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ABSTRACT

Regardless of the vast amount of research on the topic of mathematical development, there is limited understanding on its development in different types of students. For instance, mathematics anxiety and preterm birth are risk-factors for lower mathematical achievement. However, there is a gap in research on how these individual factors affect the development of specific skills, such as rational number knowledge and skills, amongst primary school students. This is unfortunate as all students need rational number knowledge for learning more complex mathematics topics and within everyday mathematical reasoning. Students' ability to apply their mathematical knowledge in various situations, for example in everyday life or to solve novel mathematical tasks, is another aspect examined in this research. The current dissertation includes four articles that aim to provide novel insights on individual differences in the mathematical knowledge and skills of primary school students. More specifically, it expands our knowledge on how individual factors, mathematics anxiety and preterm birth, relate to the learning of rational number knowledge and flexible mathematical skills.

Studies I-III aimed to investigate how mathematics anxiety is related to a wide range of mathematical knowledge and skills of primary school students. In Study I, the participants were 412 fifth and sixth graders. Studies II and III focused specifically on two subgroups of low performing students, those with or without a tendency for a fraction misconception, referred to as the natural number bias (NNB). These studies expand our knowledge on the complex relation between mathematics anxiety and fraction understanding amongst students who are still learning about fractions. Study IV compared fifth grade mathematical skills (including rational numbers), mathematics motivation, and cognitive abilities of 11-year-old children born very preterm (n = 67) and full-term (n = 72). This study expands our understanding on the role of preterm birth related cognitive difficulties in the development of rational number knowledge and the ability to apply mathematical knowledge in everyday contexts.

Study I showed that mathematics anxiety has a negative relation with mathematics performance already in primary school. It also showed that the mathematics anxiety-performance relation differed across the mathematical tasks and the two measures of mathematics anxiety: trait and state mathematics anxiety.

For instance, lower fraction arithmetic knowledge was related to higher mathematics trait anxiety, but not state anxiety. Then again, state anxiety explained students' performance on a novel rational number arithmetic task, even after controlling for trait mathematics anxiety and relevant mathematical knowledge. Intriguingly, mathematics anxiety was not related to students' ability to notice multiplicative relations embedded in pictures. Thus, differences in task characteristics influenced the relation between mathematics anxiety and performance.

Studies II and III examined the relation between a fraction related misconception, namely the NNB, and mathematics anxiety. Study II showed that the NNB group reported lower state anxiety related to fraction arithmetic than the No-NNG group. Study III showed that the fraction state anxiety significantly increased after fraction instruction in the NNB group. These results support the notion that the NNB is a misconception. The initially low fraction state anxiety in the NNB group suggests that the students are unaware of their incorrect answers and low fraction performance. As their awareness increased, their anxiety increased. Thus, increased mathematics performance may lead to increased mathematics anxiety when overcoming a misconception. Intriguingly, the qualitative difference in fraction understanding between the two low performing groups was not captured by trait mathematics anxiety. Thus, the situational nature of state anxiety could be valuable for measuring qualitative differences and dynamic changes within mathematical understanding, for example the overcoming of a misconception. In general, the two measures of mathematics anxiety unveiled different nuances in students' mathematical understanding and task perceptions.

Study IV further emphasises that mathematical knowledge and skills should be measured in various subpopulations and across multiple tasks. The children born very preterm and full-term did not significantly differ on multiple fifth grade mathematical skills, including whole number and rational number magnitude knowledge, or in mathematics motivation. However, the children born very preterm had weaker spontaneous mathematical focusing tendencies meaning difficulties with recognising and applying their mathematical knowledge in situations that are not explicitly mathematical. This finding implies that the children born very preterm may have difficulties applying their mathematical knowledge in everyday life contexts. This difficulty could not be fully explained by group differences in cognitive abilities. Moreover, this study yields further evidence that individual factors do not necessarily affect all mathematical skills similarly.

Overall, the current dissertation sheds light on individual differences in rational number knowledge and skills, including the ability to apply mathematical knowledge in novel contexts. The findings reveal that the challenges in developing mathematical competence are multifaceted, as they are influenced by both individual factors, such as mathematics anxiety and preterm birth, but also task characteristics. This highlights the importance of examining subpopulations and various mathematical skills. These findings also emphasise the importance of acknowledging individual differences in mathematics teaching, as mathematical skills are relevant within the everyday lives of all individuals.

KEYWORDS: Fraction knowledge, primary school, mathematics anxiety, preterm birth, flexible mathematical thinking

TURUN YLIOPISTO
Kasvatustieteiden tiedekunta
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HILMA HALME: Yksilölliset erot murtolukuosaamisessa: uusia näkökulmia tutkimalla matematiikka-ahdistusta ja keskosuutta
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TIIVISTELMÄ

Oppilaiden matemaattisen taitojen kehitystä on tutkittu paljon, mutta tutkimustietoa taitojen kehityksestä erilaisten oppijoiden osalta on rajallisesti. Esimerkiksi matematiikka-ahdistus ja keskosena syntyminen ovat riskitekijöitä heikommalle matemaattiselle osaamiselle. Kuitenkaan ei tiedetä, miten nämä yksittäiset tekijät vaikuttavat yksittäisten matemaattisten taitojen kehittymiseen, kuten rationaalilukuosaamisen kehittymiseen alakouluikäisillä oppilailla. Tämä on ongelmallista, sillä kaikki oppilaat tarvitsevat rationaalilukuosaamista myöhempien matemaattisten taitojen oppimiseen ja jokapäiväiseen matemaattiseen päättelyyn. Yksilöllisten erojen merkitys oppilaiden taidoissa soveltaa matemaattista osaamistaan erilaisissa tilanteissa, kuten arjessa tai uudentyyppisten matemaattisten tehtävien ratkaisemisessa on toinen tärkeä tutkimusaihe. Tämä väitöskirja sisältää neljä artikkelia, joiden tavoitteena on tarjota uusia näkökulmia yksilöllisiin eroihin alakouluikäisten oppilaiden matemaattisessa osaamisessa tutkimalla erikseen matematiikka-ahdistusta ja keskosena syntymistä. Tämä tutkimus laajentaa ymmärrystämme yksilöllisistä tekijöistä, jotka voivat vaikuttaa rationaalilukuosaamisen ja joustavien matemaattisten taitojen kehitykseen.

Osatutkimuksissa I-III tavoitteena oli selvittää matematiikka-ahdistuksen yhteyttä matemaattisiin taitoihin alakouluikäisillä. Osatutkimukseen I osallistui 412 viides- ja kuudesluokkalaista. Osatutkimuksissa II ja III keskityttiin erityisesti kahteen oppilasryhmään: heikosti murtolukutehtävässä suoriutuneet oppilaat, joilla oli tai ei ollut murtolukuihin liittyvää virhekäsitystä (englanniksi natural number bias). Virhekäsitys liittyy oppilaiden taipumukseen yleistää luonnollisten lukujen ominaisuuksia murtolukutehtävien ratkomisessa. Tutkimusten tulokset lisäävät tietoamme matematiikka-ahdistuksen ja murtolukujen ymmärtämisen monimutkaisesta suhteesta alakouluikäisillä oppilailla, jotka vielä opettelevat murtolukuja. Osatutkimuksessa IV verrattiin pikkukeskosina (n = 67) ja täysiaikaisina (n = 72) syntyneiden 11-vuotiaiden oppilaiden viidennen luokan matemaattisia taitoja (mukaan lukien rationaalilukuosaamista), matematiikkamotivaatiota ja kognitiivisia taitoja. Tutkimustulokset lisäsivät ymmärrystämme pikkukeskosena syntymiseen liittyvien riskitekijöiden, kuten heikompien kognitiivisten kykyjen, yhteydestä rationaalilukuosaamisen kehittymiseen ja oppilaiden kykyyn käyttää matemaattisia taitojaan arjen tilanteissa.

Osatutkimus I osoitti, että matematiikka-ahdistus on negatiivisesti yhteydessä matematiikan osaamiseen jo alakouluiässä. Osatutkimus osoitti myös, että matematiikka-ahdistuksen ja matemaattisen suoriutumisen välisessä suhteessa oli eroja matemaattisten tehtävien välillä ja suhde oli riippuvainen siitä, mitattiinko yleistä vai tilannekohtaista matematiikka-ahdistusta. Esimerkiksi heikompi aritmeettisten murtolukutehtävien osaaminen oli yhteydessä korkeampaan yleiseen matematiikka-ahdistukseen, mutta ei tilannekohtaiseen matematiikka-ahdistukseen. Tilannekohtainen matematiikka-ahdistus puolestaan oli merkittävästi yhteydessä oppilaiden joustavaan rationaalilukuosaamiseen, jopa sen jälkeen, kun yleinen matematiikka-ahdistus ja monet tehtävän kannalta olennaiset matemaattiset taidot oli kontrolloitu. Matematiikka-ahdistus ei kuitenkaan ollut yhteydessä oppilaiden taitoihin havaita multiplikatiivisia suhteita erilaisissa kuvissa. Täten erot tehtävien ominaisuuksissa vaikuttivat matematiikka-ahdistuksen ja matemaattisen suoriutumisen väliseen suhteeseen.

Osatutkimuksissa II ja III tutkittiin oppilaiden murtolukuihin liittyvän virhekäsityksen vaikutusta matematiikka-ahdistukseen. Tutkimus II osoitti, että oppilaat, joilla oli virhekäsitys raportoivat vähemmän murtolukuihin liittyvää tilannekohtaista ahdistusta (murtolukuahdistus) kuin oppilaat, joilla ei ollut virhekäsitystä. Osatutkimus III osoitti, että murtolukuahdistus lisääntyi merkittävästi murtolukuopetuksen jälkeen, mutta vain virhekäsityksen omaavien ryhmässä. Nämä tutkimustulokset tukevat aikaisempia havaintoja, että kyseessä on virhekäsitys. Virhekäsityksen omaavilla alhaisempi murtolukuahdistus oletetaan johtuneen siitä, että oppilaat eivät aluksi tiedostaneet vastanneensa murtolukutehtäviin väärin. Kun heidän tietoisuutensa kasvoi, myös murtolukuahdistus kasvoi. Täten matemaattisen osaamisen lisääntyminen voi joskus johtaa korkeampaan matematiikka-ahdistukseen, esimerkiksi kun on kyseessä virhekäsityksestä poisoppiminen. Mielenkiintoista on myös se, että oppilasryhmät eivät eronneet toisistaan yleisessä matematiikka-ahdistuksessa. Tämä osoitti, että tilannekohtaisen matematiikka-ahdistuksen mittaaminen voisi olla hyödyllistä, kun halutaan tutkia laadullisia eroja matemaattisten käsitteiden ymmärryksessä, kuten virhekäsityksiä ja niistä poisoppimista. Sekä yleisen että tilannekohtaisen matematiikka-ahdistuksen mittaaminen paljasti erilaisia asioita oppilaiden matemaattisesta osaamisesta ja tehtävien hahmottamisesta.

Osatutkimus IV korosti edellisiä tutkimuslöydöksiä siitä, että matemaattista osaamista on syytä tutkia nykyistä laajemmin monenlaisilla oppijoilla ja käyttäen erilaisia tehtäviä. Oppilaat, jotka olivat syntyneet pikkukeskosina, suoriutuivat useista viidennen luokan matematiikan osaamisalueista, kuten kokonaislukujen ja rationaalilukujen suuruuden ymmärtämisetä, yhtä hyvin kuin täysiaikaisena syntyneet. Myöskään matematiikkamotivaatiossa ei ollut ryhmien välisiä eroja. Keskosena syntyminen vaikutti kuitenkin oppilaiden taitoon kiinnittää spontaanisti huomiota matemaattisiin piirteisiin, sillä heillä oli vaikeuksia tunnistaa ja käyttää matemaattista tietämystään tehtävissä, jotka eivät olleet selkeästi matemaattisia. Tämä havainto viittaa siihen, että heillä voi olla vaikeuksia matemaattisen osaamisen soveltamisessa arkielämän tilanteissa. Tämä ei selittynyt ryhmien välisillä eroilla kognitiivisissa taidoissa. Osatutkimus vahvisti tämän väitöskirjatutkimuksen näyttöä siitä, että yksilölliset tekijät eivät välttämättä vaikuta samalla tavalla kaikkiin matematiikan taitoihin.

Kokonaisuudessaan tämä väitöskirja tuo uutta tietoa yksilöiden välisistä eroista rationaalilukuosaamisessa, varsinkin murtolukuosaamisessa, ja taidoissa soveltaa matemaattista osaamista uusissa tilanteissa. Tulokset osoittavat, että matemaattisen osaamisen kehittämisen haasteet ovat moninaisia ja niihin vaikuttavat sekä yksilölliset tekijät, kuten matematiikka-ahdistus ja pienkeskosena syntyminen, että matemaattisten tehtävien ominaisuudet. Tämä korostaa erilaisten oppijoiden ja erilaisten matemaattisten taitojen tutkimisen tärkeyttä. Yksilölliset erot matematiikan oppimisessa on hyvä huomioida myös matematiikan opetuksessa, sillä matemaattiset taidot ovat tärkeä osa jokaisen ihmisen arkea.

ASIASANAT: Murtolukuosaaminen, alakouluikäinen, matematiikka-ahdistus, keskosuus, joustava matemaattinen ajattelu

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Helsinki, 21.4.2024
Hilma Halme

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List of Original Publications

The present dissertation is based on the following original publications, which are referred to in the text by their Roman numerals. The

Study I Halme, H., Trezise, K., Hannula-Sormunen, M., & McMullen, J. (2022). Mathematics anxiety and performance across adaptive and routine tasks. *Journal of Numerical Cognition*, *8*(3), 414-429. https://doi.org/10.5964/jnc.7675

Study II Halme, H., Van Hoof, J., Hannula-Sormunen, M., & McMullen, J. (2023). Not realizing that you don't know: Fraction state anxiety is reduced by natural number bias. *British Journal of Educational Psychology*. https://doi.org/10.1111/bjep.12637

Study III Van Hoof*, J., Halme, H.*, Hannula-Sormunen, M., & McMullen, J. (submitted). When anxiety grows with knowledge: the role of the natural number bias. Preprint available: https://doi.org/10.31234/osf.io/9pt7d

Study IV Halme, H., McMullen, J., Nanu, C., Nyman, A., Pipari Study Group, & Hannula-Sormunen, M. (2022). Mathematical skills of 11-year-old children born very preterm and full-term. *Journal of Experimental Child Psychology*, *219*, 1-18. https://doi.org/10.1016/j.jecp.2022.105390

*Co-first author

In all publications, Halme contributed to the conceptualisation, data analysis, and writing of the manuscripts. Halme was also responsible for designing and collecting the data for Studies I-III.

1 Introduction

Mathematics is a difficult subject known for its cumulative basis of knowledge building. For instance, fraction knowledge builds upon students' understanding of proportional reasoning and yields towards students' acquisition of algebra knowledge (Booth & Newton, 2012; Hansen et al., 2015). Regardless of the vast amount of research on the topic of mathematical development, there is limited understanding on differences across mathematical skills in different types of students. More specifically, there is a gap in research on how certain individual differences, specifically mathematics anxiety and preterm birth, relate to rational number knowledge. This is problematic as all students should acquire sufficient rational number proficiency to be able to manage more complex mathematical subjects and even for reasoning about quantitative relations in everyday life (DeWolf et al., 2015; McMullen et al., 2017). In general, a variety of mathematical skills, ranging from routine to non-routine and formal to informal, are required for mathematical proficiency in today's society (e.g., Gravemeijer et al., 2017). The current research examines students with varying individual differences, focusing on mathematics anxiety and children born preterm, to gain better understanding of factors that hinder students' mathematical attainment. This should reveal new insights especially into the complexity of rational number knowledge development and why some students struggle particularly with fractions.

Mathematics anxiety is one important factor to consider when it comes to understanding the development of mathematical knowledge (Foley et al., 2017; OECD, 2013). Mathematics anxiety is defined as anxiety when performing mathematics and within situations related to mathematics, regardless of whether it is an academic or non-academic context (Richardson & Suinn, 1972). It has been shown that individuals with high mathematics anxiety perform less accurately and slower than those with less mathematics anxiety (Ashcraft & Kirk, 2001; Chan & Tang, 2020). It is theorised that mathematics anxiety depletes cognitive resources needed for high mathematics achievement, such as working memory and attention (Ashcraft & Moore, 2009; Ramirez et al., 2018). However, most studies examining mathematics anxiety in primary school students conduct mathematics achievement tests or only examine whole number knowledge. This limited focus is problematic because differences in task characteristics are shown to influence the relation between mathematics anxiety and performance (e.g., Demedts et al., 2022; Trezise & Reeve, 2018). Additionally, mathematics anxiety research has mainly examined explicitly mathematical contexts, such as whole number arithmetic (e.g., Sorvo al., 2017; Punaro & Reeve, 2012; Vukovic et al., 2013), while less examined are non-routine mathematical tasks with less explicit solutions. Thus, mathematics anxiety research should be expanded to examine a range of mathematical skills relevant for primary school students, such as rational number related knowledge and skills.

