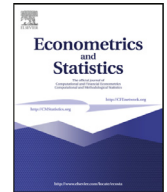




Contents lists available at ScienceDirect

Econometrics and Statistics

journal homepage: www.elsevier.com/locate/ecosta

Risk-return trade-off in international stock returns: Skewness and business cycles

Henri Nyberg^a, Christos S. Savva^{b,*}^a Department of Mathematics and Statistics, 20014, University of Turku, Finland^b Department of Commerce, Finance and Shipping, Cyprus University of Technology, P.O Box 50329, 3603, Limassol, Cyprus

ARTICLE INFO

Article history:

Received 23 December 2022

Accepted 25 February 2023

Available online xxx

JEL classification:

C22

C58

E44

G12

G15

Keywords:

Regime switching model

GARCH-in-mean model

SGT distribution

ICAPM

ABSTRACT

The fundamental risk-return relation is examined with a flexible regime switching model combining the impact of skewness and business cycle regimes in stock returns. Key methodological and empirical findings point out the need for a highly nonlinear and non-Gaussian model to get a reliable picture on the risk-return relationship. With an international dataset of major countries to global financial markets, the empirical results show that accounting especially for skewness patterns leads to the expected positive risk-return relation, which is importantly also maintained over different business cycle conditions.

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1. Introduction

Despite theoretical grounds showing that the trade-off between risk and expected stock market return should be positive (Merton, 1973), various empirical studies have shown mixed evidence of positive and negative signs. These representative studies contain, for instance, French, Schwert and Stambaugh (1987), Campbell and Hentschel (1992), Glosten, Jagannathan and Runkle (1993), Theodossiou and Lee (1995), Scruggs (1998), Bali and Peng (2006), Lanne and Saikkonen (2006), Lundblad (2007), Rossi and Timmermann (2015) and Liu (2017), Aslanidis, Christiansen and Savva (2021), showing different signs and only occasionally statistically significant relationships. Recently, it has been shown, however, that acknowledging explicitly skewness and kurtosis effects in asset returns are key ingredients behind forming an adequate model for the risk-return relation. Conventional econometric approaches fail to incorporate these non-Gaussian features in stock returns, and when taking them appropriately into account, the risk-return relationship becomes mostly positive and statistically significant (see similar type of evidence in Lanne and Saikkonen (2007), Theodossiou and Savva (2016), Liu (2017), Savva and Theodossiou (2018), Theodossiou et al. (2020), Theodossiou and Savva (2022) and Delis Savva, and Theodossiou (2021)).

Our contribution is to re-examine the risk-return trade-off in a more general setting than in the past studies by combining the best of two so far disjoint modelling worlds: That is, we aim to integrate well-documented and important skewness

* Corresponding author.

E-mail address: csavva@ucy.ac.cy (C.S. Savva).

<https://doi.org/10.1016/j.ecosta.2023.02.004>

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Please cite this article as: H. Nyberg and C.S. Savva, Risk-Return Trade-off in International Stock Returns: Skewness and Business Cycles, *Econometrics and Statistics*, <https://doi.org/10.1016/j.ecosta.2023.02.004>

patterns and business cycle expansion and recession regimes in excess stock returns by introducing a highly flexible non-linear model for the risk-return relationship. Specifically, we suggest a regime switching model that allows not only the parameters of the conditional mean and variance but also the parameters related to higher moments (skewness and kurtosis) to be specific for the state of the business cycle. To gain wider empirical evidence than just considering the U.S. data, as almost in all the past studies, we consider the usefulness of our flexible modelling approach in an international dataset containing multiple countries essential for global financial markets.

Our key theoretical and empirical finding is that it is necessary to use our highly flexible and hence a more realistic distributional assumption behind excess stock returns to get a reliable picture on the risk-return relationship. In addition to “mechanically” acknowledging various non-Gaussianities in returns, we also emphasize, in line with the theoretical and empirical asset pricing research, the importance of accommodating business cycle variation to such a relation. These views lead simultaneously to a more structural reasoning behind the non-Gaussian and regime switching type of nonlinear statistical behaviour that seems very much present and needs to be addressed adequately when making conclusions on the risk-return trade-off.

According to various past studies, we find substantial skewness patterns in (international) stock returns with an important effect on the estimates of the risk-return coefficients. The presence of conditional and unconditional skewness has been documented in several previous empirical studies with much more restrictive models than ours. [Campbell and Hentschel \(1992\)](#), [Harvey and Siddique \(1999\)](#) and [Lanne and Saikkonen \(2007\)](#) incorporated conditional skewness in various GARCH-M specifications, with varying conclusions on expected returns and relation to the conditional variance. The economic case for including skewness in models of risk and return is that the mean and variance are not the only factors driving the returns distribution because, *ceteris paribus*, risk-averse investors must prefer right-skewed portfolios to left-skewed ones. For instance, in line with the findings of [Lustig and Verdelhan \(2012\)](#), adding assets that decrease a portfolio’s skewness (and thus making them more left-skewed) should result in higher expected returns. Skewness is consequently a key element in models of risk and return. During the periods of excess volatility, investors realize that their expected returns will not materialize and then switch to less risky assets, thereby introducing increased negative skewness to the price of risk (see [Delis et al., 2021](#)). In case of downturns (troughs), such as business cycle recessions, it is hence of paramount importance to examine the role of skewness in models that incorporate risk and return.

Past studies emphasizing business cycle variation in the risk-return relation include, for example, the work of [Campbell and Cochrane \(1999\)](#) who develop a theoretical habit formation framework that supports a countercyclical behaviour of the risk-return trade-off. [Lustig and Verdelhan \(2012\)](#) examine the impact of business cycle variation on the risk-return trade-off using the U.S. and other OECD countries. Their findings support the view that equity Sharpe ratios and the risk-adjusted cost of capital are higher in recessions than in expansions. [Lettau and Ludvigson \(2010\)](#) suggest, after surveying several techniques and approaches, that the risk-return relation should be further linked to macroeconomic conditions. That is the key objective in this study combining business cycle expansion and recession regimes with the skewness, and partly also kurtosis patterns.

In the past empirical studies, the risk-return relationship is typically considered by using different variants of the GARCH-in-mean (GARCH-M) model. Since the contribution by [Engle, Lilien and Robins \(1987\)](#), there have been various extensions and modifications (see, e.g., [Campbell and Hentschel, 1992](#); [Glosten et al., 1993](#); [Lanne and Saikkonen, 2007](#); [Theodossiou and Savva, 2016](#)), but regime switching specifications have been scant. Concentrating specifically on the typical binary classification of expansion and recession business cycle regimes, we consider a regime switching model where the key parameters of the risk-return relationship are determined by the observed binary time series measuring the state of the economy and modelled with a binary response model. This follows the ideas in [Nyberg \(2012\)](#), but we are now substantially extending that by allowing much more flexible and regime-specific distributional parameters, which turns out an important and necessary modification to accurately establish the expected positive risk-return trade-off. The dependence on the observable regime indicator makes the regime switching model computationally much easier to handle compared with the alternative approaches, such as extracting (latent) regimes using Markov switching models (see, e.g., [Hamilton and Lin, 1996](#); [Kim and Lee, 2008](#)), with importantly no guarantees that the regime switching mechanism is really determined by business cycle variation. That is, in Markov switching models, regime extraction is based purely on statistical grounds, proving arguably (statistical) smoothness to the results over time, but simultaneously we cannot be sure the real driving force behind the regimes. With the U.S. data, [Nyberg \(2012\)](#) finds evidence of statistically significant business cycle-specific regime switching behaviour in the U.S. excess stock returns and a stronger positive risk-return relationship during recessions. When considering other asymmetries than strict business cycle regimes, past findings are again quite inconclusive: For example, [Rossi and Timmermann \(2010\)](#) find evidence of a non-monotonic relation between risk and expected portfolio returns. The asymmetries of risk-return trade-off in response to negative and positive realizations in financial markets are further supported by [Bonomo et al. \(2011\)](#), [Rossi and Timmermann \(2015\)](#) and [Feunou, Jahan-Parvar and Okou \(2018\)](#), among others.

Our regime switching model is in its part based on the skewed generalized t-distribution (hereafter SGT) to accommodate fat tails, peakness and skewness, all typically observed in asset returns. In addition to the commonly used GARCH-M model variants with Gaussian innovations, there have been past studies using the t-distribution (see, e.g., [Lanne and Saikkonen, 2006](#)) and the skewed t-distribution of [Hansen \(1994\)](#) (see, e.g., [Nyberg, 2012](#); [Theodossiou and Savva, 2016](#); [Delis et al., 2021](#)) in connection with the risk-return modelling. All of these distributions are special cases of our general SGT framework, hence providing a highly appropriate and flexible building block for more general examination combined with regime switching behaviour. [Liu \(2017\)](#) have recently considered the role of time-varying skewness and kurtosis parametrization

to the risk-return relation in 12 international financial markets at various investment horizons. His findings suggest countercyclical effects in the U.S. and emerging European markets over the three-month horizon and procyclical relation over the 12-month horizon. However, in his model specification the data distribution cannot be defined in terms of finite set of parameters, whereas we obtain parameter estimates in one step without using estimated variables.

Our empirical results are based on an international dataset containing the monthly observations of eight countries between the years 1960–2020. This sample period also contains the recent pandemic recession, which impact on the risk-return relation has not yet been examined to the best of our knowledge. These eight countries are essential for the international financial markets, and importantly we also have well-established business cycle turning point chronologies available for these countries to enable the estimation of our regime switching model. Our main empirical conclusions emphasize the importance of modelling business cycle regimes and associated skewness in asset pricing. In particular, we show that accounting for skewness leads to the theoretically expected positive risk-return relation and it is maintained over business cycle regimes. Aside from finding large volatility increases in the business cycle downturns, the skewness price of risk is negative throughout the sample and drops (becomes more negative) during the recession periods. These decreases show the immense reaction of investors during crisis periods, as well as the rapid changes in sentiment regarding portfolio risk and the investment horizon. Evidently, the negative reactions are deeper during the downturns.

The plan of the paper is as follows. [Section 2](#) introduces our regime switching model, containing a description of the SGT distribution and modelling the business cycle regime. Empirical results of our international dataset are reported in [Section 3](#), first containing the main results and then several additional and robustness analyses. Finally, [Section 4](#) concludes and presents a discussion on our main findings.

2. Model

2.1. Starting point

Following, e.g., [Theodossiou and Savva \(2016\)](#), the starting point in the risk-return relation examination has typically been the GARCH-in-mean (Generalized Autoregressive Conditional Heteroskedasticity-in-mean, GARCH-M) model of [Engle et al. \(1987\)](#), and its subsequent variants and extensions. The idea in the basic GARCH-M model is that a portfolio's excess return r_t is specified as

$$r_t = \alpha + c\sqrt{h_t} + u_t, \quad (1)$$

where $h_t = \text{var}(r_t | I_{t-1})$ is the conditional variance of r_t and I_{t-1} is the available information set at time $t-1$ (not including r_t). The main interest is in the GARCH-in-mean coefficient c linking the conditional standard deviation $\sqrt{h_t}$ to the conditional mean $\mu_t \equiv \alpha + c\sqrt{h_t}$. In the basic GARCH-M setup, the error term u_t consists of $u_t = \sqrt{h_t}e_t$, where e_t is an independent and identically distributed (iid) error term with zero mean and unit variance. Specifically, the common assumption has been to assume $e_t \sim \text{nid}(0, 1)$, i.e., assuming the Gaussianity of the underlying innovations, leading to the (Gaussian) GARCH-M model, and the use of the Gaussian log-likelihood function. Following the usual quasi-maximum likelihood (QML) inference, the evident divergence from the normality have typically been acknowledged (implicitly) in the asymptotic distribution of the QML estimator and the use of robust standard errors in empirical analysis.¹ The QML estimator is consistent under mild conditions and, in finite samples, its efficiency is close to the correctly specified ML estimator.

