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# Pygmies, Bushmen, and savage numbers – a case study in a sequence of bad citations

Antti J.V. Tuominen *

*Department of Mathematics and Statistics, University of Turku, 20014 Turun Yliopisto, Finland*

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**Abstract**

There is a prevalent claim in the literature examining the history of numbers and the development of number words that some African group ("Bushmen" or "Pygmies") counts in a particular way, where their numerals are of the form 1, 2, 3, 2+2, 2+2+1, etc. Numerous forms of this claim are traced back to their original sources through an extensive search of the available literature. The author argues that the different forms can be traced back to two early sources, which have been misquoted and bastardized along the way.


**Abstrakti**

Numeraalien historiaa ja kehitystä käsittelevässä kirjallisuudessa esiintyy usein väite, että jokin afrikkalainen kansa ("bushmanit" tai "pygmit") laskee siten, että heidän numeraalinsa ovat muotoa 1, 2, 3, 2+2, 2+2+1, jne. Monia väitteen eri muotoiluja jäljitetään alkuperäislähteisiin tutkimalla laajasti saatavilla olevaa kirjallisuutta. Kirjoittaja argumentoi, että väitteen eri muodot voidaan jäljittää kahteen varhaiseen alkuperäislähteeseen, joiden sanoma on matkalla vääristynyt.


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*Keywords:* Counting; Africa; Bushmen; Development of counting; Misconceptions; Ethnomathematics

## 1. Introduction

Academics have commonly repeated stories to students and colleagues, which rarely if ever get checked for accuracy. Often this allows for mistakes and misinterpretations to creep in through embellishment (Rekdal, 2014). Such stories have been documented in varied fields like physics (Hon and Goldstein, 2013),

* Correspondence to: Rekolanpolku 6 A 9, 21500 Piikkiö, Finland Proper, Finland.
*E-mail address:* ajvtuo@utu.fi.

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nutrition (Rekdal, 2014), linguistics (Overmann, 2020), and the study of ants (Wetterer, 2006), although many more could be doubtlessly listed. There are even cases where entire theoretical frameworks thrive in academia, even with available counter evidence (Lévy-Bruhl, 1910; Tatsioni et al., 2007). In this paper I examine one often repeated claim about number words in African languages.

A cautionary remark on terminology will be given here. In this study the choice was made to use the terms "African pygmies" and "Bushmen", since these are the words used to this day in various forms of the claim in question and thus not mentioning them would make it harder for this study to be located by those also interested in the topic. However, the reader should not take this to mean the terms are fully appropriate terminology. "African pygmies" refers to a collection of hunter-gatherer tribes in Central Africa such as Mbuti and Twa. These groups mostly speak related languages and have historically been grouped together. There seems to be no universally accepted replacement for the term "pygmy", even though it is considered derogatory by most of these peoples. Generally speaking, specific tribe names are preferred, but some researchers use the term "Central African foragers" (Dembner, 1996; Hewlett et al., 1998). Equally, the term "Bushman" has a troubled and complex history and the term San is used instead by many modern organizations. The acceptance of the term San has not been universal, due to it also originating as a pejorative name given to them by their neighbors, although it has been gaining ground as the preferred term, especially in academia (Guenther, 2006; Mountain, 2003). Generally, specific group names are preferred, but in this case the study will reveal that the group referred to is not known, so even this compromise cannot be taken.

Already in 1973 Claudia Zaslavsky published her pioneering work, *Africa Counts*, where she collects information about numeracy, number systems, and mathematical games and practices in Africa (Zaslavsky, 1973). Her goal was to collect this information together as a resource for educators, but importantly she also includes critiques of some authors' treatment of African number systems and culture. Authors often dismiss African numerals with a few lines, stopping only to use them as examples of the most primitive number systems. This gives an impression of the continent as an uncivilized hinterland (and not just when it comes to mathematics), which Zaslavsky rejects with reason. After all, even if we disregard Egypt, Africa has many languages with complex number systems, like Yoruba and Edo, particularly in areas where cowrie shells were used as currency (Zaslavsky, 1973, pp. 204-212).

In a series of articles published in the 1980s, John Harris examined the history of using number lists from usually unnamed Australian aboriginal and Torres Strait islander languages as examples of "primitive number systems". He also discussed how this use perpetuated the idea that aboriginals were unable or simply too stupid to count up to three. This was not actually the case, but a false impression based on a lack of unique words for three or four. Their ability to determine how many sticks were in a bundle, for instance, was evidently not hindered by this. On the other hand, sometimes even when they expressed a number like 105 as "five men and a hand" (that is "five twenties and five") it was dismissed as "not real numbers" (Harris, 1982, 1987).

In 2014 Michael Barany published an article (Barany, 2014) about how "savage numbers", which he defines as "number-like or number-replacing concepts and practices attributed to peoples seen as civilizationally inferior", were used in Victorian scientific literature. Barany tells how an insignificant tangent in the diary (Galton, 1853). Francis Galton kept during his African expedition has been used to show how "primitive people" thought about numbers. Galton wrote that the Damara he traveled with insisted on trading sheep for "sticks of tobacco" one at a time instead of all in bulk. Although Galton did not even speak their language, his report morphed into an amusing anecdote to be spread the world over about how the Damara are in fact incapable of comprehending numbers in the first place. This 1853 story is still being used to this day in history of math textbooks (e.g. Burton (2011), quoted in section 2). Curiously, they leave out the part where Galton likens the Damara's understanding of counting to that of his spaniel, Dinah. They also leave out the relevant context that Galton would later go on to write many of the foundational texts of eugenics and even coined the term (Aubert-Marson, 2009).

Both Harris and Barany identify the works of John Crawfurd as a significant link in the spread of these anecdotes and examples. In 1863 Crawfurd published the article *On the Numerals as Evidence of the Progress of Civilization* (Crawfurd, 1863). In it he attempts to link the "rudeness" of number systems in various languages with the level of civilization of the speakers of those languages. Barany points out that savage numbers from sources like these were often used to argue for the primitivity of the "lower races", thus underpinning Victorian scientific racism (Barany, 2014). The word lists Crawfurd collected were quoted by popular early historians of mathematics (e.g. Conant (1896)), leading to their popularization in the 1900s through later works. While, one hopes, the examples are not intentionally used to promote such beliefs in the present, their unquestioned repetition can only serve to obfuscate true human history.

Explorers of the nineteenth century were rarely if ever linguists, and their transcriptions of languages they did not speak were inconsistent and seldom explained. Taking any example of the words of some small indigenous language at face value is likely to end in failure. Even a relatively well known European language with a well-established orthography like Finnish was reported by E. B. Tylor to have *lokke* as its word for ten in 1871 (Tylor, 1871, p. 238), but the word is and was in fact *kymmenen*.[1] It is clearly crucial to be extremely source-critical when reading books with examples from languages which you can assume the author does not speak.

Having an understanding of these facts caused skepticism, when the author took a course in the History of Mathematics in his university and was presented with lecture notes prepared by the teacher,[2] which included a suspiciously perfect example of a 2-count number system. 2-count here means that the numerals are formed recursively by addition by using the words for one and two. According to these lecture notes, "African Pygmies" counted *a*, *oa*, *ua*, *oa-oa*, *oa-oa-a*, *oa-oa-oa* and so on. The word for three is *ua* not *oa-a*, but otherwise the progression is neat and tidy, the words are so short that it would be hard to imagine shorter ones, and it comes from Africa, a place often stereotyped as "primitive". The reference to "African pygmies" is broad and there is no citation nor any mention of a particular language or people.

Research soon revealed that history of mathematics literature holds three main versions of this claim. A number of formulations of the claim are summarized in Figure 1. In this paper these will be categorized broadly into the "a-oa"-form, "xa-t'oa"-form and "one, two, many"-form of the claim depending on the numerals involved. In addition to this, there were some oblique forms of the claim that omitted any actual number words and merely associated the general recursive form of the numerals with some African group. The African group was almost exclusively "Bushmen from South Africa". It became clear this was the more authentic description. The search for the original sources of the claims leads to two places. The sources of those who described the language as having "one, two, many"-type numerals eventually lead to the early research of Wilhelm Bleek (1866, 1869), a pioneer in the field of Khoesan language studies. The more substantial, but evidently less accurate, form of the claim originates in a 1824 travel diary by George Thompson (Thompson, 1827), who was searching for land suitable for colonial expansion.

