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New directions in Stochastic Multicriteria Acceptability Analysis

by

Tommi Tervonen

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University of Turku
Department of Information Technology
20014 Turku, Finland

and

University of Coimbra
Faculty of Economics
3004-512 Coimbra, Portugal

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Supervisors

Risto Lahdelma
Department of Information Technology
University of Turku
Finland

José Rui Figueira
Department of Engineering and Management
Instituto Superior Técnico, Technical University of Lisbon
Portugal

Pekka Salminen
School of Business and Economics
University of Jyväskylä
Finland

Reviewers

Annika Kangas
Department of Forest Resource Management
University of Helsinki
Finland

Salvatore Greco
Faculty of Economics
University of Catania
Italy

Opponents

Jyrki Kangas
Metsähallitus
Finland

João Paulo Costa
Faculty of Economics
University of Coimbra
Portugal

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Abstract

Decisions taken in modern organizations are often multi-dimensional, involving multiple decision makers and several criteria measured on different scales. Multiple Criteria Decision Making (MCDM) methods are designed to analyze and to give recommendations in this kind of situations. Among the numerous MCDM methods, two large families of methods are the multi-attribute utility theory based methods and the outranking methods. Traditionally both method families require exact values for technical parameters and criteria measurements, as well as for preferences expressed as weights. Often it is hard, if not impossible, to obtain exact values.

Stochastic Multicriteria Acceptability Analysis (SMAA) is a family of methods designed to help in this type of situations where exact values are not available. Different variants of SMAA allow handling all types of MCDM problems. They support defining the model through uncertain, imprecise, or completely missing values. The methods are based on simulation that is applied to obtain descriptive indices characterizing the problem.

In this thesis we present new advances in the SMAA methodology. We present and analyze algorithms for the SMAA-2 method and its extension to handle ordinal preferences. We then present an application of SMAA-2 to an area where MCDM models have not been applied before: planning elevator groups for high-rise buildings. Following this, we introduce two new methods to the family: SMAA-TRI that extends ELECTRE TRI for sorting problems with uncertain parameter values, and SMAA-III that extends ELECTRE III in a similar way. An efficient software implementing these two methods has been developed in conjunction with this work, and is briefly presented in this thesis. The thesis is closed with a comprehensive survey of SMAA methodology including a definition of a unified framework.

Extended abstract in Finnish

Nykypäivänä monet organisaatiot kohtaavat päivittäin tilanteita, joissa niiden tulee tehdä päätöksiä ottaen huomioon useita vaikutuksia. Tämän tyyppiset päätökset vaihtelevat tehtaan sijoituskohteesta pilvenpiirtäjän hissiryhmän tyypin valintaan. Molemmat edellämainituista tilanteista sisältyvät tärkeään ryhmään päätöksenteko-ongelmia, jotka koostuvat äärellisestä määrästä vaihtoehtoja joiden hyvyyttä mitataan usealla kriteerillä. Näiden mittausten perusteella paras vaihtoehto voidaan valita, tai vaihtoehdot voidaan järjestää suosittavuuden mukaan, tai ne voidaan lajitella kategorioihin, jotka ovat etukäteen määriteltyjä ja järjestettyjä paremmuuden mukaan.

Tässä väitöskirjassa käsiteltävät päätöksenteko-ongelmat ovat yllä mainittua tyyppiä. Kriteerit joilla vaihtoehtoja mitataan voivat olla ordinaalisia tai kardinaalisia. Ordinaalisilla kriteereilla ainoastaan vaihtoehtojen suosittavuusjärjestys voidaan määrittää. Kardinaalisilla kriteereillä vaihtoehtoja kyetään mittaamaan numeerisilla arvoilla. Päätöksentekomenetelmät yhdistävät eri kriteerien arvot ottaen huomioon päätöksentekijöiden preferenssit pyrkimyksenä rakentaa preferenssirelaatio joka ratkaisee kyseessä olevan ongelman. Useimmissa käytännön sovelluksissa tätä ratkaisua ei tulisi tulkita kirjaimellisesti, vaan käyttää lähtökohtana syvemmälle analyysille. Jatko-analyysi voi mahdollisesti koostua mallin arvojen tarkentamisesta tai muuttamisesta.

Saavutetut ratkaisut eivät ole riippuvaisia ainoastaan kriteerimittauksista ja päätöksentekijöiden preferensseistä, vaan myös mallin tyypistä ja sen teknisistä parametreista. Tämän vuoksi ongelmaan on suositeltavaa soveltaa useaa eri päätöksentukimenetelmää ja verrata niiden antamia tuloksia, jos vain mahdollista. Vertailu tulee tehdä kuitenkin siten, että päätöksentekijät ja analysoija ymmärtävät sekä edellytykset mallin käytölle, että mallin hyvät ja huonot puolet.

Monikriteerinen päätöksenteko on tieteellisenä kenttänä suhteellisen nuori ja jakautunut eri koulukuntiin. On olemassa useita menetelmiä jotka kukin ottavat huomioon käytännön ongelmissa kohdattavia erityispiirteitä. Kaksi suurta menetelmäperhettä ovat moniattribuuttiseen hyötyteoriaan perustuvat menetelmät ja outranking-menetelmät. Tässä työssä keskitytään outranking-menetelmistä ELECTRE-menetelmäperheeseen. Hyötyteoria antaa pohjan vanhimmille vielä käytössä oleville päätöksentekomenetelmille. Sen menetelmillä on aksiomaattinen pohja toisin kuin ELECTRE-menetelmillä. Nämä sallivat kuitenkin monimuotoisemman preferenssien mallintamisen kynnysfunktioiden avulla. Kynnysfunktiot ja muut ELECTRE:n käsitteet saattavat joillekin päätöksentekijöille olla helpommin ymmärrettävissä kuin hyötyteoria.

Viimeisinä vuosina on tullut selväksi, että päätöksentekomenetelmien tulisi kyetä ottamaan huomioon mallin parametrien arvojen epävarmuudet ja epätarkkuudet. Perinteiset moniattribuuttiseen hyötyteoriaan pohjautuvat menetelmät tai ELECTRE-menetelmät eivät tähän kykene. 1990-luvulla syntyi uusi menetelmäperhe, stokastinen monikriteerinen arvostusanalyysi (SMAA), joka eksplisiittisesti sallii epävarmuuksien mallintamisen. Eri SMAA-menetelmät soveltuvat kaiken tyyppisille päätöksenteko-ongelmille ja mahdollistavat epävarmojen, epätarkkojen, ja puuttuvien parametriarvojen käytön. Ensimmäinen SMAA-menetelmä ja sen SMAA-2--laajennus pohjautuvat moniattribuuttiseen hyötyteoriaan ja käyttävät sitä parhaan vaihtoehdon valintaan (SMAA) tai vaihtoehtojen järjestämiseen (SMAA-2). SMAA-menetelmiä voidaan käyttää myös sellaisissa päätöksenteko-ongelmissa, joissa ei ole ollenkaan preferenssitietoa saatavilla. Myös epätarkkaa preferenssitietoa voidaa käyttää SMAA:ssa. Samoin kriteerimittaukset ja muut mallin parametrit voivat olla epätarkkoja.

SMAA-menetelmät käyttävät simulaatiota laskeakseen ongelmaa kuvaavia indeksejä. Perheen eri menetelmät tuottavat erilaisia indeksejä. Näistä useasti tärkeimpiä ovat preferenssien, kriteerimittausten ja muiden parametrien osuus, joka sijoittaa vaihtoehdon tietylle lajittelusijalle tai tiettyyn kategoriaan. Nämä indeksit lasketaan teoriassa moniulotteisina integraaleina, mutta käytännössä niitä arvioidaan Monte Carlo-simulaatiotekniikalla. Kuten tässä väitöskirjassa myöhemmin näytetään, SMAA:n algoritmit ovat nopeita ja tarpeeksi tarkkoja käytettäväksi kaikissa käytännön kokoa olevissa päätöksenteko-ongelmissa.

Tämä väitöskirja koostuu artikkeleista jotka käsittelevät useita SMAA-menetelmäperheen osa-alueita. Tutkimuksen ensimmäisessä osassa analysoitiin SMAA-2 ja SMAA-O menetelmien algoritmit sekä teoreettisesti että käytännön testeillä. Teoreettinen osa johti laskennallisen kompleksisuuden määrittämiseen sekä algoritmien tarkkuuden laskemiseen. Tarkkuuslaskelmien pohjalta voidaan määrittää tarvittavien Monte Carlo simulaatioiden määrä, jotta saavutetaan riittävän pienet luottamusvälit ko. indekseille. Tässä osassa kuvaamme myös SMAA-2 ja SMAA-O algoritmit pseudokoodina.

Toinen osa sisältää realistisen sovelluksen, jossa SMAA:ta on sovellettu hissisuunnitteluun. Tässä sovelluksessa tutkittiin SMAA:n soveltuvuutta hissiryhmän valintaan korkeiden rakennusten suunnittelussa. Tutkimus tehtiin yhdessä KONE-yhtiön työntekijän kanssa. Käytimme KONE:en rakennussimulaattoria tuottamaan mittauksia suoritustehokkuuskriteereille. Muodostimme näistä monimuuttujisen normaalijakauman, jossa epävarmuudet suoritustehokkuuskriteereiden kesken olivat riippuvaisia. Tämän lisäksi käytimme ordinaalista kriteeriä hinnan määrittämiseen (tarkkoja arvoja ei ollut saatavilla), sekä kardinaalista epätarkkaa kriteeriä hissiryhmän vaatiman lattiapinta-alan mittaamiseen. Mallin avulla kykenimme erottamaan alkuperäisistä 10:stä vaihtoehdosta neljä mahdollisesti implementoitavaa vaihtoehtoa, joista yksi osoittautui selvästi parhaaksi kompromissivaihtoehdoksi.

Väitöskirjan kolmas osa koostuu ELECTRE-menetelmien ja SMAA:n yhdistämisestä. ELECTRE-perheen kaksi menetelmää, ELECTRE III ja ELECTRE TRI laajennettiin käyttämään epätarkkoja arvoja. Nämä laajennukset kantavat nimiä SMAA-III ja SMAA-TRI. Tässä yhteydessä laajennettiin myös SMAA-menetelmäperheen käsitettä: sen sijaan että keskitytään eri SMAA-menetelmiin, on tärkeämpää nähdä niiden idea simulaation käyttämisestä eri indeksien laskemisessa. Tällöin SMAA:ta voidaan käyttää "ulkoisten" menetelmien soveltamiseen epätarkkojen mittausten kanssa. SMAA:ta voidaan käyttää tällä tavalla myös herkkyysanalyysiin ja parametrien "herkkyyden" kvantifioimiseen.

