Reflections on the Significance of Misrepresenting Preferences

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Abstract. This paper deals with the concept of manipulation, understood as preference misrepresentation, in the light of the main theoretical results focusing on their practical significance. It also reviews some indices measuring the degree of manipulability of choice functions. Moreover, the results on complexity of manipulation as well as on safe manipulability are briefly touched upon.

Keywords: preference misrepresentation, degrees of manipulability, safe manipulability, Gibbard-Satterthwaite theorem, Campbell-Kelly theorem

1 Introduction: The Concept of Manipulation

Ever since the publication of Farquharson's seminal work in late 1960's the concept of manipulation has played an important role in the social choice and voting theory [8]. Stemming from the Latin word 'manipulus' (handful, bundle or, as a military term, maneuverable formation) it refers to 'handling or using, esp with some skill, in a process or action: to manipulate a pair of scissors' (Collins English Dictionary). It also denotes 'falsification for one's own advantage'. It is in the latter meaning that 'manipulation' is being used in the social choice theory. In short, it refers to activity whereby an individual or group gives an incorrect report on its preferences in order to change the voting outcome to his/her (hereafter his) or its advantage. It is quite common to speak about manipulation just in those cases where the intended result is achieved, i.e. when the falsification succeeds in bringing about an improvement in the outcome reached. Let us now make this concept a bit more precise.

Let X be the set of alternatives, N the set of n voters, $\mathcal{R}$ the set of n-person preference profiles over X and $F : \mathcal{R} \times A \rightarrow 2^A$, for any $A \subseteq X$, a social choice

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function. I.e. F associates with any subset A of X and preference profile over it, a subset of A called the winners or social choice set. A pair consisting of a set of alternatives and a preference profile over this set is called a situation.

Formally, F is manipulable iff there is a situation (X, R) where $R = (R_1, \ldots, R_n) \in \mathcal{R}$, an individual $i \in N$ and a pair $x, y \in X$ so that x is strictly preferred to y by i while $F(X, R_1, \ldots, R_n) = y$ and $F(X, R_1, \ldots, R_{i-1}, R'_i, R_{i+1}, \ldots, R_n) = x$ with $R'_i \neq R_i$. In other words,

Definition 1 *F is manipulable (by individuals) if and only if (hereafter iff) there is a situation and an individual so that the latter can bring about a preferable outcome for himself by preference misrepresentation than by truthful revelation of his/her preference ranking,* ceteris paribus.

More concisely, F is manipulable iff there is at least one such situation where the n-tuple of sincere voting strategies does not lead to a Nash equilibrium (in pure strategies).

Definition 2 *F is non-trivial (non-degenerate) iff for each alternative x, there is a preference profile so that x is chosen.*

Table 1 illustrates manipulation in the the widely used plurality runoff system. Here $X = \{A, B, C\}$ and $|N| = 17$. With sincere voting the runoff contestants are A and C, whereupon C wins. Should now the 2 right-most voters switch their preference between A and B, the runoff would take place between A and B, whereupon A, their favorite, would win. Note that with sincere voting the outcome is the worst for the 2 voters, with strategic voting their most preferred alternative wins.

Table 1. Manipulation in plurality runoff system

6 voters	5 voters	4 voters	2 voters
A	C	B	A
B	A	C	B
C	B	A	C

Table 2 gives another example of manipulation, this time in the case of the amendment procedure used e.g. in U.S. Congress as well as in Finland and Sweden in parliamentary decision making. Here $X = \{A, B, C\}$ and $|N| = 9$. Since the procedure is based on an agenda, we use the following agenda to illustrate manipulation: (1) A vs. B, (2) the winner vs. C. With sincere voting B wins. Suppose that the 2 right-most voters vote *as if* their preference were: $C \succ A \succ B$. Then the winner is C, their first ranked alternative.

In the definition above as well as in the examples just discussed, manipulation takes the form of misrepresentation of preferences, i.e. reporting in the ballots cast a preference order that does not correspond the preferences one holds with

Table 2. Manipulation in the amendment procedure

2 voters	3 voters	2 voters	2 voters
A	B	C	C
B	C	A	B
C	A	B	A

regard to the alternatives or candidates at hand. Manipulation in this sense does not, however, cover the entire spectrum of strategic behaviour in voting context. In particular, it does not cover manipulation through agenda control. Our primary aim is, however, to assess the significance of the results achieved in the field of preference misrepresentation.

