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# The Number Navigation Game: an overview of an iterative development process

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**Abstract** The present chapter provides an overview of the iterative development and testing of the Number Navigation Game (NNG), a game-based learning environment which aims to strengthen primary school students' flexibility and adaptivity with arithmetic problem solving. The set of studies described in this chapter asked different questions using different methodologies through the phases of game development. However, the connecting principle was placing the integration of game features and mathematical content in the center of the process and developing valid and reliable means to operationalize the expected mathematical learning outcomes. Each step in the cycle served as a source of information to be used both in the development of the NNG and in the development of different types of measures of the mathematical learning outcomes. Conclusions of these steps are discussed in relation to the iterative game design process, the importance of using an integrated game design, and questions of measurement in the broader context of educational game design.

**Keywords** game-based learning, iterative game design, arithmetic flexibility, evaluation

## Introduction

After the initial excitement regarding the potential of using games for educational purposes, it is now clear that games as such do not represent a magic bullet. While most reviews find that game-based learning can be associated with higher learning gains

when compared to more traditional methods of instruction, the difference is usually small (Boyle et al., 2016; Hainey, Connolly, Boyle, Wilson, & Razak, 2016; Wouters, van Nimwegen, van Oostendorp, & van der Spek, 2013; Young et al., 2012). The explanation regarding this discrepancy between the potential of game-based learning and the lack of strong empirical results that would substantiate these claims is usually two-fold.

Firstly, despite the affordances of technology to provide new ways of interaction with the educational content, most technology-supported educational tools are only used as side activities connected to traditional methods of content delivery (Bray & Tangney, 2017). In the specific domain of game-based learning this means that the learning content is often not delivered by the gameplay, but the gameplay is used as an incentive to make students deal with the intended educational content (Devlin, 2011; Habgood, 2007; Habgood & Ainsworth, 2011). This can create frustration and disappointment and can explain why game-based learning is often not found to be more motivating for students than traditional instructional methods (Wouters et al., 2013) and why most game-based interventions are only marginally effective (Boyle et al., 2016; Wouters & Oostendorp, 2013; Wouters et al., 2013).

Second, there is often a disconnect between the type of skills and knowledge practiced within the context of a game and the means of measuring the expected learning outcomes (All, Nuñez Castellar, & Van Looy, 2015, 2016). Measurement problems, such as using standardized tests to measure the outcomes of complex game-based training effects, relying solely on game performance measures, the overuse of quasi-experimental designs and the lack of large-scale randomized studies are often raised as problems which hinder the estimation of the real educational potential of games (All, Nunez Castellar, & Van Looy, 2014; All et al., 2016; Connolly, Boyle, Hainey, & Boyle, 2010; De Freitas & Oliver, 2006; Wouters et al., 2013; Young et al., 2012).

In order to make use of the potential affordance of games for educational purposes it is important to both carefully design the integration of learning content and game features and have appropriate measures which can capture different aspects of the intended learning outcomes. Achieving this balance requires an iterative process of design and testing with the interaction of game design and educational content in the center (All et al., 2015; Vanden Abeele et al., 2012).

The current chapter aims to describe the process of development and cycles of testing of the Number Navigation Game (NNG), a game-based learning environment which aims to promote primary school students' flexibility and adaptivity with arithmetic problem solving. This chapter summarizes the results of a set of studies which used different methodologies to answer questions at different phases of development of the NNG. As a common theme, each study explored the relationship of game features and educational content within the game, and how the learning gains associated with

gameplay can be operationalized and measured both in the game and outside the game. Furthermore, the chapter provides a reflection on the use of iterations in developing game-based learning environments and on questions related to the integration of educational content and gameplay, measurement of learning outcomes, and the application game-based learning in the everyday classroom practice.

## Background and aims

In this chapter, the argument is made that the value of a game-based learning environment is strongest, if there is a gap in teaching or learning a specific content for which the game-based format provides opportunities that were either difficult or not possible to achieve otherwise (e.g. Bray and Tangney, 2017). In case of the NNG this learning outcome was *flexibility and adaptivity with arithmetic problem solving*, which entails a strong mental representation and understanding of numerical relations and the efficient use of these relations when selecting problem solving methods across various arithmetic contexts (Baroody, 2003; Heinze, Star, & Verschaffel, 2009; Lehtinen, Hannula-Sormunen, McMullen, & Gruber, 2017; Nunes, Dorneles, Lin, & Rathgeb-Schnierer, 2016; Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009). More specifically, the main aim of NNG practice was to strengthen students' *adaptive number knowledge*, which is a component of adaptivity with arithmetic and is described by the ability to recognize opportunities to use numerical relations during arithmetic problem solving (McMullen et al., 2016, 2017). This type of mathematical expertise requires mental representation characterized by rich connections of numbers and arithmetic operations which promote the recognition of useful and efficient arithmetic strategies.