The general challenge in the empirical implementation of (1) is to find an empirical proxy for the unobserved conditional market volatility $\sqrt{h_t}$, which can be seen as a latent variable in the GARCH context and replaced by some realized volatility measure in some alternative approaches. To complete the GARCH-M model, the typical assumption concerning the conditional variance is to assume the GARCH(1,1) persistence structure

$$h_t = \nu + \delta u_{t-1}^2 + \gamma h_{t-1}, \quad (2)$$

where the conditional variance of excess returns is specified as a function of the past squared regression errors u_t and the past conditional variance.

Following the positive risk-return relation, resulting, e.g., from the Intertemporal Capital Asset Pricing Model (ICAPM) of [Merton \(1973\)](#), the relationship in (1) generally suggests a positive risk-return coefficient (i.e., $c > 0$). Even though the use of the conditional variance h_t is directly compatible with the ICAPM, the conditional standard deviation $\sqrt{h_t}$ has also commonly been used to determine the contemporaneous volatility feedback (see, e.g., [French et al., 1987](#); [Bali and Peng, 2006](#); [Ludvigson and Ng, 2007](#); [Theodossiou and Savva, 2016](#), [Savva and Theodossiou \(2018\)](#)). As all the mathematical proofs of our framework, to be presented in the next few sections (and also derived in the work of [Theodossiou and Savva, 2016](#)) are based on the use of the conditional standard deviation (as in (1)), we restrict ourselves to this case in this study.²

¹ An alternative class of models to the GARCH-M models to examine the risk-return trade-off are based on regression models where realized volatility-based conditional volatility proxies are used instead of extracting them using the GARCH-M model (see, e.g., [French et al. 1987](#); [Harvey and Siddique, 2000](#); [Christoffersen, Heston and Jacobs, 2006](#); [Guo, Wang and Yang, 2013](#); [Roon and Karehnke, 2017](#)). The recent contributions on QML estimation in GARCH models and review of past results, see, e.g., [Werge and Wintenberger \(2022\)](#) and the references therein.

² Nevertheless, the results are still qualitatively the same in the case of using conditional variance. These findings are available from the authors upon request.

2.2. Skewed generalised t (SGT) framework

As an extension to the commonly used GARCH-M model (1), with the common assumption of Gaussian innovations, here innovations follow the skewed generalised t (SGT) distribution, as employed and introduced first by [Theodossiou and Savva \(2016\)](#) to model the risk-return relation.³ Under the SGT framework, the conditional density of r_t , $f(r_t|I_{t-1})$, conditional on the information set I_{t-1} (containing relevant predictive information up to time $t - 1$), is

$$f(r_t|I_{t-1}) = \frac{k}{2} \left(\frac{n+1}{k} \right)^{-\frac{1}{k}} B\left(\frac{1}{k}, \frac{n}{k}\right)^{-1} \phi_t^{-1} \left(1 + \frac{|u_t|^k}{\left(\frac{n+1}{k}\right)(1 + \text{sign}(u_t)\lambda)^k \phi_t^k} \right)^{-\frac{n+1}{k}}, \quad (3)$$

where u_t is determined as

$$u_t \equiv r_t - m_t = r_t - \left(\alpha + c\sqrt{h_t} \right), \quad (4)$$

and m_t is the conditional mode of returns r_t and ϕ_t is a time-varying scaling parameter related to $\sqrt{h_t}$. In [Eq. \(3\)](#), the (positive) kurtosis parameters k and n and the skewness parameter ($\lambda \in (-1, 1)$) dictate the shape of the distribution. Moreover, $\text{sign}(u_t)$ is the sign function taking values -1 for $u_t \leq 0$ and 1 for $u_t > 0$ and $B(\cdot)$ is the beta function.

The SGT enables a very general class of distributions, containing various special cases used in the past risk-return trade-off studies, being hence an excellent building block for our purposes when aiming to accommodate various nonlinearities. For example, in addition to the benchmark Gaussian case, Student t and skewed t are obtained as special cases of the SGT distribution and have been used in the past risk-return trade-off studies (see, e.g., [Lanne and Saikkonen, 2006](#); [Nyberg, 2012](#)). Our contribution is to combine the SGT to a regime switching model. Due to its flexibility, SGT adjusts to the data without restrictive pre-assumptions on the shape of the distribution. According to the resulting estimation results, we can then implicitly evaluate the adequacy of the past distributional assumptions to meet important variation in the conditional higher-order moments not yet adequately addressed in the past risk-return modelling studies.

Under the SGT framework, when $n > 2$ (which guarantees the existence of the conditional variance), the conditional mean and variance of r_t are (see the proofs in [Theodossiou and Savva, 2016](#); and [Theodossiou, 1998](#)):

$$\mu_t = E(r_t|I_{t-1}) = m_t + E(u_t|I_{t-1}) = m_t + p\sqrt{h_t} \quad (5)$$

and

$$h_t = \text{var}(u_t|I_{t-1}) = (\Xi_2 - \Xi_1^2)\phi_t^2, \quad (6)$$

where

$$p = \frac{\Xi_1}{\sqrt{\Xi_2 - \Xi_1^2}}, \quad (7)$$

$$\Xi_1 = 2\lambda \left(\frac{n+1}{k} \right)^{\frac{1}{k}} B\left(\frac{2}{k}, \frac{n-1}{k}\right) B\left(\frac{1}{k}, \frac{n}{k}\right)^{-1} \quad (8)$$

and

$$\Xi_2 = (1 + 3\lambda^2) \left(\frac{n+1}{k} \right)^{\frac{2}{k}} B\left(\frac{3}{k}, \frac{n-2}{k}\right) B\left(\frac{1}{k}, \frac{n}{k}\right)^{-1}. \quad (9)$$

In [\(5\)](#), the parameter p (cf. [\(7\)](#)) is a symmetric function of the skewness parameter λ and a nonlinear function of the kurtosis parameters k and n (see a detailed illustration in [Theodossiou and Savva, 2016](#); especially discussion connected to [Fig. 1](#)).

Inserting $m_t \equiv \alpha + c\sqrt{h_t}$ into the conditional mean excess return in [Eq. \(5\)](#) yields

$$\mu_t = \alpha + c\sqrt{h_t} + p\sqrt{h_t}, \quad (10)$$

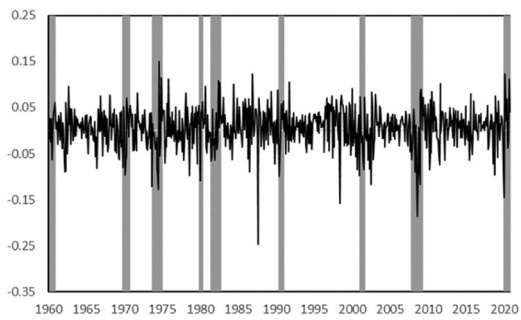
where the “pure” risk premium $c\sqrt{h_t}$ (cf. [Eq. \(1\)](#)) is expected to be positive and $p\sqrt{h_t}$ is the skewness-kurtosis premium. Therefore, the model [Eq. \(1\)](#) can be rewritten as

$$r_t = \mu_t + \varepsilon_t = \alpha + (c + p)\sqrt{h_t} + \varepsilon_t. \quad (11)$$

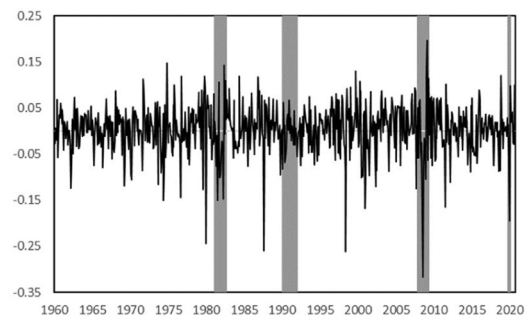
The error term $\varepsilon_t = u_t - p\sqrt{h_t}$ has a zero expectation, while the term $(c + p)\sqrt{h_t} \equiv \xi\sqrt{h_t}$ combines the impact of the “pure” and “skewness-kurtosis” risk components in the conditional mean of excess returns.⁴ This decomposition in [\(10\)](#) –

³ The SGT distribution is a very general distribution containing various other distributions as special cases (see [Theodossiou, 1998](#)) and that is also closely related to various other distributions used for financial data (see, e.g., [Bali and Theodossiou \(2008\)](#) and [Theodossiou \(2015\)](#), and the references therein).

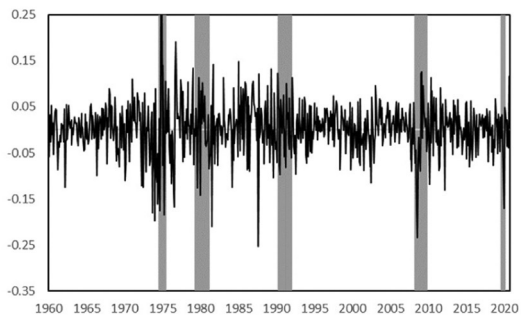
⁴ At this point, it is important to note that in the presence of skewness, the estimation of [Eq. \(1\)](#) using a symmetric log-likelihood specification (such as based on the Gaussian likelihood) will result in a biased estimator for the price of “pure risk” (the GARCH-in-mean effect) measured by c . In fact, the resulting estimator will be $\xi = c + p$, not c . Moreover, the computed standardized errors will not possess a zero mean and a unit variance, and the conditional variance will thus be misspecified.



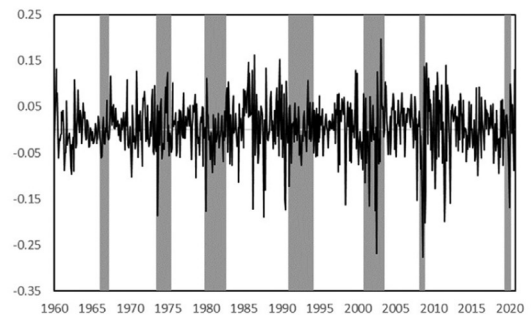
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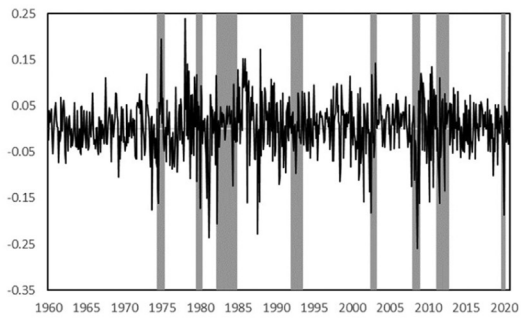
(b) Canada



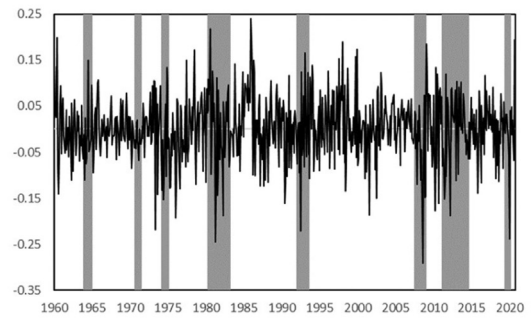
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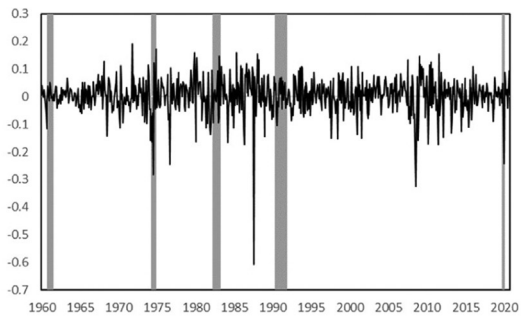
(d) Germany



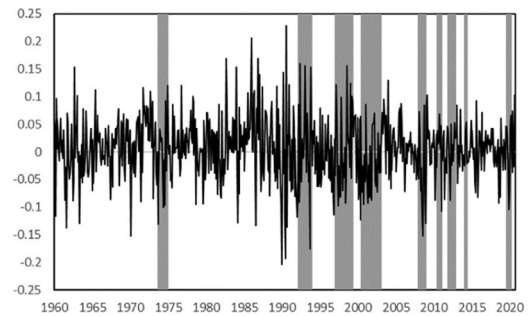
(e) France



(f) Italy



(g) Australia



(h) Japan

Fig. 1. Excess Stock Returns and Business Cycle Regimes. *Notes:* Figure 1 displays the excess stock returns r_t and the values of the business indicator y_t for the sample period from February 1960 to December 2020. The shaded areas are the recession periods determined by the NBER and ECRI.