2 Principal Results

The best-known result in manipulation literature is undoubtedly a theorem proven by Gibbard and Satterthwaite [12,17]. In contradistinction to the social choice function defined above, the theorem deals with another formal counterpart of voting rule, viz. resolute social choice function, sometimes also known as social decision function. This concept refers to mechanisms that in every situation end up with a singleton set of alternatives. I.e. in every situation one and only one alternative is specified as the winner.

Theorem 1 *([12,17]). Every universal and non-trivial resolute social choice function is either manipulable or dictatorial.*

One strategy of proof is the following [9]:

1. It is shown that any universal, non-trivial and non-manipulable SCF must satisfy the Pareto condition if the number of voters is two.
2. One goes through all 36 different preference profiles (of two voters and three alternatives) and determines the winners that are possible under Pareto principle. It turns out that the possible outcomes make either the rule manipulable at some profile or one of the voters is a dictator (the outcome is always his first ranked alternative).
3. The argument is extended to larger electorates and larger alternative sets.

The theorem is *prima facie* very damaging to the view that voting procedures always reveal 'the will of the people'. After all, what it says is that no reasonable voting rule can be expected to accomplish this under all circumstances. But does it apply to all reasonable voting rules? It does not. In fact, it applies directly to very few since very few systems are resolute. By far the most may end up in a tie between two or more alternatives. These are then broken in various ways to elect one of them. Nonetheless the rules themselves are typically not resolute.

It is, however, relatively straight-forward to show by way of examples that all systems used in practice are – while not resolute – still manipulable through preference misrepresentation. Two examples were shown in Section 1 [16]. What should be observed, though, is that in both examples above, a coordination of several like-minded voters is required for successful manipulation. By glossing over the possibility of ties in outcomes, the Gibbard-Satterthwaite theorem also overlooks the distinction one could make between procedures manipulable by individuals and those manipulable by coalitions. Similarly, it overlooks the distinction between outcomes that result from manipulation in cases where there are ties in manipulated and non-manipulated outcomes. These distinctions are taken into account in Taylor's comprehensive analysis [19]. In analyzing non-resolute choice rules one typically needs to make assumptions regarding voter preferences over subsets of alternatives. Some seem pretty natural (e.g. that a voter with ranking $a \succ b \succ c$ prefers the outcome a to a tie between a and b or the latter to the tie between all three), but others involve dwelling into the risk-postures (e.g. assuming that the voter with the above ranking prefers an $a - c$ tie to b). By assuming that the outcomes are always singleton sets, Gibbard and Satterthwaite bypass these complications. The overall conclusion, however, remains that a vast majority of voting systems is manipulable in some sense. So, the Gibbard-Satterthwaite theorem seems to extend far wider than the concept of resolute social choice function would envisage.

So, much of the dramatic effect of the Gibbard-Satterthwaite theorem is lost once one realizes that it applies directly mechanisms that are not used. In the context of this observation Gärdenfors' theorem seems a significant step forward in applied social choice theory [10].

Theorem 2 *([10]). If a social choice function is anonymous and neutral and satisfies the Condorcet winning criterion, then it is manipulable.*

The Condorcet winning criterion – it will be recalled – is satisfied by all voting systems that always result in the Condorcet winner when one exists in the observed profile. Condorcet winner, in turn, is an alternative that would defeat all others in pairwise contests with a majority of votes. Social choice functions that satisfy the Condorcet winning criterion are generally known as Condorcet extensions. A noteworthy aspect of this theorem is its wider range of applicability: it covers all social choice functions, not just resolute ones. In particular, it covers basically all voting procedures that single out a set of winners once the ballots have been cast.

Strategy of proof of this theorem is the following:

- One begins with a specific 3-voter, 3-alternative profile, where the one specific alternative is ranked first by two voters. One postulates that this specific alternative is chosen in this profile.
- Another specific 3-voter, 3-alternative profile is then focused upon and all logically possible choice from this profile are analyzed.
- For each choice from the latter profile, one shows that if this were the actual choice, then the social choice function applied would be manipulable by some voter at some other profile. Since the Condorcet winner is chosen in the first profile, the conclusion is that all Condorcet extensions are manipulable.