As studies show, direct instructional methods are insufficient in developing flexibility and adaptivity with arithmetic (Canobi, Reeve & Pattison, 2003, Hickendorff, 2018) as the definition of what “useful” or “efficient” means can depend largely on the context, problem type, or individual preferences (Heinze et al., 2009; Threlfall, 2009; Verschaffel et al., 2009). Therefore, developing methods to enhance flexibility and adaptivity with arithmetic should inherently promote an open-ended explorative type of mathematical practice. In this context, the transformative value (Bray & Tangney, 2017) of the game-based format lies in the open-ended practice opportunities. In alignment with core aspects of flexibility and adaptivity with arithmetic (Baroody, 2003; Lehtinen et al., 2017; Threlfall, 2002, 2009; Verschaffel et al., 2009), the basic structure of the NNG is open-ended, the game does not pre-define “optimal solutions”, but it only provides a structure through its' rules and design, which aims to promote reflection on the “usefulness” of various different solutions for the same problem. The main aim of the NNG game mechanics is to naturally enhance students' tendency to look for

alternative solutions which is a prerequisite of developing a strong mental representation of numerical relations (Rittle-Johnson & Star, 2007; Star & Rittle-Johnson, 2008).

In the regular mathematics instruction this type of training might require individualized instruction and extra resources from teachers. The affordances of game-based learning, such as adaptivity, immediate feedback, and the scalability of practice, can provide new opportunities for developing these complex mathematical skills and knowledge. It was expected that intensive practice with the NNG, which requires working with different combinations of numbers and operations, solving arithmetic problems in multiple ways and reflecting on solutions and their underlying arithmetical relations, would strengthen students' adaptive number knowledge and thus help to develop their flexibility and adaptivity with arithmetic problem solving.

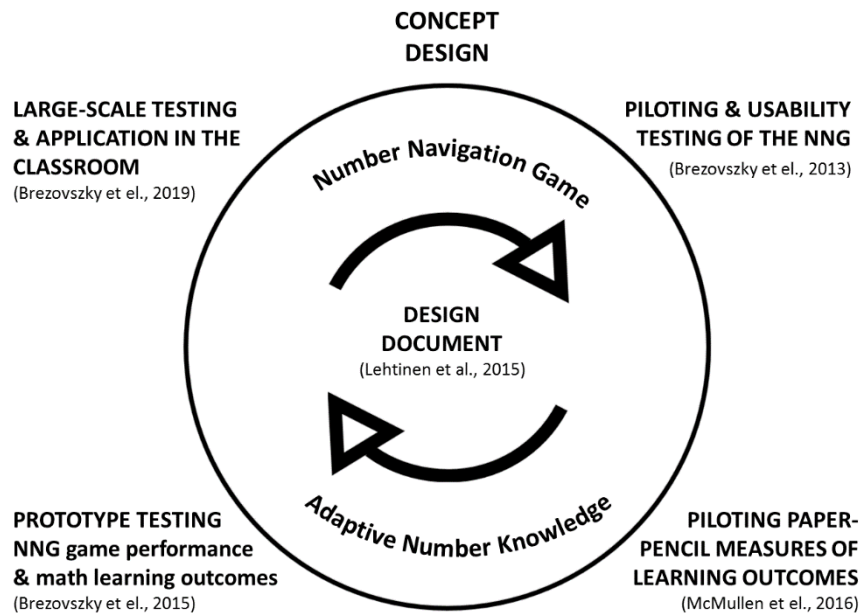
Through the set of studies presented in this chapter, three main areas of game-based learning were explored: *game design, measurement of learning outcomes, and efficiency and application of the game-based training.*

The main aims of these studies were to:

- 1) Operationalize adaptive number knowledge within a game-based learning environment and design a game for primary school students targeted primarily at developing adaptive number knowledge.
- 2) Operationalize adaptive number knowledge outside the game-based learning environment and show effects outside the game context.
- 3) Provide empirical evidence for the effects of the NNG training in developing primary school students' adaptive number knowledge and related mathematical skills first in controlled settings and later in the naturalistic classroom practice.