Väitöskirjan viimeinen osa sisältää kaiken menneen tutkimuksen yhdistämisen yksittäiseksi kehykseksi, jonka perusteella käytettävä SMAA-menetelmä voidaan helposti valita ongelman erityispiirteiden perusteella. Tämä kehys auttaa myös SMAA-menetelmien puutteiden kartoituksessa ja siten tulevan tutkimuksen suunnittelussa.

Jotta päätöksenteko-menetelmä saavuttaisi suosiota myös kehittäjäpiirinsä ulkopuolella, tarvitaan sille käyttäjäystävällinen ohjelmisto. Teoreettisen tutkimuksen lisäksi tämän väitöskirjatutkimuksen aikana allekirjoittanut kehitti ohjelmiston joka implementoi SMAA-III- ja SMAA-TRI-menetelmät. Ohjelmisto on kirjoitettu C++-kielellä ja se käyttää graafista käyttöliittymää varten gtkmm-kirjastoa. Tämän vuoksi ohjelmisto on helposti siirrettävissä uusille alustoille. Tällä hetkellä se onkin jo käännetty Windows XP:lle, Max OS X:lle ja Linuxille. Tulevaisuuden tutkimustyö tulee keskittymään menetelmäperheen laajentamiseen sekä näiden laajennusten implementoimiseen ohjelmistossa.

Extended abstract in Portuguese

As organizações actuais deparam-se diariamente com situações de tomada de decisões com base em múltiplos critérios. Estas decisões podem ir desde a selecção de um sítio para localizar uma fábrica até à escolha de um conjunto de elevadores para um arranha-céu. Ambos os exemplos mencionados têm como característica comum o facto de disporem de um conjunto finito de alternativas avaliadas a partir de um conjunto ou família coerente de critérios. Dependendo da problemática em questão, podemo-nos preocupar com a escolha da melhor ou melhores alternativas, com a ordenação das alternativas de melhor para a pior ou ainda com a classificação das alternativas em categorias predefinidas.

Os problemas de decisão tratados nesta tese são dos tipos mencionados anteriormente. Os critérios definidos para avaliar as diferentes alternativas podem ser de natureza ordinal ou cardinal. Em relação aos critérios ordinais apenas pode ser construída uma ordenação das alternativas de acordo com as preferências do decisor. Relativamente aos critérios cardinais, estes têm a vantagem de poder ser traduzidos por valores numéricos. Nesta tese partimos do princípio que a família de critérios possa comportar ambos os tipos de critérios, que são usados para modelar ou construir as preferências através de uma relação de prevalência que será explorada para "resolver" os problemas atrás mencionados. Nos problemas reais os resultados provenientes da aplicação directa de um determinado método não deverão ser interpretados literalmente como tais, ou seja, tal como nos aparecem após a aplicação do método. Uma análise mais profunda será necessária, as imprecisões, inexactidões, insuficiências, incertezas ou indeterminações nas avaliações das alternativas segundos os diferentes critérios bem como aquelas que estão associadas aos parâmetros (preferências ou técnicos) dos modelos fazem intervir uma parte de arbitrário que necessita de ser estudada de forma mais aprofundada.

O apoio multicritério à decisão é um campo da ciência relativamente novo e dispersa-se por várias escolas de pensamento. Existem vários métodos propostos na literatura, cada um privilegiando algumas características particulares encontradas nos problemas reais. Há, no entanto, duas grandes famílias de métodos que por várias razões, incluindo algumas de natureza histórica, se impuseram: métodos baseados na teoria da utilidade ou valor multicritério e métodos baseados nas relações de prevalência. Este trabalho concentra-se na família dos métodos ELECTRE, que são métodos baseados na construção de uma ou várias relações de prevalência seguida de uma exploração dessa ou dessas relações.

A teoria de utilidade dá a base para os métodos de apoio à decisão mais antigos, mas ainda em uso. Estes métodos têm fortes bases axiomáticas, o que não é o caso dos métodos ELECTRE. De qualquer maneira, os métodos ELECTRE têm uma vantagem em relação aos métodos baseados na teoria da utilidade, dado que partem do princípio que não existe uma função utilidade que por algum processo se podem determinar. Os métodos ELECTRE têm ainda a característica de trabalharem com um modelo com lineares, baseiam-se assim no chamado modelo do pseudo-critério.

Recentemente, tem-se atendido com alguma profundidade e preocupação para o facto de que os parâmetros dos modelos são incertos ou inexactos bem como os desempenhos das alternativas nos critérios. Os métodos tradicionais baseados na teoria de utilidade multicritério e os métodos de ELECTRE não o fizeram de forma sistemática. No início dos anos 90 nasceu uma nova família dos métodos, Stochastic Multricriteria Acceptability Analysis (SMAA), que explicitamente permite modelar esta imprecisão. Os vários métodos da família SMAA foram concebidos para problemas de apoio à decisão de todos os tipos, e possibilitam usar parâmetros e desempenhos imprecisos, inexactos e/ou insuficientes. O primeiro método da família SMAA e a sua extensão SMAA-2 são baseados na teoria da utilidade multicritério e forma concebidos para as problemáticas da escolha da melhor alternativa (o caso do SMAA) ou para a ordenação das alternativas (SMAA-2). Os métodos SMAA podem igualmente ser usados em problemas de decisão onde não se disponha de nenhuma informação preferencial.

Os métodos da família SMAA usam as técnicas de simulação para as medidas descritivas ou índices que servem para dar informação estatística sobre o problema. Os diferentes métodos propõem diferentes índices, dentre os mais usuais destaca-se aquele que nos permite ter informação sobre as preferências que colocam uma certa alternativa numa dada posição da ordenação. Estes índices são calculados de forma exacta a partir da teoria dos integrais múltiplos. Mas, na prática são estimadas através da simulação de Monte Carlo. Nesta tese pode-se constatar que os algoritmos dos métodos SMAA são rápidos e suficientemente exactos para serem usados em todos os problemas de decisão de dimensões aceitáveis.

Esta tese é composta de vários artigos que tratam também várias subáreas de aplicação dos métodos SMAA. Na primeira parte, fez-se uma análise teórica e experimental dos algoritmos SMAA-2 e SMAA-O. A parte teórica resultou no estudo da complexidade e no cálculo de precisão dos algoritmos. Baseado nos cálculos sobre a precisão foi possível determinar a quantidade de simulações de Monte Carlo para obter intervalos de confiança suficientemente pequenos, mas significativos para justificar os índices em questão. Nesta parte apresenta-se igualmente os algoritmos de SMAA-2 e SMAA-O em pseudo-código.

A segunda parte contém uma aplicação real do SMAA na área do planeamento da instalação de um conjunto de elevadores. Nesta aplicação foi investigada a aplicabilidade do SMAA na escolha de um conjunto de elevadores para arranha-céus. A investigação e aplicação foram efectuadas junto da empresa KONE. Usou-se o simulador de prédios da KONE para construir a matriz de desempenhos. A imprecisão relativamente a estes desempenhos foi modelada através de uma distribuição de Gauss multivariada. Para além disso usou-se um critério ordinal para modelar o preço, dado que os valores exactos não eram conhecidos, e um critério cardinal para representar a área necessária. A partir de uma análise preliminar 4 das 10 alternativas inicias puderam ser seleccionadas como potenciais opções a implementar. Seguidamente pode observar-se que uma destas 4 alternativas é claramente a melhor alternativa de compromisso.

A terceira parte da tese é consagrada à uma combinação entre os métodos ELECTRE e SMAA. Dois métodos da família ELECTRE, ELECTRE III e ELECTRE TRI, foram estendidos para usar valores inexactos. Estas extensões chamam-se SMAA-III e SMAA-TRI. O conceito de família dos métodos de SMAA foi também estendido neste contexto: em vez de nos concentrarmos em métodos diferentes, o que é importante é a ideia de usar simulação para calcular os índices. Assim, o SMAA pode-se usar para aplicar métodos "externos" com parâmetros inexactos para analisar a sua robustez.

A última parte desta tese conte apresenta uma estrutura geral onde se enquadram os métodos SMAA e que pode facilitar a escolha de um determinado método em função das características específicas do problema. Esta estrutura também é útil para construir um mapa das lacunas dos métodos SMAA e ajuda, assim, no planeamento da investigação futura.

Para que um método de apoio à decisão alcance alguma popularidade fora do círculo de seus autores e comunidade científica da área, torna-se necessário dispor de um software amigável para os utilizadores. Para além da investigação teórica, dos testes experimentais e das aplicações reais efectuados no quadro desta tese, também foi consagrado algum tempo à implementação informática e construção de uma interface amigável dos métodos SMAA-III e SMAA-TRI. Os métodos foram implementados em linguagem C++ e usa a biblioteca gtkmm necessária para a construção da interface gráfica. Tal torna possível a portabilidade para novas plataformas. Neste momento já existem versões para Windows XP, Mac OS X, e

Linux. Parte da investigação futura será consagrada a novas extensões dos métodos SMAA e sua implementação.

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The work leading into this thesis has been completed in various universities. The first year of the thesis I was working as a part-time assistant at the Department of Information Technology of the University of Turku (Finland). During this time the research was supported through my salary. In September 2004 I moved to the Faculty of Economics of the University of Coimbra, Portugal, where I worked in the INESC-Coimbra research center. This lasted until May 2006, when I moved to my current location: the Centre for Management Studies of Instituto Superior Técnico at the Technical University of Lisbon.

Although this thesis has been completed in a somewhat unorthodox fashion, by working in three different universities under supervision of three professors, I consider this being an advantage. I have enjoyed the possibility of experiencing working cultures of different universities, which is something that would never have happened by completing the whole thesis in Finland. I am grateful for all three supervisors of my thesis: Risto Lahdelma, José Rui Figueira, and Pekka Salminen. I thank them all both for their professional guidance as well as for making me feel welcome in all places I was working in.

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List of original publications

[I] Tervonen T., Lahdelma R., 2007. Implementing stochastic multicriteria acceptability analysis. European Journal of Operational Research 178(2), 500–513.

[II] Tervonen T., Hakonen H., Lahdelma R., 2008. Elevator planning with stochastic multicriteria acceptability analysis. Omega 36(3), 352–362.

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[IV] Tervonen T., Figueira J. R., Lahdelma R., Salminen P., 2007. Towards robust ELECTRE III with simulation: theory and software of SMAA-III. Working paper 1/2007 of CEG-IST. Submitted to European Journal of Operational Research.