Another important factor known to affect mathematical development is individual differences in cognitive abilities. Both domain general and domain-specific cognitive skills have been found to support the development of mathematical proficiency, including rational number knowledge (e.g., Geary et al., 2017; Ye et al., 2016). Thus, it could be expected that students with weaker cognitive abilities would struggle with the development of rational number knowledge and skills. Children born preterm are shown to have lower cognitive abilities and academic difficulties, especially in learning mathematics (Sansavini et al., 2011; Taylor et al., 2009; Twilhaar et al., 2018). However, similar to mathematics anxiety, research regarding the mathematical skills of children born preterm has been mainly conducted using standardised tests (see for example meta-analysis by Twilhaar et al., 2018). Thus, there is limited knowledge on how specific mathematical skills, such as rational number knowledge or the flexible use of mathematical skills in everyday-like contexts, are affected by preterm birth. To support educators in their work and to provide additional academic support effectively, there is need for more nuanced research on mathematical attainment in populations that may have different developmental trajectories due to cognitive difficulties.

Despite the underlying reason, shortages in mathematical competences are restricting people's success in work careers and societal activities (Foley et al., 2017; Gravemeijer et al., 2017). This is because mathematical knowledge is not confined to the classroom or the completion of textbook tasks, but it is important for applying mathematical problem solving within and outside the school context. However, mathematical features of the environment are less salient for some students, making them less likely to notice the opportunities for applying their mathematical skills in their everyday life (Hannula & Lehtinen, 2005; McMullen et al., 2014). Students need to develop flexible mathematical skills to be able to use their mathematical knowledge in various situations, for example to solve a novel mathematical task or to apply mathematical problem solving in everyday situations. While routine understanding of a topic provides a basis for flexible mathematical skills, it alone is not enough for the flexible application of mathematical knowledge in novel contexts (McMullen et al., 2020; Verschaffel et al., 2009). While there is evidence of individual differences in students' flexible mathematical skills, there is scarce knowledge on the individual factors that may explain the origins of these differences.

The current dissertation aims to contribute to our understanding on how mathematics anxiety and preterm birth are related to mathematical knowledge and skills of late primary school students. It will expand on previous research by examining these factors in the realm of rational number knowledge and skills. As rational number knowledge is a crucial part of the mathematics curriculum and everyday mathematical skills, more attention should be given to individual risk-factors, such as mathematics anxiety and preterm birth. The current dissertation also goes beyond mathematics performance within textbook-based tasks to include non-routine mathematical tasks that require flexible mathematical thinking. Researching flexible mathematical skills aligns with the increase in demand for future-oriented skills and STEM professionals (Gravemeijer et al., 2017), as mathematics constraints may discourage students from choosing certain career paths (Foley et al., 2017). The current research is a step towards understanding how to support mathematical proficiency in students at risk for mathematics learning related challenges. This includes promoting the development of flexible mathematical skills side by side routine understanding already at an early age.

1.1 Rational number knowledge and skills

Instruction on rational numbers forms a crucial part of the mathematics curriculum in elementary school. However, previous literature has shown that students struggle to understand rational numbers, especially the unique properties of fractions (e.g., McMullen & Van Hoof, 2020; Siegler et al., 2011; Vamvakoussi et al., 2018). Fractions are considered to be one of the hardest topics to learn in primary school (Siegler & Pyke, 2013) and not all students are able to learn the properties of fractions even by the end of secondary school (Van Hoof et al., 2018). This is problematic as fraction knowledge is an important steppingstone towards understanding more advanced mathematics topics, such as algebra (Booth & Newton, 2012; DeWolf et al., 2015; Siegler et al., 2011, 2012). Considering the cumulative nature of mathematics, it is important to understand the challenges that students may face in developing rational number knowledge. This includes both mathematical aspects and individual factors, in this case mathematics anxiety and preterm birth related cognitive difficulties.

Throughout the course of numerical development, a key principle is that all real numbers have magnitudes, which can be placed on a number line (Siegler & Pyke, 2013). This has been suggested to form the cornerstone for comprehending the relations between numerical concepts. The integrated theory of numerical development proposes that learning to conceptualise numbers as magnitudes is integral for processing whole numbers and fractions (Siegler & Pyke, 2013). Many students struggle to grasp that a fraction is one magnitude comprised of a part-whole relation between two whole numbers, namely the numerator (i.e., the part) and denominator (i.e., the whole). Instead students may believe that fractions should be treated as two separate whole numbers, disregarding their relational qualities (Stafylidou & Vosniadou, 2004). This is suggested to be because students face difficulties with adapting and expanding their initial number concept of natural numbers to include the unique properties of fractions (e.g., Vamvakoussi et al., 2018). This transition from whole numbers to fractions appears to be particularly challenging, and these challenges may be a reason why both adults and children have more negative attitudes towards fractions than whole numbers (Sidney et al., 2021),.

An increased awareness of the relational structure of fractions is shown to support the development of fraction knowledge (DeWolf et al., 2015). It has been suggested that developing an understanding of whole number proportional reasoning builds the foundation for reasoning about fraction magnitudes (e.g., Hansen et al., 2015; Ye et al., 2016). Fraction knowledge also relies on whole number arithmetic knowledge, such as proficiency with multiplication facts (Hecht et al., 2003). Furthermore, the ability to notice multiplicative relations is suggested to support students' understanding of equivalent fractions (Boyer & Levine, 2012). For example, the fractions 1/3 and 3/9 are equivalent in magnitude and the denominator is three times the quantity of the numerator. Research has shown also that students' attentional predisposition towards multiplicative relations is positively related to their rational number knowledge (McMullen et al., 2014, 2022; McMullen, Hannula-Sormunen, et al., 2016).

In Finnish primary schools, fraction knowledge is part of the mathematics curriculum from the third grade until the sixth grade (Opetushallitus [OPH], 2014). Fraction knowledge is typically first introduced in the third grade by building a basic understanding of fraction magnitudes, then moving onto fraction arithmetic in the fourth grade. In the fourth grade, the textbooks include addition and subtraction of fractions with the same denominator (e.g., 1/5 + 2/5). In the fifth grade, the textbooks include also fractions with different denominators (e.g., 1/3 + 2/6) and mixed fractions are introduced in the sixth grade. A large focus of the current research is on fraction arithmetic knowledge, more specifically, students' ability to solve addition and subtraction equations involving fractions. Another focus of this research expands to the flexible application of rational number knowledge in novel, non-routine contexts. The target population of the current research is late primary school students (i.e., fifth and sixth graders), because they should have a foundational understanding of rational numbers, including fractions. Yet, they are still in the process of developing their fraction understanding, allowing the examination of the relation between individual factors and knowledge development.

1.1.1 Natural number bias

Theories of mathematical knowledge development emphasise the fundamental role of prior knowledge in mathematical learning (Simonsmeier et al., 2022). According to these theories, prior knowledge can either support or hinder the learning of new concepts. In cases where there is a radical change from prior knowledge, for instance in the case of moving from natural numbers to rational numbers, prior knowledge of natural numbers can lead to misconceptions when solving rational number tasks (Merenluoto & Lehtinen, 2004).

Natural number bias (NNB) is a well-known phenomenon related to difficulties with rational number knowledge (Van Hoof et al., 2017). The natural number bias refers to the tendency to apply natural number properties in rational number tasks, even when this is inappropriate (Ni & Zhou, 2005). However, the properties of natural numbers are not sufficient to solve fraction tasks. For instance, some students assume that a fraction addition task is solved by adding the natural numbers in the numerators and denominators, resulting in systematic mistakes, for example claiming that 1/2 + 1/3 equals 2/5 (Jarrah et al., 2022; Siegler et al., 2011). Consequently, students would need to expand their number concept to include the properties of fractions and not being able to do this may cause a bottleneck in their learning of fraction knowledge. Unfortunately, previous studies have shown that some students have persistent difficulties with conceptualising the properties of fractions, with evidence of a NNB still in secondary school (e.g., Van Hoof et al., 2018).

It is important to note that having a NNB is not an all or nothing issue, as studies have found qualitatively different profiles of students in regards to their conceptual understanding of fractions (e.g., González-Forte et al., 2020; Van Hoof et al., 2018). While some students have a naïve and natural number based concept of fractions, some may already have some understanding and apply a natural number based reasoning only on some tasks, and other students may exhibit no signs of a NNB (González-Forte et al., 2020; Van Hoof et al., 2018). Consequently, there is debate whether natural number based reasoning is a naïve misconception (González-Forte et al., 2022; Merenluoto & Lehtinen, 2004) or a deliberate strategy that students use when they are unsure how to solve a new fraction task (Alibali & Sidney, 2015).

According to the dual process theories of reasoning, the correct understanding of rational numbers is suggested to coexist with an earlier, more intuitive natural number based understanding (Vamvakoussi, 2015). Consequently, to solve a rational number task correctly, students need to successfully inhibit their intuitive natural number based reasoning and apply a slower, more effortful reasoning process called analytic reasoning. The dual processing account is supported by research showing that students with an NNB profile answered fraction items faster than students without the NNB (e.g., Reinhold et al., 2020). Studies examining students' confidence levels have shown that students with a strong NNB profile are confident in their incorrect natural number based answers (e.g., González-Forte et al., 2022; Merenluoto & Lehtinen, 2004). Even when students were confronted with a correctly reasoned answer, they were reluctant to change their natural number based answer (González-Forte et al., 2022). This suggests that students with a strong natural number based answering profile are unaware that they are solving fraction tasks in an incorrect way. However, the relation between mathematics anxiety and the NNB has not been previously examined in primary school. Thus, the current dissertation research yields new insights by examining this relation.

1.1.2 Developing flexible mathematical skills

While routine understanding of a topic allows students to complete well-rehearsed tasks, the ability to flexibly apply one's mathematical knowledge is shown to require more than routine understanding. Star & Seifert (2006) suggest that with knowledge comes flexibility. They define a "flexible solver" as one who has acquired knowledge of multiple solution procedures and has the capacity to invent and create new procedures. However, not all students learn the skills to become flexible solvers. For instance, McMullen et al. (2020) examined middle school students and found that high performance on routine rational number tasks did not guarantee high performance on a novel rational number arithmetic task. This aligns with the more general idea of adaptive expertise, which suggests that routine understanding of a topic is not enough to be able to apply the knowledge in novel situations (Verschaffel et al., 2009). However, what is less clear, is how individual differences affect the development of flexible mathematical skills.

One aspect of developing flexible mathematical skills measured within the current dissertation is an individual's attention towards numerical features within everyday contexts, more specifically their spontaneous mathematical focusing tendencies (Hannula & Lehtinen, 2005; McMullen et al., 2014; McMullen, Chan, et al., 2019). For some the world is full of opportunities for practicing and using their mathematical skills, while for others mathematical features of the environment may be less salient (Hannula & Lehtinen, 2005). This was first examined in young children and it was shown that spontaneous focusing on numerosity (SFON), defined as focusing of attention and utilising exact numerosity in non-explicitly mathematical situations (Hannula & Lehtinen, 2005), relates to early mathematical skill development (e.g., Batchelor et al., 2015; Hannula et al., 2010). The main focus of the current research is on spontaneous focusing on quantitative relations (SFOR) defined as "the spontaneous (i.e., undirected) focusing of attention to quantitative relations and the use of these relations in reasoning in situations that are not explicitly mathematical" (McMullen et al., 2014). For example, noticing that your parent is twice your height or that you have a third of a glass of juice left. SFOR tendency could be considered an indicator of an individual's predisposition to pay attention to multiplicative relations in various contexts, such as everyday contexts.

Previous research has shown SFOR tendency to be relevant within the development of rational number knowledge. SFOR tendency was shown to uniquely predict students' learning of rational numbers from 3rd through 6th grades, even after controlling for other skills, including nonverbal reasoning, arithmetic fluency, whole number line estimation (McMullen, Hannula-Sormunen, et al., 2016; Van Hoof et al., 2016), and prior rational number knowledge (McMullen et al., 2017). There is also evidence on the co-development of rational number knowledge and SFOR tendency (McMullen et al., 2017). The proposed mechanism is that when students recognise multiplicative relations in their everyday environments, they engage in self-initiated practice with their mathematical skills, such as fraction knowledge, in various contexts (McMullen et al., 2014). Consequently, students develop their understanding of and reasoning with numerical relations, for instance, how whole number multiplicative relations relate to fraction magnitude representations. Importantly, individual differences in SFOR tendency are not entirely explained by the ability to recognise and name the multiplicative relation, when specifically asked to do so (McMullen et al., 2014; 2016). However, it is not well-known what causes individual differences in SFOR tendency.

Another construct related to flexible mathematical skills in this research is adaptive number knowledge, which was first examined within whole numbers (McMullen, Brezovszky, et al., 2016) and later extended to rational numbers. Adaptive rational number knowledge is measured by a student's ability to integrate their rational number knowledge and recognise relevant arithmetic relations to solve a novel arithmetic sentence production task (McMullen et al., 2020). In the task, students are given a set of numbers, including fractions and decimals, and a whole number (e.g., 0.25, ½, ¼, 0.5 and 4). Students should use arithmetic operations to combine these numbers to produce as many arithmetic sentences as possible that result in a target answer (e.g., 1). Students can use the numbers and all arithmetic operations as many times as they want. To produce multiple different solutions, students need to have a well-connected network of knowledge about rational numbers (McMullen et al., 2020). More specifically, the task requires understanding of the relations between fraction magnitudes (e.g., ¼ is less than ½), rational number representations (e.g., ¼ = .25), and arithmetic procedures (e.g., fraction addition with different denominators). However, not all students who have high levels of rational number knowledge can integrate and apply this knowledge in the novel rational number arithmetic task (McMullen et al., 2020).

There can be many reasons why students are not able to apply their routine rational number knowledge in novel tasks or contexts. For instance, students may not be acquainted with the notion of integrating the two notations of rational number knowledge within one task. One reason for this could be that rational number representations, such as decimals and fractions, are often found in separate sections of mathematics textbooks and taught separately. Notably, it was shown that higher SFOR tendency relates to higher adaptive rational number knowledge in middle school students (McMullen et al., 2022). The self-initiated practice related to SFOR tendency may support the integration of students' prior knowledge of rational number representations (Lehtinen et al., 2017; McMullen, Chan, et al., 2019). It is also possible that both SFOR tendency and adaptive rational number knowledge reflect a student's ability to see numerical relations in various contexts. Thus, spontaneous mathematical focusing tendencies, seen already at a young age (Hannula & Lehtinen, 2005), could be a proxy for a student's flexible mathematical thinking. These tendencies are even referred to as one's tendency to see opportunities to apply their mathematical knowledge in non-mathematical contexts (Hannula & Lehtinen, 2005). However, mathematics anxiety or low cognitive abilities may be hindering students from acquiring well-integrated mathematical knowledge that can be applied beyond the highly practised, routine textbook tasks. Thus, the current research examines these relations to improve our understanding on factors related to flexible mathematical thinking.

1.2 Mathematics anxiety and performance

Mathematics anxiety has been defined as anxiety that interferes with one's ability to manipulate numbers and solve mathematical problems in academic and everyday life situations (Richardson & Suinn, 1972). PISA data indicates that high mathematics anxiety is related to low mathematics achievement for students across the globe (OECD, 2013). While mathematics anxiety is mostly researched in older students, it has been found to be negatively related to mathematics performance already in primary school (Barroso et al., 2021; Dowker et al., 2016; Namkung et al., 2019). Meta-analyses have found that the strength of the relation between mathematics anxiety and performance in school-aged students is between -.27 and -.34 (Hembree, 1990; Ma, 1999; Namkung et al., 2019). The negative relation was shown to be weaker for primary school students (Namkung et al., 2019), and to increase towards adolescence (Dowker et al., 2016). Many reasons have been suggested for the age-related increase, including higher cognitive requirements of more advanced mathematics, an increased exposure to negative mathematical experiences and increases in general anxiety (Dowker et al., 2016). While there is some overlap, it is important to note that mathematics anxiety is considered to be separate from test anxiety and general anxiety (Caviola et al., 2022). Yet, there are still many unanswered questions regarding mathematics anxiety and its relation with mathematical performance.

1.2.1 Theories of the developmental relation

Longitudinal studies have shown inconsistent findings regarding the direction of the causal relation between mathematics anxiety and performance. Some studies have found that low mathematics performance leads to later mathematics anxiety (e.g., Sorvo et al., 2019; Wang et al., 2020), while others have found the opposite relation (e.g., Cargnelutti et al., 2017; Vukovic et al., 2013). Thus, two main theoretical models have been proposed as an explanation for the developmental relation between mathematics anxiety and performance (for an overview see Carey et al., 2016). The deficit theory proposes that mathematics anxiety is caused by poor mathematics achievement due to deficits in prior mathematical skills. This is supported by studies of learning disabilities (e.g., Passolunghi, 2011). Similarly, a review of intervention studies showed that interventions aimed at improving mathematics skills not only increased participants' mathematics performance, but also reduced their mathematics anxiety (Balt et al., 2022). The debilitating anxiety model proposes that mathematics anxiety debilitates performance by competing for the same cognitive resources, such as working memory, that are needed to perform the mathematical task (Ashcraft & Kirk, 2001; Ashcraft & Krause, 2007; Vukovic et al., 2013). The primary cause of low performance being emotion regulation rather than low cognitive abilities, as students with high working memory can still have low performance due to mathematics anxiety (e.g., Vukovic et al., 2013). It is important to note that these two theoretical models are not mutually exclusive.