It is well-known that not all voting systems are Condorcet extensions. Of those that are not, the theorem, of course, says nothing, but again a more detailed analysis reveals that manipulability is a pervasive property among these as well. Gärdenfors points out, however, two choice functions that are not manipulable:

– If every voter's preference ranking is linear or strict (no ties), then a social choice function that chooses the Condorcet winner when one exists and all alternatives, otherwise, is non-manipulable.
– Under the same assumption concerning voter preferences a social choice function that chooses the Condorcet winner when one exists and the set of Pareto undominated outcomes, otherwise, is also non-manipulable.

Pareto domination is defined as follows. An alternative x Pareto dominates another alternative y iff x is ranked at least as high as y by all voters, and strictly higher by at least one voter. The set of Pareto undominated alternatives consists of those that are not Pareto dominated by any others. Typically this is a very large set and, hence, the improvement in terms of discriminating power of the latter function is not typically much greater than that of the former.

The outlook for finding a system that would encourage sincere preference revelation from voters is, thus, not promising in the light of these results. On a more positive side the following theorem is worth mentioning.

Theorem 3 *([6]). Let n be the number of voters and m the number of alternatives. (i) For $n = 4$ or $n = 4k + 2$ with $k \geq 0$ and $m \geq 3$, if F is anonymous, neutral and strategy-proof social choice function on Condorcet domain, then F is the Condorcet rule (i.e. selects the Condorcet winner). (ii) For $n = 4k$ with $k \geq 1$ and $m \geq 4$, if F is anonymous, neutral and strategy-proof social choice function on Condorcet domain, then F is the Condorcet rule.*

Condorcet domain is the class of situations where there is a Condorcet winner. Campbell and Kelly's theorem thus essentially states that all Condorcet extensions are immune to manipulation – i.e. strategy-proof – as long as we allow only those profiles where a Condorcet winner exists [6]. As will be seen shortly, the restriction envisaged is important.

3 The Practical Significance of the Results

The Table 1 example shows that manipulable systems can present some of the voters with a dilemma: (1) to vote according to their true preferences, thereby contributing to their favourite's possible victory on the first round and at the same time risking its loss on the second round by not voting for a weaker contestant in the first round. Or (2) to use their vote to contribute to the success – on the first round – of a candidate that is a weaker competitor to their own favourite on the second round – assuming there is going to be one. This is a quandary that faces those voters who can reasonably expect their favourite to make it to the second round, but to fall slightly short of the 50% required for overall victory on the first round. Similar incentives are faced by small-party supporters in two-party systems: should one reveal one's true preferences in voting or should one support 'the lesser of two devils'? These dilemmas are well known.

Table 1 is instructive in a another sense as well. To wit, the two voters whose strategic behaviour has been in the focus of our interest are in fact making a choice between their best and worst alternative: with sincere voting their worst alternative wins, while by misrepresenting their preferences, *ceteris paribus*, their best alternative gets elected. It would seem that the supporters of A would have strong incentives to vote for B rather than A on the first round. Should this happen, the outcome would be the victory of B in the first round since it would get more than 50% of the votes. Not a disastrous outcome but not optimal either. To get the desired result the supporters of A need coordination in order to avoid overshooting – and ending up with B – and undershooting – and ending up with the worst possible outcome C.

One of the factors restricting the practical significance of the general manipulability results is the fact that, although the system may be manipulable, the difference between the manipulated and sincere voting result is small and certainly not of the order of magnitude of Table 1 example. Moreover, the *ceteris paribus* clause embedded into the manipulability results is to be taken seriously. The reason is simple: if the other parties get a hint that some party aims at strategic misrepresentation of its preferences, they may resort to misrepresentation counter-measures themselves. Thus, for example in Table 1, if the supporters of B suspect that the two A supporters intend to vote for B in the first round to get it defeated by A on the second one, they might strategically vote for C in the first round so that C would become the overall winner. This is better than A for the supporters of B. Thus, the counter-measures may well frustrate the efforts of the manipulators. In other words, manipulability of a system in a situation does not mean that strategic misrepresentation would be plausible or likely. Indeed, preference misrepresentation may conceivably lead to better, worse or equal outcome with respect to the sincere voting outcome. More recent research has, accordingly, focused on these aspects as will be discussed later on.