[1] Tylor's work appears to ultimately be leaning on the 1780 trilingual dictionary of Sámi by Ihre et al. (1780), where the words *lokke* (ten) and *lokket* (to count), both of which Tylor uses as examples, appear on the same page. The dictionary was clearly a source for many linguists and historians interested in comparative linguistics of Sámi, Finnish and Hungarian (Porthan, 1795, p. 42). Still, where exactly Tylor got his version of the facts is an open question. It should also be noted here that *lokke* is an antiquated spelling and the idea of one unified Sámi language is considered false. In different Sámi languages the word for ten is *luhkie* (Southern Sámi), *lúhke* (Ume Sámi), *lågev* (Pite and Lule Sámi), *logi* (Northern Sámi), *lååi* or *låå'k* (Skolt Sámi), *love* (Inari Sámi), лоагкь (Kildin Sámi), and логке (Ter Sámi).

[2] I will here leave the lecture notes anonymous for several reasons. The notes appear to have a long history of edits by various individuals, and I am not sure who the original author is. More importantly, it seems unnecessary to name (and thus shame) particular professors, who were just using the best information available to them to teach an introductory course to university students. Published books and journal articles are not only the original source of the claim discussed here, but also appropriately held to a much higher standard than Power Point presentations and lecture notes given to students.

| Author | Schmidl (1915) | Fettweis (1927) | Menninger (1969) |
|---|---|---|---|
| **People** | Bushmen | Bushmen | Bushmen |
| **Area** | Vaal and Riet rivers | Vaal and Riet rivers | South Africa |
| **1** | *‖a* | *a* | *a* |
| **2** | *\|oa* | *oa* | *oa* |
| **3** | *!uo* | *uo* | |
| **4** | *\|oa-\|oa* | *oa oa* | *oa-oa* |
| **5** | *\|oa-\|oa-‖a* | *oa oa a* | *oa-oa-a* |
| **Upper limit** | 10 | 10 | |
| **Author** | Eves (1990) | Flegg (1983) | Huylebrouck (2006) |
| **People** | Pygmies | Bushmen | Bushmen |
| **Area** | Africa | Botswana | |
| **1** | *a* | *a* | *xa* |
| **2** | *oa* | *oa* | *t'oa* |
| **3** | *ua* | *ua* | *'quo* |
| **4** | *oa-oa* | *oa-oa* | *t'oa-t'oa* |
| **5** | *oa-oa-a* | *oa-oa-a* | *t'oa-t'oa-xa* |
| **Upper limit** | | 6 | 10 |
| **Author** | Seidenberg (1960) | Ifrah (1985) | Dantzig (1930) |
| **People** | Bushmen | Bushmen and Pygmies | Bushmen |
| **Area** | Vaal and Riet rivers | Africa | South Africa |
| **1** | *xa* | ”one” | ”one” |
| **2** | *t'oa* | ”two” | ”two” |
| **3** | *'quo* | ”two-one” | ”many” |
| **4** | *t'oa-t'oa* | ”two-two” | |
| **5** | *t'oa-t'oa-t'a* | ”many” | |
| **Upper limit** | 10 | 4 | 2 |

Figure 1. Various forms of the claim present in the academic literature.

In this study, the author traced and located the original sources by finding sources that mention the claim, seeing what sources they cite and thus forming a sort of genealogy of how the claim spread through the literature (see Figures 2 and 4). In this case the method proved very successful, however, this is not the case for every claim. In some cases this method will lead into a brick wall when no citations are given at all (Rekdal, 2014). The success in this case owes to a tendency for writers in this field to paraphrase each other and refer to older well known works. However, in many cases stories like this are told as anecdotes in private conversations or even taught to a full lecture hall of university students, often gaining a bit of personal dramatization from the speaker through many re-tellings. This is a dimension that the method has not been able to capture at all, although it may well have been an important reason why this particular example continues in the literature when some others are left untold.

## 2. The *a-oa*-form of the claim

By way of comparison it became clear that the aforementioned lecture notes at my university were based on Howard Whitley Eves' (1911-2004) book called *An Introduction to the History of Mathematics* from 1953. The 1990 edition's (Eves, 1990) chapter 1.2, called *Number Bases*, was essentially identical to the lecture notes in order of presentation, terminology, and the examples used. Eves writes (emphasis by the author):

> There is evidence that 2, 3, and 4 have served as primitive number bases. For example, there are natives of Queensland who count “one, two, two and one, two twos, much,” and **some African pygmies count “a, oa,**

> **ua, oa-oa, oa-oa-a, and oa-oa-oa" for 1, 2, 3, 4, 5, and 6.** A certain tribe of Tierra del Fuego has its first few number names based on 3, and some South American tribes similarly use 4. (Eves, 1990, p. 12)

Since the connection with the notes was so strong, Eves was chosen as a launching off-point for looking for older sources. Eves does not use citations in his textbook, but does give a bibliography at the end of the chapter. One of these books is Karl Menninger's (1898-1963) book *Number Words and Number Symbols, A Cultural History of Numbers* (Menninger, 1969). In the book Menninger considers the existence of quinary or base-5 number systems, which he also calls gradations of 5. In this section Menninger is explicitly concerned with "pure" gradations, i.e. if a number system does not have a specific word for $5^2$, it is not considered quinary by Menninger, even though some researchers may disagree with this analysis (Salzmann, 1950). Menninger writes (emphasis by the author):

> Some have attempted to see [a quinary] system in the fact, for example, that the **Bushmen of South Africa have only the number word *a*, "one," and *oa*, "two," and that they say "four" as *oa oa* and "five" as *oa oa a*.** But this is an instance of mistaking one numerical principle for another: for this is not a gradation but merely an ordering, a much earlier phase, especially if it occurs in speech. (Menninger, 1969, p. 64)

While the change from "Pygmies" to "Bushmen" might at first seem alarming, it is not surprising. Central African forager peoples do not have a number system like the one Eves describes. The languages of these peoples belong in the large Niger-Congo and Central Sudanic language families (Bahuchet, 2006, p. 9) and even going back to proto-language reconstructions, these languages still have unique numerals up to five or more in both Niger-Congo (Pozdniakov, 2018) and Central Sudanic (Güldemann, 2018b) languages. In addition, even in Figure 1 most older versions of the claim use the term "Bushmen", which implies the reference to "Bushmen" is the original form. In fact, Eves' book is the oldest source found by the author, which names "African pygmies" in this context. Eves may thus be the originator of this particular mistake.

Menninger's quote also reveals he is criticizing previous writers, so the claim must precede him. The version of Menninger's book quoted is a translation of the German 1958 edition (Menninger, 1958). That edition too is a newer edition of the original 1934 edition, which also included the "'Bushman" claim (Menninger, 1934, p. 42). However, the first edition did not specify the speakers of the language, just gave the numerals.[3] Unfortunately, Menninger's book does not use citations either, so we must pick another book from Eves' bibliography.

As Eves' book was updated for various editions, new books appear in his bibliography, which in turn cite older versions of Eves' book. Some of these diversions are given at this opportunity because they innovate on the claim about "Bushmen" in some way. The claim has relatively recently been used in a 2011 edition of *The History of Mathematics: An Introduction* by David Burton, where the claim is in the form (emphasis by the author):

> Anthropologists tell us that there has hardly been a culture, however primitive, that has not had some awareness of number, though it might have been as rudimentary as the distinction between one and two. [...] **A similar system has been reported for the Bushmen of South Africa, who counted to ten** $(10 = 2 + 2 + 2 + 2 + 2)$ **with just two words; beyond ten, the descriptive phrases became too long.** It is notable that such tribal groups would not willingly trade, say, two cows for four pigs, yet had no hesitation in exchanging one cow for two pigs and a second cow for another two pigs. (Burton, 2011, pp. 1-2)

[3] From (Menninger, 1934, p. 42): "Man hat einen Ansatz darin sehen wollen, daß ein Stamm nur die zwei Zahlworte a ‚eins', oa ‚zwei' hat, und ‚vier' bildet: oa oa, ‚fünf': oa oa a."