It has been proposed that the relation between mathematics anxiety and performance could be bidirectional (Carey et al., 2016), meaning poor mathematics performance can be both the cause and the outcome of mathematics anxiety. This is supported by empirical evidence that found a reciprocal relation between mathematics anxiety and mathematics achievement in first and second grade students (Gunderson et al., 2018). Similarly, Pekrun et al. (2017) followed students from fifth to ninth grade and found mathematics anxiety, and other mathematics achievement emotions (e.g., enjoyment), to develop reciprocally with mathematics achievement (Pekrun et al., 2017). It is possible that students with poor early mathematical skills develop mathematics anxiety, which then results in avoidance behaviour and reduced mathematics attainment, eventually leading to a vicious cycle (Dowker et al., 2016; Maloney & Beilock, 2012). Due to the limited number of longitudinal studies on mathematics anxiety, there is not enough empirical evidence for causal claims. It is also important to note that the studies examining longitudinal relations between mathematics anxiety and performance amongst primary school students have mainly examined whole number knowledge and trait mathematics anxiety (e.g., Cargnelutti et al., 2017; Gunderson et al., 2018; Mononen et al., 2022; Sorvo et al., 2019). Thus, there is scarce research on developmental relations in other mathematics topics or other measures of mathematics anxiety, such as situational measures, that could explain the ambiguous longitudinal findings.

1.2.2 Trait and state mathematics anxiety

Recent studies have distinguished between two aspects of mathematics anxiety, trait mathematics anxiety and state anxiety. The former refers to a general anxiety towards mathematics and the latter to a temporary situational anxiety during a mathematics-related situation (Conlon et al., 2021). Trait and state mathematics anxiety components are mutually dependent, meaning that individuals with high trait mathematics anxiety tend to also perceive mathematics situations as more anxiety inducing (Conlon et al., 2021; Demedts et al., 2022; Orbach et al., 2020). A major difference between state and trait achievement emotions is suggested to be the level of generalisation over time (Pekrun, 2006). Trait mathematics anxiety is generally measured with questionnaires that rely on retrospective memory of mathematical experiences, while state anxiety assessments occur in real-time during a mathematical situation (Bieg et al., 2015). Studies have shown that the relation that state and trait mathematics anxiety have with mathematical performance can differ depending on the measure of mathematics anxiety (Goetz et al., 2013; Orbach et al., 2019, 2020) and content of the task (Demedts et al., 2022; Punaro & Reeve, 2012; Trezise & Reeve, 2018). Thus, these two measures of mathematics anxiety may reveal different insights into the relation between mathematics anxiety and performance. Nevertheless, most mathematics anxiety research examines only trait mathematics anxiety, and less is known about state anxiety.

A meta-analysis showed that the strength of the relation between mathematics anxiety and performance can differ depending on the dimensions of trait mathematics anxiety measured within the questionnaire (Barroso et al., 2021). More specifically, measuring the cognitive component of worry or mathematics learning anxiety (i.e., anxiety related to learning situations) were more strongly related to mathematics achievement than studies measuring evaluation anxiety (i.e., anxiety related to mathematics tests). Similarly, Sorvo et al. (2017) found that primary school students' arithmetic fluency was rather related to anxiety about mathematics related situations than anxiety about failure in mathematics. It is possible that evaluation-related anxiety is less specific to the mathematics domain but rather related to all academic topics. Another issue with certain items on mathematics anxiety questionnaires is that they may relate to other forms of anxiety, such as test and social anxiety (e.g., items measuring anxiety during a mathematics test or when explaining problems in front of the class). However, Núñez-Peña et al., (2014) examined multiple mathematics anxiety questionnaires and found that equivalent results to longer questionnaires can be obtained by asking only one question: "On a scale from 1 to 10, how math anxious are you?". Thus, long questionnaires, which may tap into other anxiety domains, are not necessary for the measurement of mathematics anxiety.

Previous research has also defined questionnaire statements that include statements about anxiety in mathematics related situations to measure state-like anxiety (e.g., Balt et al., 2022; Orbach et al., 2020). These state-like anxiety measures were found to have a stronger relation with performance than measures of trait mathematics anxiety (e.g., Sorvo et al., 2017). However, often the statements in the state-like anxiety questionnaires are asked before the mathematics task in a format that relies on retrospective memory of previous mathematical events, such as "I get anxious when I start doing math" (Sorvo et al., 2017). Thus, these questionnaires could still be considered a trait measure rather than a state measure. A situational measure of anxiety (i.e., state anxiety) should measure real-time emotions specific to a task or activity, such as solving a mathematics task (Bieg et al., 2015). As state anxiety is a real-time assessment of a situation, it is suggested to be a more accurate indicator of mathematics anxiety and less subject to recall bias or subjective beliefs, such as gender stereotypes (Bieg et al., 2015; Goetz et al., 2013). The same cannot necessarily be said about state-like anxiety. As few studies have investigated real-time state anxiety reactions and their influence on mathematical performance, the differences between trait and state measures of mathematics anxiety have not been established.

Previous studies examining real-time state anxiety have also shown a stronger relation between mathematics performance and state anxiety than trait mathematics anxiety (Orbach et al., 2020; Punaro & Reeve, 2012). In addition, state anxiety responses have been found to vary when task characteristics, such as problem difficulty or time pressure, were manipulated (Demedts et al., 2022; Punaro & Reeve, 2012; Trezise & Reeve, 2018). For instance, Punaro & Reeve (2012) showed that primary school students had higher state anxiety for difficult mathematics problems than easy problems. This notion that task characteristics influence state experiences aligns with research in another motivation related framework, situational interest. Multiple factors have been shown to affect students' situational interest towards a mathematical task, including individual interest, task difficulty, and competence beliefs (e.g., Høgheim & Reber, 2019; Koskinen et al., 2023). Subsequently, more research should investigate how task characteristics affect the relation between state anxiety and mathematics performance. This may reveal insights into task-specific challenges within the learning of mathematics.

1.3 Mathematics anxiety and fraction knowledge

Meta-analyses have found that mathematics anxiety has a negative relation with mathematics performance across various mathematical skills (Barroso et al., 2021; Caviola et al., 2022; Namkung et al., 2019). Furthermore, mathematics anxiety was found to have a stronger relation with performance on advanced mathematical topics than foundational ones (Caviola et al., 2022; Namkung et al., 2019). This is suggested to be partially due to the higher cognitive requirements and reliance on prior mathematical skills. However, research on mathematics anxiety in primary school has mainly focused on whole number knowledge or examined mathematics achievement tests (e.g., Orbach et al., 2020; Sorvo et al., 2017), with limited knowledge on other topics relevant within the primary school curriculum. For instance, fraction knowledge is an important part of the mathematics curriculum, especially in primary school, and considered more difficult to learn than whole numbers (Siegler & Pyke, 2013). Difficulties with fraction learning could be a cause and consequence of mathematics anxiety. Before the current research, few studies had examined the relation between mathematics anxiety and fraction knowledge, for instance Starling-Alves et al. (2022) showed a negative relation between trait mathematics anxiety and fraction knowledge in late primary school. To expand prior research, the current dissertation investigated the relation between mathematics anxiety and performance across routine and non-routine mathematical skills relevant within the primary school curriculum, including fraction knowledge. Furthermore, it measured both trait and state mathematics anxiety to examine potential task-specific differences in experiences of mathematics anxiety.

Students with mathematics anxiety may struggle to adapt their initial number concept from whole numbers to also include the properties of fractions. Trait mathematics anxious individuals may avoid the effortful learning of mathematics (Ashcraft & Moore, 2009), such as fraction knowledge, due to the anxiety induced even by the anticipation of a mathematical situation (Lyons & Beilock, 2012). Furthermore, it is proposed that mathematics anxiety may indirectly affect mathematical attainment by interfering with other factors that support the learning of mathematical knowledge, such as executive functions (Ashcraft & Kirk, 2001; Ashcraft & Moore, 2009; Suárez-Pellicioni et al., 2016). For instance, mathematics anxiety may interfere with performance on more demanding mathematics tasks by overloading cognitive resources, such as working memory resources needed for strategy selection (Ramirez et al., 2013, 2016). If students do not have the cognitive capacity to apply effortful problem solving strategies, they may instead use fast intuitive reasoning and be more prone to natural number based reasoning. Thus, the current research examined whether there is a relation between mathematics anxiety and the presence of the NNB, especially in primary school students who are in the process of learning fractions. This would yield knowledge on factors that may contribute to difficulties with fraction learning.

The NNB became a concept of interest in the current dissertation also as a factor that could influence the relation between fraction knowledge and state anxiety. As previously mentioned, research on confidence ratings have suggested that in students with a strong NNB profile, the NNB is a naïve misconception (González-Forte et al., 2022). The current dissertation examines whether these findings are supported by students' state anxiety measured after the fraction task. If students with a strong NNB profile report low fraction state anxiety, this would be further evidence for students' unawareness of their incorrect answers and for the notion that the NNB is a misconception. This is because poor understanding or low performance typically relates to high anxiety (Namkung et al., 2019). Thus, examining state anxiety could yield new insights into the debate whether the NNB is a strategy students use or a naïve misconception (Alibali & Sidney, 2015; Merenluoto & Lehtinen, 2004). This would improve our understanding of the development of fraction knowledge in primary school students.

1.3.1 Mathematics anxiety and flexible mathematical skills

Mathematics anxiety may not be only relevant within the development of routine mathematical skills, but also a cause for individual differences in students' ability to apply those skills in various contexts. Previously it has been theorised that affective factors may explain individual differences in adaptive expertise with mathematics (Verschaffel et al., 2009) and mathematical flexibility (Ramirez et al., 2018). Yet, there is a lack of research on the role of mathematics anxiety in flexible mathematical skills, such as those relevant within the realm of rational number proficiency.

Mathematics anxiety could be one reason for the individual variation found in students' SFOR tendency. It has been theorised that mathematics anxiety may reduce engagement in informal mathematics experiences (Ramirez et al., 2018). Thus, mathematics anxious students may be less likely to pay attention to the mathematics around them. It is also possible that students with a predisposition for SFOR tendency are less likely to develop mathematics anxiety. Seeing the world through mathematical lenses creates opportunities to practise and hone one's mathematical skills in variable contexts (McMullen et al., 2019). This in turn may develop self-efficacy with mathematics and support students in understanding the value of mathematics outside the school context. It has been shown that improved competence with mathematical skills reduces mathematics anxiety (Balt et al., 2022; Geary et al., 2019; Gunderson et al., 2018) and higher self-efficacy relates to lower mathematics anxiety (e.g., Lee, 2009). In addition, Supekar et al. (2015) found that sustained exposure to mathematical stimuli through intensive cognitive tutoring can reduce mathematics anxiety. This is unsurprising as exposure-based therapy is successfully used to treat various anxiety disorders. Yet, the relation between SFOR tendency and mathematics anxiety has not been previously researched. Examining this relation could yield knowledge on individual differences in SFOR and whether frequent exposure to mathematics in various situations, for instance through self-initiated practise, could be a key to reducing mathematics anxiety.

There is also a lack of research on the role of mathematics anxiety in students' ability to make the complex connections between numerical characteristics required for adaptive rational number knowledge. On the one hand, high competence in mathematics generally relates to lower trait mathematics anxiety (Namkung et al., 2019) and high competence with rational numbers is needed for adaptive rational number knowledge (McMullen et al., 2020). Thus, it is possible that trait mathematics anxiety is not a component within adaptive rational number knowledge. On the other hand, it is one thing to succeed in well-rehearsed mathematical tasks and another to apply one's mathematical knowledge in a novel task. Another possibility is that encountering a non-routine mathematical task leads to experiences of heightened state anxiety, even in some students with adequate routine mathematical skills. In this situation, state anxiety may be induced by the perceived complexity of the task (Trezise & Reeve, 2018), required flexible mathematical thinking, such as creation of multiple solution strategies, and/or unease with the novelty of the situation. Thus, the current research examined both trait and state anxiety to ascertain what influences students' ability to apply their mathematical knowledge in various situations.

As there is no prior empirical evidence on the relation between mathematics anxiety and adaptive rational number knowledge, it is not known if the influence of mathematics anxiety is direct or indirect. Trait mathematics anxiety may have an indirect relation with adaptive rational number knowledge, if it hinders the development of mathematical skills that support adaptive rational number knowledge. These skills include SFOR tendency (McMullen et al., 2022), guided multiplicative relations (McMullen et al., 2022), routine whole number arithmetic knowledge (Bakker et al., 2022), and routine fraction knowledge (McMullen et al., 2020). For example, routine rational number knowledge builds the basis for adaptive rational number knowledge, while SFOR tendency may be relevant in the integration of rational number knowledge, such as procedural, magnitude, and representational knowledge. The latter would be expected to occur through the self-initiated practise with routine rational number knowledge in everyday situations (McMullen, Chan, et al., 2019). If students with high mathematics anxiety have weaker SFOR tendency, they may be less supported in the integration of their rational number knowledge. This emphasises the accumulative nature of mathematics knowledge and how mathematics anxiety may hinder the development of flexible mathematical skills in many ways, such as reducing prior knowledge attainment or the tendency to notice mathematical relations (e.g., SFOR tendency).

One objective of the current research is to fill the gap of limited knowledge on how mathematics anxiety manifests across mathematical skills relevant for primary school students. The current research expands prior findings by examining the role of mathematics anxiety in fraction arithmetic knowledge and flexible mathematical skills, such as SFOR tendency and adaptive rational number knowledge. It is also important to note that most studies examining the relation between mathematics anxiety and performance, especially in primary school, have examined trait mathematics anxiety. As previously mentioned, research has shown that the relation between state anxiety and performance may vary across task characteristics (Trezise & Reeve, 2018). Furthermore, state anxiety may have a different relation with performance than trait mathematics anxiety (e.g., Orbach et al., 2019; 2020). Thus, the current research also expands our limited knowledge on the potentially nuanced relations the two aspects of mathematics anxiety have with mathematics performance.

1.4 Preterm birth: The influence of cognitive abilities on mathematical skills

Cognitive abilities are important in the development of mathematical skills and shown to predict mathematical achievement (e.g., Best et al., 2011; Bull & Scerif, 2001). Executive functions, for instance, are involved in problem solving, strategy selection, and the inhibition of task irrelevant information or behaviour (Best et al., 2011; Miyake et al., 2000). Executive functions can be separated into three distinct, but interrelated variables: working memory, inhibition, and cognitive flexibility (Lehto et al., 2003; Miyake et al., 2000). Executive functions are suggested to support the activation of mental schemas, which are not automatically activated by the task (Agostino et al., 2010). Within rational number tasks, executive functions could support individuals in shifting from and inhibiting a previously relevant cognitive set or mental schema (i.e., natural numbers) and strengthen the activation of a currently relevant one (i.e., rational numbers). According to previous research, rational number knowledge is positively related to executive functions, especially inhibition (e.g., Avgerinou & Tolmie, 2020; Gómez et al., 2015) and working memory (e.g., Hecht & Vagi, 2010; Jordan et al., 2013; Siegler & Pyke, 2013). Executive functions have been shown to be relevant also for relational reasoning, such as reasoning about quantity (Blair et al., 2008; Bull & Scerif, 2001). The relational reasoning component of SFOR is likely to require executive functions for maintaining information while shifting attention between multiple aspects (e.g., colour, size, and quantity) to compare objects within a picture (i.e., in SFOR tasks) or in everyday environments. However, there is limited knowledge on the relation between cognitive abilities and less formal aspects of mathematical development, such as SFOR tendency.

Examining subpopulations of students that are known to have difficulties with mathematics due to lower cognitive abilities may yield novel insights into causes of individual differences in SFOR tendency and rational number proficiency. Recent reviews have found a higher prevalence of mathematical learning difficulties amongst children born preterm compared to full-term (Johnson et al., 2011; Sansavini et al., 2011; Taylor et al., 2009; Twilhaar et al., 2018). The lower mathematical attainment of children born preterm is shown to be associated with weaker cognitive abilities (Johnson et al., 2011; Sansavini et al., 2011; Simms et al., 2013; Taylor et al., 2009). However, studies examining the mathematical knowledge of children born preterm commonly conduct standardised achievement tests. This approach limits our knowledge on the relation between specific mathematical skills and cognitive abilities (Johnson et al., 2009). This is problematics because the impacts of cognitive abilities on the development of mathematical knowledge may be dependent on the skills measured. As previously mentioned, rational number knowledge, for instance, is an important cornerstone of later mathematical development and crucial within everyday life situations. However, the effect of preterm birth on rational number on or the ability to recognise mathematical features in everyday life (i.e., spontaneous mathematical focusing tendency) has not be examined. The current research expands prior findings on whether the impact of preterm birth on the development of mathematical knowledge differs across mathematical skills and depends on the cognitive abilities affected by preterm birth.