(11), enabled by the SGT framework, allows to identify the impact of skewness on the risk-return trade-off through the sign and size of p .

Another important perspective of (11) is the natural inclusion of the constant term α . [Scruggs \(1998\)](#), [Lanne and Saikkonen \(2006\)](#) and [Nyberg \(2012\)](#) have estimated different GARCH-M models, with and without the constant, finding much stronger and statistically significant risk-return trade-off without the constant. Despite these theoretical arguments coming from the pure version of the ICAPM, the constant term has automatically been included in the large majority of past studies and employed models. [Hedegaard and Hodrick \(2016\)](#) have argued that the inclusion of the constant is necessary to test whether the expected risk premium is dynamically linked to the conditional variance. Importantly, in our model and the resulting decomposition (10) – (11), the constant α can be connected to the skewness premium part, meaning that the model Eq. (11) is in line with the implication of the (pure) ICAPM but also incorporating the arguments of [Hedegaard and Hodrick \(2016\)](#).

As in the benchmark GARCH-M model in [Section 2.1](#), in this study the conditional variance will be primarily modelled with the usual parsimonious GARCH(1,1) persistence structure, generating $\sqrt{h_t}$, with two major modifications. In particular, in the next section, we allow the conditional variance to follow a regime switching specification, incorporating potential asymmetric effects over business cycles. Moreover, as a typical and commonly used extension in the past studies, an asymmetric GARCH specification (GJR-GARCH(1,1) model of [Glosten et al., 1993](#)) is also employed:

$$h_t = \nu + (\zeta i_{t-1} + \delta) \varepsilon_{t-1}^2 + \gamma h_{t-1}, \quad (12)$$

where $i_t = 0$ for $\varepsilon_t \geq 0$ and $i_t = 1$ for $\varepsilon_t < 0$. Parameter ζ captures asymmetric volatility. Positive values of ζ imply that volatility is higher (lower) when negative (positive) shock of the same magnitude occurs in a stock market.

2.3. QR-SGT with dependence on business cycles

The essential aim and novelty of this study is to extend otherwise already very flexible SGT framework ([Section 2.2](#)) to incorporate the state of the business cycle regime in the risk-return relation. Regime switching dynamics is introduced by an observable binary time series determining the state of the economy. In practice, in [Section 3](#) we specify the binary variable y_t such that $y_t = 1$ denotes a recession state (with connected parameters labeled with 1) and $y_t = 0$ denotes the expansion state of the business cycle at time t .

A (marginal) model for the binary variable, conditional on the information set I_{t-1} , can be specified by assuming that y_t follows the Bernoulli distribution:

$$y_t | I_{t-1} \sim B(P_t), \quad (13)$$

where P_t is the conditional expectation of y_t , $E_{t-1}(y_t)$, which is equal to the conditional probability of the outcome $y_t = 1$ (i.e., $P_{t-1}(y_t = 1)$). Thus,

$$P_t = E_{t-1}(y_t) = P_{t-1}(y_t = 1) = \Phi(\pi_t), \quad (14)$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function and π_t is a linear function of information (variables) included in the information set I_{t-1} . Expressions (13) – (14) define a probit model, which is now part of the regime switching model to be introduced next.

To complete the model specification of y_t , the linear function π_t should be determined. Following [Nyberg \(2012, 2018\)](#) and [Fokianos and Moysiadis \(2017\)](#), and the references therein, we assume the following first-order autoregressive persistence structure in π_t :

$$\pi_t = w + \rho \pi_{t-1} + x_{t-1} \varphi, \quad (15)$$

where w is an intercept term and $|\rho| < 1$ is the first order coefficient. The (row) vector x_{t-1} contains the predictive variables and φ is a related (column) vector of parameters. The probit model based on (15) has been found particularly successful in recession prediction (forecasting) (see, e.g., [Nyberg, 2010, 2012, 2018](#)). Following these studies, in [Section 3](#), we use financial variables in x_t (i.e., the term spreads between the long-term and short-term interest rates and the lagged stock returns), which have been found useful in business cycle prediction.

As the observable binary variable y_t defines the regime and regime switching parameters, and now combined with the SGT in (3) – (4), we call the model as the “QR-SGT”, where QR stands for the qualitative response (here specifically the binary response) variable y_t providing the regime switching part tied with the flexible SGT framework. This all allows us to examine “the best of the two worlds” emphasized in the past research: That is allowing for business cycle variation via regimes but simultaneously also acknowledging (higher moment) non-Gaussian features in stock returns to the risk-return relation. Specifically, in the QR-SGT case, the excess stock return r_t follows a regime switching model where the regime is defined by the observable expansion and recession states determined by the value of y_t . The regime switching structure of the conditional mean can hence be expressed as (cf. (11))

$$r_t = (1 - y_t) \left[(c_0 + p_0) \sqrt{h_t} + \alpha_0 \right] + y_t \left[(c_1 + p_1) \sqrt{h_t} + \alpha_1 \right] + \varepsilon_t, \quad (16)$$

where $\varepsilon_t \sim$ SGT and the outcome of $y_t = j$ ($j = \{0, 1\}$) defines the regime and hence also regime-specific parameters, including the pure risk coefficients c_0 and c_1 and skewness risk parameters p_0 and p_1 .

In (16), the dependence on the contemporaneous y_t implies that to obtain the fitted values of r_t , i.e. $E_{t-1}(r_t)$, requires to specify a binary response model for the state of the economy, as defined in Eqs. (14) – (15). However, in the risk-return considerations, the main interest is typically in the regime-specific parameters and their implications to the risk-return relation. As the joint conditional density function of (r_t, y_t) can be factored as

$$g(r_t, y_t | I_{t-1}) = f(r_t | y_t, I_{t-1}) P(y_t | I_{t-1}), \quad (17)$$

we see that in $f(r_t | y_t, I_{t-1})$ we are conditioning on the (observable) state of the business cycle, while the term $P(y_t | I_{t-1})$ provides the (conditional) regime probabilities based on the information available at time $t - 1$. Notice that these two conditional densities and components have no parameters in common (see Eqs. (14) – (15) and the SGT parametrization). The form of (17) is similar to the Markov switching models, with the essential difference that here y_t is now an observable time series, instead of a latent state, explicitly controlled by the econometrician.⁵ Therefore, the factored conditional density function (17) implies that the resulting log-likelihood function can be written as a sum of the binary response and regime switching components.

For most of our purposes and hypotheses of interest, only the first component $f(r_t | y_t, I_{t-1})$ is of interest and the model for the state of the business cycle is highly secondary. However, one context where the model for the business cycle regime is required is when obtaining proxies for (ex-ante) expected returns. Assuming the conditional independence between y_t and ε_t yields

$$E_{t-1}(r_t) = (1 - P_t) \left[(c_0 + p_0) \sqrt{h_t} + \alpha_0 \right] + P_t \left[(c_1 + p_1) \sqrt{h_t} + \alpha_1 \right], \quad (18)$$

where P_t is obtained as in Eq. (14). This can be interpreted as the time-varying weight between the business cycle regimes when computing the fitted values (i.e., the conditional expectations of returns, interpreted as expected returns implied by the model).

In addition to the regime switching conditional mean (16), we extend the SGT framework (Section 2.2) by regime switching dynamics in all the parameters. That is, the functions of the SGT parameters are now allowed to be business cycle specific through Ξ_1 and Ξ_2 (see (8) – (9)) functions:

$$\Xi_1 = (1 - y_t) \Xi_{1,0} + y_t \Xi_{1,1} \quad (19)$$

and

$$\Xi_2 = (1 - y_t) \Xi_{2,0} + y_t \Xi_{2,1} \quad (20)$$

Moreover, the conditional regime-dependent volatility process is now also allowed to follow a regime-dependent GJR-GARCH model, indicating that the volatility dynamics (may) follow different dynamic patterns (i.e., asymmetric features) over the business cycle regimes (Halunga and Savva, 2019):

$$h_{jt} = \nu_j + (\delta_j + \zeta_j I_{t-1}) \varepsilon_{t-1}^2 + \gamma_j h_{t-1}, \quad (21)$$

where $j = \{0, 1\}$, implying that

$$h_t = (1 - y_t) h_{0t} + y_t h_{1t} \quad (22)$$

Compared with the time-invariant SGT parametrization, our regime-dependent QR-SGT model, defined in Eqs. (16) – (22), emphasize the fact that the excess stock return r_t is now highly dependent on the state of the business cycle. This concerns also higher conditional moments than just conditional mean and variance: See, e.g., Eq. (18) where the flexible dynamics of the QR-SGT model allow the expected pure risk and kurtosis-skewness premiums to be dependent on regime-specific parameters c_j and p_j ($j = \{0, 1\}$, see Eq. (16)), which in turn are dependent on the regime-specific skewness and kurtosis parameters (cf. Eqs. (7) – (9) and (19) – (20)).

3. Empirical results

3.1. Dataset

Throughout this study, we use monthly data on eight countries (U.S., Canada, U.K., Germany, France, Italy, Australia and Japan), downloaded from Global Financial Data and for the period from January 1960 to December 2020. The first 12 observations are used as initial values in estimation. The monthly excess stock return series (r_t) are constructed as the difference between the monthly return on the market index and the return on the risk-free interest rate such as the three-month T-bill rate in the U.S. Monthly data frequency has been a typical and a natural selection, and that is also the case here as international business cycle turning points, and hence the values of y_t , are generally available on monthly basis.⁶

⁵ In practice, the structure in (17) eases substantially all the computations in parameter estimation (finding (quasi) maximum likelihood estimates) when comparing with, for example, the Markov switching-based models, with unobserved regimes, such as Markov switching GARCH models (see, e.g., Wee, Chen and Dunsmuir, 2022).

⁶ The codes from the data source (Global Financial Data - <https://globalfinancialdata.com/>) are as follows: 3- month treasury bills – ITXX3D, 10-year government bonds – IFXX10D, Stock returns – GFPRXXSTD. Here XX represents country's identity, where AUS stands for Australia, CAN for Canada, DEU for Germany, FRA for France, GBR for the UK, ITA for Italy and, USA for the USA.