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Contents

Chapter 1

Introduction

Organizations of all sizes often face situations in which decisions have to be taken based on multi-dimensional data. This type of decisions range from siting a factory to choosing a type of elevator system for a high-rise building. Both of these belong to an important class of decision making problems characterized by being composed of a finite set of alternatives evaluated on the basis of several criteria. Based on this evaluation the best alternative can be chosen, or the alternatives can be ranked, or they can be sorted into pre-defined and ordered categories. These types of problems are termed multiple criteria choosing, ranking, and sorting problems, respectively (Figueira et al., 2005). This type of decision making is called Multiple Criteria Decision Making (MCDM) or Multiple Criteria Decision Analysis (MCDA). Sometimes MCDA is used to abbreviate Multiple Criteria Decision Aiding as well (Belton and Stewart, 2002). In this thesis we use the term MCDM.

1.1 Multiple Criteria Decision Making

The problems considered in this thesis consist of a finite set of alternatives evaluated on basis of a set of criteria. The criteria can be ordinal, in the case that only the ranking of alternatives with respect to the criteria is available, or cardinal, if they can be measured on numerical scales. MCDM methods then aim to aggregate these values in a way that takes into account the preferences of the Decision Makers (DMs), in order to construct preference relations that solve the problem. In most real-life cases the solution should not be taken *per se*, but only as a starting point for a more through analysis. This includes possibly refining the model and re-performing the analysis in an iterative way.

The solutions are dependent not only on the criteria measurements and the preferences of the DMs, but also on the type of model and its technical parameters. Therefore, if several methods can be applied to the problem setting in question, it is advisable to compare their results. For all the methods applied, the analyst as well as the DMs should acknowledge the prerequisites for its use, as well as the advantages and disadvantages the method has.

MCDM as a scientific field is relatively young and quite dispersed in different schools of thought. There exists a large amount of methods each designed to tackle certain specificities of real-life MCDM problems. Two large families of methods are the multiattribute utility theory (MAUT) based methods (see e.g. Keeney and Raiffa, 1976) and the outranking methods. Some well-known outranking methods are the ELECTRE methods (Roy, 1996), PROMETHEE methods (Brans and Mareschal, 2005), and the SIR method (Xu, 2001). In this thesis we concentrate in ELECTRE methods from the outranking approach. The utility theory gives a basis for the oldest MCDM methods still in use today, and is mathematically more firmly based than the ELECTRE methods. Nevertheless, the ELECTRE methods allow more versatile modelling of preferences in terms of thresholds. These might be more easy for some DMs to understand than the concepts of MAUT.

In the recent years it has become more than apparent that MCDM methods should be able to take into account uncertainty and imprecision in the parameters. The classical methods applying both MAUT and outranking model do not accomplish this. In the 1990s emerged another family of MCDM methods, the Stochastic Multicriteria Acceptability Analysis (SMAA) that explicitly allows to model uncertainty.

1.2 Stochastic Multicriteria Acceptability Analysis

SMAA is a family of methods that allows to handle various types of MCDM problems having uncertain, imprecise, or missing values for the model. The original SMAA method (Lahdelma et al., 1998) and its extension SMAA-2 (Lahdelma and Salminen, 2001) applied MAUT in order to choose the best alternative (SMAA) or to rank the alternatives (SMAA-2). SMAA methods can handle decision making situations with completely missing preference information. If some preference information is available, it can be incorporated to the model. Also criteria measurements as well as other parameters of the model can be imprecise.

SMAA methods apply simulation in order to provide the DMs with indices describing the problem. Different methods of the family produce different indices, but the most important ones are usually the share of weights, criteria measurements, and other parameters that assign an alternative to a certain rank or category. These indices are calculated in theory through multidimensional integrals, but in practice Monte Carlo simulation is used to computed approximations for the values. As shall be shown later on, the SMAA algorithms are fast and accurate enough to use in all decision making problems of sizes encountered in practice.

1.3 Contributions

This thesis is composed of contributions in various parts of the SMAA methodology. We have analyzed the classical SMAA and SMAA-2 algorithms to give bounds on computational complexity and the amount of Monte Carlo iterations needed to obtain sufficient accuracy for the analysis. In this work we also described the algorithms for SMAA-2 and SMAA-O in pseudo-code.

A realistic case study of applying SMAA to elevator planning was made by us. In this study we applied the KONE building simulator in order to generate measurements from which a multivariate Gaussian distribution could be defined. This had to be done, because the measurements were highly correlated. Otherwise the results would have contained biases.

Another part of the work comprises of combining ELECTRE methods with the SMAA methodology. This work resulted in two new methods, SMAA-III and SMAA-TRI, that allow ELECTRE III and ELECTRE TRI, respectively, to be used with imprecise values. An important direction explored in these works was the usage of SMAA as an "external" method for performing robustness analysis with third party MCDM methods.

We have combined all the important past research into an integrated SMAA framework. This allows to get a complete picture of the current state of SMAA as well as its shortcomings. This framework allows to easily choose the SMAA variant to use based on the particularities of the decision making problem in question.

1.4 Outline of the thesis

We begin this thesis by giving a brief introduction to two classical MCDM methodologies considered in the contributions: MAUT and the outranking model as applied in ELECTRE methods. We then continue, in Chapter 3, by presenting the basic SMAA methodology. We introduce the basic SMAA method, SMAA-2, and an extension to handle ordinal criteria. We review some computational results, and present an application to elevator planning. We continue on the theory of SMAA by presenting outranking-based SMAA methods in Chapter 4. We define a unified SMAA framework in Chapter 5. In Chapter 6 we introduce the software produced in conjunction to this research. The software implements the two new methods, SMAA-III and SMAA-TRI. We summarize the publications in Chapter 7 before proceeding to give concluding remarks in Chapter 8.

Chapter 2

Multiple Criteria Decision Making

In this chapter we briefly present two major MCDM methodologies: MAUT and the outranking model. We will concentrate in MAUT from the point of view that it will be used for ranking the alternatives, and in outranking model as applied in the ELECTRE methods. The small introduction to these two approaches is given because they are extended in SMAA approaches considered in the contributions. For more information and references on both approaches, see Belton and Stewart (2002). A detailed description of MAUT can be found in Keeney and Raiffa (1976), and one of ELECTRE methods in Roy (1996).

2.1 Multiattribute utility theory

Unidimensional utility theory is based on the concept that each alternative, when evaluated with respect to uncertain conditions, is assigned an expected utility value. These values describe the "goodness" of alternatives taking into account the preferences of the DM. The alternative with the highest expected utility is the most preferred one, or "best" in the considered problem setting.

The expected utility values are formed based on lotteries. These are defined as follows: consider a set of consequences $c_1, \ldots, c_n$, which are ordered so that c_n is the most preferred one and c_1 the least preferred one. Then consider two alternatives, x_1 and x_2, that each have for it assigned a probability p_i^1 or p_i^2 that when the alternative x_* is implemented, it results in consequence c_i with probability p_i^*. Now suppose that the DM asserts that for each i, he is indifferent between the two options:

1. Certainty. Receive c_i.

2. Risky. Receive c_n (the best consequence) with probability π_i and c_1 with the probability $1 - \pi_i$.

Then the expected values of the π's can also be used to numerically scale probability distributions over the c's (Keeney and Raiffa, 1976). That is, risks associated with consequences happening are used to calculate expected utilities of the alternatives. The DM's attitude towards risk defines which alternatives obtain the highest utilities. From these we can form utility functions that map the values into utility scores in an arbitrary, non-linear way that take into account the DM's attitude towards risk.

MAUT extends the unidimensional utility theory so that alternatives are considered with respect to several attributes, that is, criteria. In MAUT the utility functions of individual criteria are combined with scaling factors. These describe trade-offs the DM accepts to be consistent with his/her preferences. The weights of SMAA and SMAA-2 models that apply utility theory are these very scaling factors.

Another way of presenting the utility theory is to consider the utility functions to represent a complete preorder. This is defined with strongly complete and transitive binary relation based on trade-offs between criteria. By presenting utility theory in this way the difference between it and the outranking model considered in the next section comes clearer: the outranking model is based on the outranking relation that is weak, simply reflexive and neither strongly complete nor transitive.

2.2 Outranking model

Unlike MAUT, outranking model does not have an axiomatic basis, but instead relies on the intuition of how "goodness" of the alternatives is judged. The basic idea is that small differences between alternatives are indifferent, and differences over some certain magnitude do not bring any additional value. For example, when buying a car, it does not make a difference for most of the DMs whether the car costs 10000 euros or 20 more. In analogy, if one car costs 10000 and two others 2000000 and 3000000, probably there is no difference between preferability of the first over the second one to the first over the third one. Both of the latter ones are considered "bad" with respect to the price of the first one.

One of the largest families of outranking methods are the ELECTRE methods. It includes ELECTRE I (Roy, 1968), II (Roy, 1971; Roy and Bertier, 1973), III (Roy, 1978), IV (Roy and Hugonnard, 1982), TRI (Yu, 1992b), and 1S (Roy and Skalka, 1984). The two above mentioned characteristics of outranking models are modelled in ELECTRE methods as thresholds. That is, as an indifference threshold defining the difference until which the values are considered indifferent, and a preference threshold for over which the differences do not bear additional value no matter how big they are. The thresholds can be defined as constant ones or, for example, as a percentage of the value. When a criterion is defined with two such thresholds, it is called a *pseudo-criterion*. All the ELECTRE methods extended in this thesis use pseudo-criteria.

Outranking methods are called such, because instead of aggregating their criteria values to a single attribute describing goodness of the alternative, they form an outranking relation between alternatives. An alternative is said to outrank another if it is considered *as good as or better*. The outranking methods then exploit these outranking relations, for example, to form a ranking of the alternatives (as in ELECTRE III, see Roy, 1978), or to assign the alternatives into categories (as in ELECTRE TRI, see Yu, 1992b). The weights in ELECTRE methods are not scaling factors as in MAUT-based models, but interpreted as votes for the criteria (Vincke, 1992).

2.3 Imprecision

The basic methods of both of the above-mentioned approaches require exact values to be defined for the model. In MAUT this means that although the attitude towards risk should take uncertainty into account, the attitude must be strictly defined with numerical values. The scaling factors (weights) between pairs of criteria must be exact. In ELECTRE methods the situation is similar: deterministic weights are needed, as well as exact values for the thresholds. In both methodologies the basic methods require exact values for the cardinal criteria measurements.

Some MAUT extensions and the ELECTRE methods allow to use also poorer, ordinal scales. In these only the ranking of alternatives is required. But through the years it had become apparent that more free modelling of imprecision is needed. This applies to all parameters of the models: preferences, criteria measurements, as well as to the technical parameters. A new approach that allows explicitly to account for uncertainties and imprecision as well as missing values in all parameters is the Stochastic Multicriteria Acceptability Analysis.