Children born preterm experience difficulties when there is an increase in the cognitive requirements of a task, for example the demand for executive functions (Wehrle et al., 2016). Thus, they may also have difficulties learning advanced mathematical topics due to the increased cognitive demands. Previous research has shown that the lower mathematical skills of children born preterm are related to their lower cognitive abilities, such as naming speed (Hannula-Sormunen et al., 2017; Korpipää et al., 2019; Rose et al., 2011), visuospatial processing (Clayton et al., 2021; Hannula-Sormunen et al., 2017; Simms et al., 2015), and executive functions (Clayton et al., 2021; Mulder et al., 2010; Rose et al., 2011; Simms et al., 2015). For instance, lower basic numerical skills of children born very preterm have been shown to be related to lower working memory and visuospatial processing (Simms et al., 2015). Similarly, previous studies on the same cohort of children born preterm, as examined within this dissertation, found lower cognitive abilities related to lower basic numerical skills at preschool age (Hannula-Sormunen et al., 2017), and lower basic numerical and arithmetic skills at the beginning of first grade (Korpipää et al., 2019). Based on these findings, lower domain-general cognitive abilities could also affect more advanced mathematical skills, such as the learning of rational number knowledge and the development of SFOR tendency in children born preterm.

Whole number skills may mediate the relation between cognitive abilities, such as working memory, and rational number knowledge (Hecht et al., 2003). For instance, whole number line estimation and arithmetic fluency predict rational number magnitude knowledge in primary school (Hansen et al., 2015; Jordan et al., 2013; Namkung & Fuchs, 2016; Vukovic et al., 2014; Ye et al., 2016). Previous research on children born preterm has found lower arithmetic fluency in earlier school years (Clark & Woodward, 2015; Pritchard et al., 2009; Short et al., 2003), including a study from the current cohort (Korpipää et al., 2019). Consistent with lower arithmetic skills, weaker counting skills have been found at primary school age (Korpipää et al., 2019; Pritchard et al., 2009; Simms et al., 2015) and even secondary school age (Clayton et al., 2021). In contrast, Rose et al. (2011) showed that children born preterm compared to full-term had lower performance on applied mathematical problems than whole number arithmetic computation. Children born preterm appear to catch up with children born full-term on whole number line estimation skills, as well as symbolic and non-symbolic whole number comparison skills by the age of eight years (Clayton et al., 2021; Guarini et al., 2014, 2020; Simms et al., 2015). Thus, it is possible that primary school aged children born preterm may have less difficulties with highly practised routine knowledge, such as whole number knowledge, but struggle with more complex mathematical knowledge, such as SFOR and rational number knowledge.

It is important to note that lower SFOR tendency might not only be an outcome of preterm birth, but also a cause of lower rational number development in children born preterm. SFOR tendency has been shown to be an important contributor to the development of rational number knowledge (McMullen et al., 2017; McMullen, Hannula-Sormunen, et al., 2016; Van Hoof et al., 2016). As previously stated, a higher tendency for spontaneous mathematical focusing is proposed to facilitate mathematical learning through self-initiated practice with mathematical skills in everyday situations (McMullen et al., 2017; McMullen, Chan, et al., 2019). As there is limited research on SFOR tendency, it is unclear whether lower cognitive abilities may hinder the development of SFOR tendency. The current research on children born preterm could also expand our understanding on whether similar cognitive underpinnings are related to the development of both SFOR tendency and rational number knowledge.

Overall, the current dissertation research will contribute to our understanding of under researched individual factors that may have unique contributions to the development of rational number knowledge. While the two factors, preterm birth and mathematics anxiety, may seem unrelated, they together highlight the complex nature of mathematical development and the multitude of involved factors, including affective and cognitive factors. Furthermore, research on both factors has mainly examined standardised achievement tests or been limited to only a few mathematical skills, especially when examined in primary school children. Thus, the current research expands prior research on mathematical challenges related to both individual factors, while also expanding our knowledge on potential challenges related to rational number development. In addition, the current research builds a basis for future research to examine interactions between cognitive and affective factors within the development of rational number knowledge and skills. For instance, mathematics anxiety is shown to have an indirect effect on mathematics achievement through its influence on executive functions, such as working memory and inhibition (e.g., Ashcraft & Kirk, 2001; Ramirez et al., 2013). Thus, the current research yields information for developing interventions that aim to improve rational number knowledge and/or the ability to apply mathematical knowledge in various situations, such as everyday environments (e.g., Määttä et al., 2022; 2024; McMullen, Hannula-Sormunen, et al., 2019). In general, research that examines subpopulations of students that are known to have difficulties with mathematics is valuable, as it may yield novel insights into individual differences in the development of mathematical proficiency.

2 Research aims

The current dissertation contributes to the field of mathematics education and educational psychology by expanding our knowledge on factors that may influence students' capabilities to build strong mathematical skills. The objective of this research is to examine how individual factors, specifically mathematics anxiety and preterm birth, relate to the mathematical skills of primary school students. While the general approach of the research is to understand individual differences in mathematical skills, the practical approach is to examine populations known to struggle with mathematics. First, the dissertation contributes to the scientific understanding of mathematics anxiety in mathematical development by expanding research to fraction arithmetic knowledge and non-routine tasks. Second, it yields knowledge on whether preterm birth related lower cognitive abilities influence the development of mathematical knowledge, including rational number knowledge and SFOR tendency. Together, these also contribute to our understanding of the challenges that students may have with flexible mathematical skills. Thus, the present dissertation consists of four studies that aim to:

1) Investigate the relation between mathematics anxiety and fraction knowledge related skills (Study I, II, III)

Mathematics anxiety is consistently found to correlate with lower mathematics achievement (Namkung et al., 2019), but this relation has been less researched in the domain of rational numbers, especially amongst primary school students. This is unfortunate as the challenging nature of fractions could be a tipping point for some students in the direction of mathematics anxiety. To shed light on how mathematics anxiety affects mathematics performance, it is important to consider differences between mathematical sub-domains. Study I examined mathematics anxiety across a wide range of tasks ranging from non-routine to routine and whole numbers to rational numbers. Surprisingly, Study I showed that the relation between mathematics anxiety, specifically state anxiety, and fraction arithmetic knowledge was not equivalent to the other tasks. Thus, Studies II and II examined whether the presence of a NNB may influence how mathematics anxiety manifests in fraction tasks by comparing two groups of students with low performance: those with and without a NNB.

2) Examine whether preterm birth related factors influence the development of rational number knowledge related skills (Study IV)

Children born preterm are known to known to struggle with mathematics and their lower mathematical abilities have been previously shown to be related to their lower cognitive abilities (Sansavini et al., 2011; Simms et al., 2015; Taylor et al., 2009). Nevertheless, there is limited knowledge on whether the mathematical deficits are global or specific to certain mathematical skills. For instance, preterm birth related lower cognitive abilities and early mathematical deficits may influence the development of later mathematical skills. However, the influence of preterm birth on the development of rational number knowledge has not been previously examined. Thus, Study IV compares children born very preterm and full-term on a wide range of mathematical and cognitive skills relevant within the development of rational number knowledge. As mathematics is cumulative in nature, it is important to understand the origins of mathematical challenges faced by students to avoid hindrances in later achievement.

3) To advance our understanding on factors influencing flexible mathematical skills. (Study I, IV)

To address the aim of flexible mathematical skills, it was investigated whether individual differences in affect and cognitive abilities play a role in students' flexible mathematical skills. The chosen flexible mathematical skills, more specifically SFOR and adaptive rational number knowledge, are relevant within the realm of rational number proficiency. Study IV expands our knowledge on the role of cognitive abilities and mathematics motivation in students' SFOR tendency. Study I examines the relation between both state and trait mathematics anxiety and performance on these flexible, non-routine tasks. These tasks also included a parallel version of the SFOR task, where the mathematical nature of the task is made explicit. This allows the examination of the role of mathematical explicitness in mathematics anxiety responses. In addition, it was not known if mathematics anxiety has a direct or indirect influence on flexible rational number knowledge. Thus, Study I also examines whether state and/or trait mathematics anxiety explain variance in performance on the adaptive rational number knowledge task, after accounting for relevant mathematical skills and knowledge. In general, the ability to apply one's mathematical knowledge beyond textbook related tasks should be a sought after outcome of primary education for all students.

3 Methods

The present dissertation consists of four studies that address individual differences in the rational number knowledge of primary school students. Studies I to III were part of a larger longitudinal research project referred to as ATOM, which is abbreviated from the Finnish words “Ajattelutaidot ja oppimismotivaatio” (a.k.a. Thinking skills and study motivation). The ATOM project was conducted as part of a larger research project Growing Mind, which is a consortium between the University of Turku, the University of Helsinki, and the University of Tampere. This research was funded by the Strategic research council of the Academy of Finland. The ATOM project examines the relation between mathematical skills and mathematics anxiety in fifth and sixth grade students over a period of one school year. Study I is a cross-sectional study including the whole sample and all measurement data, while Study II is a cross-sectional study focusing on a subset of the sample. Both studies used data collected at measurement point 1. Study III is a longitudinal follow-up study including data collected at measurement points 1 and 3 (six months apart). Study IV is part of the PIPARI research project, which follows children born preterm and full-term from birth to adulthood (https://sites.utu.fi/pipari/en). Study IV is a cross-sectional design examining how mathematics skills and related cognitive abilities have developed in children born very preterm compared to children born full-term by the age of 11 years. It also included retrospective analysis of group differences in early mathematical skills and cognitive abilities at the age of five years. All studies took place in Finland. The following subsections give an overview of the study samples, procedures, and analyses methods used in the four studies. These are also summarized in Table 1.

Table 1. Overview of the methods in the studies.

	Aims	Participants	Procedure and measures	Analyses
Study I	Examine how trait and state MA relate to performance across routine and non-routine mathematical tasks	*n* = 406, 50% female, mean age 11.89 years Fifth grade (*n* = 188) Sixth grade (*n* = 218)	Students completed a digital test battery with the following measures: • Trait & state MA • Mathematics tasks: SFOR, guided multiplicative relations, whole number arithmetic, fraction magnitude, fraction arithmetic, and adaptive rational number knowledge	• Between grade *t*-tests • Between groups repeated measures ANOVA (state anxiety) • Correlations and regression analysis for relations between MA and performance
Study II	Investigate how the presence of a NNB influences fraction state anxiety responses	*n* = 119, subset of the participants from Study I NNB group (*n* = 60) No-NNB group (*n* = 59)	Same procedure as Study I, but only the following measures: • Trait & state MA • Mathematics task: whole number and fraction arithmetic knowledge	• Between group *t*-tests • Two-way ANOVA (group x task type) for state anxiety
Study III	Explore how fraction state anxiety develops in students with and without a NNB	*n* = 334 at follow-up of Study I *n* = 83 for groups from Study II NNB group (*n* = 37) No-NNB group (*n* = 45)	A follow-up of Study II (six months later) on the following measures: • Performance and state anxiety for fraction arithmetic knowledge	• Two-way ANOVA (group x time) for state anxiety & performance • Paired *t*-tests for change over time
Study IV	Compare 11-year-old children born very preterm and full-term across a range of 5th grade mathematical skills and cognitive abilities relevant for rational number development	*N* = 136, fifth graders Preterm group (*n* = 64): 41% female and mean age 11.81 years Full-term group (*n* = 72): 56% female and mean age 11.76 years	Participants were individually tested on the following measures: • Mathematics: spontaneous mathematical focusing, arithmetic fluency, whole number and rational number magnitude knowledge, and mathematics motivation • Cognitive: visuospatial processing, verbal working memory, naming speed, inhibition, and shifting	• Between group *t*-tests • ANCOVA for group differences (controlling for cognitive abilities) • Correlations for performance between the mathematics tasks • Retrospective analysis of measures at age five years

3.1 Ethical considerations

Studies I, II, and III were conducted according to the ethical permission granted to the Growing Mind project by the ethics committee of the University of Helsinki. These three studies were part of the same longitudinal data collection. Students' parents received a detailed description of the structure of the study and parents' written consent was a prerequisite for a student's participation. However, the parent permission did not state anything about mathematics. Instead, the study was described to the parent as "Ajattelutaidot ja oppimismotivaatio" (a.k.a. Thinking skills and study motivation). As nothing in the study could be considered outside the realm of students' everyday classroom mathematical tasks and normal study motivation related questions, this was not considered unethical. Even though the study measured mathematics anxiety, it was not expected to induce any more anxiety than a normal school related mathematical testing situation. Furthermore, teachers and students participated voluntarily, and they were allowed to withdraw their participation at any time. Students were informed that participation or lack of would not affect them or their grade in any way. If students did not agree to participate, the teacher organized some other activity for the students to do during the time of the testing.

The data management in Studies I, II and III was the same, as they were all from the same longitudinal data collection. In all documents, student names are coded as participant code numbers. Prior to any analysis, the main researcher (in this case Hilma Halme) combined the personal data and measurement data behind the student's participant code, which was a participant number. Birthdates were coded to ages and rounded to avoid possibility of figuring out the original birthday, and classes had been given numbers from which the specific classroom cannot be identified (only whether it is a 5th or 6th grade class and there were at least 10 classes from each grade). Furthermore, gender data was included as it was not considered to be enough to identify a student with only gender and grade data available. Afterwards, personal information data and measurement data were stored in separate storage locations on protected servers with separate access codes for each folder. Only researchers who have signed the confidentiality contract have access to original documents or any material, which has personal data on individual teachers or students. All the original documents are stored on protected servers, which abide by the EU data protection protocol (i.e., GDPR). During the analysis, only the participant numbers were used. After the main publications, the data will be annotated and opened in electronic form for the research community.

Study IV was accepted by the Ethics Review Committee of the Hospital District of Southwest Finland Ethics Committee of University of Turku, and all required research permissions were gathered by the PIPARI study group, as part of their longitudinal data collection of children born preterm. Permission forms were gathered at two time points, when the longitudinal research project started and when the data for this specific study was collected. This included written consent from the participants and their parents. During coding of the data, all identifying information had been removed or coded in an unidentifiable way. For instance, participants' birthdates were coded as the age rounded to two decimals. The participants' personal data (i.e., neonatal medical information) and measurement data was stored behind their participant number. As medical information is not available to the general public, it cannot be used to narrow down and identify a participant. Only the testers, who collected the data, and the PIPARI study group has knowledge of the participants identities. Due to the PIPARI study protocol and the ongoing longitudinal nature of the research, the data used within this study was not opened to the research community.

In general, the dissertation abided by the TENK guidelines and the University of Turku policies (https://www.utu.fi/fi/tutkimus/avoin-tiede/aineistot).

3.2 Participants

Participants in all the studies were primary school students attending school in Finland. In Finland, children start primary school in the fall of the year when they turn seven years. Primary school consists of grades one through six.

Studies I, II, and III were based on a longitudinal dataset, which included 412 primary school students, with 188 students attending the fifth grade (50% female) and 218 attending the sixth grade (51% female). The students were from 27 classrooms with Finnish as the teaching language. In both grades, approximately one-third of the students had completed the ongoing academic year's fraction instruction before the first measurement point. The studies differed slightly on the number of participants depending on the focus of each study and the number of participants with missing data from the dependent variables. A detailed description of the participants can be found in Table 1.

Study II and III focused on two specific subgroups of participants, specifically those who exhibited to have a clear NNB (NNB groups) or no signs of a NNB (No-NNB group). The categorisation was done according to the participants' answers on the fraction arithmetic task. The NNB group consisted of the participants who has answered all the fraction arithmetic items with a natural number based procedure. The No-NNB group consisted of the participants who had low performance but had not answered any fraction arithmetic items with a natural number based procedure. Only 13 participants in the No-NNB group had all eight items incorrect, so the criteria for the No-NNB group was expanded to allow the two same denominator items (i.e., 1/3+1/3 & 3/4 - 1/4) to be correct. Thus, the participants in the No-NNB group had a maximum of two out of eight fraction items correct and all the other items had to be incorrect due to some other reasoning than the NNB. Given that diverse developmental trajectories of anxiety have been found in different subgroups of students (e.g., Klee & Miller, 2019), the dichotomous distinction between learners with and without a NNB was considered a necessary strength of the study.

In Study III, the same NNB and No-NNB groups were followed up as in Study III. However, this study focused on students' fraction knowledge development towards a more correct understanding of fraction knowledge (i.e., overcoming the NNB misconception). Thus, the NNB group included only the participants who had reduced in their number of NNB answers between the first and last measurement points. The No-NNB group remained the same. In both groups, some participants did not have data from the follow-up measurement due to not finishing the test, being sick on the day of measurement, or not wanting to participate. Analysis of the whole sample was also included in study III to examine the general development of fraction arithmetic knowledge and fraction state anxiety.

Study IV was based on a dataset collected as part of a longitudinal research project PIPARI. The participants were Finnish speaking children who had participated in mathematics-related testing sessions at the age of five years (i.e., before primary school), as part of the research project. From the 11-year-old sample, we excluded children born very preterm who had been retained to ensure that there were no grade-related differences in knowledge. In addition, we excluded children with a full-scale intelligence quotient (FSIQ) below the average range (i.e., less than 70), because children with a very low FSIQ are more likely to have global rather than specific deficits. For more specific details on inclusion and exclusion criteria, see Halme et al. (2022). A detailed description of the participants can be found in Table 1.