In this study, we follow the business cycle turning points determined by Economic Cycle Research Institute (ECRI) to obtain the values of y_t .⁷ ECRI determines the business cycle turning points along the same principles as the National Bureau of Economic Research (NBER) for the U.S. economy. The recession definition that the NBER follows is generally considered as the “official” business cycle dating algorithm for the U.S. economy. They importantly use information of various coincident indicators on the state of the business cycle (than just, say, real GDP growth). We prefer ECRI’s international business cycle chronologies over the corresponding ones that, for example, the OECD provides (see, e.g., [Lustig and Verdelhan, 2012](#)). The latter leads to a much greater number of turning points, also in the case of the U.S. economy, while ECRI’s international business cycle chronologies are much better in accordance with the NBER. Without the loss of generality, the values of y_t , i.e., zeros and ones, can be either way, but here we denote the recession state by $y_t = 1$ (regime 1) and expansion state $y_t = 0$ (regime 0). Concerning the value of y_t at turning point months, there is no exact convention used in the past research: We classify both peak and trough business cycle turning point months as recessionary months.

Our dataset also contains the most recent recession period, which is partly connected to the current pandemic recession circumstances. In most countries, according to ECRI, the recession already started during the year 2019 and ended either in April or May 2020. In the U.S. case, the business peak (February 2020) was already identified by the NBER in the summer 2020 and the trough (April 2020) recently in July 2021.

[Fig. 1](#) depicts the excess stock return series with the recession periods. These eight countries have been selected mainly due to their major impact on international financial markets and due to an access (by ECRI) on their monthly business cycle turning point chronologies up to the beginning of the 1960s. The available business cycle turning points also simultaneously define the starting point of our sample period. This dataset hence contains a well-representative collection of countries, also considered in a few existing studies examining the risk-return trade-off in international case (see, e.g., [Cohray and Tourani-Rad, 1994](#); [Theodossiou and Lee, 1995](#); [Liu, 2017](#); [Savva and Theodossiou, 2018](#)).

The variables included in the vector x_t (see (15)) are financial leading indicators which have been found to contain predictive power for the state of the business cycle. We restrict ourselves to financial variables mainly for two reasons. First, they are continuously available in real time without subsequent revisions: See, e.g., the related discussion on the importance of real time availability in the risk-return relation examination in [Lustig and Verdelhan \(2012\)](#). Second, with the data sources described above, we are able to obtain a balanced panel of observations of these leading indicators for each country. In line with the past empirical findings in business cycle prediction and forecasting literature, we concentrate on the following three variables: (Domestic) term spread, lagged stock return and the foreign term spread. The term spread (“yield spread”), that is the difference between the long-term and short-term interest rates, specifically 10-year and three-month rates, has widely been found the best combination to lead real activity.⁸ As a foreign term spread, we consider the U.S. term spread, while for the U.S. case, that is the German term spread. The lagged (excess) market return is simply the lagged return r_t . [Estrella and Mishkin \(1998\)](#), [Nyberg \(2010, 2012, 2018\)](#) and [Bluedorn, Decressin and Terrones \(2016\)](#), among others, provide evidence in favour of predictive power of stock returns for the (binary-valued) state of the business cycle y_t , especially when included in together with the (domestic) term spread.

Descriptive statistics of excess stock market returns in different countries are presented in [Table 1](#). We report the means, standard deviations, skewness and kurtosis statistics for the whole sample and separately for the returns realized during the expansion and recession states of the business cycle. The mean of excess stock market returns is positive, but the returns also exhibit clear business cycle variation: As expected, during the recession periods the returns have mainly been negative. The standard deviation is also higher in the recession regime, compared with the expansion regime, indicating that volatility is time-varying and dependent on the state of the business cycle as typically found in the past studies ([Schwert, 1989](#); [Hamilton and Lin, 1996](#); [Bloom et al., 2018](#); among others). Excess kurtosis and negative skewness in almost all the cases lead to the result that the normal distribution does not describe returns adequately. Similarly, Kolmogorov-Smirnov test rejects the normality hypothesis. Regarding the expansion periods negative skewness is observed for all markets, except Japan, while during the recession periods, skewness is negative for all the countries except from the UK and Japan.

3.2. Main results

In this section, we present our main empirical findings, starting with the conventional and commonly used GARCH(1,1)-M and GJR-GARCH(1,1)-M models (see [Eq. \(1\)](#) and [Eq. \(12\)](#), respectively) and assuming Gaussian innovations. Nevertheless, the Gaussianity assumption implies inability to account for (typically negative) skewness and (excess) kurtosis effects in the data and subsequently their impact on the risk-return trade-off. Therefore, we move next to the flexible SGT distribution, determining the conditional distribution $f(r_t|y_t, I_{t-1})$. This is the key step towards our novel QR-SGT specification with business cycle-specific regimes and skewness effects. Throughout this section, our main interest and hypothesis of interest is whether the apparent nonlinear risk-return trade-off is arising from the business cycle regimes or just due to non-Gaussian features in returns, or both.

⁷ The monthly business cycle turning points determined by ECRI are available at <https://www.businesscycle.com/ecri-business-cycles/international-business-cycle-dates-chronologies>.

⁸ As in [Nyberg \(2012\)](#), one issue with the term spread is that the first available lag (i.e., the most recent available value) is not necessarily optimal predictor for the future state of the business cycle. That is, the leading time is longer than one month. Throughout this study, along the past evidence, we employ the sixth lag for the domestic term spread and the third for the foreign term spread.

Table 1
Descriptive statistics.

	USA	Canada	UK	Germany	France	Italy	Australia	Japan
Whole Sample								
Mean	0.530	0.377	0.322	0.409	0.035	0.349	0.363	0.520
Variance	18.626	28.266	33.480	33.493	47.063	35.410	41.602	31.506
Skewness	-0.671	-0.993	0.085	-0.630	-0.210	-0.479	-1.741	0.019
Kurtosis	5.473	7.218	8.761	5.230	4.092	4.806	15.593	3.975
Observ.	719	719	719	719	719	719	719	719
Kol. Smir.	0.362	0.368	0.363	0.371	0.357	0.372	0.382	0.357
Expansion ($y_t = 0$)								
Mean	0.741	0.554	0.354	0.843	0.508	0.473	0.451	0.778
Variance	14.844	24.092	26.397	29.291	30.721	40.035	39.569	28.313
Skewness	-0.709	-0.957	-0.564	-0.351	-0.237	0.017	-1.841	0.003
Kurtosis	6.333	6.623	5.026	4.050	4.596	3.832	17.405	4.456
Observ.	625	650	632	555	595	549	667	567
Kol. Smir.	0.369	0.383	0.356	0.400	0.369	0.378	0.384	0.363
Recession ($y_t = 1$)								
Mean	-0.867	-1.286	0.087	-1.058	-0.415	-1.378	-0.765	-0.443
Variance	41.932	65.419	85.779	45.171	57.632	67.501	67.572	42.502
Skewness	-0.105	-0.536	0.922	-0.907	-0.756	-0.329	-0.880	0.200
Kurtosis	2.727	5.053	7.672	5.868	3.920	3.617	5.028	2.920
Observ.	94	69	87	164	124	170	52	152
Kol. Smir.	0.386	0.432	0.413	0.361	0.378	0.470	0.377	0.423

Notes: Descriptive statistics of monthly excess stock returns (sample period 1960–2020). The excess returns are scaled by 100, indicating that they are percentages. Kol. Smir. refers to the Kolmogorov-Smirnov test statistic, examining whether the data are normally distributed (H_0). Normality is rejected if the test statistic is greater than the critical value, which is calculated as $1.63/\sqrt{T}$ for the 1% significance level where T is the sample size (number of observations). For example, for the whole sample, the critical value at the 1% significance level is 0.061.

As described in Section 2.3 and Eq. (17), our main interest is in the estimated parameter coefficients related to the regime switching part, generated by the conditional density $f(r_t|y_t, I_{t-1})$. As discussed in connection to Eq. (18), to obtain, for example, forecasts or fitted values of excess stock returns and hence proxies for expected returns, the model for the binary-valued regime switching variable, determining $P_t = P(y_t|I_{t-1})$, is also needed. Eq. (18) hence also shows how our regime switching model specifies ex-ante (based on the information at time $t - 1$) time-varying risk and kurtosis-skewness premium where the (conditional) recession probability P_t is an essential driver between the regimes.

3.2.1. Modelling the state of the business cycle

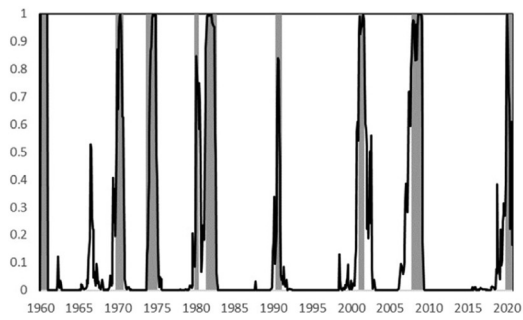
Before concentrating on the (regime switching) risk-return coefficients, in Table 2 we report the results of the estimated models (see Eqs. (14) – (15)) for the state of the business cycle in different countries. Given the inference in (17), the country-specific estimation results are the same for all the QR-SGT specifications used in our analysis.

In accordance with the findings of Nyberg (2010, 2012), we find that the estimated coefficients of (lagged) domestic and foreign term spreads and stock market returns are negative and statistically significant, implying that low values of the term spreads and stock returns predict a high probability of recession. The values of the pseudo- R^2 (Estrella, 1998) in Table 2 show that the models are predicting the state of the business cycle (very) accurately in all the countries. That can be seen also in $CR_{50\%}$ statistics, denoting the percentages of correct predictions, which are very high ranging from 0.848 to 0.982.⁹

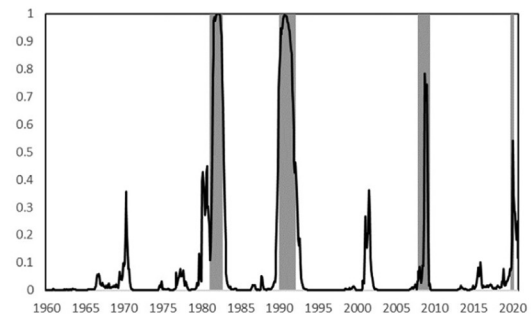
Fig. 2 depicts the estimated recession probabilities (see (14)), which are also the estimated mixing weights between the two regimes of y_t . Given the factorization (17), the resulting business cycle regime probabilities in Fig. 2 are identical irrespective of the subsequent selection of the regime switching SGT specification. As it can be inferred from the figures, together with the evidence in Table 2, the recession probabilities match well with the recession and expansion periods for all the countries, with the recession probability being high during the recession periods and close to 0 during the expansions.¹⁰

⁹ A binary prediction takes the value of 1 if the probability is greater than 0.5 and 0 otherwise.