## Methods

The choice for a game-based learning environment brings some inherent questions regarding design decisions, the integration of game features and learning content, measurement, and applicability of the training. To address these issues, the design was implemented as an iterative cycle of testing and development as it is presented on Fig. 1.



**Fig. 1.** Iterative development and testing process of the NNG (Brezovszky, 2019, p. 22).

As Fig. 1 shows, in the piloting phase, an early prototype of the NNG was tested from a usability perspective focusing on the relationship of game features and the mathematical learning content (Brezovszky, Lehtinen, McMullen, Rodriguez, & Veermans, 2013) and a series of pilot studies were conducted that aimed to develop and validate measures of adaptive number knowledge outside the game context (McMullen et al., 2016). The NNG was developed into a working prototype taking into account results from the initial play testing. This was followed by a small-scale pilot study using a second prototype of the NNG which study focused on the relationship of students' game performance and the development of their mathematical learning outcomes (Brezovszky et al., 2015). Findings from the piloting phase were used to develop both the NNG and in-game and paper-pencil measurements of the mathematical learning outcomes. The game design process and the relationship of the mathematical content and game features was documented and described in a design document (Lehtinen et al., 2015), which served as a guideline in the following stages (see Fig. 1 in the center). In the final, evaluation stage a large-scale randomized control study was conducted using a final prototype of the NNG in three different grade levels (Brezovszky et al., 2019).

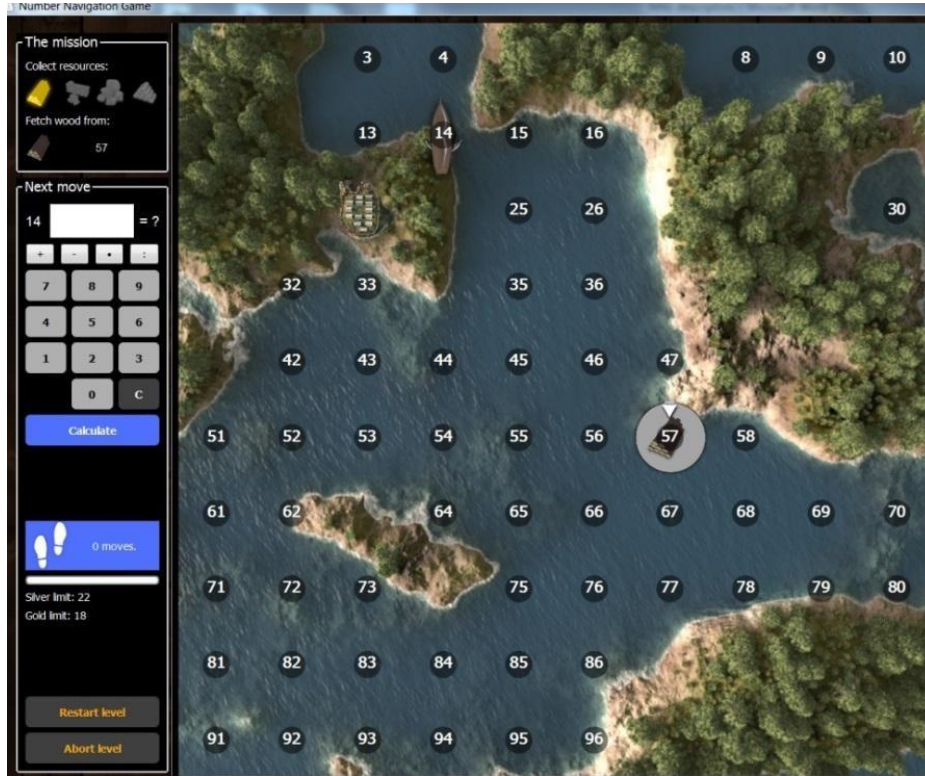
## Results

### Relationship of game design and mathematical learning content

To achieve complex learning outcomes using game-based learning, an *integrated game design* process is needed where the educational content is delivered through the core game mechanics (Habgood, 2007; Salen & Zimmerman, 2003). Accordingly, in the NNG the aim was to integrate the open-ended, explorative mathematical practice within the core game structure so that each game feature triggers mathematically relevant actions, and the greater percent of students' interaction with the game equals to time on task with the intended learning content (Habgood, 2007; Habgood & Ainsworth, 2011; Salen & Zimmerman, 2003; Vanden Abeele et al., 2012).