Chapter 3

Stochastic Multicriteria Acceptability Analysis

One way to overcome the weaknesses of the utility theory based approach is through an inverse method: instead of asking parameter values and giving an answer to the problem in question, the values resulting in different outcomes are described. The Stochastic Multicriteria Acceptability Analysis (SMAA) (Lahdelma et al., 1998; Lahdelma and Salminen, 2001) methods include computing multidimensional integrals over feasible parameter spaces in order to support DMs with descriptive measures. The methods solve various problems encountered in the traditional approach by allowing to use parameters with ignorance on the values. For example, usually different weight elicitation techniques produce different values, and therefore deterministic weights are harder to justify than, for example, weight intervals.

There have been similar approaches before SMAA. The first one was the comparative hypervolume criterion by Charnetski (1973) and Charnetski and Soland (1978). Rietveld (1980) and Rietveld and Ouwersloot (1992) presented similar methods for problems with ordinal criteria and ordinal preference information. Bana e Costa (1986, 1988) presented the overall compromise criterion. We note that the probability distributions used in SMAA are not the only possibility for modelling uncertain parameter values. Other possible approaches include entropy methods (Abbas, 2006; Jessop, 1999), rough sets (Greco et al., 1999, 2000, 2001, 2002; Pawlak and Słowiński, 1994), fuzzy sets (Roubens, 1997), interval methods (Mustajoki et al., 2006, 2005), and Dempster-Shafer theory (Beynon, 2002; Beynon et al., 2001a,b, 2000).

We will describe here the SMAA-2 (Lahdelma and Salminen, 2001) and SMAA-O (Lahdelma et al., 1998) methods, as well as our new ELECTRE-based SMAA methods, SMAA-TRI ([III]; Tervonen et al. (2007)) and SMAA-

III ([IV]). Other methods/extensions of the SMAA family not presented here are a technique for handling dependent criteria (Lahdelma et al., 2006a,b), cross confidence factors (Lahdelma and Salminen, 2006a), SMAA-D (Lahdelma and Salminen, 2006b) for data envelopment analysis, SMAA-P (Lahdelma and Salminen, 2003) applying prospect-theory, and Ref-SMAA (Lahdelma et al., 2005) for using reference points in SMAA. A method similar to Ref-SMAA has been presented by Durbach (2006). Different SMAA methods have been applied in various real-life cases: harbour citing (Hokkanen et al., 1999), waste treatment facility citing (Lahdelma et al., 2002), determining the implementation order of a general plan (Hokkanen et al., 1998), choosing a clearer for polluted soil (Hokkanen et al., 2000), forest planning (Kangas et al., 2006, 2003a; Kangas and Kangas, 2003; Kangas et al., 2005), elevator planning ([II]), and designing a framework for an oil spill response effectiveness (Linkov et al., 2007). For a complete survey on SMAA, see [V].

3.1 SMAA-2

The discrete decision-making problem considered in SMAA-2 (Lahdelma and Salminen, 2001) refers to a set of m alternatives $X = \{x_1, \ldots, x_i, \ldots, x_m\}$, that are evaluated on the basis of n criteria $\{g_1, \ldots, g_j, \ldots, g_n\}$. The evaluation of alternative x_i on criterion g_j is denoted $g_j(x_i)$. Without loss of generality we assume that all the criteria are to be maximized. The model considers multiple DMs, each having a preference structure representable through an individual weight vector w and a real-valued utility function $u(x_i, w)$ that has a commonly accepted shape. The most commonly used utility function is the linear one:

$$u(x_i, w) = \sum_{j=1}^{n} w_j g_j(x_i). \tag{3.1}$$

The weights will be assumed non-negative and normalized. Therefore the feasible weight space will be:

$$W = \left\{ w \in R^n : w \geq 0 \text{ and } \sum_{j=1}^{n} w_j = 1 \right\}.$$

The feasible weight space of a 3-criteria problem with no preference information is illustrated in Figure 3.1.

The SMAA methods are developed for situations where criteria values and/or weights or other model parameters are not precisely known. Uncertain or imprecise criteria values are represented by stochastic variables

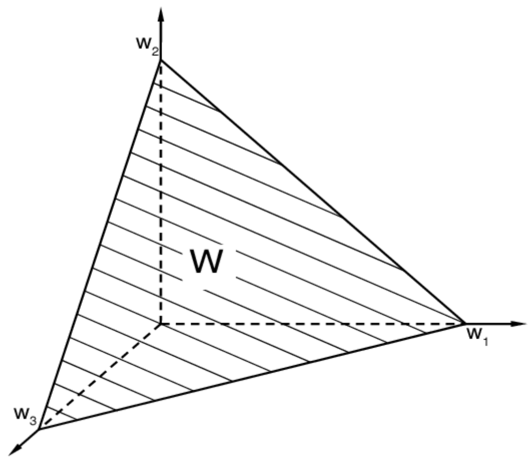

Figure 3.1: The feasible weight space of a 3-criteria problem.

ξ_{ij} corresponding to the deterministic evaluations $g_j(x_i)$ with density function $f_\chi(\xi)$ in the space $\chi \subseteq R^{m\times n}$. In principle, arbitrary distributions can be used, but in practice a uniform distribution in a certain interval or a Gaussian distribution is often used.

Similarly, the DMs unknown or partially known preferences are represented by a weight distribution with a joint density function $f_W(w)$ in the feasible weight space W. Total lack of preference information on weights is represented by the uniform weight distribution in W:

$$f_W(w) = 1/\mathrm{vol}(W).$$

As for the utility function based approaches, one should note that the weights are defined as scaling factors: the weights rescale the values of partial utility functions in such a way that the full swing in the scaled function indicates the importance of the criterion (see Belton and Stewart, 2002, Sect. 5.4).

The fundamental idea of SMAA is to provide decision support through descriptive measures calculated as multidimensional integrals over stochastic parameter spaces. Approximations for these measures are computed through Monte Carlo simulation. This means that they might contain errors, but the error margins are so small that usually they do not have to be taken into account (when the number of Monte Carlo iterations is large enough, see Section 3.4). SMAA-2 (Lahdelma and Salminen, 2001) defines three main types of descriptive indices for decision support: rank acceptability indices, central weight vectors, and confidence factors. These measures do not give definite answers, but rather provide DMs with more insight into the decision making problem. In order to introduce these indices we first need to define a ranking function as follows:

$$rank(i, \xi, w) = 1 + \sum_{k \neq i} \rho\Big(u(\xi_k, w) > u(\xi_i, w) \Big),$$

where $\rho(true) = 1$ and $\rho(false) = 0$. Note that $rank(i, \xi, w) \in \{1, \ldots, m\}$. Let us also define the sets of favourable rank weights $W_i^r(\xi)$ as follows,

$$W_i^r(\xi) = \{w \in W : rank(i, \xi, w) = r\}.$$

3.1.1 Rank acceptability index

The rank acceptability index b_i^r describes the share of parameter values granting alternative x_i rank r. It is computed as a multidimensional integral over the criteria distributions and the favourable rank weights as follows,

$$b_i^r = \int_{\xi \in \chi} f_\chi(\xi) \int_{w \in W_i^r(\xi)} f_W(w)\, dw\, d\xi.$$

The most acceptable (best) alternatives are those with high acceptabilities for the best (smallest) ranks. Evidently, the rank acceptability indices are within the range [0,1], where 0 indicates that the alternative will never obtain a given rank and 1 indicates that it will obtain the given rank always with any choice of weights.

Rank acceptability indices can be used to classify alternatives into stochastically efficient ($b_i^1 >> 0$) or inefficient ones (b_i^1 near zero, for example, < 0.05). A zero first rank acceptability index means that an alternative is never considered the best with the assumed preference model. For stochastically efficient alternatives, the index measures the strength of the efficiency considering simultaneously the uncertainties on the criteria measurements and the DMs' preferences.

Scaling of the criteria affects the rank acceptability indices. Therefore scaling must not be done abritrarily when trying to classify the alternatives on the basis of rank acceptability indices (Lahdelma and Salminen, 2001). For example, if the minimum and maximum criterion values are chosen as the corresponding scaling points, the possible introduction of a new alternative might change these values and, therefore, also the rank acceptability indices to a large extent (Bana e Costa, 1988).

3.1.2 Central weight vector

The central weight vector w_i^c is defined as the expected center of gravity of the favourable weight space. It is computed as a multidimensional integral over the criteria and weight distributions as

$$w_i^c = \frac{1}{b_i^1} \int_{\xi \in \chi} f_\chi(\xi) \int_{w \in W_i(\xi)} f_W(w) w \, dw \, d\xi.$$

The central weight vector describes the preferences of a typical DM supporting this alternative with the assumed preference model. By presenting the central weight vectors to the DMs, an inverse approach for decision support can be applied: instead of eliciting preferences and building a solution to the problem, the DMs can learn what kind of preferences lead into which actions, without providing any preference information.

3.1.3 Confidence factor

The confidence factor p_i^c is defined as the probability for an alternative to be the preferred one with the preferences expressed by its central weight vector. It is computed as a multidimensional integral over the criteria distributions as follows,

$$p_i^c = \int_{\substack{\xi \in \chi : u(\xi_i, w_i^c) \geq u(\xi_k, w_i^c) \\ \forall k=1,\dots,m}} f_\chi(\xi) \, d\xi.$$

The confidence factors measure whether the criteria measurements are accurate enough to discern the efficient alternatives. If the problem formulation is to choose an alternative to realize, the ones with low confidence factors should not be chosen. If they are deemed as attractive ones, more accurate criteria data should be collected in order to make a reliable decision.

3.2 Preference information

In most decision-making problems it is possible to elicit some preference information from the DMs. This information can possibly be imprecise and uncertain. Although SMAA methods allow preference information to be represented with an arbitrary density function, it is usually easier to elicit the preferences as constraints for the weight space. Then, the density function is defined with a uniform distribution in the restricted weight space W' as

$$f_{W'}(w) = \begin{cases} 1/\mathrm{vol}(W'), & \text{if } w \in W', \\ 0, & \text{if } w \in W \setminus W'. \end{cases}$$

In particular, we can have the following types of constraints (Lahdelma and Salminen, 2001):

1. Intervals for weights ($w_j \in [w_j^{min}, w_j^{max}]$).

2. Intervals for weight ratios (trade-offs) ($w_j/w_k \in [w_{jk}^{min}, w_{jk}^{max}]$).

3. Linear inequality constraints for weights ($Aw \leq c$).

4. Nonlinear inequality constraints for weights ($f(w) \leq 0$).

5. Partial or complete ranking of the weights ($w_j > w_k$).

Figure 3.2 illustrates the feasible weight space of a 3-criteria problem with interval constraints for weight w_1. Figure 3.3 illustrates the feasible weight space of a 3-criteria problem with complete ranking of the weights.