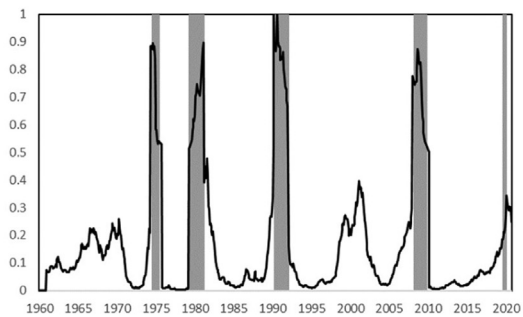
¹⁰ Recent pandemic crisis (COVID-19) affects the business cycle dynamics. Some of the notable cases include Italy, where the market's negative reaction to the spread of the coronavirus began in mid-February 2020, and the first large stock market crash occurred on February 21. Moreover, the U.S. equity markets hit a low on March 23, following the World Health Organization's declaration of a pandemic on March 11, the exponential increase in daily new COVID-19 cases worldwide and government decisions to enact lockdowns. In Fig. 2, we can see that recession probabilities for this last recession episode are rather high in most countries.



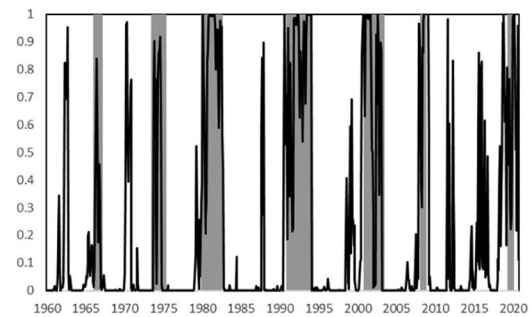
(a) USA



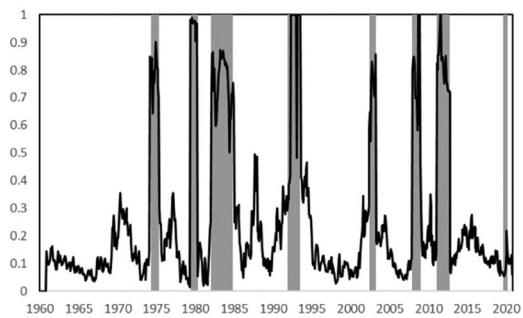
(b) Canada



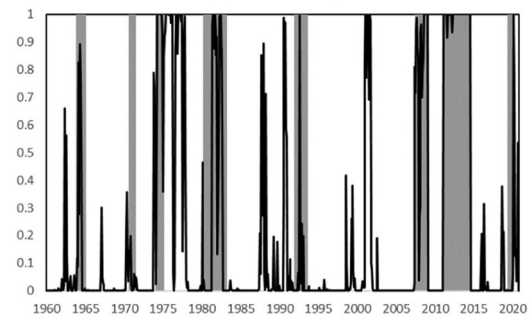
(c) UK



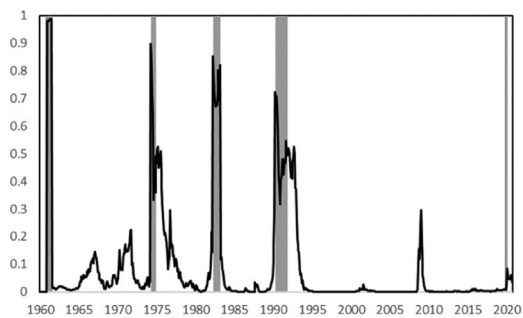
(d) Germany



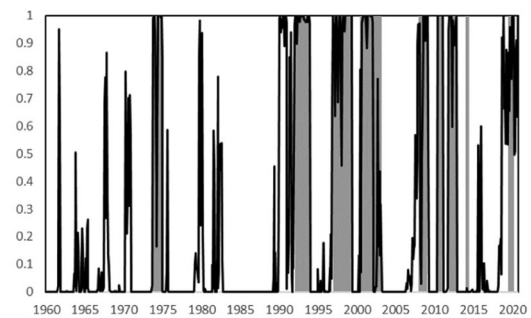
(e) France



(f) Italy



(g) Australia



(h) Japan

Fig. 2. Estimated Recession Probabilities in Different Countries. *Notes:* This figure shows the estimated recession probabilities in different countries implied by the probit model (14) – (15), whose estimation results are presented in Table 2. The shaded areas are the recession periods determined by the NBER and ECRI.

Table 2
Estimation results of the probit model for the business cycle regimes.

	USA	Canada	UK	Germany	France	Italy	Australia	Japan
W	0.158 (0.027)	-0.019 (0.016)	-0.032 (0.014)	0.083 (0.014)	-0.537 (0.235)	-0.057 (0.015)	0.026 (0.023)	0.022 (0.011)
P	0.886 (0.009)	0.917 (0.006)	0.914 (0.013)	0.933 (0.006)	0.539 (0.208)	0.919 (0.010)	0.976 (0.016)	0.963 (0.005)
ϕ_1 (stock returns)	-0.129 (0.015)	-0.050 (0.007)	-0.012 (0.005)	-0.040 (0.006)	-0.026 (0.008)	-0.015 (0.004)	-0.026 (0.004)	-0.041 (0.005)
ϕ_2 (dom. spread)	-0.183 (0.025)	-0.116 (0.012)	-0.025 (0.008)	-0.033 (0.006)	-0.091 (0.055)	0.015 (0.004)	-0.032 (0.005)	-0.018 (0.002)
ϕ_3 (foreign spread)	-0.070 (0.012)	-0.024 (0.009)	-0.038 (0.008)	-0.048 (0.006)	0.118 (0.056)	-0.018 (0.006)	-0.025 (0.006)	-0.010 (0.004)
Pseudo- R^2	0.952	0.950	0.866	0.766	0.762	0.661	0.939	0.816
CR _{50%}	0.943	0.964	0.973	0.851	0.971	0.818	0.960	0.876
Log lik.	-278.82	-227.30	-265.25	-386.08	-330.57	-393.25	-186.66	-370.87

Notes: The estimation results of the probit model for the business cycle regime in different countries, as described in Eqs. (14) – (15), and based on the full sample period (effectively 1960 M2-2020 M12). The coefficient ρ represents the autoregressive coefficient, ϕ_1 represents the returns at lag 1, ϕ_2 the country's term spread, and ϕ_3 the foreign country's term spread. Robust standard errors are given in parentheses. Bold numbers denote statistically significant coefficients at least at the 5% significance level. The pseudo- R^2 is the counterpart to the coefficient of determination used in models with continuous (real-valued) dependent variables. The CR_{50%} is the percentage of correct signal predictions when the 0.50 threshold is applied for probability predictions (14).

Table 3
Estimation results of the (gaussian) GARCH-M model.

	USA	Canada	UK	Germany	France	Italy	Australia	Japan
Mean Equation (Eq. (1))								
α	0.024 (0.017)	0.021 (0.013)	0.015 (0.045)	0.136 (0.007)	0.016 (0.013)	0.005 (0.009)	0.007 (0.019)	0.009 (0.165)
c	0.262 (0.303)	0.042 (0.305)	0.027 (0.156)	0.047 (0.147)	-0.124 (0.471)	-0.215 (0.441)	0.157 (0.497)	0.283 (0.506)
Volatility Equation: GARCH(1,1) (Eq. (2))								
V	0.973 (0.458)	6.004 (6.188)	0.670 (0.333)	1.549 (0.805)	0.855 (0.496)	1.021 (0.658)	1.697 (3.512)	1.247 (0.694)
Δ	0.137 (0.036)	0.198 (0.143)	0.101 (0.035)	0.120 (0.036)	0.087 (0.029)	0.079 (0.028)	0.126 (0.082)	0.085 (0.037)
Γ	0.819 (0.041)	0.591 (0.034)	0.882 (0.037)	0.838 (0.047)	0.892 (0.036)	0.903 (0.034)	0.842 (0.167)	0.876 (0.052)
AIC	4245.66	4546.18	4852.34	5011.54	5131.70	5489.60	4865.38	4982.82
BIC	4268.55	4569.07	4875.23	5034.43	5154.59	5512.49	4888.27	5005.71
Log lik.	-2117.83	-2268.09	-2421.17	-2500.77	-2560.85	-2739.80	-2427.69	-2486.41

Notes: As in Table 2, the full sample period (1960 M2 - 2020 M12) is used in estimation. Robust standard errors are given in parentheses. The reported values of the log-likelihood function and information criteria (AIC and BIC) are for the whole QR-SGT model including the (independent) probit model for the business cycle indicator y_t for comparison reasons (see the country-specific results in Table 2).

3.2.2. Regime switching risk-return relation over business cycles

As discussed above, the main interest in our analysis is in (potentially) regime switching risk-return coefficients connected to the risk-return relationship where the regime is based on the state of the business cycle. Our focus is to allow and evaluate especially the impact of business cycle-specific skewness to this relation. The fact that recession periods have been rare means that it is somewhat questionable to rely very short subsample periods. Therefore, in this section we rely on the full sample period starting from the year 1960, while in Section 3.3 we report robustness checks for our main findings with a subsample starting from January 1985. In addition, in Section 3.3, we also consider the impact of business cycle-specific kurtosis along with nonlinearities originating from skewness and the U.S. business cycle regimes as a proxy for global business cycles.

In Table 3, we start with the estimated coefficients of the conventional (Gaussian) GARCH(1,1)-M model (see Eqs. (1) and (2)), without dependence on the state of the business cycle. That is the model specification commonly considered in the past studies. The estimation results for the conditional volatility are largely similar for each country: Especially the estimated GARCH coefficient γ is relatively large and highly statistically significant, showing strong volatility clustering. The main interest is anyway in the GARCH-in-mean parameter c , which is mostly statistically insignificant and both positive

Table 4
Estimation results of the SGT model (QR-SGT model without regime switching).

	USA	Canada	UK	Germany	France	Italy	Australia	Japan
Mean Equation (Eq. (1))								
A	-0.059 (0.082)	-1.113 (0.110)	0.141 (0.075)	-0.710 (0.112)	-1.042 (0.117)	-0.511 (0.122)	-1.487 (0.122)	0.534 (0.116)
C	0.538 (0.023)	0.591 (0.024)	0.331 (0.018)	0.441 (0.023)	0.519 (0.023)	0.173 (0.021)	0.575 (0.022)	0.052 (0.024)
P	-0.394 [-0.480 -0.294]	-0.281 [-0.346 -0.220]	-0.297 [-0.368 -0.241]	-0.224 [-0.285 -0.166]	-0.279 [-0.356 -0.199]	-0.089 [-0.179 0.002]	-0.244 [-0.312 -0.176]	-0.055 [-0.124 0.037]
Volatility Equation (Eq. (12))								
v	1.727 (0.068)	4.240 (0.155)	0.685 (0.033)	1.980 (0.102)	1.375 (0.073)	0.884 (0.058)	5.069 (0.177)	1.054 (0.059)
δ	0.007 (0.004)	0.096 (0.005)	0.059 (0.004)	0.136 (0.005)	0.087 (0.004)	0.089 (0.003)	0.121 (0.006)	0.070 (0.003)
ζ	0.226 (0.008)	0.113 (0.007)	0.049 (0.004)	-0.016 (0.006)	0.002 (0.004)	-0.010 (0.014)	0.067 (0.007)	0.025 (0.004)
γ	0.784 (0.005)	0.692 (0.008)	0.895 (0.003)	0.821 (0.006)	0.877 (0.004)	0.904 (0.003)	0.732 (0.006)	0.885 (0.004)
Distribution Parameters (Eq. (3))								
n	6.912 (0.299)	8.786 (0.707)	10.781 (1.093)	5.075 (0.249)	5.078 (0.196)	5.767 (0.284)	3.579 (0.076)	100 (.)
k	2.057 (0.039)	1.584 (0.032)	1.825 (0.040)	2.474 (0.062)	2.453 (0.054)	2.301 (0.053)	2.711 (0.056)	1.593 (0.035)
λ	-0.265 (0.006)	-0.193 (0.005)	-0.197 (0.006)	-0.148 (0.007)	-0.186 (0.006)	-0.059 (0.007)	-0.171 (0.006)	-0.036 (0.006)
AIC	4182.78	4475.94	4819.78	4987.46	5106.24	5475.84	4771.42	4982.20
BIC	4223.98	4517.14	4860.98	5028.66	5147.44	5507.04	4812.62	5003.40
Log lik.	-2082.39	-2228.97	-2400.89	-2484.73	-2544.12	-2728.92	-2376.71	-2482.10
LR test	70.88	78.24	40.56	32.08	33.46	21.76	101.97	8.61

Notes: Estimation results for the full sample period. Robust standard errors are given in parentheses. Confidence interval for the skewness price of risk (p) is constructed based on the bootstrapping procedure of Rapach and Wohar (2009). The reported values of the log-likelihood function and information criteria (AIC and BIC) are for the whole QR-SGT model including the (independent) probit model for the business cycle indicator y_t for comparison reasons (see the country-specific results in Table 2). Likelihood ratio (LR) test compares the GJR-SGT specification to the (Gaussian) GARCH specification - critical value for $\chi^2_{4, 0.05} = 9.488$.

and negative signs occur. Large standard errors confirm the inconclusive conclusions on the sign, as emphasized in the past literature (e.g., 95% confidence intervals contain both negative and positive signs).