3.3 Measures

Studies I, II, and III were part of the same data collection and all measures were conducted using a digital test battery. In Study IV, all the mathematical and cognitive tasks were paper and pen tasks, unless otherwise specified. From Study IV, in this section will only be described the mathematics and cognitive measures reported from the testing at the age of 11 years, as these are mainly discussed in this dissertation. The testing at the age of five years included different mathematics and cognitive measures and the detailed descriptions can be found in Halme et al. (2022).

Both studies I and IV included measures of spontaneous mathematical focusing, either spontaneous focusing on numerosity (SFON) and/or spontaneous focusing on multiplicative relations (SFOR). The most important factor in the measurement of spontaneous mathematical focusing is ensuring that the participants are not aware of the mathematical nature of the tasks prior to testing and are not externally made aware of it during testing. Without explicit knowledge of the mathematical nature of the task, it can be assumed that the mathematics (e.g., multiplicative relations) described by the participants in the task are self-initiated by the participant in a spontaneous manner (Hannula & Lehtinen, 2005; McMullen et al., 2014; McMullen, Chan, et al., 2019).

3.3.1 Mathematics anxiety measures in studies I, II and III

Mathematics trait anxiety (only Study I and II) was examined with a questionnaire that included nine statements of anxiety within mathematics situations. There were six statements about general mathematics anxiety that were adapted from Sorvo et al., (2017) (i.e. I feel anxious about mathematics; I get anxious when I have to do mathematics exercises; I get anxious when I have to do mental arithmetic; I get anxious when I have to start mathematics homework; I feel tension, when I have to do mathematics tasks; I am worried that I will not learn mathematics) and three equivalent statements modified to be about fractions (i.e. I am anxious about fractions; I get anxious when I have to do fraction tasks; I am worried that I will not learn fractions). These statements were selected as they should not have characteristics that overlap with social or test anxiety. The faces scale used for measuring trait mathematics anxiety was continuous from 1 to 5 (i.e., not at all to very much) with faces that changed from less anxious to more anxious, see Figure 1. The faces scale was adapted from Punaro & Reeve (2012).

As all the nine items loaded onto one component within the whole sample and in the two groups (NNB and No-NNB), the average score for the nine items was used as the measure for trait mathematics anxiety. Cronbach's alpha was 0.94 for the whole sample.

State anxiety was assessed after each task (except fraction magnitude knowledge) with a prompt to evaluate one's level of anxiety "How anxious were you while doing the previous task?" The question was adapted from Trezise & Reeve (2018). This was measured on the same continuous scale as trait mathematics anxiety (see Figure 1).

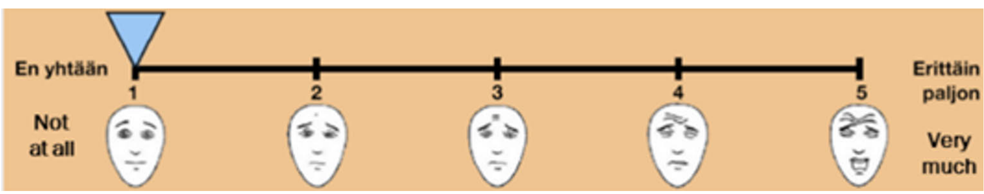

Figure 1. The faces scale used for measuring mathematics anxiety.

3.3.2 Mathematics measures in studies I, II, III

The measures were conducted in the order they are listed. To ensure that the participants were not aware of the mathematical nature of the tasks, SFOR was the very first measure of the digital test batter, even before trait mathematics anxiety.

SFOR tendency (only Study I) measured a student's ability to recognize and describe multiplicative relations embedded in everyday-like context, when not guided to do so. It was measured using two items from the teleportation task and two items from the plate task (McMullen et al., 2016). The participant had exactly two minutes for each item. Each item included objects that need to be compared (Figure 2) e.g., "*Describe how the objects have changed [or meals differ]. Describe in as many ways as possible.*" Students could focus their attention on multiple features within each picture (e.g., colour, shape, object type, quantity). One point was given for each description of a multiplicative relation (e.g., three times as many cans or 1/4 is pizza) whether it was mathematically accurate or not. Cronbach's alpha was 0.60.

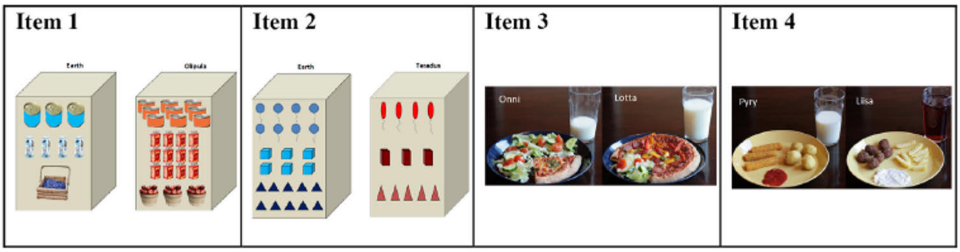

Figure 2. The items used for measuring SFOR tendency.

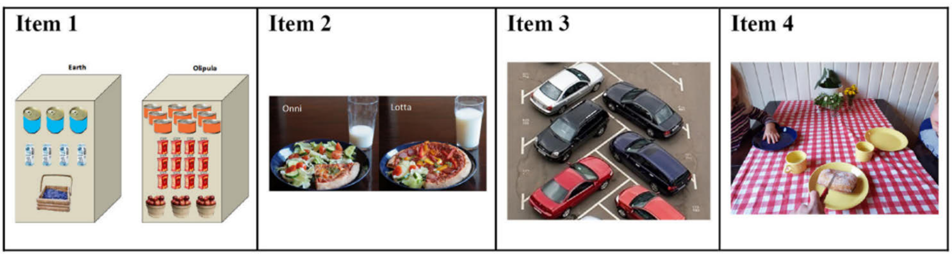

Figure 3. The items used for measuring guided multiplicative relations.

Guided multiplicative relations (only Study I) measured the students' ability to apply their knowledge of multiplicative relations (including fractions) in an everyday context when guided towards the mathematical nature of the task. The items included one teleportation task and one plate task item from the SFOR task, and two novel items of photos of everyday situations, shown in Figure 3. The task instruction guided students towards the mathematics, but not specifically multiplicative relations, for example "*Compare mathematically how the objects have changed…*”. The participant had exactly 90 seconds for each item. The scoring was identical to the SFOR task, with not maximum limit per item (obtained scores 0 to 6 points). Cronbach’s alpha was 0.53.

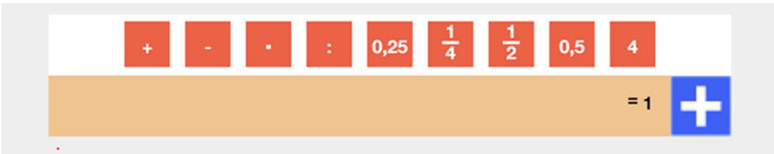

Figure 4. An example item from the task measuring adaptive rational number knowledge.

Adaptive rational number knowledge (only Study I) measures students ability to recognise numerical characteristics and arithmetic relations between rational numbers (McMullen et al., 2020). It was assessed with a rational number version of the arithmetic sentence production task (McMullen et al., 2016; 2020; 2022). Participants first had a practice item with only whole numbers and then two test items with rational numbers. The test items included two pairs of equivalent fractions and decimals (e.g., ½ and 0.5, ¼ and 0.25), a single whole number (e.g., 4), and a target number (e.g., 1), with an example item shown in Figure 4. The participant could use each number and operation repeatedly. The participant had exactly 120 seconds to form as many valid arithmetic sentences as they can that produce a target number. One point was given for each mathematically correct arithmetic sentence. The digital test did not allow an identical arithmetic sentence to be produced twice. There was no maximum limit of answers per item. The used measure was the sum score for the two items, with a Cronbach’s alpha of 0.80.

Fraction magnitude knowledge (only Study I) was assessed using an ordering task. There were five items in which students ordered three fractions from smallest value to largest. The items were all shown at the same time. The participant had exactly 120 seconds to complete the task, receiving one point for each correct answer. Cronbach’s alpha was 0.87.

Whole number arithmetic knowledge (only Study I and II) was assessed with a fill-in-the-blank task, which is considered a routine whole number task for Finnish students of this age. There were six items in which the student had to solve what number was missing (e.g., 27-2 = 50 / x). The items were all shown at the same time. The participant had 150 seconds to complete the task, receiving one point for each correct answer. Cronbach’s alpha was 0.86 for the sample in Study I.

Fraction arithmetic knowledge included eight items of addition and subtraction with fractions, including two same denominator items and six different denominator items, see Figure 5. The items were all shown at the same time. The participant had 150 seconds to complete the task, receiving one point for each correct answer. Cronbach's alpha was 0.84 for the sample in Study I.

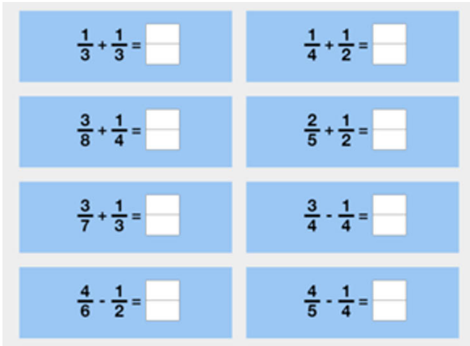

Figure 5. The items used for measuring fraction arithmetic knowledge.

3.3.3 Mathematics related measures in study IV

The following mathematical measures were conducted at the age of 11 years (i.e., fifth grade) in the order they are listed. As aforementioned, it was crucial that the measures of spontaneous focusing (i.e., SFON and SFOR) were conducted first to ensure that the participants were not aware of the mathematical nature of the tasks. As aforementioned, descriptions of the mathematical skills measured at the age of five years can be found in Halme et al., (2022).

SFON was assessed using a modified version of the Model task (Hannula & Lehtinen, 2005). There were three trials in the task: a zebra, a leopard, and a crocodile. In each trial, the tester drew 11 stripes, 13 dots or 12 teeth respectively on an A4-sized outline of the animal. The tester's drawing was the model that was shown to the participant and then turned upside down on the table. Then, the participant drew their animal to look exactly like the model one. Afterwards, the participant was interviewed on what he or she had paid attention to on the task. The maximum score for the task was three points, with one point from each trial. Cronbach's alpha was 0.68

SFOR was measured using the teleportation task (McMullen et al., 2016), see Figure 2 items 1 and 2. In the task, the participant was asked to describe how a set of three common objects changed during a transformation (i.e., teleportation to another planet). The materials all change colour, shape, and number of items in a uniform way. On the first trial, participants were asked to describe the transformation. On the second trial, there was a different amount of the same original objects. This time the participant was asked to draw what they expected would arrive based on the previous trial. This procedure was repeated with a new set of three objects. Points were given for each multiplicative relation mentioned or drawn in the four trials with a maximum of three points per trial and 12 points for the task. Cronbach's alpha was 0.74.

Guided focusing on quantitative relations was measures to assess whether participants were able to describe the multiplicative relations imbedded in the SFOR tasks, when asked to do so. It was completed after the SFOR task. Participants were given the first item from the teleportation task and instructed to "describe how the items were divided" (McMullen et al., 2016). This task was used to ascertain that the participants had the skills needed for the SFOR task.

Arithmetic fluency was measured using the Woodcock-Johnson standardised mathematics fluency subtest (Woodcock et al. 2001). Participants had three minutes to complete as many single-digit basic arithmetic problems (i.e., addition, subtraction, and multiplication) out of a two-page set of 160 items. One point was given for each correct answer, with the sum score used as the measure.

Mathematical achievement was assessed with the fifth grade version of the Lukilasse (Häyrinen, Serenius-Sirve & Korkman, 1999). It is a curriculum-based standardized mathematics test consisting of various types of problems, including items with fractions and decimals. The maximum score was 21 points for the test. Cronbach's alpha was 0.68.

Rational number magnitude knowledge was assessed with a 12-item rational number test adapted from Stafylidou & Vosniadou (2004), consisting of six items of fraction and decimal comparisons and six ordering items (e.g., ''Circle the larger fraction. If the numbers are equal circle both: 5/8, 4/3"; ''Put the numbers in order from smallest to largest: 6/8, 2/2, 1/3"). The maximum score was 12 points. Cronbach's alpha was 0.70.

Whole number line estimation measured proportional reasoning with whole numbers. It was measured with the number line estimation task (Siegler & Opfer, 2003), in which the participant estimated the place of 22 numbers on a number line ranging from 0 to 1000. The absolute error (%) score was calculated from the difference between the estimated and actual position of the number. Cronbach's alpha was 0.62 with the 22 items, and it would have improved with fewer items, but not changed the results. Thus, the average absolute error (%) score calculated from all items was used.

Mathematics motivation was assessed with a shortened Finnish version of the Fennema–Sherman Mathematics Attitude Scales (Metsämuuronen, 2009). Participants answered statements related to four aspects of mathematical attitudes: liking mathematics (5 items), self-concept in mathematics (5 items), utility in mathematics (5 items), and mathematics anxiety (3 items). The scale included 18 items with scoring from 1 to 5 points (i.e., disagree to agree). The average score across all items was calculated. Cronbach's alpha for the scale was 0.90.

3.3.4 Cognitive abilities measured in Study IV

The following cognitive measures were used to examine whether lower mathematical skills in the children born very preterm at the age of 11 (i.e., fifth grade) could be a consequence of lower cognitive abilities. As aforementioned, descriptions of the cognitive abilities measured at the age of five years can be found in Halme et al., (2022).

Naming speed, inhibition, and shifting were assessed using the NEPSY-II inhibition subtest (Korkman et al., 2008). In the task, the participant was given a paper with a series of white and black circles and squares. The participant was asked to either name the shapes (naming speed), the opposite shape (inhibition) or shift between alternating responses (shifting), as quickly as possible. For naming speed, the time for completion was used. For inhibition and shifting, the time for completion and number of errors were recorded and a standardised score was calculated according to the NEPSY-II guidelines.

Verbal working memory was measured using the backwards digit span subtest from the WISC-IV (Wechsler, 2003). In the backwards condition, a series of digits were read by the tester, and the participant was asked to repeat the digits in reverse order. The used measure was the total number of series recalled correctly.

Visuospatial processing was assessed with the block recall subtest from the WMTB-C (Pickering & Gathercole, 2001). The tester tapped on identical wooden blocks that are attached to a board. The participant repeated the sequences. The total number of correct sequences was recorded.

3.4 Statistical analyses

This section describes the statistical analyses used within this dissertation. The studies all consisted of quantitative data that was analysed using IBM SPSS Statistics (version 27 or higher). Table 2 outlines the specific statistical tests used, the studies the test was used in, and the purpose of the statistical test.

Table 2. Overview of the statistical analyses used in this dissertation research.

Statistical test	Study	Purpose
Chi-square test	II, IV	Compare group differences on demographic variables (e.g., gender)
Cronbach's alpha	I, II, III, IV	Determine internal consistency of measures: trait MA questionnaire (Study I, II), mathematical knowledge measures (Study I, II, III, IV), mathematics motivation questionnaire (Study IV)
Independent samples *t*-test	I, II, IV	Compare group differences on dependent variables: 5th vs 6th (Study I), NNB vs No-NNB (Study II), children born very preterm vs full-term (Study III)
Paired samples *t*-test	III	Examine change over time for fraction performance and fraction state anxiety
Cohen's *d*	I, II, III, IV	Determine effect size for group differences (Study I, II, IV) or change over time (Study III)
Pearson correlation	I, III	Examine the relation between state and trait MA (Study I), and their relation to mathematical skills (Study I, III)
	IV	Examine the relation between mathematical measures
Two-way ANOVA	I	Examine differences in state anxiety across the mathematical tasks between two groups (5th and 6th grade)
	II	Examine differences in state anxiety (fraction vs whole number) between groups (NNB vs No-NNB)
	III	Examine group differences (NNB and No-NNB) in the development of fraction state anxiety and fraction performance (separately)
ANCOVA	IV	Compare groups on mathematical tasks while controlling for cognitive measures
Hierarchical regression analysis	I	Examine whether state anxiety explained unique variance in performance on a mathematical task after controlling for trait MA and background variables
	I	Examine whether state and/or trait MA explained unique variance in performance on ARNK after controlling for performance on other measured mathematical tasks and background variables

Note. MA = mathematics anxiety, NNB = natural number bias, ANOVA = analysis of variance, ANCOVA = analysis of covariance. To account for familywise error rate, alpha adjustments were done using the Bonferroni-Holm procedure (Holm, 1979) for Pearson correlations, univariate ANOVAs, and two-way ANOVAS to correct for multiple comparisons.

4 Overview of studies

4.1 Study I

Halme, H., Trezise, K., Hannula-Sormunen, M. M., & McMullen, J. (2022). Characterizing mathematics anxiety and its relation to performance in routine and adaptive tasks. *Journal of Numerical Cognition*, *8*(3), 414–429. https://doi.org/10.5964/jnc.7675

Mathematics anxiety and performance relations have been shown to vary depending on the content of the mathematics task (Orbach et al., 2020; Trezise & Reeve, 2018). Yet, little is known how mathematics anxiety manifests in some subdomains relevant for primary school mathematical development, such as fraction arithmetic knowledge. Furthermore, there is limited knowledge on the role of mathematics anxiety in tasks that vary from routine textbook tasks, such as in SFOR or adaptive rational number knowledge (McMullen et al., 2019). Thus, the aim of this study is to investigate the relation between trait and state mathematics anxiety across five different tasks, ranging from whole numbers to rational numbers and routine to non-routine knowledge.