Worth mentioning as derivative of Menninger and possibly Eves is Georges Ifrah (1947-2019), who discusses 2-count number systems and argues they cannot exist. In his book *From One to Zero: A Universal History of Numbers* (Ifrah, 1985) he writes (emphasis by the author):

> **Early in [the 20th] century there were still peoples in Africa, Oceania, and America who could not clearly perceive or precisely express numbers greater than 4.** To them, numbers beyond that point were vague, general notions related to physical plurality. [...] Among other examples of the same kind, we can mention the Indians of Tierra del Fuego, the Abipones in Paraguay, **the Bushmen and Pygmies in Africa**, and the Botocoudos in Brazil. [...] Since these people were able to use combinations of their two basic number words for the next two numbers - “two-one” for 3, “two-two” for 4 - one might wonder why they did not use “two-two-one” for 5, “two-two-two” for 6, “two-two-two-one” for 7, and so on. T**hat would be overlooking the fact that to express 3 and 4, numbers that they can recognize by direct perception**, people at this stage “simply pair 1 and 2, then 2 and 2; for them, **these are still pairs,** though for us they are whole numbers when we designate them as 3 and 4. Being able to conceive, recognize, and name only a single element or a pair of elements, how could these people, on their own, express 5 as 2 + 2 + 1, or 6 as 2 + 2 + 2, when each of those expressions would contain three elements?” (Ifrah, 1985, pp. 6-7)

Ifrah also repeats this idea in his later book *A Universal History of Numbers* (Ifrah, 2000, pp. 5-6). It is curious that Ifrah thinks this is the case. On one hand, putting “Bushmen and Pygmies” together like he does makes it seem like Ifrah has seen several forms of the claim and simply smashed them together to be sure. On the other hand, this does not explain why Ifrah thinks the numerals end at four, when almost all other sources list more numbers than four (see Figure 1). In his bibliography he even cites Menninger’s book which mentions a “Bushman” word for five. To support this section Ifrah cites the writings of Lucien Lévy-Bruhl (1857-1939) and Lucien Gerschel. In his work *How Natives Think* (Lévy-Bruhl, 1910), Lévy-Bruhl describes the differences between how “primitive people” think and how “modern people” think. Lévy-Bruhl used the different numeral systems of different peoples and cultures as one line of evidence of a fundamental difference on the level of logic between them, but does not mention the “Bushmen” or “Pygmies” and does not interpret the data as Ifrah does. For further discussion on Lévy-Bruhl’s theories, see for example (Segal, 2007).

Lucien Gerschel, on the other hand, appears to be a researcher of religions and numbers, although it is difficult to find anything biographical about him. His articles do not seem to hold more information on the topic of the “Bushmen”. However, in his article “La conquête du nombre : des modalités du compte aux structures de la pensée”, Gerschel presents the hypothesis that number systems with only two numerals cannot count beyond four, for reasons outlined by Ifrah in the above quote (Gerschel, 1962). This explains why Ifrah thinks a 2-count system is simply logically impossible.

Eves also cites Graham Flegg’s (1924-2015) 1983 book *Numbers: Their History and Meaning* (Flegg, 1983) where Flegg writes:

> The Bushmen of Botswana are perhaps the oldest people in all Africa. Their spoken number words proceed

| | | | |
|---|---|---|---|
| one: | a | four: | oa-oa |
| two: | oa | five: | oa-oa-a |
| three: | ua | six: | oa-oa-oa |

> Beyond six, they use a word meaning many. (Flegg, 1983, pp. 19-20)

Flegg appears to be the earliest source referring specifically to the “Bushmen of Botswana”, likely based on the idea that many San live in Botswana. Flegg does not give a citation, but his bibliography refers back

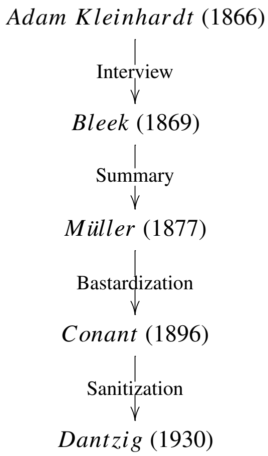


Figure 2. A description of the trajectory of the claim that "Bushmen" have inarticulate numbers, and they are of the "one, two, many"-type. Arrows indicate direct citation and the text characterizes what was done to the information.

to Eves and Menninger. Most likely Eves and Flegg have referred to each other throughout many editions of their books.

The 1953 edition of Eves' book has the exact same quote as presented above (Eves, 1953, p. 8). In this edition, Eves' much shorter bibliography includes a reference to Ore Øystein's (1899-1968) book *Number Theory and Its History* from 1948 (Øystein, 1948). The book is clearly the source of Eves' early chapters, however; it only claims that "certain African tribes use basic groups of 3 and 4" (Øystein, 1948, p. 2). Thus, while critical for Eves, this is not the source of the claim nor really even an example of the claim itself.

## 3. "One, two, many"-form of the claim

In his work Howard Eves also cites Tobias Dantzig's (1884-1956) book *Number: The Language of Science* from 1930 (Dantzig, 1930), which seems promising as a potential source as it predates all of the other material presented so far. He does talk about the San, but the claim is markedly different (bold emphasis the author's).

> [Anthropological studies] reveal that those savages who have not reached the stage of finger counting are almost completely deprived of all perception of number. [...] **The Bushmen of South Africa have no number words beyond one, two and many, and these words are so inarticulate that it may be doubted whether the natives attach a clear meaning to them.** (Dantzig, 1930, pp. 4-5)

Among Dantzig's (and Eves') sources we also find the book *The Number Concept: Its Origin and Development*, by Levi Leonard Conant (1857-1916) (Conant, 1896). It was first published in 1896, and is an influential early work on different number systems. In many ways Conant's book is a perennial classic of the history of mathematics, being cited over and over again even today. In the book we find the claim that Dantzig is clearly interpreting (bold emphasis by the author):

> Only the more intelligent of the Andamans can count at all, many of them seeming to be as nearly destitute of the number sense as it is possible for a human being to be. **The Bushmen of South Africa have but two numerals, the pronunciation of which can hardly be indicated without other resources than those of the English alphabet.** Their word for 3 means, simply, many, as in the case of some of the Australian tribes. (Conant, 1896, pp. 28-29)

When discussing the difference between counting and number words he adds:

> The Bushman has no number word that will express for him anything higher than 2; but with the assistance of his fingers he gropes his way on as far as 10. (Conant, 1896, p. 31)

Here we can clearly see how the "Bushmen" are used as an example of the lowest ranks of numerical understanding and the language that the "Bushmen" speak is described in an incredibly dismissive way. Dantzig goes so far as to suggest that "Bushmen" don't actually understand the few numerals they do have. While both authors pay lipservice to the idea that perhaps "we" are not so different from the "rudest tribes" after all, both also link the amount of numerals in the language to a people's level of civilization and intelligence. This is the exact problematic connection noted by Barany in Victorian texts (Barany, 2014).