The NNG was developed in the larger context of the studies presented in this chapter at the University of Turku Mathematical Learning Research Group. The game aims to improve primary school students' adaptive number knowledge by providing opportunities to engage in strategic work with various combinations of numbers and operations. The hundred-square (see Fig. 2) was selected as a basic external representation of natural numbers in the game, as it can support the development of number sense and numerical understanding (Beishuizen, 1993; Siegler & Ramani, 2009; Wilson et al., 2009). Additionally, the ten by ten square structure was selected because it allows for both horizontal (number line) and vertical (base-10 system) representation and movement within those representations. Combining these two into diagonal movements can support decomposing and recombining numbers in various ways.

The game narrative uses the metaphor of navigation with a ship through different landscapes of land and sea where a map represents a basic game unit (Fig. 2). The players' goal is to collect different raw materials in order to build a settlement. Navigation is done by strategically selecting and executing different calculations to move the ship from one number to the next. An important aspect of the NNG design is that it makes reflection about alternative arithmetic solution methods a means of making progress within the game. The different game modes and rules aim to encourage players to use a large variety of number-operation combinations in their calculations, as well as look for different numerical relations and key numbers (i.e., numbers with several factors) within the natural number system.



**Fig. 2.** Screenshot of an NNG game map (harbor at number 14, target at number 57).

In terms of game design this type of game behaviour is triggered for example by the different layout of each map where numbers covered by land cannot be used, by the different game modes which require players to adapt their strategies and use different restrictions in their calculations (e.g., reduce the number of moves between two numbers), or by random events like “pirate ships” which can appear to redirect players’ routes and prompt them to use alternative number-operation combinations (for a more detailed description of the game features see Lehtinen et al., 2015). This variability makes reflection on the efficiency of each move (equation) a natural part of the gameplay, which is important because it is in line with the in-situ, context-dependent definition of adaptivity and flexibility with arithmetic (Threlfall, 2002, 2009; Verschaffel et al., 2009) and because reflection on the educational content is a much desired but hardly achieved part of educational game design (Veermans & Jaakkola, 2019).

When the NNG was tested in observational studies (Brezovszky et al., 2013), results showed that the integrated game design was efficient in triggering the desired mathematical problem-solving during gameplay. For example, even during a 1.5 hours long play testing session, players executed around 200 mental calculations, and importantly they also rejected even more alternative solutions while thinking about how to progress in the game. Players discussed and reflected frequently on possible solution options, and different game modes were efficient in prompting players to adapt their solutions strategies accordingly. These results suggest that in line with the idea of an integrated educational game design, both actual time on task and discussion around the game were mathematically relevant experiences.

### **Operationalization and measurement of learning outcomes**

In the initial stage efficiency in the NNG was operationalized by the extent to which the game is able to enhance the type of mathematically relevant behavior in players (Brezovszky et al., 2013). This could include using a variety of numbers in their equations, using reflection and discussion when making moves, etc. (Brezovszky et al., 2013). Based on these results, it was assumed that the different NNG game features were efficiently triggering the expected mathematical behavior and the amount of gameplay was considered as a measure of game performance. Thus, in future quasi-experimental (Brezovszky et al., 2015) and experimental studies (Brezovszky et al., 2019) game performance was measured by the total number of maps completed in the NNG.

For better validity and reliability, it is suggested that learning outcomes related to game-based trainings are measured using a combination of different methods and types of measurements such as game-based and paper-pencil measures, near- and far-transfer tasks, and standardized tests (All et al., 2014, 2016; Cheung & Slavin, 2013; Oprins & Korteling, 2014). The main learning outcome associated with NNG is adaptive number knowledge, which was measured by the Arithmetic Production Task developed in the larger context of studies presented in this chapter (Brezovszky et al., 2015, 2019; McMullen et al., 2016, 2017).