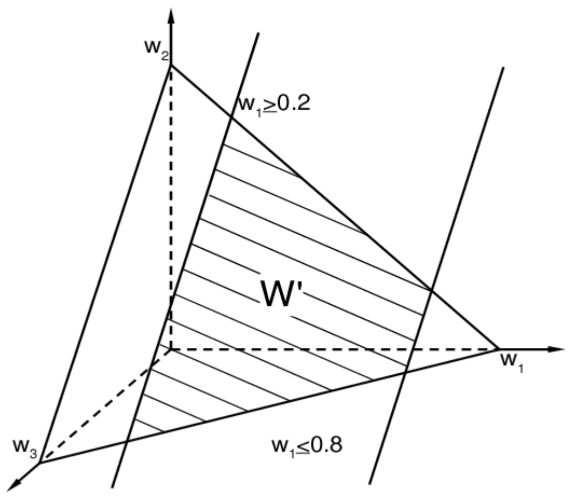

Figure 3.2: The feasible weight space of a 3-criteria problem with constraints on w_1.

When there are multiple DMs, the constraints have to be aggregated before applying. Possible non-interactive aggregation techniques include forming union or intersection, or averaging weight space density functions of different DMs. There exists also a technique based on belief functions for eliciting and aggregating the preference information, see Tervonen et al. (2004a,b).

3.3 Ordinal criteria (SMAA-O)

SMAA-O (Lahdelma et al., 2003) extends SMAA to consider ordinal criteria measurements, meaning that the DMs have ranked the alternatives according to each (ordinal) criterion. In SMAA-O, the ordinal information is mapped to cardinal without forcing any specific mapping. This means that nothing is assumed about the weights of criteria ranks in the piecewise linear mapping.

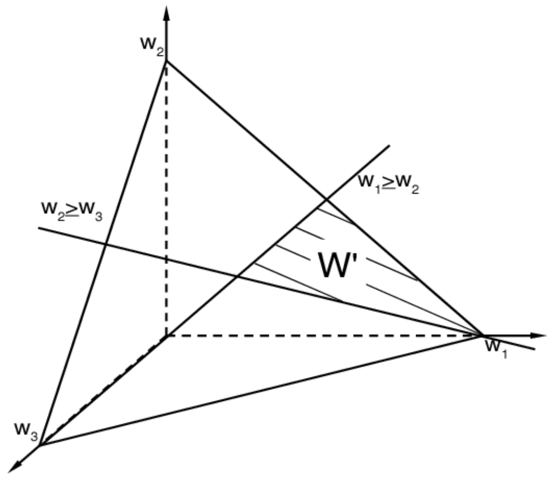

Figure 3.3: The feasible weight space of a 3-criteria problem with complete ranking of the weights.

The possibility of using ordinal measurements has its advantages. Usually the experts defining criteria measurements can rank alternatives with respect to each criterion faster than if they use cardinal measurements. Therefore, if ordinal measurements provide sufficient accuracy for the decision-making problem in question, savings can be obtained.

Ordinal criteria are measured by assigning for each alternative a rank level number $r_j = 1, \ldots, j^{max}$, where 1 is the best and j^{max} the worst rank level. Alternatives considered equally good are placed on the same rank level and rank levels are numbered consecutively. On an ordinal scale, the scale intervals do not contain any information, and should be therefore treated as such without imposing any extra assumptions. However, some mapping can be assumed to underlie the ordinal information. In SMAA-O, all mappings that are consistent with the ordinal information are simulated numerically during Monte Carlo iterations. This means generating random cardinal values for the corresponding ordinal criteria measurements in a way that preserves the ordinal rank information. Figure 3.4 illustrates a sample mapping generated in this way.

The MAUT-based SMAA methods can be used with any kind of utility function jointly accepted by the DMs, but if we have an additive utility function, the shape of the function will be considered unknown. In this case, the DMs partial utility functions are simulated in the same way as the ordinal to cardinal mappings. However, simulation is not necessary for ordinal criteria, because the simulated cardinal values can be interpreted directly as partial values on a linear scale. Therefore, if the DMs accept

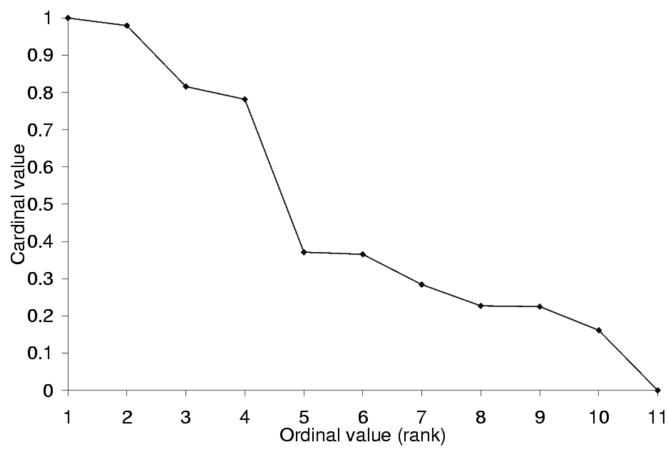

Figure 3.4: A sample ordinal to cardinal mapping of SMAA-O. (Lahdelma et al., 2003)

an additive utility function, it is not necessary for the DMs to agree on a common shape of the partial utility functions for the ordinal criteria.

SMAA-O has been combined with the so-called SWOT methodology in the work of Kangas et al. (2003b). For an alternative technique for applying ordinal criteria in simulation-based approaches, see Leskinen et al. (2004).

3.4 Simulation

The various distributions applied in the integrals of SMAA vary according to the application and can be arbitrarily complex. Usually the integrals have high dimensionality as well. Numerical integration techniques based on discretizing the distributions with respect to each dimension are infeasible, because the required effort depends exponentially on the number of dimensions. Therefore, instead of trying to obtain exact values for the integrals, Monte Carlo simulation is applied to obtain sufficiently accurate approximations. In this section we address the simulation technique, accuracy of the computations, and the complexity issues. For a full description of the algorithms, see [I].

3.4.1 Simulation technique

Monte Carlo simulation is applied in computation of the integrals. For all the acceptability index-type measures, a similar technique is applied: in each iteration, measurements for the parameters (criteria measurements, weights, ...) are drawn from their corresponding joint distributions, and a ranking or a classification is built based on these values. After this, counters for the corresponding ranks or classes with respect to the alternatives are increased. After a number of iterations, the indices are obtained by dividing the counters with the number of iterations. The central weights are computed in a similar fashion, so that in each iteration, when an alternative obtains first rank, the weight vector is added to its "summed weight vector". This vector is divided component-wise in the end by the number of iterations to obtain the central weight vector.

Weight generation is an important part of the simulation technique. If there is no preference information available, the n uniform distributed weights are generated as follows: first $n-1$ independent random numbers are generated from the uniform distribution within the range $[0, 1]$, and sorted into ascending order $(q_1, \ldots, q_{n-1})$. After that, 0 and 1 are inserted as the first (q_0) and last (q_n) numbers, respectively. The weights are then obtained as intervals between consecutive numbers $(w_j = q_j - q_{j-1})$.

If there exists preference information, the weight generation technique must be altered. In the case of complete ordinal preference information, the weights can simply be sorted according to the ranking. Lower bounds for weights can be handled by using a simple transformation technique, because the lower-bounded feasible weight space is homomorphic with the original one. The lower bounded weights are defined by generating the random numbers from interval $[0, 1-s]$, where s is the sum of all lower bounds, and adding to them the corresponding lower bounds.

Upper bounds for weights cannot be handled with a similar technique, but instead a simple rejection technique is applied, in which the weight vectors not satisfying the upper bounds are rejected. The tip of the simplex cut off by the upper bounds has relatively small area compared to the one of lower bounds. Therefore the increase in computational complexity due to upper bounds is relatively low. In addition, lower bounds might even render some of the upper bounds redundant. Consider for example a 3-criteria problem with lower bounds of 0.2 for all weights. The maximum value that any weight can obtain is $1 - 0.2 - 0.2 = 0.6$, and therefore all upper bounds higher than 0.6 are redundant. The amount of weights rejected due to upper bounds can be estimated in the following way: if we consider all weights to have a common upper bound w^{max}, the probability for the largest of the generated weights to exceed the upper bound is

$$\begin{aligned} P[max\{w_j\} > w^{max})] =& n(1-w^{max})^{n-1} - \binom{n}{2}(1-2w^{max})^{n-1} \\ &+ \cdots (-1)^{k-1}\binom{n}{k}(1-kw^{max})^{n-1} \cdots , \end{aligned}$$

where the series continues as long as $1 - kw^{max} > 0$ (David, 1970).

3.4.2 Accuracy of computations

Accuracy of computations can be calculated by considering the Monte Carlo simulations as point estimators for the descriptive measures. To achieve accuracy of A with 95% confidence for the rank acceptability indices, we need the following number of Monte Carlo iterations K (Milton and Arnold, 1995):

$$K = \frac{1.96^2}{4A^2}.$$

For example, to achieve 95% confidence on error limits of ± 0.01 for the rank acceptability indices, we need to execute 9604 Monte Carlo iterations. The accuracy of confidence factors depends on the accuracy of central weight vectors in a complicated manner, but if we disregard this source of error, the same equation for accuracy applies. The accuracy of the central weight vectors depends on the acceptability indices, and the required amount of iterations is calculated as follows:

$$K = \frac{1.96^2}{b_i^1 4A^2}.$$

It should be noted that the accuracy of the computations does not depend on the dimensionality of the problem, but only on the number of iterations.

3.4.3 Complexity issues

The required number of Monte Carlo iterations in typical SMAA applications is fairly high, and therefore for having practical applicability the complexity of SMAA computations should not be too high with respect to the number of criteria and alternatives. The complexity of SMAA-2 and SMAA-O has been analyzed in [I]. The complexity of computing the acceptability indices and central weight vectors with independent criteria measurements and cardinal criteria is $O(K \cdot (n \log(n) + m \cdot n + m \log(m)))$. The complexity of computing the confidence factors is $O(K \cdot m^2 \cdot n)$. In these formulas K is the number of Monte Carlo iterations, m the number of alternatives, and n the number of criteria.

The use of ordinal criteria adds to the complexity with a factor of $\log(m)$. In practice this has very little effect. What has a larger impact to the running times is the handling of preference information. The formulas above assume that there are no constraints on the weights, which in practice is usually not the case. As described in Section 3.4.1, lower bounds for weights do not affect the complexity of the weight generation, but upper bounds might have a great impact on it.