Unlike the other writers we have mentioned, Conant cites a specific source: "Müller, Sprachwissenschaft, I. ii. p. 29" (see Müller, 1877, p. 29). This refers to Friedrich Müller (1834-1898), who wrote the book series *Grundriss der Sprachwissenschaft* between 1876 and 1888 where he tried to describe languages of the world, organized by what he saw as the world's different races organized by hairtype. He writes the following about the number system of the "language of the Bushmen" (Die Sprachen der Buschmänner):

> The number expressions actually only go up to "two" (*ǀ̣ku* or *ǀ̣u*). One is pronounced *ǀ̣oai*. The expression for "three" *ǀ̇oaya* actually means "many" and is used in connection with finger language for all numbers up to ten, so in connection with five raised fingers [*ǀ̇oaya*] means five, in connection with seven raised fingers it means seven.[4] (Müller, 1877, pp. 26-27)

Here Müller explains what the actual numerals are and that the "Bushmen" can count at least to ten using their fingers. Reading Müller's book also shows that what was meant by the numbers being "inarticulate" was that they contain click consonants. Using modern click notation, the numbers reported by Müller are *!oai*, "one", *!(k)u*, "two", and *ǂoaya*, "many".[5] Additionally, Müller gives a detailed citation for his information:

> Glossary recorded in 1866 in Cape Town Jail from Adam Kleinhardt, the son of a ǀ̣Kora and a Bushman woman (kindly given [to us] by Dr. Bleek).[6] (Müller, 1877, p. 28)

The cited German linguist Wilhelm Heinrich Immanuel Bleek (1827-1875) was one of the first to work on "Bushman languages" as they were called during the field's inception. A more modern term corresponding to Bleek's usage would be *Tuu and Kx'a languages* or *non-Khoe Khoesan languages*.[7] This terminology is not purely linguistic, rather South Africa was seen as containing multiple different native races of people who spoke different languages, had different ways of life, and different mental characters. The main four

[4] In the original: "Die Zahlenausdrücke reichen eigentlich nur bis „zwei" (*ǀ̣ku* oder *ǀ̣u*). Eins lautet *ǀ̣oai*. Der Ausdruck für „drei" *ǀ̇oaya* bedeutet eigentlich „viel" und wird in Verbindung mit der Fingersprache für alle Zahlen bis Zehn hinauf gebraucht, so dass er in Verbindung mit aufgehobenen fünf Fingern so viel als wie fünf, in Verbindung mit aufgehobenen sieben Fingern so viel als wie Sieben bedeutet."

[5] According to Müller ǀ̇ is the palatal click (Palataler Schnalzlaut) and ǀ̣ is the alveolar click (Cerebraler (lingualer) Schnalzlaut), which correspond to modern ǂ and ! respectively.

[6] In the original: "Wörterverzeichniss, aufgenommen im Jahre 1866 im Gefängnisse der Capstadt durch Adam Kleinhardt, den Sohn eines ǀ̣Kora und einer Buschmannsfrau (durch gütige Vermittlung des Dr. Bleek)."

[7] At least at this point in history. Later Khoe languages were also called "Central Bushman languages" or Central Khoesan language as an attempt to find higher order family relationships. However in the 1870s Bleek was still trying to argue for this relationship, which Müller, for example, did not support.

races often mentioned are “Bushman”, “Hottentot”, “Kafir” and “Negro”, where “Bushman” loosely corresponds to San, “Hottentot” to Khoekhoe and “Kafir” to Xhosa and other Bantu language speaking peoples. Depending on the author “negro” might refer to other African black people than the three aforementioned, be a synonym of “Kafir”, or even something more specific. It is important to note here that “Hottentot” and especially “Kafir” are seen as slurs (Mbowa, 2020).

Bleek pioneered the study of languages in South Africa and interviewed many speakers of |Xam as is also the case here. While Bleek’s work is invaluable to modern researchers and there is much to commend in his fieldwork, it should here be noted that he is not a wholly unproblematic figure. Much of his interest in “the Bushman language” came from the fact he perceived it as primitive. Bleek even theorized that the “Bushman” might be the closest to the gorilla and other non-human apes, having “retained” the click consonants in their language from prehistory (Bank, 2000). On the other hand, Bleek also held them up as the “noble savages” of South Africa, uniquely capable of telling European style moral fables, while the Bantu speaking peoples were preoccupied by “mere” ancestor worship. He has even been credibly accused of editing some of the stories he received in interviews to remove references to the scatological and erotic to further this image (Wittenberg, 2012).

Müller does not cite a specific publication by Bleek, but the source can be determined by comparison to Bleek’s earlier work. The examples and words are lifted straight from Bleek’s early observations on the language, published in 1869 as a report titled *The Bushman Language* (Bleek, 1869). The report was published in a collection of essays about the Cape area. The ultimate source for these numerals are Bleek’s interviews of Adam Kleinhardt, who was the son of a San woman and a !Kora man. In his report Bleek writes (bold emphasis by the author):

> **Of the numerals, the second (*!ku* or *!u*), at least, offers no resemblance either to the same numeral in the Bântu languages, or in Hottentot; and beyond two, every higher number is *ǂoaya* “many,”** although the Bushman may indicate with his fingers to some extent the exact number. *e.g.*, *ǂoaya*, showing four fingers, *i.e.*, “as many as four,” will indicate four, and *ǂoaya*, showing seven fingers, seven. (Bleek, 1869, pp. 282-283)

In addition, while discussing adjectives, Bleek gives some example sentences clearly showing the number *!oai* to be one, although he himself leaves it unstated:

| | |
|---|---|
| *n !kăχͨen n !u gan ǁu \|en* | ”my two sisters are small;” |
| *n !kăχu !oai gan \|eri* | ”my one sister is small.” (Bleek, 1869, p. 277) |

Since Bleek’s original notebooks have all been digitized (Bleek, 1866), we can even look one layer deeper for further context. The notebooks show Bleek’s transcription of |Xam was still being refined. Firstly, the sentence “my two sisters are small” has two different transcriptions *n ǂǂkăχͨen n !u gan ǁu |en̄* (Bleek, 1866, p. 17, lines 10-16), and *ṅ !kapen n !u gan ǁu |en̄́* (Bleek, 1866, p. 40, lines 10-11). Secondly the word *ǂoaya* is observed to mean “many” and “three” (e.g. (Bleek, 1866, p. 18, lines 12-13)), “*m pͨhͨānde gan ǂoaya*, my children are many (three, four, &)”). Thirdly, the two forms given for two can be explained by Bleek doubting his earlier transcriptions of numerals upon further interviews of Kleinhardt. Bleek writes

| | | |
|---|---|---|
| *\|weĩn* | louse; | |
| *\|weĩnya !koai* | one louse; | |
| *\|weĩnya ǁku* | two lice; | (!(k)u) |
| *\|weĩnya !ko!koanya* | many lice; | |

(Bleek, 1866, p. 53, lines 16-19)

Additionally, we have the phrases *|ke !koi kan ǂoaya*, “these men are so many (three, four, five, six, lifts my (sic) as many fingers as Indicate the numbers)” and *|ke kan ǂnoana*, “these men are ten” (Bleek, 1866, p. 48, lines 7-13).

In fact, Bleek’s notebook also contains a word for three. Bleek gives the translation “my three daughters” for “*m▱kaχai enǂnoãna*”[8] (Bleek, 1866, p. 19, line 3). This note appears to be from a later interview, most likely with a different “Bushman”, on 31st of August 1870. Since Bleek wrote his report in 1869, Müller had no way of knowing about the word for three.

However, in the fourth part of his series *Grundriss der Sprachwissenschaft*, Müller gives a number of corrections and additions, including an expanded grammar of the |Xam language (Müller, 1888). Müller’s source for this information is a set of handwritten notes given to him by Johannes Theophilus Hahn (1842-1905), who collected the linguistic data from his |Xam servant ‖Khaba |Hĩ (Müller, 1888, p. 1). Based on Hahn’s notes Müller now writes that not only is there a word for three, there is even a paraphrastic way to express four and five:

> These [numbers] reach only to three; from then on the expressions are compounded when there is a need for it.
> 1 *!oai*
> 2 *!kū*
> 3 *!noɣ̃a*
> 4 by compounding, i.e.: *!eie hĩṅ !ū !eie hĩṅ !ū* “there are two people, there are two people”.
> 5 also by compounding, i.e.: *!eie hĩṅ !ū !eie hĩṅ !ū !kui a haṅ !oai* “there are two people, there are two people, there is one man”
> viel = *|ɣhoaya*, applied to any number greater than three.[9] (Müller, 1888, pp. 12-13)

There is also a footnote that explains that *!kui* or “man” has the plural *!eie* or *!ē*.

Now we can surmise that Conant’s explanation is based on incomplete information about |Xam, which is the modern name of the language in question. |Xam has been dead since around the early 1900s, but has become a big cultural symbol in South Africa (Jones, 2019).

Be that as it may *!oai*, *!kū*, *!noɣ̃a*, *!eie hĩṅ !ū !eie hĩṅ !ū*, and *!eie hĩṅ !ū !eie hĩṅ !ū !kui a haṅ !oai*, or however one might properly render that in a more modern orthography, is quite far from *a*, *oa*, *ua*, *oa-oa*, and *oa-oa-a*. It is evident that |Xam cannot be the point of origin for Eves’ claim.