The participants were 406 primary school students attending the 5th grade (n = 188, 51% female, mean age 11.42 years) and 6th grade (n = 218, 50% female, mean age 12.39 years). The participants completed a digital test battery with the measures presented in the following order: SFOR tendency, trait mathematics anxiety, guided multiplicative relations, adaptive rational number knowledge, fraction magnitude knowledge, whole number arithmetic knowledge, and fraction arithmetic knowledge. State anxiety was measured after each task except for fraction magnitude knowledge.

The results showed that state anxiety responses across tasks were lower for the SFOR task than all the other tasks. State anxiety responses did not differ between the whole number and fraction arithmetic tasks. The results in Table 3 show that trait mathematics anxiety was negatively correlated with state anxiety for each measure. However, the relation between task performance and mathematics anxiety was not similar for the two aspects of mathematics anxiety. For SFOR tendency and guided multiplicative relations, trait mathematics anxiety and task-specific state anxiety were not related to task performance. Performance on the fraction arithmetic task was negatively related to trait mathematics anxiety, but it was not significantly related to task-specific state anxiety. Both trait mathematics anxiety and task-specific state anxiety were negatively related to performance on whole number arithmetic knowledge and adaptive rational number knowledge. In addition, regression analysis showed that state anxiety explained unique variance in performance on whole number arithmetic knowledge and adaptive rational number knowledge, even after controlling for trait mathematics anxiety.

Table 3. Pearson correlations between trait mathematics anxiety, state anxiety, and performance for each task (*N* = 406) with the examined relations in bold.

	1	2	3	4	5	6	7	8	9	10
1. Trait anxiety	-									
2. SA SFOR	**.35*****	-								
3. SA Guide	**.53*****	.54***	-							
4. SA Whole number	**.51*****	.33***	.50***	-						
5. SA Fraction	**.44*****	.35***	.49***	.65***	-					
6. SA ARNK	**.57*****	.47***	.64***	.66***	.54***	-				
7. SFOR	**-.03**	**.01**	.06	-.06	.03	-.08	-			
8. Guided	**-.08**	-.02	**-.10**[a]	-.17***	-.05	-.14**	.47***	-		
9. Whole number	**-.20*****	-.04	-.03	**-.43*****	-.01	-.23***	.24***	.36***	-	
10. Fraction	**-.17*****	-.04	-.02	-.14**	**-.06**	-.12*	.07	.26***	.37***	-
11. ARNK	**-.16****	-.03	-.06	-.26***	-.04	**-.27*****	.25***	.33***	.50***	.41***

Note. SA = state anxiety; Guided = guided multiplicative relations; Whole number = whole number arithmetic knowledge; Fraction = fraction arithmetic knowledge; ARNK = adaptive rational number knowledge. *p < 0.05 ** p < 0.01 ***p < 0.001; a not statistically significant after Bonferroni correction.

Regressions analyses were also conducted to examine whether state or trait mathematics anxiety explained unique variance in students' adaptive rational number knowledge, after controlling for performance on relevant mathematical measures (i.e., SFOR, guided multiplicative relations, whole number arithmetic knowledge, fraction magnitude knowledge and fraction arithmetic knowledge). Trait mathematics anxiety did not explain unique variance beyond relevant mathematical knowledge, but state anxiety did, even if trait mathematics anxiety was included in the model.

These results confirm that mathematics anxiety responses and their relations with performance differ across mathematical tasks in primary school students. This may be due to differences in task characteristics. For instance, mathematics anxiety was related to performance on tasks with mathematically explicit content and tasks with symbolic quantities rather than non-symbolic quantities. Furthermore, the unexpected finding that fraction arithmetic performance was not negatively related to state anxiety could be due to the NNB. NNB answers accounted for over a third of the answers across the fraction arithmetic items. These students may not have realised their fraction answers were incorrect leading to low performance and low state anxiety. This possible incongruity in some students could have caused the non-significant relation between fraction performance and state anxiety. Thus, the role of the NNB was examined in Study II and Study III.

In addition, mathematics anxiety, especially state anxiety, may explain why some students are unable to apply their mathematical knowledge in a novel, flexible mathematical task, such as the adaptive rational number task. The unique contribution of state anxiety beyond prior mathematical knowledge may reflect students' difficulties with integrating conceptual and procedural knowledge of fractions and decimals (McMullen et al., 2020). Thus, supporting previous research that state anxiety may reflect concurrent task appraisals, such as anxiety induced by task difficulty (Trezise & Reeve, 2018). Then again, state anxiety may reduce students' ability to apply complex procedural strategies that require working memory resources (Caviola et al., 2017; Ramirez et al., 2016). Due to the correlational nature of the study, the current findings cannot claim a causal direction for the relation between mathematics anxiety and performance.

4.2 Study II

Halme, H., Van Hoof, J., Hannula-Sormunen, M., & McMullen, J. (2023). Not realizing that you don't know: Fraction state anxiety is reduced by natural number bias. *British Journal of Educational Psychology*. https://doi.org/10.1111/bjep.12637

The natural number bias (NNB) is a common fraction misconception referring to the tendency to incorrectly overgeneralise natural number reasoning to fraction tasks (e.g., Ni & Zhou, 2005). If students with a strong NNB profile are not aware that they are answering fraction items incorrectly, they may exhibit lower state anxiety on a fraction task (referred to as fraction state anxiety). This could explain why Study I found no significant correlation between fraction arithmetic performance and state anxiety (Halme et al., 2022). The main aim of this study was to examine the relation between mathematics anxiety (trait and state) and the NNB. More specifically, whether the NNB could explain the anomalous relation between fraction performance and fraction state anxiety. This relation has not been previously examined in primary school students.

The participants in this study were a subsample of Study I, consisting of 119 participants with or without a NNB related answering profile on the fraction arithmetic task. The students were categorized as belonging to an NNB group (n = 60) if all fraction arithmetic items were answered with an NNB answer. The No-NNB group (n = 59) consisted of low performing students with a maximum of two correct and all other answers incorrect due to some other reasoning then the NNB (see participants section for specifics). The following measures were included in this study: trait mathematics anxiety, fraction arithmetic performance and state anxiety, and whole number arithmetic performance and state anxiety.

Independent samples t-tests showed that the NNB group had lower fraction state anxiety than the No-NNB group (p = .006, d = .52). There were no group differences in trait mathematics anxiety. A two-way ANOVA showed a significant group (NNB vs NO-NNB) by task (fraction vs whole number) interaction for state anxiety. The NNB group reported significantly lower state anxiety for fraction arithmetic than the whole number arithmetic (p = .001), while the No-NNB group had the opposite (p = .04), as shown in Figure 6. Importantly, the groups did not differ on their performance or state anxiety on the whole number arithmetic task. This emphasises how the NNB only affected the relation between performance and state anxiety on the fraction task. In addition, the non-significant relation found between fraction performance and state anxiety in the whole group was re-examined. After excluding all students with any level of NNB (i.e., inclusion criteria NNB score = 0), there was a small, yet significant negative correlation between fraction performance and state anxiety. Notably, most of the students in the NNB group had not had the ongoing year's fraction instruction.

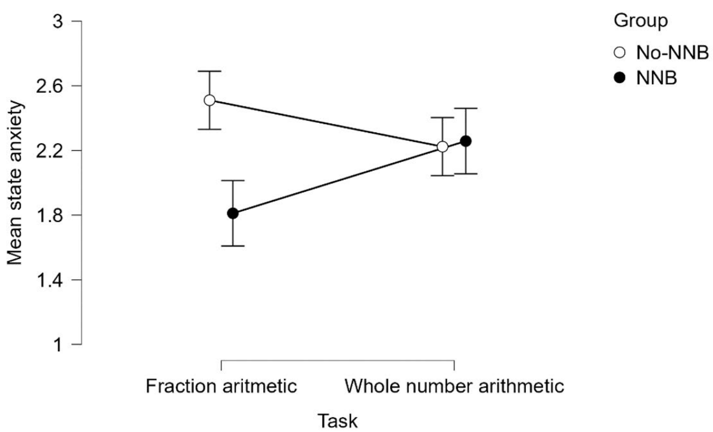

Figure 6. The group by task type interaction for state anxiety. The error bars represent 95% confidence intervals.

The results confirm that subgroups of fraction learners with qualitatively different reasons for low performance can differ in their fraction state anxiety. While performance and state anxiety on the whole number task were equivalent between the groups, the NNB group had significantly lower fraction state anxiety than the No-NNB group. Having a fraction performance score of zero and low fraction state anxiety, as in the case of the NNB group, contrasts with what would be expected within mathematics anxiety research. These results suggest that students with the NNB appear to be unaware that they are answering the fraction task incorrectly, in line with previous research (González-Forte et al., 2022; Merenluoto & Lehtinen, 2004). If the NNB group had lower state anxiety on both task than this could imply a low anxiety answering profile, but this was not the case. The reason for the lower anxiety could be a lack of negative experiences with fractions due to not having had the concurrent academic year's fraction instruction, further supporting a more naïve understanding. As trait mathematics anxiety levels did not differ between the groups, the results indicate that state anxiety can reveal qualitative differences in low performers and their task-related perceptions that cannot be captured by trait mathematics anxiety. In addition, the findings suggest that the development of fraction state anxiety may differ in students with an NNB profile. This was examined in Study III.

4.3 Study III

Van Hoof*, J., Halme, H.*, Hannula-Sormunen, M., & McMullen, J. (preprint, 2023). When anxiety grows with knowledge: the role of the natural number bias. https://osf.io/preprints/psyarxiv/9pt7d

Study II showed cross-sectionally that the NNB may affect students' state anxiety responses on a fraction arithmetic task. However, the longitudinal development of fraction state anxiety in students with an NNB profile had not been previously examined. Thus, the aim of this longitudinal study was to examine the relation between the NNB and the development of fraction state anxiety, especially in students who (partially) overcome the NNB. To examine whether a change in fraction state anxiety in students with a NNB profile is related to overcoming a misconception or just generally low performance, the study compares two groups of low performers with or without an NNB profile.

This study is a follow up of the students from study II. The participants were 334 fifth and sixth grade students, with a specific focus on the same two low performing subgroups of students: the NNB group (n = 37) and the No-NNB group (n = 45). To examine students who at least partially overcame the misconception, the NNB group only consisted of students who reduced in their number of NNB answers over time.

The measures included in this study were students' fraction arithmetic performance and fraction state anxiety measured in November and May of the same school year.

A two-way ANOVA showed that there was an interaction between group (NNB and No-NNB) and time (time point 1 and 2) for fraction state anxiety, as depicted in Figure 7. More specifically, the NNB group significantly increased in fraction state anxiety between the measurement points, while the No-NNB group significantly decreased in fraction state anxiety (both $p < .001$). There was a main effect of time for fraction performance, but no interaction, with the pairwise comparisons showing that both groups increased equivalently in their fraction performance, as depicted in Figure 7. Thus, the two groups had similar fraction performance development, but opposite directions in their state anxiety development. In addition, the study examined the general development of fraction state anxiety and performance (separately) in the whole group of students ($n = 334$). Paired samples t-tests showed that fraction performance significantly increased over time, but fraction state anxiety did not significantly decrease. Excluding the NNB group from the whole group analysis led to a significant decrease in anxiety over time. Last, correlation analyses showed that while fraction performance and state anxiety did not correlate at time point 1 in the whole sample analysis, a significant negative correlation was found at time point 2.

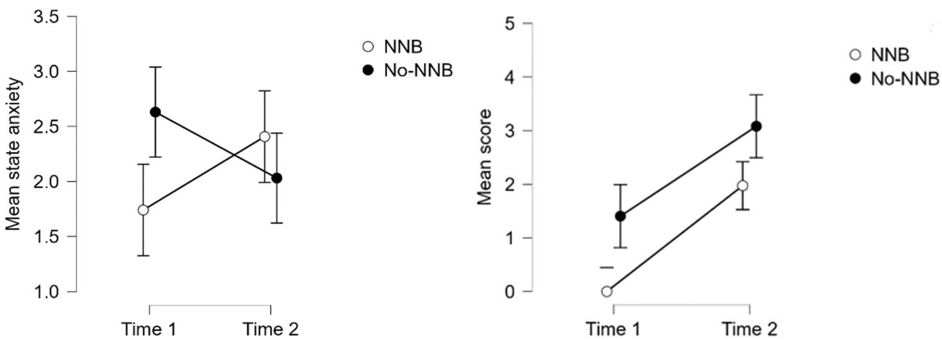

Figure 7. The development of fraction state anxiety (left) and performance (right) for the two low performing groups. The error bars represent 95% confidence intervals.

The findings show that a better understanding of a mathematics topic may sometimes increase mathematics anxiety, in contrast to previous research (Balt et al., 2022). The increase in fraction state anxiety was specifically related to overcoming a misconception, and not a consequence of prior low performance, as indicated by the No-NNB group. This confirms the hypothesis that the NNB group was initially unaware of their misconception and thought they were answering correctly, and when they became aware of their misconception their state anxiety increased. The proposed explanation is that after the concurrent academic years fraction instruction, the students in the NNB group learn that a natural number based answer is not sufficient for solving fraction tasks. As they realised that fractions are more difficult than they thought, their fraction state anxiety increased. This led to students' fraction state anxiety levels becoming equivalent across low performers at time point 2, resulting in a negative relation between fraction performance and state anxiety within the whole sample. This further emphasises that the presence of the NNB influences the performance-anxiety relation. Likewise, the decrease in state anxiety over time in the whole group was hidden, because of the increase in state anxiety in students with the NNB. These findings have implications for intervention research as improved performance may not always lead to reduced mathematics anxiety.

In general, the results of Study III further support the findings from Study II that 1) the NNB partially explains the lack of a relation between fraction performance and state anxiety, 2) the proposition that students' subjective performance perceptions influence their state anxiety responses, 3) the importance of examining subgroups with qualitative and not only quantitative performance differences, as these may change the interpretations of whole group analysis.

4.4 Study IV

Halme, H., McMullen, J., Nanu, C. E., Nyman, A., & Hannula-Sormunen, M. M. (2022). Mathematical skills of 11-year-old children born very preterm and full-term. ***Journal of Experimental Child Psychology*****,** ***219*****, 105390. https://doi.org/10.1016/j.jecp.2022.105390**

Preterm birth is a known risk-factor for poorer academic achievement, especially in mathematics (Taylor et al., 2009; Twilhaar et al., 2018). Most studies examining primary school children born preterm conduct standardised achievement tests (Twilhaar et al., 2018) and only a few studies have measured specific mathematical skills that could be affected by preterm birth (e.g., Guarini et al., 2014, 2020; Simms et al., 2015). For instance, rational number knowledge or spontaneous mathematical focusing tendencies have not been previously examined. This study aimed to expands knowledge on 1) mathematics skills affected by preterm birth, and 2) whether weaker mathematical skills are explained by lower cognitive skills or mathematics motivation.

This study compared 11-year-old children, with an FSIQ above 70, born very preterm ($N = 64$) and full-term ($N = 72$) on a range of 5th grade mathematical skills and cognitive abilities related to rational number development. The mathematics measures included SFON, SFOR, arithmetic fluency, mathematics achievement, number line estimation, rational number magnitude knowledge. The cognitive measures included visuospatial processing, inhibition, shifting, verbal working memory, and naming speed. Furthermore, mathematics motivation was assessed, as difference is motivation are known to affect mathematics achievement. Additional analysis included retrospective group comparisons of the early mathematical skills and cognitive abilities at the age of five years.

The results showed that the children born very preterm compared to the children born full-term had significantly lower arithmetic fluency, SFON, and SFOR. However, the children born preterm had peer-equivalent rational number (including fraction and decimal items) and whole number magnitude knowledge, mathematics achievement, and mathematics motivation. There were significant group differences in cognitive abilities, specifically in visuospatial processing and naming speed. After controlling for cognitive abilities, only group differences in SFON and SFOR remained significant. Retrospective comparisons of the samples at the age of five years showed large group differences in early mathematical skills and cognitive abilities, for instance, the children born preterm had significantly lower counting skills and visuospatial processing.

The children born very preterm had grade equivalent performance in many mathematical skills by the age of 11 years, despite their lower early mathematical skills and lower visuospatial processing, present already at the age of five years. The current results support previous evidence on peer-equivalent whole number magnitude skills (Guarini et al., 2014, 2020; Simms et al., 2015) and expand to show that rational number magnitude knowledge is also peer-equivalent at the age of 11 years (i.e., fifth grade). However, the children born preterm were less likely to focus on mathematical features embedded in everyday contexts (i.e., lower SFON and SFOR). The lower spontaneous mathematical focusing tendencies may cause children born very preterm to be less likely to apply their mathematical knowledge in everyday situations (Hannula and Lehtinen, 2005; McMullen et al., 2014). Nevertheless, the children born very preterm are proposed to have reached peer-equivalent formal mathematical due to their age-equivalent verbal executive functions and the additional academic support they have received in Finland (Nyman et al., 2019). This supports previous studies that showed a relation between lower mathematical skills and lower executive functions in children born preterm and full-term (e.g., Bull & Scerif, 2001; Simms et al., 2015, Twilhaar et al., 2020). However, it is important to note that the current study examined only those children with an intelligence quotient above 70 who had not been retained; thus, these results may not generalise to all children born very preterm.