Of the results discussed in the preceding section, the theorem of Campbell and Kelly is certainly the most positive one. On closer inspection it is, however, of very restricted applicability [14,15]. Consider the example devised by Alexander Mayer of the Copeland rule applied to the following pair of profiles (Table 3):

On the left, C is the Condorcet winner and is thus elected by Copeland's rule (and by Condorcet's rule). The right-side is a result of first person's manipulation. There A, his first ranked alternative, wins with Copeland. Thus the manipulation is beneficial to the voter. Note, however, that the right profile is not in the Condorcet domain. So, by excluding profiles without Condorcet winner, the theorem in fact disregards the most obvious ways of manipulating Condorcet extensions. This, of course, doesn't undermine the validity of the result itself.

Table 3. Manipulation of Copeland's rule

1 voter	1 voter	1 voter		1 voter	1 voter	1 voter
A	B	E		A	B	E
C	C	D	$\Rightarrow$	D	C	D
B	A	C		B	A	C
D	E	A		E	E	A
E	D	B		C	D	B

4 Difficulty of manipulation

Anyone who has worked on providing examples of various criterion violations in social choice theory knows that coming up with such examples can, in cases they are theoretically possible, be exceedingly difficult for some criteria and procedures, while for others it can be relatively straight-forward. The same applies to demonstrating the manipulability of voting rules: for some rules it is easy to find profiles where voters can benefit from preference misrepresentation, while for other rules such profiles are more difficult to find. This intuitive observation suggests that perhaps it would make sense to consider the manipulability of voting rules as a matter of degree rather than dichotomy. Various ways of measuring the degree have, indeed, been devised.

- Kelly's index: $K = \frac{d_0}{(m!)^n}$, where d_0 is the number of profiles that are manipulable by at least one voter [13].
- Kelly index as modified by Aleskerov and Kurbanov: let λ_k = number of profiles that precisely k voters can manipulate [3]. Then $J_k = \frac{\lambda_k}{(m!)^n}$ is the share of profiles manipulable by k voters. The Aleskerov-Kurbanov index is the vector $J = (J_1, \ldots, J_n)$. Note that $K = \sum_j J_j$.
- three indices of freedom of manipulation $I+$, I^0, and $I-$. [1,2]

In any profile of m alternatives, each voter has $m! - 1$ possibilities for preference misrepresentation. Let k_{ij}^+ be the number of cases where misrepresentation improves the outcome to the voter i in profile j. Similarly, k_{ij}^0 = the number of cases where misrepresentation makes no change in the outcome for voter i in profile j and k_{ij}^- = the number of cases where preference misrepresentation makes the outcome worse for i in profile j [3].

- $I+ = \frac{\sum_{j=1}^{(m!)^n} \sum_{i=1}^{n} k_{ij}^+}{(m!)^n \times n \times (m!-1)}$
- $I^0 = \frac{\sum_{j=1}^{(m!)^n} \sum_{i=1}^{n} k_{ij}^0}{(m!)^n \times n \times (m!-1)}$
- $I^- = \frac{\sum_{j=1}^{(m!)^n} \sum_{i=1}^{n} k_{ij}^-}{(m!)^n \times n \times (m!-1)}$

Suppose that with sincere voting the outcome occupies k'th position in individual i's ranking. After i's misrepresentation the outcome occupies the position

s in his ranking. Let $\theta_j = k - s$, for $j = 1, \ldots, k_{ij}^+$. The variable θ_j thus shows how much – in terms of ranks – difference i's misrepresentation has made for him in a single case j. Summing up these θ_j's over cases and dividing the sum by k_{ij}^+ (the number of successful misrepresentations by i in profile j) one obtains Z_{ij}. This is then used to define efficiency index

$$I_2 = \frac{\sum_{j=1}^{(m!)^n} \sum_{i=1}^{n} Z_{ij}}{(m!)^n \times n}$$

Let $Z_{ij}^{max} = max(\theta_1, \ldots, \theta_{k_{ij}^+})$. Then

$$I_3 = \frac{\sum_{j=1}^{(m!)^n} \sum_{i=1}^{n} Z_{ij}^{max}}{(m!)^n \times n}$$