3.5 Application: elevator planning

In modern high-rise buildings workers and inhabitants are transported between floors mainly by means of multiple elevators. Elevators are usually operated by elevator group control systems in order to provide efficient transportation. When a high-rise building is designed, a suitable configuration for the elevator group has to be designed. The DMs should consider performance as well as price and other non-performance criteria of alternative elevator group configurations. Because analytical methods are limited to the up-peak traffic situation and cannot evaluate the effect of a group control algorithm, the performance has to be measured using computer simulation, which produces stochastic measurements for the performance criteria of alternative configurations. The performance of an elevator group can be measured using several criteria, such as the average waiting time or the average ride time of the passengers. The price and other non-performance criteria can usually be assessed with sufficient accuracy or by ranking the alternatives. We present here an application of SMAA in elevator planning. For full details of the application, see [II]. For more on the history of elevator planning, see e.g. Basset (1923); Browne and Kelly (1968); Morley (1962); Parlow (1966); Phillips (1966); Pinfold (1966); Strakosch (1967); Tregenza (1971).

The goal in elevator planning is to find a suitable elevator group to serve the traffic of a high-rise building. Because the buildings do not exist at the planning stage, the traffic must be estimated by using the building specifications: the number of floors, their heights, the floor area and the building type. The travel height can be calculated from the number of floors and their heights, and the total population can be estimated according to the type of building and the floor area. Building types have characteristic *traffic profiles*. For example, office buildings typically have up-peak traffic in the morning when employees enter the building, intense two-way or inter-floor traffic during the lunch time, and down-peak traffic when employees exit the building (Siikonen and Leppälä, 1991).

The performance of a group of elevators is mainly determined by the number and size of the cars and their speed. Also acceleration, door types and the group control algorithm affect performance. Usual performance criteria are the handling capacity and the interval calculated in the up-peak situation. The *up-peak handling capacity* is the percentage of population per five minutes that can be transported from the lobby to the upper floors. It is assumed that elevators are filled to 80% of rated load (although it is possible to fill elevator up to rated load that does not happen in practice). The (up-peak) *interval* is an interval between two starts from the lobby. The interval is also related to the waiting time. The up-peak is used since it is the most demanding situation considering elevator handling capacity at least in office buildings, and because there are analytical formulas for calculating the up-peak handling capacity and interval (Barney and dos Santos, 1985). The usual recommendations state that the up-peak handling capacity for an office building should be 11-17% and interval 20-30s (Barney et al., 1998).

Non-performance criteria, such as cost and occupied floor area should also be considered. The *cost* of an elevator system consists of build and maintenance costs. The *floor area* occupied by the elevator group consists of the shaft space and the waiting area for passengers. In high-rise buildings the population is large and distances are long, thus the portion of shafts is large compared to the total floor area. This means more costs, since the rentable area is reduced. In some cases the building design constraints the occupied area, sometimes there is more freedom to use space. The elevator planning is not independent of building design; the architect should take advice from the elevator planner.

Instead of considering only up-peak traffic, we take into account the entire daily traffic and consider all criteria simultaneously. In this study the following 6 criteria are considered. The cost and area criteria take into account the building owners point of view. Passengers point of view is taken into account by waiting time, journey time, the percentage of waiting times exceeding 60s, and the percentage of journey times exceeding 120s. The waiting time is measured from the moment a passenger enters the waiting area to the moment he/she enters the elevator. The journey time is the total time from entering the waiting area to exiting the elevator. The last two criteria measure unsatisfactory service, which may happen especially in intense traffic peaks.

To obtain stochastic criteria measurements for the performance criteria, we executed simulations with the KONE Building Traffic Simulator (Hakonen, 2003; Leinonen, 1999). The simulation model consists of the elevator model and traffic generation. For more details of the model, see [II]. The simulated building has a lobby floor and 19 populated floors. The estimated number of people is 60 per floor.

Figure 3.5 shows the intensities of incoming, outgoing and inter-floor passengers during the day from 7 a.m. to 7.15 p.m. The traffic profile is measured from an office building. The profile shows typical morning, lunch time and afternoon traffic peaks. When passengers are generated according to the traffic profile, the expected number of passengers is 11502. Since total population of the building is uncertain, the traffic is varied between 80% and 120% of forecasted traffic. With these parameters, we generated 21 traffic situations according to the traffic profile. The same passengers were used for all 10 alternatives in order to reduce the covariance between the measurements of different alternatives.

The number of elevators in the alternatives varied between 6 and 8,

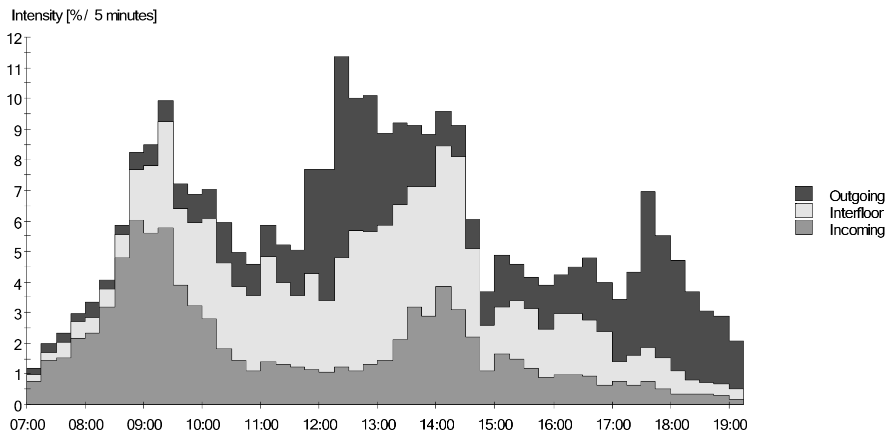

Figure 3.5: Traffic profile of the simulated building. Siikonen and Leppälä (1991)

rated load from 13 to 24 and speed from 3.5 m/s to 5 m/s. Area is the shaft space plus waiting area space. The exact costs were unknown, but alternatives could be ranked with respect to the cost. All alternatives were feasible with respect to up-peak handling capacity and interval.

The uncertainties of the performance criteria were assessed based on the simulations for each of the 10 configurations. Based on the simulation results we estimated the parameters for a multivariate Gaussian distribution, i.e. the expected value of each criteria measurement and the covariance matrix for the uncertainty dependencies. The uncertainties of the performance criteria were quite dependent, with multivariate correlations in the interval [0.8,1]. The cost was modelled as an ordinal criterion (see Section 3.3), because exact price information was not available. The required floor area was measured on a cardinal scale with 5m^2 uncertainty for all alternatives.

Preference information was added to the model in form of weight bounds to the model; weights for cost and shaft space were constrained to be in the interval [0.1,1]. The preference information was added to the model because of the strong dependencies between performance criteria, which shows that they all ultimately measure a single criterion, performance from the passengers point of view. Because of the additivity of weights, the performance would obtain too high significance in the analysis without balancing accomplished by using weight constraints.

We analyzed the model using 100 000 Monte-Carlo iterations, which gives error limits ≤ 0.01 ([I]). For results of the SMAA computations, see

[II]. Rank acceptability indices are illustrated graphically in Figure 3.6, and central weights as stacked columns in Figure 3.7. The analysis of this application allowed directly to eliminate half of the alternatives based on their confidence factors. The rank acceptability indicated four good choices for the alternative to implement. The trade-offs between the four alternatives could be stated based on the central weight vectors. From these four alternatives, one was recommended as a good compromise solution.

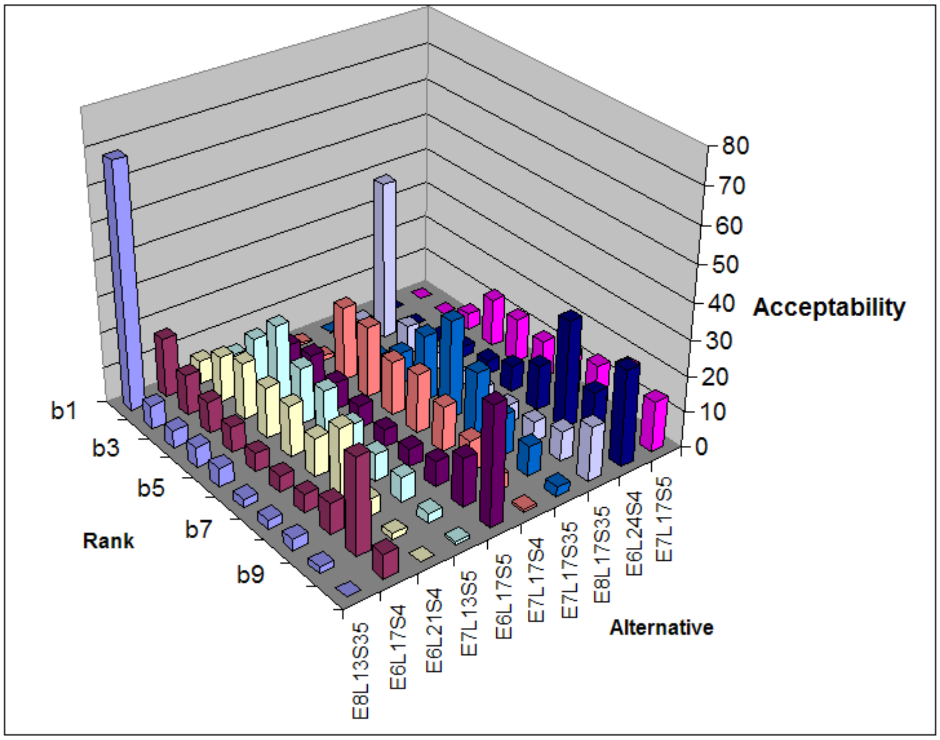

Figure 3.6: Rank acceptability indices of the study.

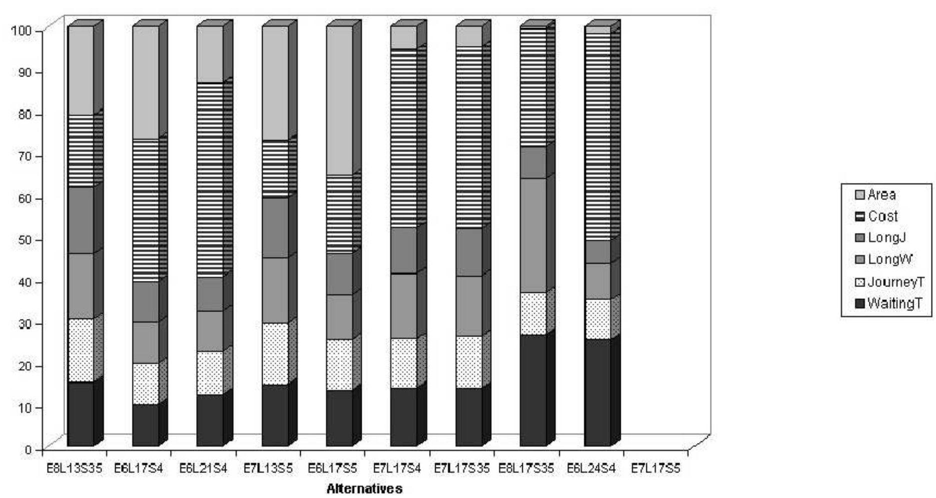

Figure 3.7: Central weight vectors of the study.