The above starting point of our empirical analysis in Table 3 is repeated using the GJR-GARCH(1,1) model (12) where the leverage effect is also acknowledged in the conditional variance. Following a large number of past findings in volatility modelling, it is evident that the GJR specification is superior to the benchmark models presented in Table 3, based on, e.g., the likelihood ratio (LR) tests between the symmetric (2) and asymmetric (17) volatility specifications.¹¹ Therefore, the rest part of the analysis is based on the GJR volatility model. Nevertheless, the estimates of the parameter c (the risk-return trade-off parameter) in the GJR-GARCH-M model, under the normality assumption of innovations, remain statistically insignificant in the same way as in the GARCH-M specifications in Table 3.

Moving to the flexible SGT framework (Section 2.2), we then explicitly acknowledge the non-Gaussian features in excess stock returns via the generalized t distribution due to its ability in modelling fat tails (peakness and skewness). These features are already visible in descriptive statistics (Table 1) and will now be integrated to be part of the risk-return trade-off analysis. Similarly, as in Table 3, the estimated coefficients in Table 4 contain the estimated mean Eq. (5) and volatility Eq. (12), and due to the SGT distribution, also the distribution parameters, including in this case especially the skewness parameter λ . In line with the U.S. evidence in Theodossiou and Savva (2016), not containing the impact of the pandemic recession 2020, we find that the estimated risk-return coefficient c is throughout in different countries statistically significant and positive. For Japan, the estimated coefficient is also positive but smaller in magnitude than in other countries and statistically significant at the 5% significance level. This all suggest that by allowing for the negative skewness coefficient (see the estimates of λ) by the SGT distribution largely leads to the expected positive risk-return relation internationally.

In Table 4, the reported values of the likelihood ratio (LR) test between the SGT and the Gaussian GARCH models point out the superior performance of the SGT. In other words, in line with the findings of Theodossiou and Savva (2016, 2022), the

¹¹ The detailed estimation results for the GARCH-M model with the GJR-GARCH(1,1) are not reported to conserve space. They are available from the authors upon request. Moreover, recall that the Likelihood Ratio (LR) test is defined as $LR = 2[\text{Log lik}(\text{unrestricted}) - \text{Log lik}(\text{restricted})]$, including the two values of the estimated log-likelihood functions based on the unrestricted and restricted models of interest. Under the null hypothesis, leading to the restricted model, the LR test follows an asymptotic χ^2 -distribution with the degrees of freedom corresponding the number of restrictions.

Table 5
Estimation results of the QR-SGT model (with different asymmetry parameters).

	USA	Canada	UK	Germany	France	Italy	Australia	Japan
Mean Equation (Eq. (16))								
α_0	-0.729 (0.088)	-1.729 (0.147)	0.132 (0.073)	-0.794 (0.101)	-1.326 (0.109)	0.052 (0.121)	-1.887 (0.132)	-0.244 (0.104)
c_0	0.722 (0.026)	0.742 (0.032)	0.371 (0.018)	0.501 (0.022)	0.519 (0.023)	0.098 (0.022)	0.632 (0.024)	0.240 (0.022)
p_0	-0.336 [-0.424 -0.241]	-0.264 [-0.329 -0.196]	-0.324 [-0.377 -0.263]	-0.183 [-0.260 -0.104]	-0.038 [-0.113 0.044]	-0.191 [-0.284 -0.097]	-0.231 [-0.299 -0.161]	-0.052 [-0.125 0.043]
α_1	-9.311 (0.499)	-2.352 (0.161)	-7.127 (0.356)	-4.146 (0.296)	-1.327 (0.314)	-4.692 (0.253)	-0.280 (0.254)	2.893 (0.487)
c_1	1.752 (0.089)	0.812 (0.036)	1.275 (0.057)	0.967 (0.052)	0.755 (0.052)	0.683 (0.036)	0.560 (0.048)	-0.663 (0.084)
p_1	-0.369 [-0.457 -0.263]	-0.424 [-0.511 -0.333]	-0.310 [-0.361 -0.250]	-0.394 [-0.482 -0.301]	-0.542 [-0.622 -0.459]	-0.094 [-0.191 -0.008]	0.036 [-0.062 0.141]	0.115 [0.022 0.203]
Volatility Equation (Eq. (21))								
v_0	2.087 (0.070)	8.200 (0.258)	0.449 (0.021)	1.054 (0.060)	0.514 (0.027)	0.630 (0.039)	6.185 (0.210)	1.519 (0.082)
δ_0	0.004 (0.004)	0.084 (0.006)	0.067 (0.004)	0.139 (0.005)	0.069 (0.003)	0.075 (0.003)	0.116 (0.006)	0.116 (0.004)
ζ_0	0.205 (0.007)	0.118 (0.008)	0.007 (0.004)	-0.054 (0.005)	-0.023 (0.003)	-0.039 (0.003)	0.088 (0.008)	0.017 (0.005)
γ_0	0.737 (0.006)	0.509 (0.012)	0.912 (0.002)	0.857 (0.004)	0.930 (0.002)	0.924 (0.002)	0.702 (0.006)	0.826 (0.005)
v_1	11.743 (0.675)	0.002 (0.260)	6.768 (0.509)	6.184 (0.335)	7.159 (0.544)	4.181 (0.268)	0.005 (0.551)	4.282 (0.465)
δ_1	0.058 (0.010)	0.002 (0.026)	0.004 (0.004)	0.049 (0.006)	0.000 (0.009)	0.101 (0.006)	0.000 (0.024)	0.000 (0.006)
ζ_1	0.142 (0.013)	0.333 (0.030)	0.195 (0.010)	0.101 (0.008)	0.153 (0.009)	0.015 (0.006)	0.160 (0.027)	0.036 (0.007)
γ_1	0.594 (0.018)	0.903 (0.013)	0.782 (0.012)	0.746 (0.011)	0.721 (0.017)	0.843 (0.007)	0.928 (0.015)	0.886 (0.014)
Distribution Parameters								
n	8.013 (0.311)	8.155 (0.572)	22.921 (5.084)	6.774 (0.511)	6.286 (0.288)	10.584 (1.307)	3.465 (0.072)	15.068 (2.371)
k	2.058 (0.035)	1.643 (0.032)	1.736 (0.038)	2.187 (0.056)	2.342 (0.050)	1.946 (0.050)	2.802 (0.059)	1.828 (0.042)
λ_0	-0.222 (0.006)	-0.180 (0.005)	-0.213 (0.006)	-0.120 (0.007)	-0.124 (0.007)	-0.025 (0.007)	-0.162 (0.007)	-0.034 (0.006)
λ_1	-0.237 (0.017)	-0.457 (0.023)	-0.210 (0.016)	-0.306 (0.010)	-0.386 (0.013)	-0.151 (0.010)	-0.380 (0.028)	0.079 (0.014)
AIC	4168.66	4465.80	4815.60	4977.05	5097.68	5467.81	4768.91	4981.95
BIC	4201.91	4509.04	4808.84	5000.30	5140.93	5521.05	4812.15	5000.20
Log lik.	-2068.33	-2216.90	-2391.80	-2472.53	-2532.84	-2717.90	-2373.45	-2474.98
LR test	4.01	5.96	4.00	5.99	8.43	4.98	4.61	4.43

Notes: See the notes to Table 4. Likelihood ratio (LR) test compares the GJR-SGT with regime switching asymmetry parameters to the GJR-SGT specification (Table 4) - critical value for $\chi^2_{1, 0.05} = 3.84$.

flexibility of the SGT distribution, leading to the “pure” risk and skewness-kurtosis premium decomposition (10) - (11), is a necessary part to establish the risk-return relation accurately. The conventional GARCH-M, with the Gaussianity assumption and even with the GJR conditional variance specification (12), is clearly not adequate to find this relation.

Given the past voluminous theoretical and empirical research on the importance of business cycle fluctuations in asset pricing, it is next of particular interest to examine the impact of business cycle regimes together with the non-Gaussian features allowed by the SGT framework. That is to examine the role of such features in the underlying SGT distributional assumption as a driver of the expected positive risk-return relation when the impact of business cycle regimes is also present. In Table 5, the QR-SGT model is otherwise the same as in Table 4 but now in the regime switching form, containing business cycle-specific parameters, including skewness parameters. In most of the cases, the skewness parameters λ_0 and λ_1 are statistically significant, with their magnitude varying with the regime and resulting in different skewness price of risk and “pure” price of risk estimates. More specifically, during recessions, the asymmetry parameter (λ_1) and the skewness price of risk (p_1) are more negative for most countries (except the UK, Italy and Japan) than in the expansion phases. This is possibly attributed to the investors’ behaviour that during periods of excess volatility (like recessions), investors realize that their expected returns will not materialize and switch to less risky assets, thereby introducing increased negative skewness to the price of risk.

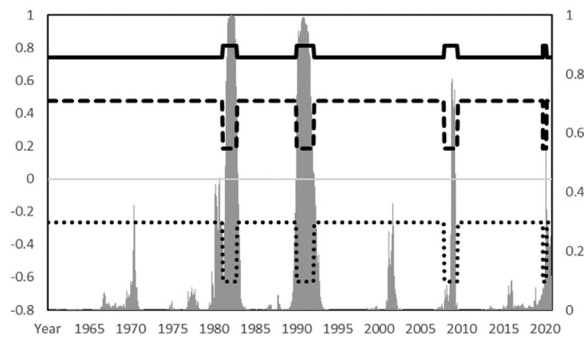


Fig. 3. Pure and Skewness Price of Risk during Business Cycle Phases: The case of Canada. *Notes:* Solid line represents the pure price of risk, dotted line the skewness price of risk and dashed lines the combined effect (left axis). The shaded area depicts the estimated (ex-ante) recession ($y_t = 1$) probability P_t (right axis).

In addition to the estimated skewness parameters, in Table 5 we find that the estimated coefficients in the mean and volatility equation are generally regime specific. In particular, in the mean equation, we find that the risk premium parameters c_0 and c_1 are almost unanimously statistically significant and positive: The only exception is Japan. Given the dominant expansion regime (i.e., as in Fig. 1, the expansion state of the business cycle is highly dominant in all the countries), on average, the estimated risk-return trade-off is positive also in Japan over time.

