile, intentional experiences of exploring the world of numerosity.

This work has implications for the diagnostics of early mathematical skills. First, the findings emphasise the importance of regarding SFON as a separate, essential process also in the assessments of young children's mathematical skills. Many tasks in the current diagnostics of mathematical skills require both focusing on numerosity or numerically relevant aspects and using of mathematical skills. However, if the child does not interpret the task as mathematical, his or her performance may not reveal anything about his or her mathematical skills except for the lack of focusing on the mathematically relevant aspect. Thus, the conclusion concerning the child's skills can be misleading if the child's attention has not been clearly enough guided towards numerical aspects in the tasks. It is of educational relevance whether the child lacks the mathematical skills required or just does not focus on numerosity or numerically meaningful aspects in the task. Second, as the empirical work was successful in quantifying a child's general tendency to spontaneously focus on numerosity across different task contexts and social situations, the study suggests taking the next step towards developing standardised sets of SFON tasks for the use of practitioners. The series of SFON assessments developed in the present set of studies serves as a good starting point for such work.

5.3. Challenges for future studies

The present set of studies is the first to establish the phenomenon of SFON tendency both theoretically and methodologically, and hence it needs to be taken as an exploratory study calling for further replications with specifications and extensions. It remains to be seen whether further studies with larger samples especially in different countries and cultures, will yield similar results on individual differences in young children's SFON tendency, and whether the association between SFON and mathematical skills is as strong as it is in the Finnish samples.

In the present set of studies SFON was defined and operationalized as including both focusing on the aspect of exact number and utilization of the recognized number in action. Further conceptual elaboration might shed light on the factors affecting SFON tendency, and may also help in designing separate tasks for measuring focusing on and utilization of exact number.

On the basis of the significant indirect and direct associations between SFON and mathematical skills in *Study II* at the age of 3.5 to 6 years, the nature of the developmental relationships between SFON and number recognition skills is preliminarily suggested to be reciprocal, but this proposal needs to be validated in future longitudinal studies with experimental designs. During the targeted period of time, children's many mathematical skills and number concepts develop significantly. It is plausible that in future studies, more detailed questions on the different relationships between SFON and different early mathematical skills can be answered. In *Study V*, which aimed to study the effect of social interaction on SFON tendency, we were not able to separate training of number recognition skills sufficiently from SFON enhancement to be able to draw firm conclusions on the direction of the causality between SFON and number recognition skills. Regarding to the question of developmental priority of SFON and countig skills there is already some preliminary evidence that 2-year-old children can spontaneously focus their attention on exact numerosity and utilize it before they start to learn to determine the cardinality of the set by verbal counting (Takila, Hannula, & Pipari-Study Group, 2005), and young children can recognize and utilize small numbers before they can count (Mix et al., 2002; Wynn, 1990; 1992b), but the nature of the relationship between former (subitizing-based) number recognition skill and SFON has not yet been studied at all. It might well be that along with children's differences in SFON tendency, there might also be developmentally significant individual differences in their awareness of number words and symbols, as well as in looking for numerical patterns or relations in their everyday contexts. These broader aspects of children's spontaneous numerical orientation could be studied in the future.

The variability and only moderate level of stability in SFON scores across different task contexts and time calls for further work on constructing the SFON assessments. Our aim was to get a reliable indicator of a child's general SFON tendency across different task contexts. We used at most three different SFON tasks at one measurement point, but the validity of SFON assessments could be increased by using more SFON tasks. The aim of the indicator obtained is to capture a child's amount of focusing on numerosity, and thus the amount of practice acquired in utilization of enumeration skills in his or her surroundings. To enable the measuring of children's spontaneous behavior only novel tasks or at least tasks with novel materials and/or contexts can be used each time. This requirement compromises the comparability of SFON measures and it caused problems in interpreting the results on SFON measured at different ages, so that the development of SFON tendency as such could not be studied in the present set of studies.

Although the development of SFON tendency was not explored in the set of original studies, during designing and piloting the SFON tasks, in the course of the studies, it became evident that children, when they get older, learn to focus on number in more demanding tasks, in which other more attractive aspects and goals can be present within the task context. For this reason, in *Study IV*, we used the average of the two SFON scores measured at the age of 4 and 5 years to obtain a reliable indicator of children's spontaneous practice generalized across time and different task contexts. It is possible that in the longitudinal *Study II* the instability shown by the path models, between SFON at the age of 6 and the previous SFON measures at the ages of 4 and 5 years indicates differences in the SFON measures, or possibly different utilizing processes for enumeration in subitizable and countable set sizes. There is currently considerable agreement in the literature about different cognitive processes for subitizing- and counting-based number recognition (e.g., Dehaene & Laurent, 1994; Sathian et al., 1999). It remains be seen in future studies, whether focusing on numerosity differs in the subitizable and countable set sizes. However, this will be a challenging task, since children's large individual differences in number recognition range, their verbal comprehension skills, and working memory capacity need to be taken into account when planning SFON tasks. Large individual differences prevented the use of SFON tasks, which include numbers of items larger than two before the age of 6 years.