Chapter 4

Outranking based SMAA approaches

SMAA has been extended for using instead of utility function (3.1) an outranking-based aggregation procedure to rank alternatives. This and other approaches described in this chapter are based on using ELECTRE type pseudo-criteria. The pseudo-criteria are defined by using thresholds that are denoted as follows:

- $q_j(g_j(\cdot))$ is the *indifference threshold* for criterion g_j,
- $p_j(g_j(\cdot))$ is the *preference threshold* for criterion g_j, and, finally,
- $v_j(g_j(\cdot))$ is the *veto threshold* for criterion g_j.

By using these thresholds a *concordance index* is defined. It is computed by considering individually for each criterion g_j the support it provides for the assertion of the outranking aS_jb, "alternative a is at least as good as alternative b". The partial concordance index is a fuzzy index computed as follows, for all $j = 1, \ldots, n$:

$$c_j(a,b) = \begin{cases} 1, \text{ if } g_j(a) \geq g_j(b) - q_j(g_j(b)), \\ 0, \text{ if } g_j(a) < g_j(b) - p_j(g_j(b)), \\ \frac{g_j(a)+p_j\big(g_j(b)\big)-g_j(b)}{p_j\big(g_j(b)\big)-q_j\big(g_j(b)\big)}, \text{ otherwise.} \end{cases}$$

After computing the partial concordance indices, a comprehensive concordance index is calculated as follows,

$$c(a,b) = \sum_{j=1}^{n} w_j c_j(a,b).$$

If veto thresholds are used, a *discordance index* can be defined also. For more information on pseudo-criteria based models, see Roy (1996).

4.1 Outranking aggregation procedure (SMAA-3)

SMAA-3 (Hokkanen et al., 1998) method is a variant of the original SMAA that applies, instead of the utility function, ELECTRE type pseudo-criteria and a maximin choice procedure (see Pirlot, 1995). According to this procedure, an alternative becomes the preferred one (not necessary unique) if the following set of constraints hold:

$$\begin{gathered}\min_{l=1,\ldots,m,l\neq i} c(x_i,x_l) \geq \min_{l=1,\ldots,m,l\neq k} c(x_k,x_l),\\ k=1,\ldots,m,k\neq i.\end{gathered}$$

Based on this the favourable weights of an alternative are defined as

$$\begin{aligned}W_i = \{w \in W : &\min_{l=1,\ldots,m,l\neq i} \sum_{j=1}^{n} w_j c_j(x_i,x_l)\\ &\geq \min_{l=1,\ldots,m,l\neq k} \sum_{j=1}^{n} w_j c_j(x_k,x_l),\\ &k=1,\ldots,k,k\neq i\}.\end{aligned}$$

Based on these, the analysis is done in a way similar to SMAA, with the exception that the criteria measurements are considered to be deterministic (no integration over χ is done), and therefore no confidence factors are computed. It should be noted that now the central weight vector can lie outside the space of favourable weights of an alternative, because this preference model is non-linear. In this kind of (easily detectable) situations a favourable weight vector is chosen with a minimal distance to the central weight vector.

In the literature there exists simulation-tests of SMAA against SMAA-3. In these tests the results of SMAA-3 were found to be quite unstable with respect to the indifference threshold (Lahdelma and Salminen, 2002). Therefore, when SMAA-3 is applied in practice, great care should be put into choosing the thresholds. These test results are confirmed in [IV].

4.2 SMAA-TRI

All the SMAA variants described until here are for ranking or choosing problem statements. ELECTRE TRI (Yu, 1992a) is a method for sorting problem statements, and SMAA-TRI extends it to allow ignorance on the parameter values. There exists a large amount of work on parameter inference and robustness analysis for ELECTRE TRI, see Dias and Clímaco

(1999, 2000); Dias and Mousseau (2006); Dias et al. (2002); Mousseau et al. (2004, 2003, 2001); Mousseau and Słowiński (1998); Mousseau et al. (2000); Ngo The and Mousseau (2002).

ELECTRE TRI uses concordance and discordance indices for sorting the alternatives into pre-defined and ordered categories. Let us denote the categories in ascending preference order $C_1, \ldots, C_h, \ldots, C_k$ (C_1 is the "worst" category). These categories are defined by upper and lower profiles that consist of measurements for all criteria. In the assignment procedure alternatives are iteratively compared with the profiles. The profiles are denoted $p_1, \ldots, p_h, \ldots, p_{k-1}$. p_h is the upper limit of category C_h and the lower limit of category C_{h+1}. The profiles have to be strictly ordered, that is, they have to satisfy

$$p_1 \, \Delta \, p_2 \, \Delta \, \ldots \, \Delta \, p_{k-2} \, \Delta \, p_{k-1}, \tag{4.1}$$

where Δ is the dominance relation ($p_1 \Delta p_2$ means that p_2 dominates p_1). This dominance relation needs to be interpreted in a wide sense, because domination depends not only on the values of components of the two profiles, but also on the threshold values. We will not describe here the assignment procedure. It requires an additional technical parameter, the lambda cutting level, to be defined. The interested reader should refer to [III].

SMAA-TRI is developed for parameter stability analysis of ELECTRE TRI, and consists of analyzing finite spaces of arbitrarily distributed parameter values in order to describe for each alternative the share of parameter values that assign it to different categories. It analyzes the stability of weights, profiles, and the cutting level.

The input for ELECTRE TRI in SMAA-TRI is the following:

1. Uncertain or imprecise profiles are represented by stochastic variables ϕ_{hj} with joint density function $f_\Phi(\phi)$ in the space $\Phi \subseteq R^{(k-1)\times n}$. The joint density function must be such that all possible profile combinations satisfy (4.1). Usually the category profiles are defined to be independently distributed, and in this case the distributions must not overlap. For example, if the profile values for a criterion are Gaussian distributed, the distributions must have tails truncated as shown by the vertical lines in Figure 4.1.

2. The lambda cutting level is represented as a stochastic variable Λ with density function $f_L(\Lambda)$ defined within the valid range [0.5,1].

3. The weights and criteria measurements are represented as in SMAA-2.

4. The data and other parameters of ELECTRE TRI are represented by the set $T = \{M, q, p, v\}$. These components are considered to have deterministic values.

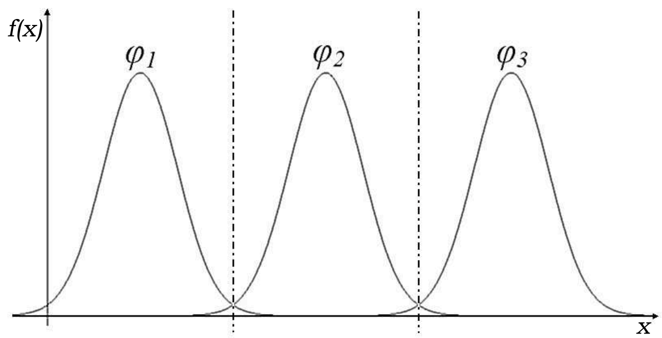

Figure 4.1: Probability distribution functions for three Gaussian distributed profile values (for a single criterion). The vertical lines show where the tails of the distributions must be truncated.

SMAA-TRI produces category acceptability indices for all pairs of alternatives and categories. The category acceptability index π_i^h describes the share of possible parameter values that have an alternative x_i assigned to category C_h. Let us define a *categorization function* that evaluates the category index h to which an alternative x_i is assigned by ELECTRE TRI:

$$h = K(i, \Lambda, \phi, w, T),$$

and a category membership function

$$m_i^h(\lambda, \phi, w, T) = \begin{cases} 1, & \text{if } K(i, \Lambda, \phi, w, T) = h, \\ 0, & \text{otherwise,} \end{cases}$$

which is applied in computing the category acceptability index numerically as a multi-dimensional integral over the finite parameter spaces as

$$\pi_i^h = \int_{0.5}^{1} f_L(\Lambda) \int_{\Phi} f_\Phi(\phi) \int_W f_W(w) m_i^h(\Lambda, \phi, w, T)\, dw\, d\phi\, d\Lambda.$$

The category acceptability index measures the stability of the assignment, and it can be interpreted as a fuzzy measure or a probability for membership in the category. If the parameters are stable, the category acceptability indices for each alternative should be 1 for one category, and 0 for the others. In this case the assignments are said to be robust with respect to the imprecise parameters.

The software presented in Chapter 6 implements SMAA-TRI, but also allows imprecision in all parameters. This means that the category acceptability indices are computed by integrating through spaces of all feasible parameter values instead of only the spaces of feasible values for lambda, profiles, and weights.

4.3 SMAA-III

ELECTRE III (Roy, 1978) is designed for solving a discrete ranking problem as it was defined for SMAA-2. Similarly to the other ELECTRE family methods, ELECTRE III is based on two phases. In the first phase, an outranking relation between pairs of alternatives is formed. The second phase consists of exploiting this relation, producing a final partial pre-order and a median pre-order.

The exploitation of the outranking relation consists of two phases. In the first phase, two complete pre-orders, Z_1 (descending) and Z_2 (ascending) are constructed with the so-called distillation procedures. In the second phase, a final partial pre-order or a complete median pre-order is computed based on these two pre-orders. In the original ELECTRE III, a median pre-order is computed based on the two complete pre-orders, Z_1 and Z_2, and the final partial pre-order.

In SMAA-III, the weights are represented as in the other SMAA methods. Imprecise thresholds are represented by stochastic functions $\alpha_j(\cdot)$, $\beta_j(\cdot)$, and $\gamma_j(\cdot)$, corresponding to the deterministic thresholds $p_j(\cdot)$, $q_j(\cdot)$, and $v_j(\cdot)$, respectively. To simplify the notation, we define a 3-tuple of thresholds $\tau = (\alpha, \beta, \gamma)$. It has a joint density function f_T in the space of possible values defining the functions. It should be noted that all feasible combinations of thresholds must satisfy $q_j(x_i) < p_j(x_i) < v_j(x_i)$.