Like the NNG, the Arithmetic Production Task was also developed through several cycles of piloting (see McMullen et al., 2016). These cycles of development were guided by the open-ended and in-situ nature of flexibility and adaptivity with arithmetic. The main aim was to transfer this theoretical concept and parts of the practical training with the NNG in a paper-pencil format that can be flexibly applied over several grade levels and on a large scale. In the Arithmetic Production Task students have to calculate a given target number by combining a set of 4-5 given numbers and the four arithmetic

operations in as many ways as possible in a limited amount of time. Responses are coded on two criteria: 1) *quantity*, where the total number of correct solutions are counted; 2) *complexity*, where the total number of multi-step equations using different types of operations are counted. Alike the NNG, this format is open-ended and does not pre-define optimal solutions, but it provides a structure so that it is applicable and scalable in the classroom.

The relation of game-performance and the paper-pencil measure of adaptive number knowledge was first piloted in a small-scale intervention study which showed a relation between the number of maps completed and students' complex solutions on the Arithmetic Production Task during post-test in grade six (Brezovszky et al., 2015). After this point, the Arithmetic Production Task was further refined with regards to the number and type of items used, and an application procedure was developed to ensure implementation fidelity on a large-scale. In a large-scale randomized intervention study, results showed a relationship between game performance and both the amount and complexity of students' solutions on the Arithmetic Production Task (Brezovszky et al., 2019). Strengthening the results of the initial observational studies, the relationship found between game performance and mathematical learning outcomes highlights the importance of an integrated game design where time on task in the NNG equaled to mathematically relevant practice that could transfer to the outside game context as well.

### **Efficiency and application of the NNG training**

The effects of the NNG training in developing flexibility and adaptivity with arithmetic problem solving were first tested in a smaller controlled quasi/experimental study (Brezovszky et al., 2015) then in a large-scale randomised intervention study in the naturalistic classroom context (Brezovszky et al., 2019). Both studies asked similar questions regarding the games' efficiency in developing adaptive number knowledge and related mathematical skills. As it was expected that students' adaptive number knowledge will be related to their general arithmetic fluency and pre-algebra knowledge (McMullen et al., 2016, 2017) these additional learning outcomes were measured by a combination of standardized and self-constructed tests. The Woodcock-Johnson Math Fluency sub-test was used to measure students' basic math fluency (Schrack, McGrew, Kevin S., & Woodcock, 2001) and a measure of pre-algebra knowledge was used as a far-transfer measure of the relational thinking and the complex, multi-step mental work with arithmetic relations that can be foundational to algebraic thinking.

Results of both small and large-scale intervention studies were promising and showed that training with the game was able to improve not only primary school students' adaptive number knowledge but also more basic arithmetic fluency and pre-algebra knowledge. Results of the large-scale intervention study (Brezovszky et al., 2019) also

revealed important grade-level differences in the development of the mathematical learning outcomes. The strongest results in developing adaptive number knowledge were observed in in grade five. In grade four, practice with the NNG was more efficient in developing more basic arithmetic skills such as math fluency, and the quantitative aspects of adaptive number knowledge. Finally, in grade six where more basic arithmetic skills are well-established, results of the training could also be noticed on far-transfer tasks such as pre-algebra knowledge. These results suggest that the design of the NNG was flexible enough to support the development of different types of mathematical skills and knowledge across different grades, which are promising results with regards to the practical use and scalability of the training.

## Conclusions

The aim of this this chapter was to present the iterative design and testing process of the NNG game-based learning environment concentrating on questions related to game design, learning outcome measurement and training application. Overall, the set of results described in this chapter show that it is possible to develop a game-based learning environment for classroom use that utilizes the affordances of gameplay to address aspects of learning (flexibility and adaptivity with arithmetic) that have proven difficult to address otherwise in the classroom. Secondly, results provide insights in important aspects of the design and evaluation of game-based learning environments that aided both the development of the NNG, and the evaluation of its outcomes.

Overall, results strongly suggest that it is worth to invest resources into the integration of the educational content and the game mechanics, and it is important to consider integration as a general guiding principle in educational game design (All et al., 2016; Devlin, 2011; Habgood, 2007; Vanden Abeele et al., 2012; Young et al., 2012). How well the content is integrated in the game, which features of the game will trigger the expected behavior or cognitive work, and how players interact with the relevant features are foundational questions as it is not the game itself but it is the design that makes game-based learning efficient or not (All et al., 2016; Bray & Tangney, 2017). The design and development process should be iterative, multidisciplinary and decisions should be based on theory and validated by empirical results.