In the current work, it is proposed that SFON tendency indicates the amount of a child's spontaneous practice in utilizing exact number recognition in natural surroundings. In order to receive a reliable indicator for a child's general SFON tendency in everyday situations, several SFON measures within different contexts are needed. However, the developed SFON assessments measured SFON in a limited number of situations. Structured observations of children's SFON in everyday surroundings in comparison with their performance on SFON tasks would show whether these two are actually related. The first indication of the actual association between observed and assessed SFON performances ($r = .55$) was accomplished in Study V, in which kindergarten personnel observed children's SFON manifestations in day-care surroundings, and these data were compared to children's SFON scores obtained in SFON tasks. These results need to be considered with caution, however, due to the very small number of children (n = 16) and the fact that the observations were made by practitioners who were taking care of the whole group of children at the same time as they were observing and guiding some children's SFON.

Observational studies on young children's mathematical activities by Ginsburg, Balfanz, and Greenes (2000) and Seo and Ginsburg (2004) have indicated that mathematical-seeming behavior appears in children's free play, although individual differences, or the real intentions of the playing children have remained unclear. The structured SFON tasks reveal young children's focusing on numerosity better than observations of free activity, since the probability of producing exactly accurate numbers in SFON tasks by accident is very low. The observational data on the specific signs of the child's considering numerosity or quantitative goals reveal SFON even when the child makes mistakes in enumeration.

Clearly, there are other factors than those explored in the current set of studies that may affect children's mathematical development and their tendency to focus on numerosity. As indicated by the findings of *Study V*, social interaction can affect SFON tendency. More broadly, other studies have shown that children's better mathematical skills are related to parents' skills in modeling mathematical thinking in everyday contexts, and in contexts involving the usual preschool material (Andersson, 2001, Blevins-Knabe & Musun-Miller, 1996; Huntsinger et al., 2000; Linnell, & Fluck, 2001; Saxe et al., 1986; Tudge & Doucet, 2004). These studies have described children's mathematical learning when a parent or an adult is present in the task situations. In the present set of studies on SFON tendency, the aim has been to obtain an indicator for a child's general tendency of self-initiated numerically meaningful practice in all situations and surroundings, not only those involving parents, or other educators. Further studies are needed to find effective ways of producing SFON enhancement in children with a very low SFON tendency. These explorations may shed light on the nature of the relationship between SFON and number recognition skills involving the essential concept of set of individuals (Spelke, 2003) at the early phases.

An interesting question which has not yet been empirically tested concerns the roles of working memory (executive component of working memory, Baddeley, 1986) and inhibitory processes (Houde, 2000) in focusing on numerosity related to early mathematical development. Focusing on a certain aspect requires simultaneous inhibition of the processing of other aspects. Inhibition has been shown to be related to working memory capacity and mathematical skills in 6-9 –year-olds (Swanson & Beebe-Frankenberger, 2004). Accordingly both domain-specific and domain-general working memory systems have been shown to mediate deficits in mathematics (Wilson & Swanson, 2001). The guided focusing tasks showed, however, that once children have the goal of quantification in their mind, they can focus their attention on numbers of items. This finding indicates that the lack of sufficient inhibitory processes or working memory capacity cannot be exclusive contributors to variance in SFON tasks, but they can still play a role in learning to focus on the aspect of number.

In future studies on SFON, the number-specificity of SFON tendency should be tested by controlling for children's general tendency to focus on and utilize non-numerical aspects of sets. Furthermore, a promising future approach could be multidiciplinary intervention studies using both behavioral measures and brain imaging methods. This approach could provide us a better understanding of developmental differences in numeracy, especially of specific processes of number recognition and focusing on numerosity.

6. REFERENCES

Aksan, N., & Kochanska, G. (2004). Links between systems of inhibition from infancy to preschool years. *Child Development, 75,* 1477-1490.

Anderson, A. (1997). Families and mathematics: a study of parent-child interactions. *Journal for Research in Mathematics Education, 28*, 484-511.

Ansari, D., & Karmiloff-Smith, A. (2002). Atypical trajectories of number development: a neuroconstructivist perspective. *Trends in Cognitive Sciences, 6,* 511-516.

Antell, S. E., & Keating, D. P. (1983). *Perception of numerical invariance in neonates*, 54, 695-701.

Aunio, P., Ee, J., Lim, S. E. A., Hautamäki, J., & Van Luit, J. E. H. (2004). Young children's number sense in Finland, Hong Kong and Singapore. *International Journal of Early Years Eduation, 12,* 195-216.