5 Main findings and discussion

The main aim was to investigate whether mathematics anxiety and preterm birth can reveal novel insights into the challenges that primary school students face in developing mathematical proficiency with rational number knowledge. Importantly, a wide range of skills is needed for mathematical proficiency ranging from routine mathematical skills to the flexible application of mathematical knowledge in novel tasks. In general, the findings of this research emphasise the importance of examining specific mathematical skills, as they reveal nuances in task-specific interactions that would not be captured by more general mathematics achievement tests. Furthermore, the findings highlight the value of examining subgroups of students with varying individual differences, as not all students have similar developmental trajectories. These novel findings provide further evidence on multiple aspects: 1) mathematics anxiety-performance relations can differ depending on the task and measure of mathematics anxiety, 2) task-specific perceptions can influence state anxiety responses, 3) the NNB as a misconception, and 4) routine mathematical knowledge does not guarantee the ability to apply the knowledge in novel mathematical tasks. The implications of these findings yield new insights and support previous evidence in the development of rational number understanding and show that not all mathematical skills are necessarily affected similarly by a specific individual factor. Consequently, these findings emphasise that more research should consider task related differences and multiple ways of measuring a concept (e.g., mathematics anxiety).

Mathematics anxiety, misconceptions, and fraction performance

One of the aims of the current research was to examine the relation between mathematics anxiety and fraction knowledge related skills. The results of Study I show that high mathematics anxiety relates to lower mathematical performance, including fraction knowledge, already in primary school. However, the relation between mathematics anxiety and performance differed across the tasks and the two measures of mathematics anxiety: trait and state mathematics anxiety. First, Study I showed that state anxiety responses differed across tasks, supporting other studies (Demedts et al., 2022; Trezise & Reeve, 2018). It also expanded previous research to show that state anxiety is significantly higher for tasks with explicit mathematical content. However, state anxiety responses were not higher for fraction arithmetic than whole number arithmetic. This contrasts previous research that has found task difficulty to affect state anxiety responses (Trezise & Reeve, 2018) or has shown that both students and adults have lower attitudes towards fractions than whole numbers (Sidney et al., 2021). Second, Study I showed that high performance on the whole number arithmetic was related to low trait and state mathematics anxiety, as was expected based on previous research on trait mathematics anxiety (e.g., Mononen et al., 2022; Sorvo et al., 2017). Unexpectedly, low performance on a fraction arithmetic task was only related to higher trait mathematics anxiety, but not state anxiety. The lack of a significant relation between fraction performance and state anxiety was proposed to be due to a fraction related misconception, namely the NNB.

The role of the NNB was researched in Studies II and III to find an explanation for the anomalous relation between fraction arithmetic performance and state anxiety. These studies focused specifically on two low performing subgroups of students, those with a NNB (NNB group) or without a NNB (No-NNB group). Based on previous research, low performance generally correlates with high mathematics anxiety (Namkung et al., 2019). Thus, it would be expected that the two low performing groups would have similar levels of state anxiety. However, the NNB group had significantly lower state anxiety than the No-NNB group. This finding of lower state anxiety is proposed to support previous evidence from confidence ratings that the NNB is a naïve misconception, and that students are unaware of their incorrect answers and low performance (e.g., González-Forte et al., 2022). This proposition is further supported by the finding that state anxiety and performance on the whole number arithmetic task did not significantly differ between the NNB and No-NNB groups. Thus, there was a performance-state anxiety discrepancy only on the fraction arithmetic task. As there was no significant group difference in trait mathematics anxiety, these findings also emphasised the relevance of examining state anxiety to capture qualitative differences in fraction understanding.

Study III was a follow-up of Study II examining whether at least partially overcoming the NNB misconception (i.e., reduction in NNB answers) after fraction instruction would influence fraction arithmetic related state anxiety. The results showed that when students in the NNB group reduced in the number of NNB answers over time, their fraction state anxiety significantly increased. To examine whether this increase is related to low performance or overcoming a misconception, the development in fraction state anxiety was contrasted with the low performing No-NNB group. Interestingly, the No-NNB group significantly decreased in their fraction state anxiety over time. This difference in state anxiety transitions could not be explained by the development in performance only, as performance increased similarly in both groups. The proposed explanation is that, after at least partially overcoming the misconception, the students in the NNB group became more aware of their incorrect answers, leading to an increase in state anxiety. This increase in state anxiety could be related to an accumulation of failure experiences during fraction instruction and a subsequent reduction in self-efficacy with fraction arithmetic (Dowker et al., 2016). Importantly, Study III yields further evidence that the initially low state anxiety is due to the NNB being a naïve misconception. In addition, the results reveal that fraction learning can increase fraction state anxiety in a group of learners who need to overcome a fraction related misconception. This finding shows that improved mathematics performance may not lead to lower mathematics anxiety, in contrast to evidence from intervention studies (Balt et al., 2022).

Investigating the role of the NNB brought new perspectives into the role of students' subjective task-specific perceptions in their state anxiety responses. Previous research has shown that state anxiety is related to task characteristics, such as task difficulty and pressure (e.g., Demedts et al., 2022; Trezise & Reeve, 2018). However, it could not be differentiated whether state anxiety interferes with performance or low performance induces state anxiety. The current results yield evidence that a student's perception of the correctness of their answers influences their state anxiety responses. Even though the NNB group had all answers incorrect on the fraction arithmetic items, they reported less fraction state anxiety than whole number state anxiety. One explanation is that natural number based addition and subtraction require less cognitive resources, such as working memory, than the whole number arithmetic task used in this study. In contrast, the No-NNB group experienced the fraction arithmetic task as more anxiety inducing than the whole number arithmetic task, in line with their performance on each task. This contrast between groups suggests that the NNB group had low initial fraction state anxiety due to an incorrect appraisal of their low performance and the demands of the task. Once students became more aware of their incorrect reasoning and that fraction arithmetic is difficult, their state anxiety increased and became more aligned with their performance. This is further supported by the result that students' state anxiety levels increase with a decrease in NNB answers, as shown in Study II. This means that changes in subjective task-specific perceptions, such as awareness of performance or task difficulty, appear to influence state anxiety.

The findings from studies II and III also emphasise the importance of accounting for subgroups when examining the mathematics anxiety-performance relations. This is especially important in tasks where qualitative understanding (i.e., conceptual understanding) and quantitative performance may be less aligned, such as in the case of misconceptions. For instance, two groups of low performing fraction learners may have similar quantitative performance but a different qualitative reason for the low performance. Studies II and III showed how whole group analysis in Study I had led to an incorrect interpretation of the mathematics anxiety-performance relation regarding fraction arithmetic. The non-significant relation between fraction performance and state anxiety found in Study I became a weak negative relation when students in the NNB group were excluded in Study II. Furthermore, Study III showed that, at the end of the school year, when the misconception was at least partially overcome by almost all students, there was a significant negative relation between fraction performance and state anxiety within the whole sample. This alignment of performance and state anxiety could have been due to the fraction instruction that occurred between the initial and final measurement time points. Thus, the anomalous fraction performance and state anxiety relation found for the whole sample in Study I was at least partially explained by the presence of student with a fraction misconception, namely the NNB. This suggests that the relation between performance and state anxiety reflects subjective task appraisals and perceptions of performance that are influenced by task-specific characteristics. The findings of these studies have implications for mathematics anxiety research and the teaching of mathematics, especially in topics that require students to adapt their conceptual understanding.

Flexible mathematical thinking

Another aspect of this dissertation research goes beyond performance in textbook adjacent tasks towards non-routine tasks that capture students' ability to apply mathematical knowledge in novel tasks and even contexts outside of the classroom. Flexible mathematical knowledge should be the sought-after outcome of mathematics education for all learners (Gravemeijer et al., 2017; Opetushallitus [OPH], 2014), regardless of their individual differences. Yet, there is limited knowledge on how individual factors, such as mathematics anxiety or lower cognitive abilities, influence the development of flexible mathematical thinking.

The current research showed that mathematics anxiety does not only negatively affect performance on routine tasks, but it also influences the ability to apply mathematical knowledge in novel situations. More specifically, Study I showed that mathematics anxiety reduced students' performance on the task measuring adaptive rational number knowledge. Students with either high trait or state mathematics anxiety had low performance on the task. However, trait mathematics anxiety did not contribute to performance beyond routine mathematical knowledge, which explained significant variance in adaptive rational number knowledge. It is likely that low mathematical performance in the routine tasks already accounted for the variance that would be explained by higher trait mathematics anxiety, implying an indirect relation. Notably, high state anxiety related to low performance, even after controlling for relevant mathematical skills. This indicates that some students may become anxious (i.e., report state anxiety) when they need to apply their mathematical knowledge in novel situations, regardless of their level of mathematical skill or trait mathematics anxiety.

Due to the correlational nature of Study I, the direction of the relation between state anxiety and task performance cannot be determined within this study. There can be various reasons for the relation between high state anxiety and low adaptive rational number knowledge. Based on the deficit theory (e.g. Carey et al., 2016), state anxiety may reflect deficits in skills due to poorly integrated conceptual and procedural knowledge of fractions and decimals. Integrated knowledge is considered a key factor for high performance on the task (McMullen et al., 2020). From the perspective of the debilitating model, state anxiety could be preventing students from using cognitively demanding problem solving strategies (Caviola et al., 2017; Ramirez et al., 2016). In this case, the novel nature of the task, a characteristic of non-routine tasks, could be the cause of the state anxiety. Nevertheless, these findings expand prior research on factors that may influence students' adaptive rational number knowledge (McMullen et al., 2020; 2022). Furthermore, the findings yield valuable insight for mathematics educators and researchers on the role of mathematics anxiety, especially state anxiety, in students' ability to use mathematical skills in novel contexts.

Examining the other non-routine tasks gave insight into the potential direction of the relation between mathematics anxiety and performance. For instance, neither state nor trait mathematics anxiety was related to performance on the SFOR task. The specific characteristics of this task require that the student is not explicitly guided towards the mathematical nature of this task, or in other words, the student is unaware that the task requires mathematics. The guided multiplicative relations task is similar to the SFOR task, but in this case, the students are explicitly asked to describe the mathematics in the pictures. The guided multiplicative relations task induced significantly more state anxiety than the SFOR task implying that the mathematical context was more anxiety inducing than the task content. In addition, in both tasks, performance is scored based on the students' descriptions of multiplicative relations, but the students are unaware of this. Therefore, the students' perception of their own performance may have misaligned with the scoring of the task. This could explain why the students' performance and state anxiety did not relate to each other on these tasks. Furthermore, the relation between state anxiety and performance was strongest for the whole number arithmetic task, which students are highly familiar with and should be able to objectively assess their own performance. This further supports that performance appraisals are a component within students' state anxiety responses. Thus, examining multiple task types, including non-routine tasks, yielded more understanding on how students' task perceptions (i.e., mathematical context) and performance appraisals relate to their state anxiety responses.

There were also nuanced relations within the mathematical skills of the children born very preterm that could not be fully captured by the measured cognitive abilities. Study IV shows that, by the fifth grade, the children born very preterm had peer-equivalent performance on many of the mathematical tasks, including whole number and rational number magnitude processing. This is noteworthy, considering the lower early mathematics skills at the age of five years; however, it supports and expands previous findings from whole number magnitude skills (e.g., Guarini et al., 2014; 2020; Simms et al., 2015). Yet, the children born preterm had lower performance on arithmetic fluency and non-routine tasks that required the application of mathematical knowledge in a novel context. More specifically, they had difficulties to spontaneously focus on mathematical features embedded in non-explicitly mathematical contexts (i.e., SFON and SFOR). The lower SFON and SFOR were not fully explained by group differences in cognitive abilities, specifically visuospatial processing and speeded naming, or mathematics motivation. Likewise, Study I showed that mathematics anxiety, an aspect related to mathematics motivation, does not have a significant relation with SFOR tendency. One reason for the lower spontaneous mathematical focusing tendencies, especially SFOR, could be difficulties with timed assessment due to slower fact retrieval (e.g., multiplication facts). This is supported by the lower arithmetic fluency, the lower early counting skills, and previous research (e.g., Clayton et al., 2021; Simms et al., 2015). While more research is needed to ascertain the cause of the lower flexible mathematical skills, these results further highlight that some students struggle to apply their mathematical skills in novel contexts. Furthermore, an important finding that not all mathematical skills are affected equivalently by an individual factor, replicating the pattern seen in the mathematics anxiety studies.

5.1 Theoretical implications

The findings have implications for educational psychology research, especially within the domain of mathematics learning. First, they contribute to mathematics anxiety research by showing the importance of examining mathematics anxiety across mathematical tasks and measures of anxiety, as the relations may differ due to task related characteristics. Not understanding this may lead to misinterpretations of findings and overseeing potentially valuable insights, such as the possibility of distinct subgroups. Thus, the findings brought out methodological considerations that should be accounted for when examining the relation between mathematics anxiety and performance. Second, the findings support and expand on previous rational number research by showing that the NNB can be a naïve misconception and that overcoming the NNB during fraction learning may induce anxiety rather than reduce it. Lastly, the findings add to previous evidence on individual differences that may be involved in students' ability to apply their mathematical knowledge in novel contexts, such as non-routine mathematical tasks. These differences can be related to individual characteristics or situational factors. In general, the findings highlight that different mathematical skills can have unique relations with other factors.

5.1.1 Implications for mathematics anxiety research

The current research shows that mathematics anxiety is related to mathematics performance already in primary school. However, the relations between mathematics anxiety and performance differed across tasks. Mathematics anxiety was related to performance on tasks with mathematically explicit content and tasks with symbolic quantities rather than non-symbolic quantities. Similarly, Starling-Alves et al. (2022) found mathematics anxiety to more relate to symbolic than non-symbolic mathematical tasks. One reason may be that symbolic quantities are more common within the formal mathematics instruction at this age (Starling-Alves et al., 2022). Together these findings indicate that mathematics anxiety is specific to the mathematical content of the task, yielding more evidence that subjective task appraisals influence responses to the task. This highlights the importance of examining mathematics anxiety across multiple tasks, as these nuanced relations would not be visible if mathematical skills are only assessed with standardised achievement tests and composite scores.

It has been proposed that state anxiety is less influenced by subjective competence beliefs than trait mathematics anxiety; thus, state anxiety is suggested as a more accurate indicator of students' mathematics anxiety (Bieg et al., 2015). The premise for the claim by Bieg et al. (2015) comes from research within gender differences in mathematics anxiety and gender stereotypes in competence beliefs. This proposition is not fully supported by the findings from studies II and III, as state anxiety responses were influenced by students' subjective competence beliefs of their fraction performance. More specifically, in some students, state anxiety reflected incorrect performance appraisals that were caused by a fraction misconception, the NNB. Previous research has also shown that state anxiety responses are influenced by concurrent task appraisals, such as task difficulty (Conlon et al., 2021; Trezise & Reeve, 2018). Furthermore, the subjective competence beliefs captured by state anxiety were not reflected in trait mathematics anxiety, indicating that the two measures of anxiety may reflect different nuances within the anxiety-performance relation. This could explain why state and trait mathematics anxiety are found to have different relations with mathematics performance (e.g., Orbach et al., 2019; 2020). While the current findings are not completely comparable with those of Bieg et al. (2015), the differences propose that several types of subjective competence beliefs may influence each measure of mathematics anxiety. However, this should be further examined before reaching conclusions.

This research cannot yield conclusive evidence towards the direction of the relation between performance and mathematics anxiety (Carey et al., 2016). However, the findings imply that a student’s subjective performance appraisals affect their state anxiety. Students with low awareness of their poor fraction performance had low fraction state anxiety, while improved awareness of their low performance led to higher fraction state anxiety. Likewise familiarity with the demands of the task led to a stronger relation between state anxiety and performance. This can be seen when comparing the relation between state anxiety and performance on the whole number task with the relation on the SFOR task. This provides evidence that performance perceptions or appraisals are the driving factor in the relation between state anxiety and performance, yielding nuances to previous research on the deficit theory (e.g., Carey et al., 2016). It is important to note that these subjective performance appraisals could be influenced by many factors other than awareness of low performance, such as perceptions of value (Pekrun, 2006), uncertainty or ambiguity tolerance (Merenluoto & Lehtinen, 2004), and even trait mathematics anxiety. To determine causality, a systems-oriented perspective might be necessary to fully understand the magnitude of components interacting within these relations (Pekrun et al., 2017), including performance appraisals, task characteristics and individual factors. Nevertheless, research on the relation between state anxiety and performance may explain some ambiguities that exist within the current theoretical models (Carey et al., 2016) that are mainly based on findings from trait mathematics anxiety.