The process of development and testing of the NNG described in this chapter is an example of such design, and highlights a number of important aspects in this process. For instance, within these iterations it is important to assess whether the design of the game triggers the intended activity. Additionally, it is important to validate whether this intended activity does indeed result in the intended learning outcomes. This may

necessitate the development of new measurement instruments as was the case here (i.e., Arithmetic Production Task). However, the added value of developing new instruments is that the process can clarify the concept to be measured and how that concept can be broken down in terms of relevant behavior (i.e., using a variety of numbers and operation, discussion regarding the efficiency of the next move). In turn, this information can be used to guide the game design process in implementing game features which can trigger the expected behavior within the game (i.e., changing game modes, various landscapes, surprise elements, etc.). In the last phase, the controlled evaluation and large-scale classroom study served to first provide a proof of concept before providing evidence that the concept can also transfer to classroom practices.

While theoretically all the described steps in the cycle could be done during one evaluation, this type of iterative design was more efficient in case of the NNG, where in line with the theoretical propositions (Baroody, 2003; Heinze et al., 2009; Threlfall, 2009) both the training and the measurements of learning gains had to be flexible and open-ended. Naturally, the iteration process should not stop, but continue during classroom use by utilizing the data of gameplay for refining the design.

## References

- All, A., Nunez Castellar, E. P., & Van Looy, J. (2014). Measuring Effectiveness in Digital Game-Based Learning: A Methodological Review. *International Journal of Serious Games*, 1(2), 3–21. <https://doi.org/10.17083/ijsg.v1i2.18>
- All, A., Nuñez Castellar, E. P., & Van Looy, J. (2015). Towards a conceptual framework for assessing the effectiveness of digital game-based learning. *Computers and Education*, 88, 29–37. <https://doi.org/10.1016/j.compedu.2015.04.012>
- All, A., Nuñez Castellar, E. P., & Van Looy, J. (2016). Assessing the effectiveness of digital game-based learning: Best practices. *Computers and Education*, 92–93, 90–103. <https://doi.org/10.1016/j.compedu.2015.10.007>
- Baroody, A. J. (2003). The development of adaptive expertise and flexibility: The integration of conceptual and procedural knowledge. In *The Development of Arithmetic Concepts and Skills: Constructive Adaptive Expertise* (pp. 1–33). Lawrence Erlbaum Associates Publishers. <https://doi.org/10.4324/9781410607218>
- Beishuizen, M. (1993). Mental strategies and materials or models for addition and subtraction up to 100 in dutch second grades. *Journal for Research in Mathematics Education*, 24, 294–323. doi:10.2307/749464
- Boyle, E. A., Hainey, T., Connolly, T. M., Gray, G., Earp, J., Ott, M., ... Pereira, J. (2016). An update to the systematic literature review of empirical evidence of the impacts and outcomes of computer games and serious games. *Computers and Education*, 94, 178–192. <https://doi.org/10.1016/j.compedu.2015.11.003>