## 4. The *xa-t’oa*-form of the claim

There is another form of the claim that must also be discussed, as seen in the tables in Figures 1 and 3. This list is usually in a table quoted below including three languages, Gumulgal, Bakairí, and “Bushman”, which appears in a 1960 article “The Diffusion of Counting Practices” by Abraham Seidenberg (1916-1988) (Seidenberg, 1960) who seems to have compiled it. Seidenberg states (bold emphasis by the author):

[8] In Bleek’s notes ▱ is a bilabial click.

[9] In the original: “Dieselben reichen blos bis drei; von da an werden die Ausdrücke, wenn Anlass dazu vorhanden ist, zusammengesetzt.
1 *!oai*
2 *!kū*
3 *!noɣ̃a*
4 wird zusammengesetzt, z. B.: *!eie hĩṅ !ū !eie hĩṅ !ū* „Menschen welche zwei, Menschen welche zwei“.
5 wird ebenso zusammengesetzt, z. B.: *!eie hĩṅ !ū !eie hĩṅ !ū !kui a haṅ !oai* „Menschen welche zwei, Menschen welche zwei, Mensch er da einer“
viel = *|ɣhoaya*, wird für jede Zahl, welche über drei hinausgeht, angewendet.”

| Number | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **"a- oa"-form** | *a* | *oa* | *ua* | *oa-oa* | *oa-oa-a* | *oa-oa-oa* |
| **"xa- t'oa"-form** | *xa* | *t'oa* | *'quo* | *t'oa-t'oa* | *t'oa-t'oa-t'a* | *t'oa-t'oa-t'oa* |

Figure 3. A table showing the differences between the "xa, t'oa"-form and the "a, oa"-form of the claim.

> Counting may seem to be an elementary process that any human being would come upon by himself, yet the **fact that many groups of savages cannot count beyond 2 makes us realize that the process is an advanced one**. Over most of aboriginal Australia one finds essentially only two number words, 'one' and 'two': many of the tribes are reported definitely as not counting beyond 2, and as indicating higher multiplicities by the word "many," while others go a little further by compounding the words - for example, expressing 3 as "two-one," 4 as "two-two," and 5 as "two-two-one." A similar method of counting, the so called 2-system, is found in New Guinea, in South America, and in South Africa (see maps 1 and 2, facing pp. 280 and pp. 284).

| | *Gumulgal (Australia)* | *Bakairi (South America)* | *Bushman (South Africa)* |
|---|---|---|---|
| 1 | urapon | tokale | xa |
| 2 | ukasar | ahage | t'oa |
| 3 | ukasar-urapon | ahage tokale (*or ahewao*) | 'quo |
| 4 | ukasar-ukasar | ahage ahage | t'oa-t'oa |
| 5 | ukasar-ukasar-urapon | ahage-ahage-tokale | t'oa-t'oa-t'a |
| 6 | ukasar-ukasar-ukasar | ahage-ahage-ahage | t'oa-t'oa-t'oa |

(Seidenberg, 1960, p. 216)

Seidenberg is using the table and the mentioned maps as an argument for his theory of numerical development, where the whole world used to be populated mostly by simple 2-count systems like the ones in the table, but after the complex decimal system was developed, most likely in the middle east, it spread to all corners of the world. The 2-count systems in the table are the remnants of this diffusion process and thus located very far from the middle east.

Seidenberg is cited in Graham Flegg's more recent book *Numbers Through the Ages* from 1989 (Flegg, 1989, p. 9), John N. Crossley's book *Growing Ideas of Number* (Crossley, 2007, p. 10) and many others. Based on this it seems that Seidenberg is the one that popularized this form of the claim.

Seidenberg later adds:

> **In South Africa, the only people reported to count by the 2-system are the Bushmen, but here we have at least one clear example from the groups on the Vaal and Riet rivers, who count to 10 by that system.** From the other groups we have lists giving words only for 1 and 2, but can deduce very little from them. (Seidenberg, 1960, p. 221)

The specific reference to Vaal and Riet rivers is of utmost interest, since it links the *a-oa*-form and the *xa-t'oa*-form together. The German mathematician Ewald Fettweis (1881-1967) mentions these rivers in relation to "Bushmen" in his 1927 book *Das Rechnen der Naturvölker* (Fettweis, 1927).[10] Fettweis was among the first people to discuss ethnomathematics and published several foundational articles in the field. Fettweis was also interested in bringing ethnographic information into school curricula to foster interest in mathematics (Rohrer and Schubring, 2011). Citing an M. Schmidl, Fettweis writes:

[10] It is noteworthy that Menninger includes Fettweis' book in his bibliography in the 1958 edition of his book (Menninger, 1958).

> **The Bushmen of Vaal and Riet rivers count:** $1 = a$, $2 = oa$, $3 = uo$, $4 = oa\ oa$, $5 = oa\ oa\ a$ etc. up to 10.[11] (Fettweis, 1927, p. 46)

This citation of M. Schmidl further indicates that the *a-oa*-form and the *xa-t'oa*-form of the claim originate at the same source. Marianne Theresie Schmidl (1890-1942) was an Austrian ethnologist and librarian, who published her thesis *Zahl und Zählen in Afrika* in 1915 (Schmidl, 1915). In her thesis, Schmidl goes through the numerals and counting systems of several language families in Africa, as known at the time. Both claims are visible in chapter E. titled *Die Pygmäensprachen*,[12] where Schmidl writes:

> As far as the language of the Bushmen is concerned, or perhaps better the languages - it by no means represents a uniform type - we can observe a pure binary system in addition to a quinary system of numbers, as told by e.g. Bleek about the tribes on the Vaal and Riet Rivers. These only have independent expressions for the first 3 numbers, with which they then form all the others by means of addition. They are[13]

The binary numbers of the Bushmen.

| | | | | |
|---|---|---|---|---|
| 1 | = | *‖a*, | | |
| 2 | = | *\|oa*, | | |
| 3 | = | *!uo*, | | |
| 4 | = | *\|oa-\|oa* | = | $2+2$ |
| 5 | = | *\|oa-\|oa-‖a* | = | $2+2+1$ |
| 6 | = | *\|oa-\|oa-\|oa* | = | $2+2+2$ and so on until 10. |

(Schmidl, 1915, pp. 194-195)

Schmidl cites Wilhelm Bleek via G. Stow and goes on to add in a footnote about number five: "Stow gives here, probably by mistake, *‖a* instead of *|a*"[14] (Schmidl, 1915, p. 195).

George William Stow (1822-1882) worked as an ethnologist and cartographer in South Africa and was an acquaintance of Wilhelm Bleek. The work cited by Schmidl, *The Native Races of South Africa* (Stow, 1905), was published posthumously in 1905. In Stow's book the numerals are in a footnote within a direct quote, which explains why Schmidl claims that the information about the numerals comes from Bleek. While Stow names Bleek's article, "On Resemblances in Bushman and Australian Mythology" (Bleek, 1874), he gives neither the information nor the date and it would have been very difficult for Schmidl to

11 In the original: "Die Buschmänner am Vaal- und Rietriver zählen: $1 = a$, $2 = oa$, $3 = uo$, $4 = oa\ oa$, $5 = oa\ oa\ a$ usw. bis 10."

12 It must here be noted, however, that the headline of the chapter is significant, and might well have trickled down to Howard Eves' book, though by what means we cannot say.