Aunola, K., Leskinen, E., Lerkkanen, M-L., & Nurmi, J-E. (2004). Developmental dynamics of math performance from pre-school to grade 2. *Journal of Educational Psychology, 96,* 699-713.

Backen Jones, L., Rothbart, M., & Posner, M. (2003). Development of executive attention in preschool children. *Developmental Science, 6,* 498-504.

Baddley, A. D. (1986). *Working memory*. London: Oxford University Press.

Baron, R. M., & Kenny, D. A. (1986). The moderator-mediator variable distinction in social psychological research: Conceptual, strategic, and statistical considerations. *Journal of Personality and Social Psychology, 51,* 1173-1182.

Baroody, A. J. (1984). Children's difficulties in subtraction: Some causes and questions. *Journal of Research in Mathematics Education, 15,* 203-213.

Baroody, A. J. (2004a). The role of psychological research in the development of early childhood mathematics standards. In D. H. Clements and J. Sarama, (Eds.). *Engaging young children in mathematics. Standards for early childhood mathematics education* (pp. 149-172). Mahwah, NJ: Lawrence Erlbaum.

Baroody, A. J. (2004b). The developmental bases for early childhood number and operations standards. In D. H. Clements and J. Sarama, (Eds.). *Engaging young children in mathematics. Standards for early childhood mathematics education* (pp. 173-220). Mahwah, NJ: Lawrence Erlbaum.

Baroody, A. J., Lai, M-L., & Mix, K. S. (in press). The development of young children's early number and operation sense and its implications for early childhood education. In O. Saracho & B. Spodek (Eds.). *Handbook of Research on the Education of Young Children* (2nd ed.). Mahwah, NJ: Erlbaum.

Bermejo, V. (1996). Cardinality development and counting. *Developmental Psychology, 32,* 263-268.

Bermejo, V., Morales, S., & deOsuna, J. G. (2004). Supporting children's development of cardinality understanding. *Learning and Instruction, 14,* 381-398.

Bialystok, E., & Codd, J. (1997). Cardinal limits: Evidence from language awareness and bilingualism for developing concepts of number. *Cognitive Development, 12,* 85-106.

Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development and Parenting, 5,* 35-45.

Briars, D., & Siegler, R. S. (1984). A featural analysis of preschoolers' counting knowledge. *Developmental Psychology, 20,* 607-618.

Camos, V., & Barrouillet, P. (2004). Adult counting is resource demanding. *British Journal of Psychology, 95,* 19-30.

Carey, S. (2002). Evidence for numerical abilities in young infants: A fatal flaw?: Comment. *Developmental Science, 5,* 202-205.

Carey, S., & Xu, F. (2001). Infants' knowledge of objects: beyond object files and object tracking. *Cognition, 80,* 179-213.

Chi, M. T. H., & Klahr, D. (1975). Span and rate of apprehension in children and adults. *Journal of Experimental Child Psychology, 3,* 93-102.

Cohen, L. B., & Marks, K. S. (2002). How infants process addition ad subtraction events. *Developmental Science, 5,* 186-212.

Cowan, N. (2001). The magical number 4 in short-term memory: A reconsideration of mental storage capacity. *Behavioral and Brain Sciences, 24,* 87-185.

Cowan, R., Dowker, A., Christakis, A., & Bailey, S. (1996). Even more precisely assessing children's understanding of order-irrelevance principle. *Journal of Experimental Child Psychology, 62,* 84-101.

Dehaene, S. (1997). *The number sense: How the mind creates mathematics.* New York: Oxford University Press.

Dehaene, S., & Laurent, D. (1994). Dissociable mechanisms of subitizing and counting: Neuropsychological evidence from simultanagnostic patients. *Journal of Experimental Psychology: Human Perception and Performance, 20,* 958-975.

Design-Based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher, 32*(1), 5-8.

Donaldson, M. (1978). *Children's minds*. Glasgow, UK: Fontana/Collins.

Enns, J. T. (1992). The nature of selectivity in early human vision. In B. Burns (Ed.), *Percepts, concepts and categories* (pp. 39-74). Amsterdam: Elsevier.

Ericsson, K. A., & Lehman, A. C. (1996). Expert and exceptional performance: Evidence of maximal adaptation to task constraints. *Annual Review of Psychology, 47*, 273-305.

Feigenson, L., & Carey, S. (2003). Tracking individuals via object-files: evidence from infants' manual search. *Developmental Science, 6,* 568-584.

Feigenson, L., Dehaene, S., & Spelke, E. (in press). Core systems of number. *Trends in Cognitive Sciences.*

Field, A. (2000). *Discovering statistics. Using SPSS for Windows.* Wiltshire: Cromwell Press.