- Bray, A., & Tangney, B. (2017). Technology usage in mathematics education research – A systematic review of recent trends. *Computers and Education*, 114, 255–273. <https://doi.org/10.1016/j.compedu.2017.07.004>
- Brezovszky, B., Lehtinen, E., McMullen, J., Rodriguez, G., & Veermans, K. (2013). Training flexible and adaptive arithmetic problem solving skills through exploration with numbers: The development of Number Navigation Game. In 7th European Conference on Games Based Learning, ECGBL 2013 (Vol. 2, pp. 626–634).
- Brezovszky, B., Rodríguez-Aflecht, G., McMullen, J., Veermans, K., Pongsakdi, N., Hannula-Sormunen, M. M., & Lehtinen, E. (2015). Developing adaptive number knowledge with the number navigation game-based learning environment. In *Describing and Studying Domain-Specific Serious Games* (pp. 155–170). Cham: Springer International Publishing. [https://doi.org/10.1007/978-3-319-20276-1\\_10](https://doi.org/10.1007/978-3-319-20276-1_10)
- Brezovszky, B., McMullen, J., Veermans, K., Hannula-Sormunen, M. M., Rodríguez-Aflecht, G., Pongsakdi, N., ... Lehtinen, E. (2019). Effects of a mathematics game-based learning environment on primary school students' adaptive number knowledge. *Computers and Education*, 128, 63–74. <https://doi.org/10.1016/j.compedu.2018.09.011>
- Brezovszky, B. (2019). Using game-based learning to enhance adaptive number knowledge (Doctoral dissertation, University of Turku, Turku, Finland). Retrieved from <https://www.utupub.fi/handle/10024/147025>
- Canobi, K. H., Reeve, R. A., & Pattison, P. E. (2003). Patterns of knowledge in children's addition. *Developmental Psychology*, 39(3), 521–534. <https://doi.org/10.1037/0012-1649.39.3.521>
- Cheung, A. C. K., & Slavin, R. E. (2013). The effectiveness of educational technology applications for enhancing mathematics achievement in K-12 classrooms: A meta-analysis. *Educational Research Review*, 9, 88–113. <https://doi.org/10.1016/j.edurev.2013.01.001>
- Connolly, T. M., Boyle, E., Hainey, T., & Boyle, J. M. (2010). A systematic literature review of empirical evidence on computer games and serious games. *Computers and Education*, 59(2), 661–686. <https://doi.org/10.1016/j.compedu.2012.03.004>
- De Freitas, S., & Oliver, M. (2006). How can exploratory learning with games and simulations within the curriculum be most effectively evaluated? *Computers and Education*, 46(3), 249–264. <https://doi.org/10.1016/j.compedu.2005.11.007>
- Devlin, K. (2011). *Mathematics education for a new era: Video games as a medium for learning*. A K Peters.
- Habgood, M. P. J. (2007). *Habgood, Matthew Peter Jacob (2007) The Effective Integration of Digital Games and Learning Content*. PhD thesis, University of Nottingham.
- Habgood, M. P. J., & Ainsworth, S. E. (2011). Motivating Children to Learn Effectively: Exploring the Value of Intrinsic Integration in Educational Games. *Journal of the Learning Sciences*, 20(2), 169–206. <https://doi.org/10.1080/10508406.2010.508029>
- Hainey, T., Connolly, T. M., Boyle, E. A., Wilson, A., & Razak, A. (2016). A systematic literature review of games-based learning empirical evidence in primary education. *Computers & Education*, 102, 202–223. <https://doi.org/10.1016/j.compedu.2016.09.001>
- Heinze, A., Star, J. R., & Verschaffel, L. (2009). Flexible and adaptive use of strategies and representations in mathematics education. *ZDM - International Journal on Mathematics Education*, 41, 535–540. <https://doi.org/10.1007/s11858-009-0214-4>
- Hickendorff, M. (2018). Dutch sixth graders' use of shortcut strategies in solving multidigit arithmetic problems. *European Journal of Psychology of Education*, 33 (4), 577–594. <https://doi.org/10.1007/s10212-017-0357-6>
- Lehtinen, E., Brezovszky, B., Rodríguez-Aflecht, G., Lehtinen, H., Hannula-Sormunen, M. M., McMullen, J., ... Jaakkola, T. (2015). Number navigation game (NNG): Design principles and game