It has been shown that trait mathematics anxiety correlates with state anxiety (Conlon et al., 2021; Demedts et al., 2022; Orbach et al., 2020), also in the current research. However, the results of the current studies combined with evidence from previous research (e.g., Conlon et al., 2021) imply that the two types of mathematics anxiety reflect different information. For instance, changes in state anxiety within the NNB and No-NNB groups appeared to be independent of the students’ initial trait mathematics anxiety, which was equivalent between the groups. Likewise, students without high trait mathematics anxiety can experience debilitating state anxiety in novel mathematical tasks. These cases of heightened levels of state anxiety are noteworthy, because it is possible that increased state anxiety eventually leads to increased trait mathematics anxiety. Spielberger et al. (1983) suggested that students who experience intense state anxiety frequently may be more likely to develop trait anxiety. If so, making sure that students have metacognitive learning strategies to deal with incongruent information or novel task types could prevent the development of mathematics anxiety (Di Leo et al., 2019). This could be especially relevant in the case of learning new mathematical topics, such as rational number knowledge. For further evidence, it should be investigated whether high task-specific state anxiety may lead to an increase in trait mathematics anxiety.

The current research also highlights methodological issues in examining mathematics anxiety-performance relations. It showed that the presence of students with an NNB interfered in the whole sample interpretation of the relation between fraction performance and state anxiety. This highlights the importance of being aware that subgroups may exist and affect the relation between mathematics anxiety and performance, especially in topics where there are known misconceptions. If the current research had only examined whole group relations, as was done in Study I, the interpretation of the relation between fraction arithmetic performance and state anxiety would have been that there is no relation. Furthermore, Study III showed that the change in state anxiety was not equivalent between the two subgroups. Thus, not accounting for qualitative differences between subgroups would have led to inaccurate interpretations of quantitative relations. This could also occur when applying longitudinal analysis, such as cross-lagged panel models, without accounting for subgroups. This is also relevant for intervention research (Balt et al., 2022), as there could be inaccurate evaluations of the accuracy of the intervention effects. For example, an intervention that improves fraction performance may lead to a decrease in mathematics anxiety in some students, but not all. Notably, this issue could be mainly with state anxiety, as trait mathematics anxiety was more related to quantitative low performance. Nevertheless, examining differences between quantitative and qualitative understanding could help unravel the underlying causal relation between mathematics anxiety and performance. What type of low performance increases mathematics anxiety and what type of mathematics anxiety reduces performance? This would support the development of learning materials, teaching, and interventions.

5.1.2 Implications for rational number research

The current findings support previous evidence that the NNB is a naïve misconception, and students are unaware of their incorrect answers (e.g., González-Forte et al., 2022; Merenluoto & Lehtinen, 2004; Reinhold et al., 2020). Previous research has shown that students' unawareness of their own poor understanding can lead to high confidence ratings (González-Forte et al., 2022) and fast reaction times (Reinhold et al., 2020), and according to the current research low state anxiety. Notably, the previous research on confidence ratings has examined older students (González-Forte et al., 2022). As the current research on state anxiety is only in students who are still learning fraction arithmetic, these results should be confirmed with older students. It is possible that the low state anxiety is only present in the initial stages of learning. Nevertheless, the current findings suggest that state anxiety could be a useful instrument for measuring changes in conceptual understanding in topics where students present with strong misconceptions, such as fraction knowledge. The situational and task-specific nature of state anxiety allows the measurement of changes that may occur within a short period of time, such as overcoming a misconception. To build a theoretical foundation, it would be valuable to examine whether the lower state anxiety is specific to fraction arithmetic tasks, or a phenomenon found across tasks with misconceptions. In addition, future studies should measure both confidence ratings and state anxiety to improve measurement validity.

The dual processing theories suggest that people tend to use fast intuitive reasoning when encountering a task and only sometimes apply slow effortful analytical reasoning (Vamvakoussi et al., 2012). When students first encounter a fraction task, they need to overcome their fast intuitive natural number based reasoning to solve the fraction task correctly. This can be an effortful process in the initial learning phase, until students learn to conceptualise fractions as one magnitude. As mathematics anxiety is suggested to overburden working memory resources (Ramirez et al., 2013), it could reduce students' ability to inhibit the incorrect, intuitive natural number based reasoning. However, Study II showed that the NNB answers were not a consequence of higher trait or state mathematics anxiety in the NNB group compared to the No-NNB group. This further supports that the NNB is related to unawareness and naïve understanding rather than a consequence of mathematics anxiety in primary school students. However, this should be confirmed with a larger sample. Furthermore, it does not rule out that the NNB could not be related to mathematics anxiety in older students. It should be researched whether mathematics anxiety is related to the persistence of the NNB or vice versa.

The findings also show that students may have different developmental pathways in reaching age-equivalent rational number knowledge in primary school. For instance, the lower SFOR in the children born very preterm did not appear to hinder their development of rational number magnitude knowledge. This contrasts with previous research that suggests SFOR tendency to relate to the development of rational number knowledge (McMullen, Hannula-Sormunen, et al., 2016). However, it has never been claimed that SFOR is a prerequisite for rational number magnitude knowledge only that it may support the development. It is possible that the developmental trajectories between SFOR tendency and rational number knowledge are different for children born preterm than the average primary school student. The children born very preterm also had lower visuospatial processing already at the age of five years, yet this did not significantly impede their whole number or rational number magnitude knowledge development. One explanation is that the currently examined preterm sample compensated with their age-equivalent verbal executive functions, such as verbal working memory. Their verbal working memory may have, for instance, supported them in benefitting from the additional academic support they received in the Finnish education system (Nyman et al., 2019). The current research supports previous findings on the pivotal role of executive functions in the mathematical development of children born preterm and full-term (e.g., Bull & Scerif, 2001; Clayton et al., 2021; Twilhaar et al., 2020), and highlights their relevance in rational number learning (e.g., Hecht & Vagi, 2010; Jordan et al., 2013).

Examining two under researched individual factors yielded novel insights into the flexible mathematical skills of primary school students. First, the ability to spontaneously focus on mathematical relations was not influenced by trait or state mathematics anxiety, but it was influenced by preterm birth. The current findings also expanded on previous research to show that individual differences in SFOR tendency were not completely explained by differences in mathematics anxiety, mathematics motivation, or the cognitive abilities measured within Study IV. However, it is important to note that the lower spontaneous mathematical focusing of children born preterm could have an underlying cognitive cause not measured within the current research. Second, it was shown that state anxiety hindered students from performing well on a novel, non-routine task measuring adaptive rational number knowledge, even in individuals without high trait mathematics anxiety. This confirms the relevance of affective factors in adaptive expertise with mathematics, as previously proposed (Verschaffel et al., 2009). However, the influence of state anxiety on performance appears to depend on the differences between the measured non-routine mathematical tasks. This provides additional empirical evidence for the prior proposition that task characteristics (Trezise & Reeve, 2018) and task perceptions (Bieg et al., 2015) are relevant factors within mathematics performance interactions. For instance, whether a task is perceived as mathematical or not, such as SFOR tasks. Third, these findings highlight that routine mathematical skills and the ability to apply mathematical knowledge in novel tasks do not necessarily develop together. To support the development of flexibility with school mathematics, further research is needed on tasks requiring flexible mathematical skills, including their cognitive requirements and the influence of affective factors, such as state anxiety.

5.2 Educational implications

The current findings support previous research that mathematics anxiety is already present in primary school and has detrimental effects on students' mathematical performance on a variety of mathematical tasks. Furthermore, they show an important yet unexpected finding that the learning of mathematical knowledge, in this case fraction arithmetic knowledge, may lead to an increase in state anxiety. It is also possible that the difficulties faced with learning fractions could in some students contribute to the development of trait mathematics anxiety. This might not be expected by teachers as previous research has suggested learning to decrease mathematics anxiety (e.g., Balt et al., 2022). Notably, not every student in the NNB group had initially low state anxiety or increased in state anxiety. It should be further researched whether this is due to teaching methods or individual differences in metacognitive strategies, for instance how one deals with frustration and challenges (Di Leo et al., 2019). Mathematics teachers and educators need this knowledge to improve formal teaching to ensure all students learn the necessary mathematical skills and avoid the development of anxiety. In addition, it should be researched whether overcoming a misconception leads to increased state anxiety in other topics and subjects known to elicit misconceptions, such as statistics or physics (e.g., Vosniadou, 2013).

Teachers and learning material developers should also be made more aware of misconceptions, such as the NNB, that may occur when students expand their mathematical understanding and number concept. Increased awareness could yield earlier identification and more support for students to overcome the NNB already in the initial stages of fraction learning. Considering that these students have been in contact with fractions since the third grade, it is highly problematic that a considerable proportion still use NNB answers in the sixth grade. This may be a curriculum issue, as same denominator fraction arithmetic problems are the first to be introduced in the fourth grade, and within these adding the numerators together is enough to solve the task, supporting natural number based reasoning. Consequently, more attention is needed on the way that fractions are taught to make sure all learners appropriately adjust their number framework when moving from whole numbers to fractions. An accumulation of successful learning experiences in mathematics improves self-efficacy and reduces mathematics anxiety (Dowker et al., 2019; Schunk & DiBenedetto, 2020), while negative teacher-related experiences may increase students mathematics anxiety (Ramirez et al., 2018). Thus, there should be more focus on improving primary school teachers' content knowledge of rational numbers (Depaepe et al., 2015). Future research should also investigate how anxiety towards a topic, such as fractions, affects learning and performance on other tasks that require knowledge from that topic, such as algebra or even physics.

The current findings emphasized that it is important to teach mathematical skills beyond the ability to solve routine textbook exercises. It was shown that students with adequate routine mathematical skills can still have difficulties with applying their mathematical knowledge in non-routine tasks. These difficulties can have different underlying causes dependent on the individual challenges. For instance, some students experience state anxiety that hinders them from applying their rational number knowledge in a novel arithmetic task. This state anxiety during novel mathematical tasks could also become a potential barrier to learning new mathematical topics. Other students may not struggle with mathematics anxiety, but they may have lower cognitive abilities. For instance, the children born very preterm caught up with their peers on most routine mathematical skills, but not in their ability to apply mathematics in everyday-like situations. As routine tasks require less cognitive resources with increased mastery, cognitive difficulties may manifest in the application of knowledge in non-routine tasks that are less practised. These findings show educators that there are a multitude of underlying challenges within learning mathematical skills and knowledge. If flexible mathematical skills are the sought-after outcome of mathematics education for all learners (Gravemeijer et al., 2017), this should also be a focus within mathematics teaching.

Avenues for supporting mathematical learning and flexible skill development within the mathematics classroom could include versatile instructional materials, teaching and allowing various solving strategies, and mixing task content, for example teaching fractions and decimals together. It has previously been shown that frequent exposure to mathematics in various situations could be a key to reducing mathematics anxiety (Supekar et al., 2015). Even though the current findings did not find higher SFOR tendency to relate to lower mathematics anxiety, it is important to note that the SFOR task did not trigger state anxiety. This is relevant information for mathematics educators, as interventions aimed at enhancing spontaneous mathematical focusing are successful in supporting both mathematical focusing tendencies as well as mathematics skills within the classroom (Hannula-Sormunen et al., 2020; Määttä et al., 2022; 2024; McMullen, Hannula-Sormunen, et al., 2019). These interventions could be particularly useful for children born preterm, to expose them to everyday mathematics. Another option could be game-based learning environments that allow students to reflect on various alternative strategies and solutions, as they are shown to support flexible mathematical skill development (e.g., Brezovszky et al., 2019; McMullen et al., 2023). This is also an interesting avenue for examining whether strengthening the ability to assess multiple solution strategies could reduce state anxiety during non-routine tasks. In general, it is important to further research how to promote routine and flexible mathematical skills in a way that does not trigger anxiety towards mathematics.

5.3 Limitations and future research

There were potential limitations in the measurement of mathematics anxiety that need to be acknowledged within this study. One limitation was the use of a researcher designed trait mathematics anxiety measure that had not been previously validated.

It was deliberately decided to only include statements that should not tap into social or test anxiety. Notably, the items chosen were adapted from items used in a previous study in Finland (Sorvo et al., 2017) and the world-wide examined PISA (OECD, 2013). Another limitation could be the use of a single item state anxiety measure. Due to the design of Study I, where state anxiety was measured across multiple tasks, it was considered more feasible to only ask one question per task. It has been shown that a one-item question is as valid of a measure of mathematics anxiety as larger questionnaires (Núñez-Peña et al., 2014) and similar one-item questions have been used as measures of state anxiety in other studies (e.g., Demedts et al., 2022; Trezise & Reeve, 2018). It was also shown in the current research that the state anxiety measured after each task correlated with the trait anxiety questionnaire indicating overlapping components. However, future mathematics anxiety research should re-examine these findings with multiple items for measuring state anxiety. A third limitation is within the use of self-report measures, as these can be subject to biases and issues with understanding the questions. Future research could supplement self-report questionnaires with interviews or examine state anxiety together with confidence ratings and task difficulty to obtain broader evidence for the validity of the current findings and their implications.

There were also limitations within the measurement of the mathematical tasks. First, spontaneous mathematical focusing tendencies (i.e., SFON and SFOR) should be measured with more tasks, including naturalistic observations (e.g., Hannula et al., 2005; Rathé et al., 2018). This would further validate that the SFON and SFOR tasks capture the real-life implications of mathematical focusing within everyday environments. Future research should also examine the longitudinal implications of the weaker spontaneous mathematical focusing tendencies of the children born very preterm. As Study IV included only a narrow rational number test with a limited number of items on ordering and comparison of fractions and decimals, there is no evidence on other rational number skills. Lower fraction arithmetic skills could explain why difficulties with algebra knowledge have been found in children born very preterm than full-term (Clayton et al., 2021). Second, the mathematics anxiety research was focused on fraction arithmetic (Studies I, II, and III) and the results may not extend to fraction magnitude knowledge or other rational number domains, such as decimals. Notably, other types of rational number knowledge are also known to elicit natural number based answers (Van Hoof et al., 2017). Further research should also examine potential procedural and conceptual rational number knowledge differences, which were not within the scope of the current research.

The current research also has implications that may need to be examined with interventions. First, intervention studies could examine whether early targeted support for realising and overcoming misconceptions would reduce the accumulation of negative experiences and subsequent anxiety. It is possible that overcoming a misconception may not be a cause for the state anxiety, but rather the negative experiences associated with the learning process. An important component to examine is feedback, which could reveal more about the relation between state anxiety and performance appraisals, as well as the process of realising the misconception. This also brings up the relevance of examining metacognitive learning processes related to dealing with the misalignment between new information and prior knowledge (Di Leo et al., 2019). Second, the current research had retrospective analysis of the mathematical skills at the age of five years; however, it was not specifically examined whether lower counting skills were related to the later difficulties with spontaneous mathematical focusing tendencies. Spontaneous mathematical focusing interventions or longitudinal studies with children born preterm could address this developmental relation. Furthermore, future studies could examine other cognitive abilities of children born preterm to rule out factors that were not examined in this study. Third, it would have been valuable to examine whether higher spontaneous mathematical focusing tendencies are a protective factor against the development of mathematics anxiety. Addressing these limitations, could further our understanding of developmental relations and ways to support students' learning of mathematics.

While a strength of the study was examining specific profiles of students, this led to relatively small samples in both the preterm sample and the subgroups of low performing students, with and without a clear NNB. This creates limitations in the generalisability of the results to the larger population. Notably, the preterm sample included only children who had an FSIQ equal to or above 70 and had not been held back in school. Thus, these results do not generalise to all children born very preterm. In addition, as many of the findings of the current research are novel, there may be limitations to the generalisability of these findings to other age groups, different cultures or educational systems, and even other mathematical topics or academic domains with misconceptions. Replication studies would be needed to verify these findings, strengthen the conclusions and implications, and yield stronger educational recommendations.

6 Conclusion

Overall, the findings show that the relation between individual factors, such as mathematics anxiety and preterm birth, and mathematics performance can differ depending on the measured mathematical skills. This emphasises the value of examining various mathematical skills to better understand difficulties within mathematical attainment. When examining mathematics anxiety, the expectation was to understand individual differences in students' fraction knowledge and flexible mathematical skills. However, the findings resulted in a broader understanding of mathematics anxiety as well. Examining subgroups in different stages of fraction understanding unveiled nuances within state and trait measures of mathematics anxiety and their relation with performance, such as the presence of subjective perceptions of performance. These subjective performance perceptions or appraisals appear to influence state anxiety, yielding nuances to the deficit theory. Furthermore, state anxiety appeared to be a valuable tool for measuring beyond quantitative performance to capture subtle qualitative differences and dynamic changes in students' fraction understanding. More specifically, the initial low state anxiety and eventual increase in state anxiety in the NNB group yielded additional evidence that the NNB is a misconception and not only a strategy that students use. In addition, the findings show that individual factors that challenge mathematical attainment, such as mathematics anxiety and preterm birth, negatively influence a student's ability to apply their mathematical knowledge in novel non-routine tasks. Together, the findings emphasise the importance of examining various tasks, measures of mathematics anxiety, and subgroups of students to better understand the intricate relations within mathematics attainment. This research is relevant for educators and researchers, as understanding causes of individual differences is the first step towards finding ways to support rational number proficiency and this should include the ability to apply rational number knowledge in novel contexts.

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