Fink, G. R., Marshall, J. C., Gurd, J., Weiss, P.H., Zafiris, O., Shah, N.J., & Zilles, K. (2000). Deriving numerosity and shape from identical visual displays. *NeuroImage, 12,* 1-10.

Freeman, N. H., Antonucci, C., & Lewis, C. (2000). Representation of the cardinality principle: early conception of error in a counterfactual test. *Cognition, 74,* 71-89.

Fuson, K. (1988). *Children's counting and concepts of number*. New York: Springer Verlag.

Fuson, K. (2004). Pre-K to grade 2 goals and standards: Achieving 21st-century mastery for all. In D. H. Clements and J. Sarama, (Eds.). *Engaging young children in mathematics. Standards for early childhood mathematics education* (pp. 105-148). Mahwah, NJ: Lawrence Erlbaum.

Fuson, K. C., Pergament, G. G., Lyons, B.G., & Hall, J. W. (1985) Children's conformity to the cardinality rule as a function of set size and counting accuracy. *Child Development, 56,* 1429-1436.

Fuson, K., Richards, J., & Briars, D. J. (1982). The acquisition and elaboration of the number word sequence. In C. J. Brainerd (Ed.), *Children's logical and mathematical cognition. Progress in cognitive development research* (pp. 33-92). New York: Springer-Verlag.

Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition, 44,* 43-74.

Gauvain, M. (2001). *The social context of cognitive development.* New York: Guilford Press.

Geary, D. C. (2000). From infancy to adulthood: the development of numerical abilities. *European Child & Adolescent Psychiatry, 9,* II, 11-16.

Gelman, R. (1990). First principles organize attention to and learning about relevant data: number and the animate-inanimate distinction as examples. *Cognitive Science, 14,* 79-106.

Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number.* Cambridge, MA: Harvard University Press.

Gelman, R., & Gallistel, C. R. (2004). Language and origin of numerical concepts. *Science, 306,* 441-442.

Gibson, E. J. (1988). Exploratory behaviour in the development of perceiving, acting, and the acquiring of knowledge. *Annual Review of Psychology, 39,* 1-41.

Gibson, E. J., & Pick, A. D. (2000). *An ecological approach to perceptual learning and development.* New York: Oxford University Press.

Gibson, E. J., & Spelke, E. (1983). The development of perception. In J. H. Flavell & E. M. Markman (Eds.), *Handbook of child psychology: Vol 3. Cognitive development* (pp. 1-76). New York: Wiley.

Ginsburg, H. P., Balfanz, R., & Greenes, C. (2000). Challenging mathematics for young children. In A. L. Costa (Ed.), *Teaching for Intelligence II: A collection of articles* (pp. 245-258). Illinois: Arlington Heights.

Ginsburg, H. P., & Goldbeck, S. L. (2004). Thoughts on the future of research on mathematics and science learning and education. *Early Childhood Research Quarterly, 19,* 190-200.

Ginsburg, H. P., Inoue, N., & Seo, K.-H. (1999). Preschoolers doing mathematics: Observations of everyday activities. In J. Copley (Ed.), *Mathematics in the early years* (pp. 88-99). Reston, VA: National Council of Teachers of Mathematics.

Ginsburg, H. P., & Seo, K-H. (1999). Mathematics in children's thinking. *Mathematical Thinking and Learning, 1,* 113-129.

Gleissner, B., Meltzoff, A. N., & Bekkering, H. (2000). Children's coding of human action: cognitive factors influencing imitation in 3-year-olds. *Developmental Science, 3,* 405-414.

Goldin-Meadow, S., Alibali, M. W., & Church, R. B. (1993). Transitions in concept acquisition: Using the hand to read the mind. *Psychological Review, 100,* 279-297.

Gordon, P. (2004). Numerical cognition without words: Evidence from Amazonia. *Science, 306,* 496-499.

Green, C. S, & Bavalier, D. (2003). Action video game modifies visual selective attention. *Nature, 423,* 534-537.

Greenes, C., Ginsburg, H. P., & Balfanz, R. (2004). Big Math for Little Kids. *Early Childhood Research Quarterly, 19,* 159-166.

Griffin, S. (2004). Number worlds: A research-based mathematics program for young children. In D. H. Clements and J. Sarama, (Eds.). *Engaging young children in mathematics. Standards for early childhood mathematics education* (pp. 325-342). Mahwah, NJ: Lawrence Erlbaum.

Hanich, L. B., Jordan, N. C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematical cognition in children with learning difficulties. *Journal of Educational Psychology, 93,* 615-626.

Hannula, M. M., & Lehtinen, E. (2001). Spontaneous tendency to focus on numerosities in the development of cardinality. In M. Panhuizen-Van Heuvel (Ed.), *Proceedings of 25th Conference of the International Group for the Psychology of Mathematics Education, 3,* 113-120. Drukkerij Wilco, The Netherlands: Amersfoort.