Fig. 3 further illustrates the positive risk-return trade-off by depicting the regime switching coefficients of the pure price of risk, the skewness price of risk and the combined effect over time in the case of Canada. The estimated (ex-ante) estimated counterparts, based on the information available at time $t - 1$, can be obtained by using the depicted (conditional) recession probability (see Eqs. (14) and (18)). Similar patterns occur also in the rest of the countries: For expositional brevity we depict only the case of Canada in Fig. 3.

In summary of our international evidence, the negative estimates of parameters p_0 and p_1 , as generated by the SGT parameters n , k and regime-dependent skewness parameters λ_0 and λ_1 , suggest that without moving to the SGT, and hence the decomposition (16), the employed econometric model is not flexible enough to produce the expected positive risk-return relationship. In the conventional GARCH-M models, the obtained risk premium estimates might easily be mixed up by the “pure” and “skewness” components, leading to the inconclusive conclusions on the risk-return relation, strongly supporting the use of the SGT. Moreover, in most cases, the estimates of c_0 and c_1 are larger than c in Table 4, highlighting a different attitude towards risk that investors may have during the business cycle regimes (expansions and recessions) compared to the average level over the cycle. That is, the level of the risk premium seems more exposed on changes in the conditional variance due to regime switching behaviour. Naturally also the regime switching constant terms and parameters p_0 and p_1 affect the expected returns.

3.3. Extensions and robustness analyses

In this section, we report some key additional analyses over the main ones presented in Section 3.2. First, we examine the role of the shape (kurtosis) parameter, and potential regime switching behaviour therein, as another source of flexibility in the QR-SGT model considered so far. Second, we consider the impact of using the U.S. business cycles regimes as a proxy of “global business cycles” instead of the domestic regimes for different countries. Third, we also consider a subsample starting from the year 1985 instead of the full sample period starting at the beginning of the 1960s.

In this section, to conserve space, we do not report the estimation results of the (regime switching) GJR-GARCH volatility equations (results are available from the authors upon request). These are essentially very much the same as we have already reported in Tables 3–5, and not of the main interest in this paper as our main objective is to consider the risk-return trade-off and the impact of skewness and kurtosis on that relation.

3.4. The role of (regime switching) kurtosis parameter

In Table 6, we examine the impact of allowing for business cycle-specific shape (kurtosis) parameters k_0 and k_1 as a part of the QR-SGT specification, bringing additional flexibility over the results reported in Tables 4 and 5. In most countries, the estimated shape parameter is higher in the recession state (except Germany and France). With this extension, the reported values of the LR tests indicate that the additional nonlinearity in country-specific examinations is not highly statistically significant over the models reported in Table 5. Now in all the countries, including also Japan, the risk premium parameters are positive in both regimes. Overall, it seems that introducing the regime-dependent skewness is a more important ingredient than allowing for the additional flexibility by regime switching kurtosis parameters. That is why we are relying on a more parsimonious QR-SGT specification considered in Section 3.2 (Table 5) as our main model.

Table 6

Estimation results of QR-SGT (with different asymmetries and shape parameters) model.

	USA	Canada	UK	Germany	France	Italy	Australia	Japan
Mean Equation (Eq. (16))								
α_0	-0.788 (0.089)	-1.901 (0.149)	0.132 (0.073)	-0.828 (0.103)	-1.195 (0.108)	0.070 (0.121)	-2.009 (0.134)	-0.100 (0.105)
c_0	0.737 (0.026)	0.768 (0.032)	0.373 (0.018)	0.542 (0.023)	0.504 (0.024)	0.093 (0.022)	0.604 (0.023)	0.198 (0.022)
p_0	-0.337 [-0.425 -0.242]	-0.255 [-0.321 -0.178]	-0.327 [-0.376 -0.268]	-0.215 [-0.292 -0.135]	-0.201 [-0.277 -0.119]	-0.037 [-0.129 0.050]	-0.214 [-0.285 -0.148]	-0.038 [-0.112 0.054]
α_1	-9.270 (0.474)	-2.521 (0.222)	-7.126 (0.362)	-9.346 (0.475)	-2.051 (0.317)	-4.726 (0.260)	-2.520 (0.284)	-2.331 (0.338)
c_1	1.723 (0.091)	0.992 (0.043)	1.277 (0.058)	1.521 (0.076)	0.877 (0.049)	0.702 (0.037)	1.075 (0.063)	0.323 (0.066)
p_1	-0.350 [-0.436 -0.245]	-0.801 [-0.892 -0.713]	-0.320 [-0.371 -0.261]	-0.242 [-0.333 -0.152]	-0.543 [-0.681 -0.459]	-0.245 [-0.347 -0.153]	-0.738 [-0.829 -0.658]	-0.006 [-0.105 0.092]
Distribution Parameters								
N	8.126 (0.352)	7.529 (0.508)	22.602 (4.906)	8.131 (0.635)	6.017 (0.234)	9.938 (1.150)	3.023 (0.053)	38.853 (18.034)
k_0	1.944 (0.035)	1.609 (0.032)	1.751 (0.039)	2.360 (0.061)	2.678 (0.060)	1.949 (0.050)	2.912 (0.061)	1.531 (0.037)
k_1	3.385 (0.131)	2.414 (0.110)	1.670 (0.055)	1.193 (0.026)	1.809 (0.047)	2.047 (0.065)	93.133 (184.016)	2.472 (0.086)
λ_0	-0.225 (0.006)	-0.176 (0.005)	-0.214 (0.006)	-0.138 (0.007)	-0.129 (0.007)	-0.024 (0.007)	-0.158 (0.007)	-0.025 (0.006)
λ_1	-0.218 (0.022)	-0.553 (0.036)	-0.211 (0.015)	-0.178 (0.009)	-0.390 (0.011)	-0.159 (0.010)	-0.540 (0.040)	-0.004 (0.017)
Log lik.	-2068.02	-2216.81	-2391.58	-2471.31	-2532.28	-2717.88	-2371.59	-2473.98
LR test	0.62	0.19	0.44	2.43	1.13	0.05	3.72	1.99

Notes: See the notes to Table 4. Confidence interval for skewness price of risk (p_0 and p_1) is calculated based on the bootstrapping procedure of Rapach and Wohar (2009). Likelihood ratio (LR) test compares the GJR-SGT with different asymmetries and shape parameters specification to the GJR-SGT with restricted shape parameters specification (see Table 5) - critical value for $\chi^2_{1, 0.05} = 3.84$. Estimation results of the volatility equations are excluded (available upon request).

3.5. U.S. business cycle regimes for all the countries

In Table 7, we report the results of the QR-SGT model corresponding the one used in Table 5, with the difference that now the U.S. expansion and recession periods determine the business cycle regimes, and hence the values of y_t , for all the countries. The major role of the U.S. onto global financial markets is the natural rationale behind this view (see, e.g., a review of the related results on the U.S. impact for predictive regressions in Rapach, Strauss and Zhou (2013)), together with the fact that the NBER business cycle dating algorithm, and the resulting official turning point chronology, is the one that ECRI aims to emulate for other (international) countries. In other words, in this section, for each country, the probit and regime switching part of the QR-SGT model is estimated based on the U.S. business cycle regimes, predicted by the domestic variables (spread and stock returns) so that the U.S term spread is employed as the foreign spread.

Table 7 shows that the main conclusions obtained in Section 3.2 are still essentially intact. Interestingly, now all the risk premium coefficients are statistically significant in both regimes. However, for Italy, the U.S. business cycle regimes changes the estimated risk-return relation so that the risk premium coefficient in the expansion regime is negative, while the skewness parameter is positive. This may suggest that the U.S. business cycles do not provide an adequate joint description with Italian stock market fluctuations. On the other hand, it is noteworthy that for Japan, for which the results in Table 5 were somewhat unclear, we now obtain coherent results when compared with other countries. The fact that ECRI has identified multiple recession periods for Japan between the years 1997 - 2020, which are not classified as recession periods in the U.S. and various other countries, may well somewhat obscure regime switching results based on the domestic regimes. Using the U.S. regimes “stabilizes” the situation and hence the major worldwide bear stock market periods in the last few decades are also better characterized in the Japanese case. On the other hand, in contrast to all the other international evidence on positive and statistically significant skewness parameter occurs in the recession regime, suggesting that the Japanese stock market behaves somewhat differently than the data of other countries implies (possibly because of the prolonged low interest rates policy).

3.6. Subsample period 1985–2020

Since the mid-1980s, the economic conditions in the U.S. have been characterized by a period of reduced macroeconomic volatility, strongly rising asset prices and falling interest rates. Whether or not this “Great Moderation” time period ended in 2007 is not of interest in this study: Instead, with this more recent subsample period, starting from the year 1985, we

Table 7
Estimation results of QR-SGT (with different asymmetry parameters) model (U.S. business cycle regimes)

	USA	Canada	UK	Germany	France	Italy	Australia	Japan
Mean Equation (Eq. (16))								
α_0	-0.729 (0.088)	-1.319 (0.139)	0.245 (0.071)	-0.504 (0.099)	-1.489 (0.116)	0.859 (0.111)	-2.629 (0.166)	0.254 (0.087)
c_0	0.722 (0.026)	0.653 (0.032)	0.310 (0.018)	0.392 (0.023)	0.569 (0.024)	-0.133 (0.020)	0.758 (0.032)	0.138 (0.019)
p_0	-0.336 [-0.424 -0.241]	-0.236 [-0.302 -0.172]	-0.261 [-0.360 -0.169]	-0.152 [-0.233 -0.071]	-0.181 [-0.269 -0.102]	0.070 [-0.008 0.149]	-0.136 [-0.224 -0.042]	-0.042 [-0.112 0.036]
α_1	-9.311 (0.499)	-8.942 (0.454)	-8.445 (0.290)	-8.151 (0.467)	-13.004 (0.604)	-5.757 (0.352)	-10.795 (0.481)	-7.531 (0.387)
c_1	1.752 (0.089)	1.575 (0.068)	1.501 (0.048)	1.819 (0.071)	2.252 (0.092)	0.983 (0.049)	1.095 (0.056)	0.872 (0.071)
p_1	-0.369 [-0.457 -0.263]	-0.493 [-0.580 -0.413]	-0.405 [-0.473 -0.331]	-0.857 [-0.967 -0.761]	-0.540 [-0.604 -0.479]	-0.513 [-0.593 -0.432]	-0.130 [-0.213 0.050]	0.120 [0.026 0.204]
Distribution Parameters								
n	8.013 (0.311)	9.473 (0.670)	100.000 (.)	6.486 (0.343)	5.786 (0.248)	10.343 (1.006)	3.376 (0.062)	55.766 (32.834)
k	2.058 (0.035)	1.665 (0.030)	1.650 (0.035)	2.494 (0.059)	2.422 (0.052)	2.012 (0.046)	3.113 (0.069)	1.667 (0.037)
λ_0	-0.222 (0.006)	-0.159 (0.006)	-0.169 (0.006)	-0.098 (0.007)	-0.118 (0.007)	0.046 (0.006)	-0.094 (0.007)	-0.027 (0.006)
λ_1	-0.237 (0.017)	-0.343 (0.013)	-0.266 (0.014)	-0.605 (0.017)	-0.366 (0.013)	-0.342 (0.015)	-0.090 (0.017)	0.077 (0.015)
Log lik.	-2070.33	-2263.40	-2307.87	-2359.94	-2364.34	-2495.89	-2370.63	-2361.74

Notes: See the notes to Tables 4-6. In these results, the U.S. recession periods determine business cycle regimes in each country.