13 In the original: "Was die Sprache der Buschmänner anbelangt, oder vielleicht besser die Sprachen - sie stellt keineswegs einen einheitlichen Typus vor -, so können wir neben einer quinaren Zahlenordnung ein reines Zweiersystem beobachten, wie es z. B. Bleek für die Stämme am Vaal- und Riet-River angibt. Diese haben nämlich nur für die ersten 3 Zahlen selbständige Ausdrücke, mit denen sie dann mittels Addition alle übrigen bilden. So ist

Die binare Zählung der Buschmänner.

| | | | | |
|---|---|---|---|---|
| 1 | = | *‖a*, | | |
| 2 | = | *\|oa*, | | |
| 3 | = | *!uo*, | | |
| 4 | = | *\|oa-\|oa* | = | $2+2$ |
| 5 | = | *\|oa-\|oa-‖a* | = | $2+2+1$ |
| 6 | = | *\|oa-\|oa-\|oa* | = | $2+2+2$ usw. bis 10." |

14 In the original: "Stow gibt hier, wohl irrtümlich, *‖a* statt *|a* an".

find it. Indeed, it is evident that Schmidl never read Bleek's article, because the numerals Stow gives are not in it. Stow quotes Bleek as saying:

> [b]oth of these nations are generally considered the lowest of the low in many points of human civilization, as, for example, in their very imperfect numerical system, the Bushmen having no numerals beyond two or three, and the Australians generally none beyond three or four. (Bleek, 1874, p. 101)

However, Bleek writes nothing in the article about paraphrastic formulations of higher numerals, nor gives any example words of any kind. The orthography used by Stow is different from Schmidl's, but Schmidl's quotation is otherwise accurate. The footnote is as follows (bold emphasis by the author):

> **These simple numerals were** (that is, among the Central and Eastern Bushmen) ***xa*, one; *t'oa*, two; *'quo*, three; any higher numbers were expressed by repetitions**, thus:

| | | |
|---|---|---|
| **T'oa-t'oa** | = | **four.** |
| **T'oa-t'oa-'ta** | = | **five.** |
| **T'oa-t'oa-t'oa** | = | **six.** |
| **T'oa-t'oa-t'oa-'ta** | = | **seven.** |
| **T'oa-t'oa-t'oa-t'oa** | = | **eight.** |
| **T'oa-t'oa-t'oa-t'oa-'ta** | = | **nine.** |
| **T'oa-t'oa-t'oa-t'oa-t'oa** | = | **ten.** |

> (Stow, 1905, p. 18)

In between paragraphs of quoting Bleek, Stow also inserts the following comment:

> It is certain, however, that the constant repetition of **the numbers three, five, and seven, their 'quo, t'oa-t'oa-'ta and t'oa-t'oa-t'oa-'ta** in their symbolic representations in **the valleys of the Gumaap and the Vaal** evidently indicate that they had a mystic or sacred meaning, now lost, but known and understood at the time by the initiated. (Stow, 1905, p. 19)

This explains how Schmidl arrived at the conclusion that the language was spoken along the Vaal and Riet rivers. Gumaap is an old precolonial name for the Riet river that appears in many of Stow's notes. Loosely, this is the area then known as the Orange Free State, in the north-eastern parts of South Africa.

Stow's book mostly lacks citations. Some of Stow's sources were never published and when an author is mentioned Stow may copy from their work for several pages (de Prada-Samper, 2016). It is possible to identify the book *Travels and Adventures in Southern Africa*, a travel diary of the expeditions of George Thompson in Southern Africa in the 1820s, as one such source. The book was published with illustrations in 1827 and then used by Stow as his source for "the Bushman number system". Let us go through Thompson's 1824 expedition for a bit of relevant context.[15]

[15] Thompson left from Cape Town on the 27th of July 1824, intending to travel northwards to Hantam, and then on to check on the lower parts of the Orange river on the west coast of the colony. He traveled on horseback, alone save for his various guides, through the Roggeveld Mountains and the Western Karoo. He found two Khoekhoe men called Witteboy and Jacob Zwart to guide him in "Groote-Toren" on the 5th of August. They passed a conical mountain called "Spioen-Berg", and north north-east from there until sunset, when they looked for a spring in a north-western direction. Having then made camp near a spring called "Adriaan's-Fonteyn", in the dusk of the 7th of August 1824, Thompson describes how a group of "Bushmen" entered their camp, enquiring about the reason of their travel. After some talking, the group had a bit of a party, where the "Bushmen" danced until midnight. It was during this party that the interview took place.

In his travels Thompson found two Khoekhoe[16] guides. Once, when a group of "Bushmen" approached their camp, Thompson was curious if their language was as "deficient in compass" as he had been told, so he had one of his guides ask questions from some of the "Bushmen". He writes of his results (bold emphasis by the author):

> The following was the result of my examination with regard to the numerals:
>
> **One, t'a; two, t'oa; three, quo.**
>
> These three sounds are the whole of their simple numerals. The others, as far as ten, are expressed by repetition and combinations of these three words, in the following manner:
>
> **Four - t'oa, t'oa.**
> **Five - t'oa, t'oa, t'a.**
> **Six - t'oa, t'oa, t'oa.**
> **Seven - t'oa, t'oa, t'oa, t'a.**
> **Eight - t'oa, t'oa, t'oa, t'oa.**
> **Nine - t'oa, t'oa, t'oa, t'oa, t'a.**
> **Ten - t'oa, t'oa, t'oa, t'oa, t'oa.**
>
> The exceeding want of invention and ingenuity displayed in their language is a striking evidence of the degraded state of intellect among them. The mere care of supporting existence seems to have engrossed their entire faculties. The intellectual nature has succumbed to the brutal. (Thompson, 1827, pp. 237-238)

Thompson gives us specific information on how and when this number system was recorded: late in the night of the 7th of August 1824, in a place known then as "Adriaan's-Fonteyn", as interpreted by Witteboy and Jacob Zwart, both Khoekhoe, not "Bushmen". There are even illustrations in the book of the "Bushmen" and of Thompson's two guides, although one can assume that suitably romantic artistic liberties were taken. Naturally, we still have to remain cautious in using Thompson's transcription of a language he was evidently hearing for the first time as a source.

It is also evident that Stow is gravely misrepresenting Thompson's account of the events. Adriaan's-Fonteyn is not on any modern map available to the author, but the Roggeveld Mountains and many of the towns Thompson mentions are. His trip took place in the western parts of the country in the edges of the semi-arid Karoo plains, while Stow claims these words would have been commonly spoken in the Orange Free State almost a thousand kilometers away. This is not just a mistake: Large parts of Thompson's diary are retold in Stow's book in a section titled "The Bushmen of the Western Karoo".

In his book Stow theorizes that there are two main groups of "Bushmen", "painter Bushmen" and "sculptor Bushmen", who are differentiated mostly by language, art, and life style. The area of the "sculptor Bushmen" around Vaal and Riet rivers is the area where the oldest and most authentic "Bushman culture" was from. To pull Thompson's numbers out of their real context, possibly because Stow was not happy about Thompson's less than charitable interpretation, only to fit them to his mythological and artistic model comes close to lying.

In addition to this, Stow misremembers or misquotes the numerals themselves turning *t'a* into *xa*, *quo* into *'quo* and *t'oa, t'oa, t'a* into *t'oa-t'oa-'ta*. Of these mistakes the first is the most significant and was

[16] In the original "Hottentot". It is here important to remember that one of the things that distinguished the San and Khoekhoe was that they spoke different languages.

Table 1
A selection of works that contain the *xa-t'oa-* and *a-oa*-forms of Thompson's numerals. Type *2+2+1* means some sort of oblique form of the claim that does not mention the numberwords themselves. AP in the notes means that the speakers are given as "African pygmies".