Hannula, M. M., & Lehtinen, E. (2004, April). Relationships between spontaneous focusing on numerosity and early mathematical skills. In J. Lobato (Chair), *Attentional Processes, Salience, and "Transfer" of Learning: Perspectives from Neuroscience, Cognitive Science, and Mathematics Education.* Symposium conducted at the Annual Meeting of the American Educational Research Association, San Diego, CA.

Hannula, M. M., & Lehtinen, E. (in press). Spontaneous focusing on numerosity and mathematical skills in young children. *Learning and Instruction.*

Hannula, M. M., Mattinen, A., & Lehtinen, E. (in press). Does social interaction influence 3-year-old children's tendency to focus on numerosity? A quasi-experimental study in day-care. In L. Verschaffel, E. De Corte, G. Kanselaar, & M. Valcke (Eds.), *Learning environments to promote deep conceptual and strategic learning.* Studia Paedagogica. Leuven: Leuven University Press.

Hannula, M. M., Räsänen, P., & Lehtinen, E. (2005). *Development of counting skills: Role of spontaneous focusing on numerosity and subitizing-based enumeration.* Manuscript submitted for publication.

Hauser, M. D., & Spelke E. S. (in press). Evolutionary and developmental foundations of human knowledge: A case study of mathematics. In M. Gazzaniga (Ed.), *The Cognitive Neurosciences, Vol. 3.* Cambridge: MIT Press.

Hiltunen, T. (2003, August). *Teacher-pupil classroom interaction & early numeracy. Comparative case study of 6-year-old children in 3 European countries, schools and preschools.* Poster presented at the 10th Biennial Conference of the European Association for Learning and Instruction, Padova, Italy.

Houde, O. (2000). Inhibition and cognitive development: object, number, categorization, and reasoning. *Cognitive Development, 15,* 63-73.

Hu, L., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modelling, 6,* 1-55.

Huntsinger, C. S., Jose, P. E., Larson, S. L., Krieg, D. B., & Shaligram, C. (2000). Mathematics, vocabulary, and reading development in Chinese American and European American children over the primary school years. *Journal of Educational Psychology, 92,* 745-760.

Jensen, E. M., Reese, E. P., & Reese, T. W. (1950). The subitizing and counting of visually presented fields of dots. *The Journal of Psychology, 30,* 363-392.

Jevons, S. (1871). The power of numerical discrimination. *Nature, 3,* 281-282.

Jöreskog, K. G., & Sörbom, D. (2001). *LISREL 8.51.* Chicago: Scientific Software International.

Karmiloff-Smith, A. (1995). *Beyond modularity. A developmental perspective on cognitive science* (2nd ed.). London: The MIT Press.

Kaufman, E.L., Lord, M.W., Reese, T. W., & Volkmann, J. (1949). The discrimination of visual number. *Journal of American Psychology, 62,* 498-525.

Keranto, T. (1981). *Lukukäsitteen kehittyminen ja kehittäminen: Matemaattis-loogiset perusteet ja luvun kognitiivinen rakentuminen.* [The acquisition and aided development of number concept: Mathematical-logical principles and cognitive structuring of number]. Acta Universitatis Tamperensis, Ser. A, Vol. 125.

Kinnunen, R., Lehtinen, E., & Vauras, M. (1994). Matemaattisen taidon arviointi [Assessment of mathematical skills]. In M. Vauras, E. Poskiparta, & P. Niemi (Eds.). *Kognitiivisten taitojen arviointi koulutulokkailla ja 1. luokan oppilailla* (pp. 55-76). University of Turku, Finland, Center for Learning Research.

Klahr, D. (1973). Quantification processes. In W. G. Chase (Ed.), *Visual information processing* (pp. 3-34). San Diego, CA: Academic Press.

Klein, A., & Starkey, P. (2004). Fostering preschool children's mathematical knowledge: Findings from the Berkeley Math Readiness Project. In D. H. Clements and J. Sarama, (Eds.). *Engaging young children in mathematics. Standards for early childhood mathematics education* (pp. 343-360). Mahwah, NJ: Lawrence Erlbaum.

Kochanska, G., Murray, K., & Coy, K. C. (1997). Inhibitory control as a contributor to conscience in childhood: From toddler to early school age. *Child Development, 68,* 268-277.

Lehtinen, E., & Kinnunen, R. (1993). Matemaattisista oppimisvaikeuksista. [About mathematical learning disabilities]. In M. Vauras (Ed.), *Oppimisvaikeudet ja opetuksen kehittäminen: Katsaus Turun yliopiston Oppimistutkimuksen keskuksen toimintaan ja tutkimukseen* (pp. 37-56). Acta Psychologica Fennica. Soveltavan psykologian monografioita 6. Rauma: Kirjapaino Oy West Point.