- description. In *Describing and Studying Domain-Specific Serious Games* (pp. 45–61). Springer International Publishing.
- Lehtinen, E., Hannula-Sormunen, M., McMullen, J., & Gruber, H. (2017). Cultivating mathematical skills: from drill-and-practice to deliberate practice. *ZDM - Mathematics Education*, 49(4), 625–636. <https://doi.org/10.1007/s11858-017-0856-6>
- McMullen, J., Brezovszky, B., Rodríguez-Aflecht, G., Pongsakdi, N., Hannula-Sormunen, M. M., & Lehtinen, E. (2016). Adaptive number knowledge: Exploring the foundations of adaptivity with whole-number arithmetic. *Learning and Individual Differences*, 47, 172–181.
- McMullen, J., Brezovszky, B., Hannula-Sormunen, M. M., Veermans, K., Rodríguez-Aflecht, G., Pongsakdi, N., & Lehtinen, E. (2017). Adaptive number knowledge and its relation to arithmetic and pre-algebra knowledge. *Learning and Instruction*, 49. <https://doi.org/10.1016/j.learninstruc.2017.02.001>
- Nunes, T., Dorneles, B. V., Lin, P.-J., & Rathgeb-Schnierer, E. (2016). *Teaching and Learning About Whole Numbers in Primary School*. Cham: Springer International Publishing. <https://doi.org/10.1007/978-3-319-45113-8>
- Oprins E., Korteling J. (2014) Transfer of Training of An Educational Serious Game: The Effectiveness of the CASHIER TRAINER. In: Cai Y., Goei S. (eds) *Simulations, Serious Games and Their Applications. Gaming Media and Social Effects*. Springer, Singapore. [https://doi.org/10.1007/978-981-4560-32-0\\_15](https://doi.org/10.1007/978-981-4560-32-0_15)
- Rittle-Johnson, B., & Star, J. R. (2007). Does comparing solution methods facilitate conceptual and procedural knowledge? An experimental study on learning to solve equations. *Journal of Educational Psychology*, 99(3), 561–574. <https://doi.org/10.1037/0022-0663.99.3.561>
- Salen, K., & Zimmerman, E. (2003). *Rules of Play: Game Design Fundamentals*. The MIT Press.
- Schrank, Fredrick A., McGrew, Kevin S., Woodcock, R. W. (2001). Technical Abstract (Woodcock-Johnson III Assessment Service Bulletin No. 2).
- Siegler, R. S., & Ramani, G. B. (2009). Playing linear number board games-but not circular ones-improves low-income preschoolers' numerical understanding. *Journal of Educational Psychology*, 101, 545-560. doi:10.1037/a0014239
- Star, J. R., & Rittle-Johnson, B. (2008). Flexibility in problem solving: The case of equation solving. *Learning and Instruction*, 18(6), 565–579. <https://doi.org/10.1016/j.learninstruc.2007.09.018>
- Threlfall, J. (2002). Flexible Mental Calculation. *Educational Studies in Mathematics*, 50(1), 29–47. <https://doi.org/10.1023/A:1020572803437>
- Threlfall, J. (2009). Strategies and flexibility in mental calculation. *ZDM*, 41(5), 541–555. <https://doi.org/10.1007/s11858-009-0195-3>
- Vanden Abeele, V., De Schutter, B., Geurts, L., Desmet, S., Wauters, J., Husson, J., ... Geerts, D. (2012). P-III: A Player-Centered, Iterative, Interdisciplinary and Integrated Framework for Serious Game Design and Development. In: De Wannemacker S., Vandercruysse S., Clarebout G. (eds) *Serious Games: The Challenge*. ITEC/CIP/T 2011. Communications in Computer and Information Science, vol 280. (pp. 82–86). Springer, Berlin, Heidelberg. [https://doi.org/10.1007/978-3-642-33814-4\\_14](https://doi.org/10.1007/978-3-642-33814-4_14)
- Veermans K., Jaakkola T. (2019) Pedagogy in Educational Simulations and Games. In: Cai Y., van Joolingen W., Walker Z. (eds) *VR, Simulations and Serious Games for Education. Gaming Media and Social Effects*. Springer, Singapore. [https://doi.org/10.1007/978-981-13-2844-2\\_2](https://doi.org/10.1007/978-981-13-2844-2_2)
- Verschaffel, L., Luwel, K., Torbeys, J., & Van Dooren, W. (2009). Conceptualizing, investigating, and enhancing adaptive expertise in elementary mathematics education. *European Journal of Psychology of Education*, 24(3), 335–359. <https://doi.org/10.1007/BF03174765>

- Wilson, K. A., Bedwell, W. L., Lazzara, E. H., Salas, E., Burke, C. S., Estock, J. L., ... Conkey, C. (2009). Relationships Between Game Attributes and Learning Outcomes. *Simulation & Gaming*, 40(2), 217–266. <https://doi.org/10.1177/1046878108321866>
- Wouters, P., & Oostendorp, H. Van. (2013). A meta-analytic review of the role of instructional support in game-based learning. *Comput. Computers & Education*, 60(1), 412–425. <https://doi.org/10.1016/j.compedu.2012.07.018>
- Wouters, P., van Nimwegen, C., van Oostendorp, H., & van der Spek, E. D. (2013). A meta-analysis of the cognitive and motivational effects of serious games. *Journal of Educational Psychology*, 105(2), 249–265. <https://doi.org/10.1037/a0031311>
- Young, M. F., Slota, S., Cutter, B., R., Jalette, G., Mullin, G., ... Yukhymenko, M. (2012). Our Princess Is in Another Castle: A Review of Trends in Serious Gaming for Education. *Review of Educational Research*, 82(1), 61–89. <https://doi.org/10.3102/0034654312436980>