| Authors | Form | Notes |
|---|---|---|
| Thompson (1827) | *t'a-t'oa* | Original form |
| Stow (1905) | *xa-t'oa* | |
| Schmidl (1915) | *‖a-\|oa* | |
| Fettweis (1927) | *a-oa* | Earliest instance of the *a-oa*-form |
| Menninger (1934) | *a-oa* | |
| Eves (1953) | *a-oa* | AP |
| Seidenberg (1960) | *xa-t'oa* | Widely cited source |
| Chapman (1966) | *a-oa* | AP |
| Zaslavsky (1973) | *2+2+1* | Claim enters ethnomathematics |
| Flegg (1983) | *a-oa* | |
| Ifrah (1985) | *2+2* | Impossible to count to 5 |
| Flegg (1989) | *xa-t'oa* | |
| Barrow (1992) | *xa-t'oa* | |
| Schmandt-Besserat (1992) | *xa-t'oa* | |
| Crossley (1994) | *xa-t'oa* | Encyclopedia |
| Subramaniam and Sahoo (1999) | *xa-t'oa* | |
| Ifrah (2000) | *2+2+1* | |
| Duffy et al. (2003) | *xa-t'oa* | |
| Huylebrouck (2003) | *xa-t'oa* | Uses *t'oa-t'oa-xa*, book |
| da Silva (2006) | *a-oa* | AP |
| Huylebrouck (2006) | *xa-t'oa* | Uses *t'oa-t'oa-xa*, book summary |
| Crossley (2007) | *xa-t'oa* | |
| Burton (2011) | *2+2+1* | |
| Bello (2012) | *2+2+1* | High-school textbook |
| Comin and dos Santos (2013) | *a-oa* | AP |
| Mol (2013) | *a-oa* | |
| Descaves (2016) | *a-oa* | AP. Uses *oa-oa*, *oa-ua*, *ua-ua*. Aimed at teachers |
| Phatshwane and Mbekomize (2017) | *xa-t'oa* | Added numerals |
| Owens et al. (2018) | *2+2+1* | Influential work in number systems |
| do Vale and Melo (2019) | *a-oa* | AP |
| Singh (2020) | *a-oa* | |

copied forward, as we have seen. It is possible that some of these mistakes were introduced during the posthumous editing process, but this does not change the legacy of the published book.[17]

## 5. The extent of the spread of the claims

Having now established where these claims come from, we take a look into their current popularity. The position that we can simply dismiss the numbers of the "Bushmen" as too difficult to pronounce, as

[17] In a preface to the book, George Theal describes how he received a rough draft of a manuscript from Lucy Lloyd. He made numerous edits and especially liberal cuts and omissions of large quotes that Stow had included for himself, possibly as reference material. At the very least, mistakes like turning *t'a* into *xa* might have been introduced by Theal sloppily copying Stow's writing. Theal, as a historian, would definitely have been capable of adding the footnote, but it is questionable if he would have had a reason to. Lloyd as one of the foremost experts on the topic would hardly have chosen to quote Thompson's ancient diary. Equally, however, this same expertise begs the question: why did she not have it edited out or revised? It is possible that Lloyd, while described as editing the text, mostly gave final opinions and had little involvement in the editing process itself.

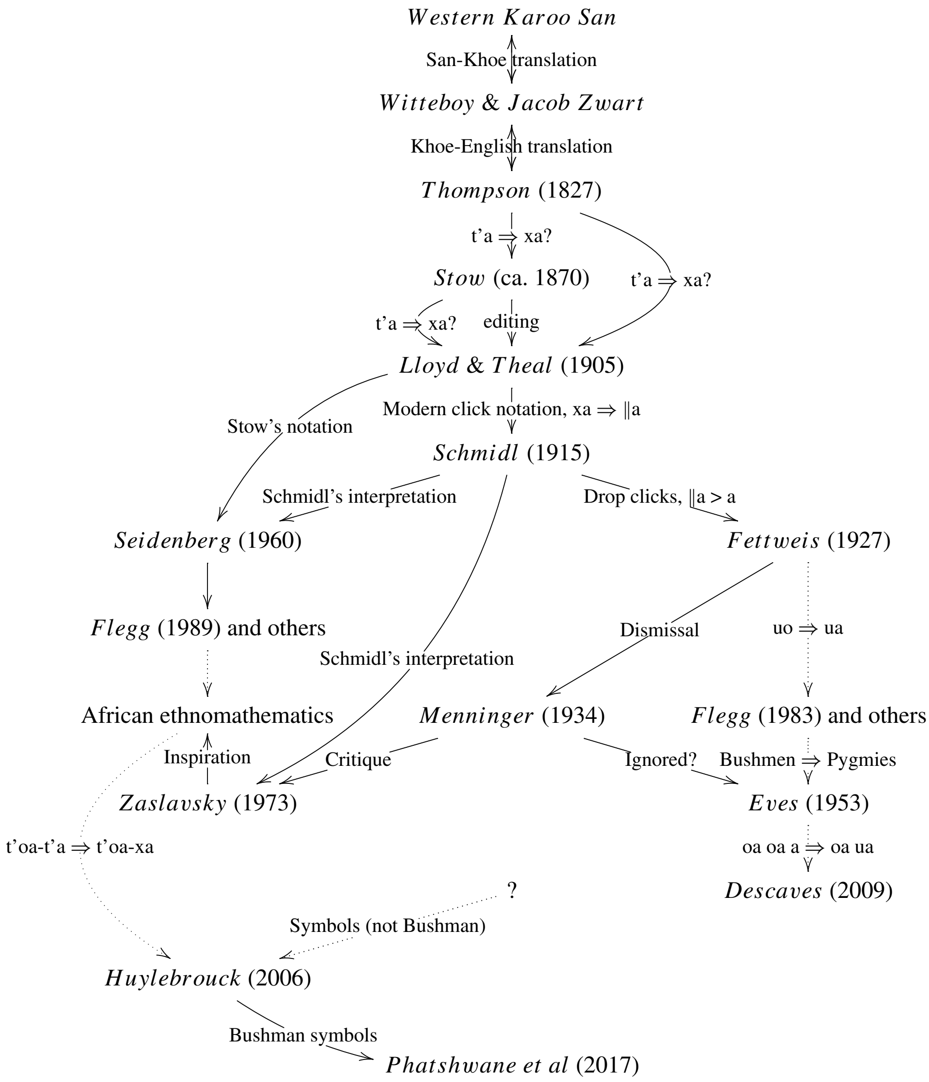


Figure 4. A graph showing the relationships between most of the books mentioned. *Stow* here refers to his original manuscript and *Lloyd & Theal* to their edited and published version. Solid arrows indicate direct citation, dashed arrows indicate indirect citation or otherwise condensed paths.

Conant (1896) and Dantzig (1930) did, seems to have been left to the dustbin of history where it belongs (see Figure 2). Unfortunately the same cannot be said for Thompson's numerals. In the interest of brevity, a sampling of the claims has been compiled into Table 1 and the relationships of key pieces are summarized in Figure 4. However, a few key instances should be pointed out, as they show further evolution of the claim.

The *Objectif CRPE: Maths* series by Alain Descaves has changed the *a*-*oa*-claim in a fundamental way. Descaves tells us that "certain African Pygmies"[18] count *a*, *oa*, *ua*, *oa-oa*, *oa-ua*, *ua-ua*, simply changing the numbers to a different additive paradigm (Descaves, 2016, p. 20). This means that not only are the speakers misidentified and the numerals wrong, they no longer even conform to the general principle that

[18] In the original: "certains Pygmées d'Afrique".

everyone else is trying to demonstrate. This annual series is especially important as it is aimed at future teachers, and further propagation of this novel form of the claim is to be expected.

Another point of interest is the scholar of ethnomathematics Dirk Huylebrouck, who has used the claim in his book (Huylebrouck, 2003, pp. 52-53) and article (Huylebrouck, 2006, p. 138) about it. Huylebrouck has slightly altered numerals and instead uses "*t'oa-t'oa-xa*" and so on. In the 2006 article summarizing his earlier book (Huylebrouck, 2006), Huylebrouck adds the following sentence right after listing the "Bushman numerals": "The use of 2 sometimes led to the following notations for 6 ||| |||, 7 |||| |||, and 8 |||| ||||". Reading Huylebrouck's book makes it clear that this refers to 2-count systems in general and the sentences are actually separated by a chapter headline. However, the article obscures this point. This has mislead others, such as Percy Phatshwane and Christian Mbekomize, who attribute both things to "the mental arithmetic skills of the Bushmen" (Phatshwane and Mbekomize, 2017, p. 12).

Something should also be said about the role of encyclopedias. In addition to the specialist encyclopedia *Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences* (Crossley, 1994, p. 152), a German patent (Schloeser, 2000) cites the claim from *Brockhaus Enzyklopädia* from 1974, which the author has been unable to access. In addition to these more traditional editions, the online encyclopedia Wikipedia also includes the claim on several pages (Username: Gubbubu, 2009; Username: J Hokkanen, 2012). Easily accessible formats like these have likely also had an effect on the spread of the claim.