Lemer, C., Dehaene, S., Spelke, E., Cohen, L. (2003). Approximate quantities and exact number words: dissociable systems. *Neuropsychologia, 41,* 1942-1958.

Levine, S. C., Jordan, N. C., & Huttenlocher, J. (1992). Development of calculation abilities in young children. *Journal of Experimental Child Psychology, 53,* 72-103.

Lindahl, M., & Samuelsson, I. P. (2002). Imitation and variation: reflections on toddlers' strategies for learning. *Scandinavian Journal of Educational Research, 46,* 25-45.

Linnell, M., & Fluck, M. (2001). The effect of maternal support for counting and cardinal understanding in pre-school children. *Social Development, 10,* 202-220.

Lobato, J. (2003). How design experiments can inform a rethinking of transfer and vice versa. *Educational Researcher, 32,* (1), 17-20

Lobato, J., Ellis, A. B., & Munoz, R. (2003). How "focusing phenomena" in the instructional environment support individual students' generalizations. *Mathematical Thinking and Learning, 5,* 1-36.

Mandler, G., & Shebo, B. J. (1982). Subitizing: An analysis of its component processes. *Journal of Experimental Psychology: General, 111,* 1-22.

Marton, F., & Booth, S. (1997). *Learning and awareness*. Mahwah, NJ: Erlbaum.

Mattinen, A. (2005). *Vaikuttaminen lukumäärien havaitsemiseen päiväkodissa.* [Influencing children's focusing on numerosity in day care]. Manuscript in preparation. University of Turku, Finland.

McCrink, K, & Wynn, K. (2004). Large-number addition and subtraction by 9-month-old infants. *Psychological Science, 15,* 776-781.

Meck, W. H. & Church, R. M. (1983). A mode control model of counting and timing processes. *Journal of Experimental Psychology: Animal Behavior Processes, 9,* 320-334.

Miller, K., & Stigler, J. W. (1987). Counting in Chinese: Cultural variation in a basic cognitive skill. *Cognitive Development, 2,* 279-305.

Mix, K. S. (2002). The construction of number concepts. *Cognitive Development, 17,* 1345-1363.

Mix, K. S., Huttenlocher, J., & Levine, S. C. (2002). *Math without words: Quantitative development in infancy and early childhood.* New York: Oxford University Press.

Nieder, A., Freedman, D. J., & E. K. Miller. (2002). Representation of the quantity of visual items in the primate prefrontal cortex. *Science, 297,* 1708-1711.

Nunes, T., & Bryant, P. (1996). *Children doing mathematics*. Cornwall, Great Britain: T.J. Press.

Ohlsson, S., & Lehtinen, E. (1997). Abstraction and the acquisition of complex ideas. *International Journal of Educational Research, 27,* 1-88.

Oliver, B., Harlaar, N., Hayiou1 Thomas, M. E., Kovas, Y., Walker, S. O., Petrill, S., Spinath, F. M., Dale, P. S., & Plomin, R. (2004). A twin study of teacher-reported mathematics performance and low performance in 7-year-olds. *Journal of Educational Psychology, 96,* 504–517.

Peterson, S. J., & Simon, T. J. (2000). Computational evidence for the subitizing phenomenon as an emergent property of the human cognitive architecture. *Cognitive Science, 24,* 93-122.

Piaget, J. (1965). *The child's conception of number*. New York: Norton.

Piazza, M., Giacomini, E., Le Bihan, D., & Dehaene, S. (2003). Single-trial classification of parallel preattentive and serial attentive processes using functional magnetic resonance imaging. *Proceedings of Royal Society of London, 270,* 1237-1245.

Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science, 306,* 499-503.

Pick, A. D., & Frankel, G. W. (1974). A developmental study of strategies of visual selectivity. *Child Development, 45,* 1162-1165.

Porter, J. (1998). The understanding of counting in children with severe learning difficulties and nursery children. *British Journal of Educational Psychology, 68*, 331-345.

Plude, D. J., Enns, J. T., Brodeur, D. (1994). The development of selective attention: A life-span overview. *Acta Psychologica, 86,* 227-272.

Pylyshyn, Z. (1999). Is vision continuous with cognition? The case for cognitive impenetrability of visual perception. *Behavioral and Brain Sciences, 22,* 341-423.

Resnick, L. B., & Greeno, J. G. (1990). *Conceptual growth of number and quantity.* Unpublished manuscript.

Rittle-Johnson, B., & Siegler, R. (1998). The relation between conceptual and procedural knowledge in learning mathematics: a review. In C. Donlan (Ed.) *The development of mathematical skills* (pp. 75-110). Hove: Erlbaum.