Table 8
Estimation results of QR-SGT (with different asymmetry parameters) model (sample period 1985 - 2020).

	USA	Canada	UK	Germany	France	Italy	Australia	Japan
Mean Equation (Eq. (16))								
α_0	-0.166 (0.087)	-0.845 (0.155)	-0.268 (0.136)	0.219 (0.144)	-0.396 (0.129)	-4.801 (0.314)	-0.024 (0.138)	-0.652 (0.098)
c_0	0.712 (0.026)	0.557 (0.032)	0.545 (0.036)	0.531 (0.030)	0.590 (0.029)	0.973 (0.049)	0.437 (0.026)	0.347 (0.023)
p_0	-0.413 [-0.491 -0.324]	-0.260 [-0.324 -0.191]	-0.339 [-0.411 -0.267]	-0.383 [-0.463 -0.307]	-0.346 [-0.416 -0.183]	-0.112 [-0.201 -0.028]	-0.347 [-0.412 -0.272]	-0.109 [-0.185 -0.023]
α_1	-14.594 (0.954)	-2.647 (0.181)	-4.390 (0.543)	2.195 (0.182)	1.904 (0.354)	-6.871 (0.421)	0.306 (1.192)	4.292 (0.531)
c_1	2.561 (0.163)	0.996 (0.041)	1.048 (0.088)	0.086 (0.034)	0.069 (0.058)	1.284 (0.063)	1.181 (0.240)	-0.941 (0.091)
p_1	-0.757 [-0.840 -0.674]	-0.602 [-0.688 -0.513]	-0.541 [-0.667 -0.437]	-0.615 [-0.736 -0.482]	-0.472 [-0.579 -0.358]	-0.549 [-0.634 -0.453]	-1.154 [-1.282 -1.042]	0.204 [0.110 0.290]
Distribution Parameters								
n	7.002 (0.314)	7.636 (0.640)	7.799 (0.424)	7.798 (0.967)	100.000 (.)	100.000 (.)	6.922 (0.277)	26.226 (12.058)
k	2.080 (0.047)	1.634 (0.042)	2.578 (0.068)	2.192 (0.079)	1.798 (0.021)	1.532 (0.054)	1.960 (0.041)	1.787 (0.061)
λ_0	-0.277 (0.008)	-0.178 (0.007)	-0.217 (0.009)	-0.252 (0.009)	-0.222 (0.008)	-0.073 (0.008)	-0.234 (0.008)	-0.070 (0.009)
λ_1	-0.542 (0.031)	-0.439 (0.025)	-0.353 (0.025)	-0.418 (0.012)	-0.307 (0.017)	-0.372 (0.015)	-0.998 (0.153)	0.132 (0.015)
Log lik.	-1228.68	-1355.83	-1370.00	-1497.43	-1502.79	-1612.58	-1409.27	-1521.51

Notes: See the notes to Tables 4-6. In these results, the sample period is 1985 M1 – 2020 M12.

aim to examine whether the empirical evidence on the risk-return trade-off is still essentially the same than obtained with the full sample period. This subsample period contains the four major business cycle recession periods, including the recent pandemic recession in 2020. In Table 8, the results and business cycle regimes are country-specific (i.e., using the domestic business cycle regimes as so far throughout this study, except Table 7).

Table 8 shows that the risk premium and skewness parameters are again statistically significant in both regimes, strengthening the evidence obtained already with the full sample period. When comparing to the risk premium coefficients in Table 5, we can see that there is no systematic increase or decrease in either of the regimes over the countries. Japan is

Table 9

Estimation results of QR-SGT (with different asymmetries) model, lagged business cycle regimes (lag=3).

	USA	Canada	UK	Germany	France	Italy	Australia	Japan
Mean Equation (Eq. (16))								
c_0	0.925 (0.024)	1.082 (0.037)	0.434 (0.019)	0.318 (0.020)	0.555 (0.023)	-0.062 (0.022)	0.722 (0.024)	0.218 (0.023)
p_0	-0.352 [-0.439 -0.256]	-0.280 [-0.346 -0.224]	-0.359 [-0.407 -0.296]	-0.162 [-0.242 -0.099]	-0.204 [-0.274 -0.117]	-0.004 [-0.091 0.095]	-0.217 [-0.289 -0.149]	-0.025 [-0.082 0.059]
c_1	0.954 (0.067)	1.279 (0.043)	1.377 (0.043)	1.303 (0.076)	2.329 (0.122)	0.654 (0.033)	0.664 (0.063)	0.348 (0.066)
p_1	-0.351 [-0.437 -0.245]	-0.809 [-0.8929-0.721]	-0.307 [-0.362 -0.248]	-0.369 [-0.452 -0.275]	-0.604 [-0.711 -0.495]	-0.387 [-0.487 -0.295]	-1.015 [-1.182 -0.858]	-0.172 [-0.261 -0.071]
Distribution Parameters								
n	8.719 (0.393)	9.388 (0.799)	29.666 (8.323)	5.185 (0.255)	5.758 (0.248)	6.183 (0.320)	3.301 (0.065)	76.580 (22.693)
k	1.890 (0.031)	1.533 (0.031)	1.719 (0.038)	2.419 (0.060)	2.442 (0.054)	2.298 (0.053)	2.950 (0.065)	1.589 (0.035)
λ_0	-0.235 (0.006)	-0.192 (0.005)	-0.236 (0.006)	-0.107 (0.007)	-0.134 (0.006)	0.003 (0.007)	-0.154 (0.007)	-0.016 (0.005)
λ_1	-0.216 (0.023)	-0.529 (0.017)	-0.204 (0.015)	-0.248 (0.012)	-0.413 (0.014)	-0.257 (0.011)	-0.999 (0.001)	-0.112 (0.016)
Log lik.	-2096.73	-2226.09	-2386.63	-2489.26	-2546.70	-2721.02	-2381.22	-2497.85

Notes: See the notes to Table 4. In this table, the regime switching parameters are based on the lagged state of the business cycle y_{t-3} . Confidence interval for skewness price of risk (p_0 and p_1) is calculated based on the bootstrapping procedure of Rapach and Wohar (2009).

the only country (out of eight) where we do not find the positive risk-return relation in both regimes (in Japan we find that the risk premium parameter is negative in the recession regime). This is likely related to the high estimate of the constant term (α_1) in the recession regime. All in all, following the arguments made in Section 3.2, as the expansion is clearly the dominant regime, on average the risk-return relation is positive (ex ante and ex post) over time also in the Japanese case.

3.7. Real-time availability and lagged business cycle regimes

As argued in Sections 2.3 and 3.2, our main interest is in the estimation results of the component $f(r_t|y_t, I_{t-1})$ in decomposition (17). One can, however, argue that the business cycle regimes y_t are not known in real time due to information lags and subsequent revisions in the economic statistics determining such regimes. This is important especially if we aim to construct forecasts or fitted values for stock returns with our regime switching model (see Eq. (18)). In that case, a model for y_t needs to be specified, like we have proceeded via the probit model above where we have not yet considered the real-time data availability of the values of y_t .

Even though forecasting is not our main objective in this paper (as we are concentrating on the regime-specific risk premium and skewness parameters), let's consider the impact of using the lagged state of the business cycle, y_{t-l} , for some lag l , instead of y_t to empirical results. This lagging operation means that the business cycle regimes are then arguably somewhat more certain in real time than without such lagging. This change also means a subsequent modification to factorization (17) where we do not concentrate on specifying $P(y_{t-l}|I_{t-1})$ more detail: It would require a separate treatment, such as to determine the coincident, even lagged, indicators to nowcast the past state of the business cycle instead of using the term spread and stock returns as leading indicators in Eq. (15)).

In Table 9, we report the results for the selection $l=3$. It turns out that the conclusions on the risk premium and skewness coefficients are essentially the same as obtained with using the values y_t in Table 5. The main results are also robust to other selections of the lag length l (such as $l=6$, not explicitly reported in Table 9 but available upon request) and therefore the key conclusions on the risk-return trade-off and connected asymmetries are still intact.

4. Conclusions and discussion

We introduce a synthesis between well-documented skewness patterns and the state of the business cycle when examining the risk-return trade-off in excess stock returns. The past empirical findings on this fundamental relationship have been highly inconclusive. We show that building upon the flexible skewed generalised t (SGT) distribution, the resulting decomposition between the risk and skewness risk premium components generally leads to the expected positive risk-return relationship in our international dataset containing eight major countries. These two components are mixed in the conventional econometric approaches, leading to the conflicting empirical results due to misspecified models.

The SGT framework contains the commonly used Gaussian and, e.g., t and skewed t (Hansen, 1994) distributions as special cases. Hence the SGT provides us a highly flexible modelling context, now combined with business cycle regimes. Our international empirical results show that the SGT-based GARCH-in-mean models, without and with the dependence on the

business cycle regimes or their future expectations, the pure risk premium is almost unanimously positive in both regimes, and statistically significant, in most countries. Given that the expansion regime dominates (i.e., recessions are generally rather rare), the time-varying risk-return relation is on average positive over the business cycles and our regime switching model hence gives important insights where this time variation is coming from. Moreover, it turns out that the skewness parameter is typically specific to the state of the business cycle and more negative in the recession regime.

Our modelling approach resolves multiple “tensions” in the past literature. First of all, due to the resulting decomposition to the pure risk and skewness price of risk components, enabled by the use of the SGT distribution, the positive and statistically significant risk-return relation can generally be confirmed in international context. This main result has been highly debatable in the past empirical and econometric modelling approaches, which have seemingly been mostly too simple and not adequately acknowledging the non-Gaussian features of excess returns and (business cycle) regimes when extracting the risk-return relation. In addition to these modelling selections, [Hedegaard and Hodrick \(2016\)](#) emphasize the inclusion of the constant term to the mean equation of the GARCH-M models to consider the validity of the ICAPM, which is the traditional theoretical starting and building block for the risk-return analyses. However, this inclusion is not in line with the pure ICAPM and various studies have found a positive and (highly) significant risk-return relation when excluding the constant. Here the SGT-based regime switching model yields the positive relationship, along the ICAPM, even when the constant term is also incorporated into the econometric model, showing in its part the adequacy of our model.

Our analysis can be extended in various ways. Considering some other regimes (such as bear and bull stock market phases) than business cycles may well bring interesting complementary insights on the behaviour of the risk-return relation over such regimes. Similarly, combining mixed frequency data information on returns and variables determining regimes using mixed-data sampling (MIDAS) type of models (see, e.g., [Ghysels, Sinko and Valkanov \(2007\)](#) and [Ghysels and Qian \(2019\)](#), and the references therein) and to develop a multivariate model to account for spillovers between countries can be interesting further extensions to be built upon the advancement made in this paper.

Declaration of Competing Interest

Both authors declare no conflict of interest.

CRediT authorship contribution statement

Henri Nyberg: Conceptualization, Methodology, Formal analysis, Investigation, Writing – original draft, Writing – review & editing, Funding acquisition. **Christos S. Savva:** Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing.

Acknowledgements

We thank the Editor, two anonymous referees and conference participants at the 15th International Conference on Computational and Financial Econometrics (CFE 2021, London) for useful comments and discussions. Financial support (Nyberg) from the Emil Aaltonen Foundation (grant [180287](#)) and the Academy of Finland (grant [321968](#)) is gratefully acknowledged.

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