On the basis of the results of Aleskerov and Kurbanov regarding 3-alternative settings the following conclusions can be made [3]:

- the likelihood of a manipulable profile depends on the assumptions regarding extended preferences (over subset of alternatives)
- for small number of voters and alternatives, threshold rule and Borda count seem most manipulable
- for medium range, plurality gets highest values of the index
- Black's procedure has the smallest values over most of the range of voters
- some index values (esp. for Black) depend on the parity of the number of voters

To a large extent the same conclusions extend to 4- or 5-alternative settings [2].

The main problem related to practical use of the above measures of the degree of manipulability is the fact that typically not all preference profiles are equally likely. This restricts the applicability of these measures as direct guidelines for selecting voting rules. This problem pertains also to the other main approach to measuring difficulty: the computational complexity of manipulation. This approach builds on and expands the results of algorithmic complexity theory, a well-established field within computer science [11]. The basic classification of computational tasks is the following:

- computationally tractable problems: those that can be computed by polynomial time algorithms of order $O(n^k)$, where k is a fixed constant and n the size of input (e.g. number of alternatives and voters). This class of problems is denoted by P.
- problems in NP (nondeterministic polynomial time): no polynomial time algorithm is known, but given a solution proposal, its correctness can be verified in polynomial time.
- NP-complete problems: if any of these are shown to be computable in polynomial time algorithm, all others can be similarly computed. Then $P = NP$.

It is generally believed – although this hasn't been proven – that $P \neq NP$.

Now, computational complexity relates to voting rules in several ways. Firstly, the computation of the election results once the ballots have been cast may, depending on the rule being applied, require varying amounts of computing resources (time, memory-space). This problem was first addressed by Bartholdi et al. in the context of Dodgson's rule [5]. More specifically, the problem addressed was: given the set C of candidates, the set V of preference rankings over C and a positive integer K, is the Dodgson score of candidate c in C less than or equal to K? It was proven that the Dodgson score is NP-complete. Proof is by reducing the score problem to another problem known to be NP-complete, viz. exact cover by 3-sets. A related problem, viz. Dodgson ranking problem is the following: given sets C and V as above with two distinguished members c and c' in C, one asks: did c defeat c' in the election? The result is that Dodgdon ranking is NP-hard, i.e. easy for a good guesser, but in general not solvable in plynomial time. In contradistinction to the Dodgson score problem this one is not NP-complete, i.e. does not imply anything with respect to the canonical quandary: is $P \neq NP$? In addition to these now classic problems, Bartholdi et al. prove similar results for the Kemeny rule, i.e. Kemeny score is NP-complete, Kemeny ranking and Kemeny winner NP-hard.

Complexity theory has also applications in the study of preference misrepresentation. In this context the problem takes the following form: given a profile Π of votes cast by everyone else but the manipulator, and a preferred alternative x, is there a vote that the manipulator can cast so that x wins? This problem is typically in NP as the yes or no answer can be checked (normally) in polynomial time. Sometimes (e.g. plurality voting) even the solution can be computed in polynomial time (in which case even the problem is in P) [7]. Bartholdi et al. prove the following important theorem [4].

Theorem 4 *([4]): the manipulation problem can be solved in polynomial time for all rules that satisfy the following:*

1. *the rule can be run in polynomial time*
2. *the rule is scoring rule*
3. *the following type of monotonicity holds, i.e. for all profiles Π and Π' and for all $a \in X$ and for all $i \in N$: $\{b : a \succ_i b\} \subseteq \{b : a \succ'_i b\}$ implies that $S(\Pi, a) \leq S(\Pi', a)$.*

It should be emphasized that the type of monotonicity featuring in the theorem is not equivalent to the standard concept of monotonicity. This can be seen e.g. in the following example (Table 4) where it turns out that while the Borda count satisfies the latter, it does not satisfy the former.