Rothbart, M. K., Posner, M.I., & Boylan, A. (1990). Regulatory mechanisms in infant development. In James, T. Enns, (Ed.). *The development of attention. Research and theory* (pp. 47-66). Amsterdam: Elsevier.

Ruff, H. A., & Rothbart, M. K. (1996). *Attention in early development: Themes and variations.* New York: Oxford University Press.

Salonen, P., Lepola, J., Vauras, M., Rauhanummi, T., Lehtinen, E., & Kinnunen, R. (1994). *Diagnostiset testit 3. Motivaatio, metakognitio ja matematiikka* [Diagnostic tests 3. Motivation, metacognition and mathematics]. University of Turku, Finland, Centre for Learning Research.

Sarnecka, B. W., & Gelman, S. A. (2004). Six does not just mean a lot: preschoolers see number words as specific. *Cognition, 92,* 329-352.

Sathian, K., Simon, T. J., Peterson, S., Patel, G. A., Hoffman, J. M., & Grafton, S. T. (1999). Neural evidence linking visual object enumeration and attention. *Journal of Cognitive Neuroscience, 11*, 36-51.

Saxe, G. B., Guberman, S. R., & Gearhart, M. (1987). Social processes in early number development. *Monographs of the Society for Research in Child Development, 52*, Serial No. 216.

Schaffer, B., Eggleston, V. H., Scott, J. L. (1974). Number development in young children. *Cognitive Psychology, 6,* 357-379.

Scholl, B. J. (2001). Objects and attention: the state of the art. *Cognition, 80,* 1-46.

Scholl, B. J., & Pylyshyn, Z. W. (1999). Tracking multiple items trough occlusion: clues to visual objecthood. *Cognitive Psychology, 38,* 259-290.

Schumacker, R. E., & Lomax, R. G. (1996). *Beginner's guide to structural equation modelling.* Mahwah, NJ: Lawrence Erlbaum.

Seo, K-H., & Ginsburg, H. P. (2004). What is developmentally appropriate in early childhood mathematics education? Lessons from new research. In D. H. Clements and J. Sarama, (Eds.). *Engaging young children in mathematics. Standards for early childhood mathematics education* (pp. 91-104). Mahwah, NJ: Lawrence Erlbaum.

Simon, T. J. (1997). Reconceptualizing the origins of number knowledge: A "non-numerical" account. *Cognitive Development, 12,* 349-372.

Skwarchuk, S-L., & Anglin, J. M. (2002). Children's acquisition of the English cardinal number words: a special case of vocabulary development. *Journal of Educational Psychology, 94,* 107-125.

Sobel, M. E. (1982). Asymptotic confidence intervals for indirect effects in structural equation models. In S. Leinhart (Ed.), *Sociological methodology 1982* (pp. 290-312). San Francisco: Jossey-Bass.

Sophian, C. (1987). Early developments in children's use of counting to solve quantitative problems. *Cognition and Instruction, 4,* 61-90.

Sophian, C. (1988). Limitations on preschool children's knowledge about counting: Using counting to compare sets. *Developmental Psychology, 24,* 634-640.

Sophian, C. (1995). Representation and reasoning in early numerical development: Counting, conservation, and comparisons between sets. *Child Development, 66,* 559-577.

Sophian, C. (1998). A developmental perspective on children's counting. In Chris Donlan (Ed.), *The development of mathematical skills* (pp. 27-41). Hove, UK: Taylor & Francis.

Sophian, C., & Kailihiwa, C. (1998). Units of counting: developmental changes. *Cognitive Development, 13,* 561-585.

Sophian, C., Wood, A. M., Vong, K. I. (1995). Making numbers count: The early development of numerical inferences. *Developmental Psychology, 31,* 263-273.

Spelke, E. (2003). What makes us smart? Core knowledge and natural language. In D. Gentner and S. Goldin-Meadow (Eds.), *Language in mind.* Cambridge, MA: MIT Press.

Spelke, E., & Tsivkin, S. (2001). Language and number: A bilingual training study. *Cognition, 78,* 45-88.

Starkey, P., & Cooper, R. G. (1995). The development of subitizing in young children. *British Journal of Developmental Psychology, 13,* 399-420.

Statistics Finland. (1997). *The educational level of Finns aged 15 years and over by age and gender in 1997.* Retrieved December 1st, 2004, from http://www.stat.fi/tk/he/edufinland/edut.html

Swanson, H. L., & Beebe-Frankenberger, M. (2004). The relationship between working memory and mathematical problem solving in children at risk and not at risk for serious math difficulties. *Journal of Educational Psychology, 96,* 471-491.

Takila, P., Hannula, M. M., & Pipari-Study Group (2005, April). *Spontaneous focusing on numerosity and premathematical skills of premature and full term infants at the age of two years.* Poster presented at the Biennial Meeting of the Society for Research on Child Development, Atlanta, GA.