The claim has even made its way into popular culture: in the historical romance novel *The Salt of My Desire*, Joan Schrauwen has a "Bushman" named 'Ki waggle his fingers and say "t'oa-t'oa'-t'a?" to indicate "five" to Grietje Nordman (Schrauwen, 2012, p. 140). It also appears in Daniel Tammet's autobiography *Embracing the Wide Sky: A Tour Across the Horizons of the Mind*, in a section where he is giving a simple history of "Counting Around the World" (Tammet, 2009, p. 140).

## 6. Discussion

We have looked at a complicated history of changes and distortions in what sorts of numerals the San use. Ultimately we have seen that no input from a speaker of these languages since 1866 has been a part of the discussion. It is a natural question to ask, then, if these numerals are at all accurate.

The author has found no discussion of the numerals presented by Thompson in the academic literature on these languages. Even though the claim is of dubious origin, the specific location should help modern linguists classify and analyze the sample. Now that the original source has been identified, perhaps this can happen.

Unfortunately, Khoesan languages in general are poorly documented. The San population has faced enslavement, war, and colonization. The result is that most of these languages are extinct and the only records of them exist in archived field notes of researchers from the mid 1800s to the early 1900s. Within South Africa all Tuu and K'xa languages, with the exception of !Xun, are moribund with under ten speakers (Jones, 2019). Consequently the same lack of knowledge applies to the numeral systems themselves, even though they are the sort of "basic" vocabulary that researchers do often document. There is particularly little discussion that the author has been able to find on paraphrased higher numerals, even when numerals are discussed (Collins and Gruber, 2014; Güldemann, 2014, 2018a; Heine and König, 2013). The tragic truth is that so much culture was eradicated that a question as simple as "who spoke these words" might never be answered. Perhaps we will have to instead ask: "who are the closest linguistic relatives of these speakers".

While a full analysis is beyond the scope of this paper, at least something should be said about the research into small number systems and what the findings of this study say about how the current literature in the history of mathematics may be borrowing from very problematic older sources. As alluded to in the introduction writers like John Crawfurd, John Lubbock, and Edward B. Tylor wrote a lot on the number systems of the world in the mid to late 1800s. It was taken as a given that the complexity of a number system could be taken as a proxy for the level of intelligence and thus evolutionary development of different peoples.[19] We have seen this much earlier in Thompson's diary from 1824, where his first instinct on how to quantify the San intellect is to inquire how they count. It is hardly surprising that this same understanding is reflected in later works, like Conant's, who cite these earlier works.

This becomes problematic when more modern histories pick up stories from these works and with them their narrative. For example, when David Burton writes: "Anthropologists tell us that there has hardly been a culture, however primitive, that has not had some awareness of number" (Burton, 2011, pp. 1-2), and then illustrates this by giving the reader examples like the "Bushmen of South Africa" and Galton's meeting with the Damara, he is unwittingly repeating the words of anthropologists of the 1800s. Indeed the progression of examples is almost a paraphrased version of Levi Conant's chapter 2, filtered through a century of new authors giving the most illustrative examples to new readers. This is the case even though Conant and Burton are using different versions of the "Bushman" numeral claim.

It could even be argued that the streamlining evident from Conant to Burton is perhaps worse than using Conant more extensively. After all, although Conant speaks dismissively of the "primitive people" he goes to a considerable amount of effort to give examples of as many different number systems as possible from all around the world. Indeed, Conant gives examples of many different number systems from Africa, both simple and complex, whereas the modern historian of mathematics chose to only include the simplest of them all. The casual reader gets the impression that (outside of Egypt) Africans have largely been incapable of understanding numbers.

This is not to suggest that using Conant as a source is recommended; merely that giving an understanding of the full complexity of number systems is better than listing only the simple ones. For example, recent studies exist on the languages of Papua New Guinea and Oceania (Owens et al., 2018), Micronesia (Bender and Beller, 2021) and Australia (Bowern and Zentz, 2012). There are also studies on "rare bases" all over the world (Hammarström, 2010) and specifically into whether hunter gatherer societies have smaller number systems than agricultural peoples (Epps et al., 2012). The field is experiencing a boom period, especially after it was suggested that the Pirahã of Amazonia do not have any number words at all (Everett, 2005).

More modern problematic models can also be repeated. Many histories of mathematics currently base their understanding of the development of number systems partially or entirely on Seidenberg's diffusion theory from 1960, which has in recent years been heavily criticized, especially when it comes the history of the languages of Australia and the Pacific (Owens et al., 2018, pp. 167-192). For example, languages do not necessarily progress linearly towards the decimal system, instead numerals can be added or even lost. There is some evidence that Bardi, a Nyulnyulan language spoken in north-western Australia, might have once used forms constructed much like Thompson's "Bushman numerals". However, the remaining modern speakers uniformly reject these forms as ungrammatical, instead using English numerals if they need higher numerals for something (Bowern, 2012).

## 7. Conclusion

In this paper we have discovered the history of a claim about the numerals of "the Bushmen" found in many resources on the history of mathematics and numeral systems. We have also established that all forms in modern literature distort the original sources and disregard research on these languages done since the 1870s.

A minority of these claims can be traced back to the original fieldwork of Wilhelm Bleek with |Xam prisoners in 1866. Because these authors are ultimately discussing his early work, which had not yet found a word for three, they claim that no word for three existed at all. Further, these authors seem to take the stance that the |Xam language is uniquely inarticulate because it uses click consonants.

19 For a more detailed history see e.g. Barany (2014) and Zaslavsky (1973, pp. 9-15).

The majority of different forms of the claim in modern literature can be traced back to a list of numbers collected by George Thompson in 1824. All later versions of the claim distort Thompson's original list to various extents, from changing the word for one from *t'a* to *xa* all the way to omitting all consonants from the numerals all together.

In addition, the speakers themselves have widely been misidentified or at best identified only in the broadest of terms. Thompson met the San near a place called "Adriaan's-Fonteyn" in the Western Karoo desert of modern South Africa. Most authors just use the term "Bushman", which is widely considered old-fashioned and sometimes derogatory and also gives the impression that all San spoke the same language, which is inaccurate. Without much exaggeration this is akin to using a misspelled list of numbers from Navajo and saying that it comes from "the Indian language of America".

Even if one were to quote Thompson directly and accurately, multiple caveats would still apply. Thompson did not speak the language himself and was using two translators who were themselves non-native speakers. He was transcribing a language that did not have any sort of a standardized orthography or other transcription method, so at best he was using his own ear and intuition to come up with one. It is also significant that he was specifically writing these words to demonstrate the "degraded state of intellect" of the San. This should not inspire confidence in any historian who intends to use it as an example.

We have also shown that the claim is still continuing to evolve in the literature in various ways. This can take the form of changing the numerals themselves or even attributing other groups' number symbols to the San. It has also spread from the academic literature to more popular media like romance novels and Wikipedia.

There are, however, still open questions raised by the study. Firstly, it would be important for Thompson's numerals to be analyzed by an expert in Khoesan languages or more specifically Tuu and K'xa languages. The languages of the San, once widely spoken in South Africa, are either endangered, moribund or extinct - though some related languages remain in Botswana and Namibia. Despite their dubious origin these words might thus prove a valuable source of information. In addition more discussion on the various periphrastic numerals in Tuu and K'xa language families would be a valuable addition to scholarship.

Secondly, many histories of mathematics adopt quite simple models of the emergence and development of numbers. For example, Seidenberg's diffusion model has been criticized, but some forms of it appear to form a basis for many histories. Many histories can also give the impression that the decimal system is the final form or a linear process of improvement that all number systems are developing towards. Research should be done to see which problematic theories are propagated in these works.

More should be done specifically to challenge old theories. Of course there are different sorts of number systems in world's languages, some simpler and some more complex, but can we actually say one will always develop towards the other? Will a more complex system always be appropriated by the speakers of a simpler one or even replace the simpler one as is? There is also a tendency to use extant number systems as stand-ins for "primitive" and ancient number systems. Is this truly appropriate or simply our best approximation?

Finally, more inquiries into the origins and histories of examples like the "numerals of the Bushmen" should be undertaken. While these stories are individually of small import, the journey of discovery into their history can be quite revelatory.

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