In the 3-person profile in the left, the subset of alternatives regarded inferior to d by all is $\{a, b\}$, and in the right-hand profile $\{b\}$. So, the Bartholdi monotonicity would require that the score of d is larger on the left than on the right profile. This is not the case if the Borda count is applied: the score of d is 8 on the left and 9 on the right. Hence, it would seem that conditions listed in the theorem are sufficient, but not necessary for polynomial time manipulability.

Table 4. Two concepts of monotonicity

1 voter	1 voter	1 voter	1 voter	1 voter	1 voter
c	e	e	c	d	d
e	d	d	a	e	e
d	c	a	e	c	a
b	b	b	d	b	b
a	a	c	b	a	c

From the practical point of view the complexity results should be understood in their proper role: they are based on worst-case settings. In other words, if a result implies that manipulating a given system is computationally intractable, this does not mean that this should always or even in a majority of situations be so. It only says that there are situations in which manipulating successfully confronts the voter with an computationally intractable problem. These kinds of situations may be extremely rare in practice.

5 Safe and unsafe manipulation

Preference misrepresentation does not always succeed. The most obvious explanation for a failure is that the *ceteris paribus* condition that is used in defining manipulability did not hold in the situation at hand. Other participants may have resorted to counter-measures so that the preference misrepresentation backfired. Obviously the possibility of such failures plays a significant role in the calculus of any voter pondering upon the choice of the voting strategy. Consider the following example devised by Slinko and White [18] where uncoordinated manipulation may backfire.

Table 5. Manipulation of Borda count may backfire

1 voter	1 voter	1 voter	1 voter
a	a	b	c
b	b	c	b
c	c	a	a

With sincere voting b wins in Borda count. If either of the two left-most voters votes $a \succ c \succ b$ and ties are broken alphabetically a wins. However, if they both manipulate, c (their worst) wins. The necessity (and precariousness) of coordination is even more evident in Table 6, also devised by Slinko and White [18] .

The Borda count yields b as the sincere voting outcome. If the 4-8 of the first 17 voters vote $a \succ c \succ b$, ceteris paribus, a wins. If 10-17 of the same voters vote as indicated, the winner is c.

Table 6. Precariousness of manipulation

17	15	18	16	14	14
a	a	b	b	c	c
b	c	a	c	a	b
c	b	c	a	b	a

These considerations motivate the introduction of the concept of safe manipulation [18].

Definition 3 *A strategic vote L is safe, iff for any subset of like-minded (identical preferences) voters the outcome resulting from their choosing L (rather than their true preference) is no worse and for some subset even strictly better (in terms of their true preferences) than the outcome of sincere voting.*

In other words, manipulation is safe whenever no harm is done to the voter by resorting to it. One could say that the manipulation is the weakly dominant strategy for the voter. In line with the standard definition one again assumes that outside the group of would-be manipulators the behaviour remains fixed, i.e. no counter-measures are envisaged.

Theorem 5 *([18]). Let a nondictatorial and resolute social choice function F be applied to a choice set of at least three alternatives. Then there exists a profile and an individual so that the individual can safely manipulate F in the profile.*

This theorem quashes the hopes of finding a reasonable sub-class of voting rules that would be immune to the Gibbard-Sattertwaite result when the additional condition that manipulation be safe is imposed. Thus, manipulability – even safe manipulability – seems to be a pervasive feature of voting rules.

Lest too drastic conclusions be drawn, it is worth emphasizing that the Slinko-White theorem is an existence result. It states that for each nondictatorial and resolute rule a situation can be found where it is safely manipulable barring counter-measures. No estimate of the probability of such situations is given in the theorem.

Finally, an important assumption underlying the above manipulability results should be made explicit: the results assume that the voters have complete information about the preference profile. Together with the assumption of no counter-measures by other voters the complete information requirement glosses over many considerations that the real world manipulability would seem to depend upon. Which is another way of saying that the theoretical results are precisely what they should be, viz. theoretical.

6 Conclusions

Thus we can briefly summarize the preceding remarks as follows:

– manipulability is a pervasive property among choice rules
– its practical importance hinges on several things
– information requirements of successful misrepresentation can be very demanding
– suitable situations may not be common
– computational requirements may be unrealistic, but these results are worst-case ones
– misrepresentation may backfire
– nonetheless the Gibbard-Satterthwaite theorem applies to safe manipulation as well

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