Tomasello, M., Carpenter, M., Call, J., Behne, T., & Moll, H. (in press). Understanding and sharing intentions: The origins of cultural cognition. *Behavioral and Brain Sciences.*

Torbeyns, J., van den Noortgate, W., Ghesquière, P., Verschaffel, L., van den Rijt, B. A. M., van Luit, J. E. H. (2002). Development of early numeray in 5- to 7-year-old children: A comparison between Flanders and The Netherlands. *Educational Research and Evaluation, 8,* 249-275.

Trick, L., Enns, J. T., & Brodeur, D. A. (1996). Life span changes in visual enumeration: The number discrimination task. *Developmental Psychology, 32,* 925-932.

Trick, L., & Pylyshyn, Z. W. (1994). Why are small and large numbers enumerated differently? A limited-capacity preattentive stage in vision. *Psychological Review, 101,* 80-102.

Tudge, J. R. H., & Doucet, F. (2004). Early mathematical experiences: observing young Black and White children's everyday activities. *Early Childhood Research Quarterly, 19,* 21-39.

Tuholski, S. W., Engle, R., W., & Baylis, G. C. (2001). Individual differences in working memory capacity and enumeration. *Memory & Cognition, 29,* 484-492.

Uller, C., Carey, S., Huntley-Fenner, G., & Klatt, L. (1999). What representations might underlie infant numerical knowledge? *Cognitive Development, 14,* 1-43.
Ullman, S. (1984). Visual rooutines. *Cognition, 18,* 97-159.
Van de Rijt, B. A. M., Godfrey, R., Aubrey, C., Van Luit, J. E. H., Ghesquière, P., Torbeyns, J., Hasemann, K., Tancig, S., Kavkler, M., Magajna, L., & Tzouriadou, M. (2003). The development of early numeracy in Europe. *Journal of Early Childhood Research, 1,* 155-180.
Van de Rijt, B. A. M., & Van Luit, J. E. H. (1999). Milestones in the development of infant numeracy. *Scandinavian Journal of Psychology, 40*, 65-71.
Van Luit, J. E. H., Van de Rijt, B. A. M., & Pennings, A. H. (1994). *Utrechtse Galalbegripp Toets*, UGT [Utrecht Numeracy Test]. Doetinchem: Graviant.
Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems.* Lisse, The Netherlands: Swets & Zeitlinger.
Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological thought processes.* M. Cole, V. John-Steiner, S. Scriber, & E. Souberman (Eds.), Cambridge, MA: Harvard University Press.
Wender, F. K., & Rothkegel, R. (2000). Subitizing and its subprocesses. *Psychological Research, 64,* 81-92.
West, S. G., Finch, J. F., & Curran, P. J. (1995). Structural equation models with nonnormal variables: Problems and remedies. In R. H. Hoyle (Ed.), *Structural Equation Modeling. Concepts, issues, and applications* (pp. 56-75). London: Sage.
Wiese, H. (2003). Iconic and non-iconic stages in number development: the role of language. *Trends in Cognitive Sciences, 7,* 385-390.
Wilson, K. W., & Swanson, H. L. (2001). Are mathematics disabilities due to domain-general or domain-specific working memory deficits? *Journal of Learning Disability, 34,* 237-248.
Wolters, G., van Kempen, H., & Wijlhuizen, G.-J. (1987). Quantification of small numbers of dots: Subitizing or pattern recognition? *American Journal of Psychology, 100,* 225-237.
Wynn, K. (1990). Children's understanding of counting. *Cognition, 36*, 155-193.
Wynn, K. (1992a). Addition and subtraction by human infants. *Nature, 358*, 749-750.
Wynn, K. (1992b). Children's acquisition of the number words and the counting system. *Cognitive Psychology, 24,* 220-251.
Wynn, K. (1992c). Evidence against empiristic accounts of te origins of numerical knowledge. *Mind & Language, 7,* 315-332.
Wynn, K. (1998). Numerical competence in infants. In Cris Donlan (Ed.). *The development of mathematical skills* (pp. 3-25). East Sussex, UK: Psychology Press.
Wynn, K., & Chiang, W-C. (1998). Limits to infants' knowledge of objects: The case of magical appearance. *Psychological Science, 9,* 448-455.
Xu, F., Spelke, E. S., & Goddard, S. (in press). Number sense in human infants. *Developmental Science.*
Yen, M., & Lo, L-H. (2002). Examining test-retest reliability. An intra-class correlation approach. *Nursing Research, 51,* 59-62.
Zur, O., & Gelman, R. (2004). Young children can add and subtract by predicting and checking. *Early Childhood Research Quarterly, 19,* 121-137.