Traditionally the thresholds in ELECTRE models have been used to model preferences of the DMs (e.g. differences deemed significant) as well as imprecision in the data. But it has been shown that the indifference threshold does not correspond to a linear imprecision interval (Lahdelma and Salminen, 2002). Therefore, in SMAA-III thresholds are used only to model preferences (together with weights). Imprecision in the criteria measurements is modelled with stochastic variables as in SMAA-2 (see Section 3.1).

Incomparabilities between alternatives can be present in the final results of ELECTRE III. This is one of the main features of ELECTRE methods in comparison with the methods applying classical multi-attribute utility theory (see Keeney and Raiffa, 1976). Incomparability is considered by the researchers and practitioners of ELECTRE methods as one of the strongest points of the methodology because it avoids to force comparison of very heterogenous alternatives. In the late seventies, it was considered a very important theoretical advance. But when dealing with practical situations, incomparabilities in the final result are sometimes inconvenient. This aspect was soon observed (Roy et al., 1986) and complete pre-orders and median pre-orders were proposed to be used in side of the partial pre-orders. SMAA-III applies median pre-orders in computing rank acceptability indices. The only information lost in using the median pre-order as the primary measure of ranking is the incomparability. It will be retained by representing it with another index.

Monte Carlo simulation is used in SMAA-III to compute three types of descriptive measures: rank acceptability indices, pair-wise winning indices, and incomparability indices. In order to compute these indices, let us define a *ranking function* that evaluates the rank r of the alternative x_i with the corresponding parameter values:

$$rank(i, w, \xi, \tau).$$

The evaluation of this function corresponds to executing ELECTRE III and returning rank of the corresponding alternative in the resulting median pre-order.

4.3.1 Rank acceptability index

The rank acceptability index, b_i^r, measures the share of feasible weights that grant alternative x_i rank r in the median pre-order by taking into account simultaneously imprecisions in all parameters and criterion evaluations. It represents the share of all feasible parameter combinations that make the alternative acceptable for a particular rank, and it is most conveniently expressed percentage-wise.

The rank acceptability index b_i^r is computed numerically as a multidimensional integral over the spaces of feasible parameter values as

$$b_i^r = \int_{W:rank(i,w,\xi,\tau)=r} f_W(w) \int_X f_X(\xi) \int_T f_T(\tau)\, dT\, dw\, d\xi.$$

The rank acceptability index has the same meaning as in SMAA-2.

4.3.2 Pair-wise winning index

The pair-wise winning index (Leskinen et al., 2006), o_{ik}, describes the share of weights that place alternative x_i on a better rank than alternative x_k. An alternative x_i that has $o_{ik} = 1$ for some k always obtains a better rank than alternative x_k, and can thus be said to *dominate* it.

The pair-wise winning index o_{ik} is computed numerically as a multidimensional integral over the space of weights that give alternative a lower rank than for another.

$$o_{ik} = \int_{w \in W:rank(i,w,\xi,\tau)<rank(k,w,\xi,\tau)} f_W(w) \int_X f_X(\xi) \int_T f_T(\tau)\, dT\, dw\, d\xi.$$

The pair-wise winning indices are especially useful when trying to distinguish between the ranking differences of two alternatives. Because the number of ranks in the median pre-order of different simulation runs varies, two alternatives might obtain similar rank acceptabilities although one is in fact inferior. In these cases looking at the pair-wise winning indices between this pair of alternatives can help to determine whether one of the alternatives is superior to the other or if they are equal in "goodness".

4.3.3 Incomparability index

Because median pre-orders are used in computing the rank acceptability indices, it is not anymore possible to model incomparability. As some DMs might be accustomed to make decisions also based on incomparabilities, another index is introduced. Incomparability index ρ_{ik} measures the share of feasible parameter values that cause alternatives x_i and x_k to be incomparable. For this reason, we define the incomparability function:

$$R(i,k,\xi,\tau) = \begin{cases} 1, & \text{if alternatives } x_i \text{ and } x_k \text{ are judged incomparable,} \\ 0, & \text{if not.} \end{cases}$$

This function corresponds to a run of ELECTRE III with the given parameter values and checking if the alternatives are judged incomparable in the final partial pre-order. In practice we do not compute the final partial pre-order, because this information can be extracted from the two partial pre-orders Z_1 and Z_2 as shown in [IV]. By using the incomparability function, the incomparability index is computed numerically as a multidimensional integral over the feasible parameter spaces as

$$\rho_{ik} = \int_W f_W(w) \int_X f_X(\xi) \int_T f_T(\tau) R(i,j,\xi,\tau)\, dT\, dw\, d\xi.$$

Chapter 5

Framework

We define now a SMAA framework to decide a method to choose on a specific decision making context. The first question to ask is whether we are dealing with a ranking or a sorting problem. If we are dealing with a sorting one, the only method of the SMAA family we can use is SMAA-TRI. With ranking problems, we have to choose the type of preference model we have: whether it is based on weights or on reference points. If we have a weight-based model, we have to choose the type of aggregation procedure: utility function or outranking method. With the reference point approach we use Ref-SMAA (see [V] or Lahdelma et al. (2005)). For utility function we use SMAA-2. With outranking model we can choose between SMAA-3 and SMAA-III. With all this information, we can choose whether to apply SMAA-2, SMAA-3, SMAA-III, or Ref-SMAA for the ranking problem. Depending on the method to apply, we obtain as output different descriptive measures that can be used to derive "second-order" aggregate measures. Choice of the method is presented as a decision-tree in Figure 5.1.

Other way to choose the method for a ranking problem is to question what kind of information is not available. Are the DMs willing to provide a shape for the utility function? If not, SMAA-2 can not be applied. Same type of questions can be posed with respect to other parameters of the methods in order to find out which method would be the most suitable.

In the context of this framework, we should notice that all other methods than Ref-SMAA, which is based on reference points, can be used with arbitrary weight information. This means that we can apply them with no preference information at all, as well as with mixed information of ordinal and cardinal types. In practice, the most useful ones are (partial) ordinal information and cardinal weight constraints. Complex weight constraints might be hard for the DMs to understand, and therefore by using more complex distributions the possibility for the information to contain uncertainty increases. If the DMs have problems understanding the underlying

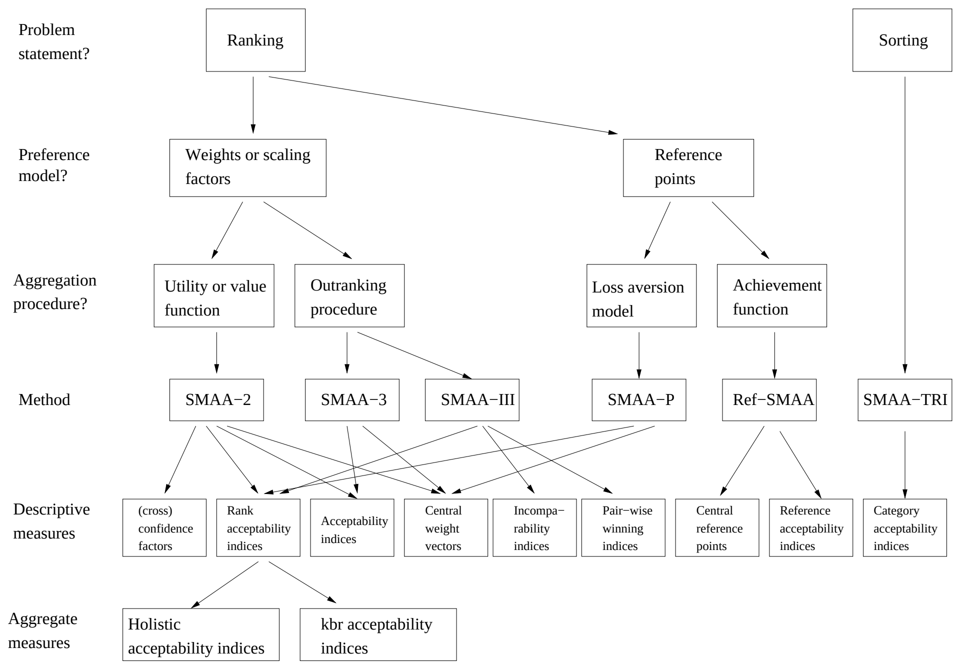

Figure 5.1: Decision-tree to choose the SMAA variant.

preference model, the achievement function based approach (Ref-SMAA) might be more suitable.

The shortcoming of the utility-function based approach (SMAA-2) is that the scaling has large effect on the results, and the meaning of the weights is based on the scale. Therefore, if the shape of the utility function is hard to define, it might be more suitable to use SMAA-3 or SMAA-III instead.

Arbitrarily distributed imprecise or uncertain criteria can be applied in all methods of the family except SMAA-3 that requires criteria measurements to have imprecision defined through thresholds. It should be noted that SMAA-O is not a stand-alone method, but rather a computational technique to handle ordinal criteria measurements. The possibility of using external sampling and the following generalisation to use SMAA with external methods can be considered a great advantage. For example, the approach applied in SMAA-TRI and SMAA-III can probably be applied to other methods as well, to use them with ignorance on the parameter values in order to analyze the stability of the results.

Chapter 6

Software

A user-friendly software is of crucial importance if an MCDM method is to enjoy a wide audience. A software was developed to allow users less accustomed in the field of numerical computation to use the new methods developed in this thesis. It was programmed in the C++ language and utilizes the gtkmm graphical user interface library (http://www.gtkmm.org) to be portable to various operating systems. Currently there exists versions for Linux, Mac OS X, and Windows XP.

The software implements SMAA-TRI and SMAA-III methods. It allows the SMAA-III model to be defined with uniform distributed, Gaussian distributed, or ordinal criteria. Ordinal criteria are not allowed for SMAA-TRI models. Ordinal criteria are modelled in SMAA-III through discrete rank values and setting indifference and preference thresholds to 0 and 1, respectively. Thresholds for cardinal criteria can have exact values or can be defined as intervals that can be absolute or a percentage of the criterion measurement in question. Criteria input screen is shown in Figure 6.1.

Criterion measurements and criteria uncertainties input screens are shown in Figures 6.2 and 6.3, respectively. The software allows to automatically set uncertainties to 5, 10, or 20 percentages of the corresponding measurement values. This allows an easy way to set up the model when using the method for an automated robustness or parameter stability analysis. The software allows three types of preferences: exact ones (expressed as exact weight values), upper and lower bounds for weights, and ordinal preferences (ranking of the criteria). The results are presented in a tabular form. While the software computes the various indices, the progress is shown interactively. Figure 6.4 presents an example of results from the model used in the case study of [III].

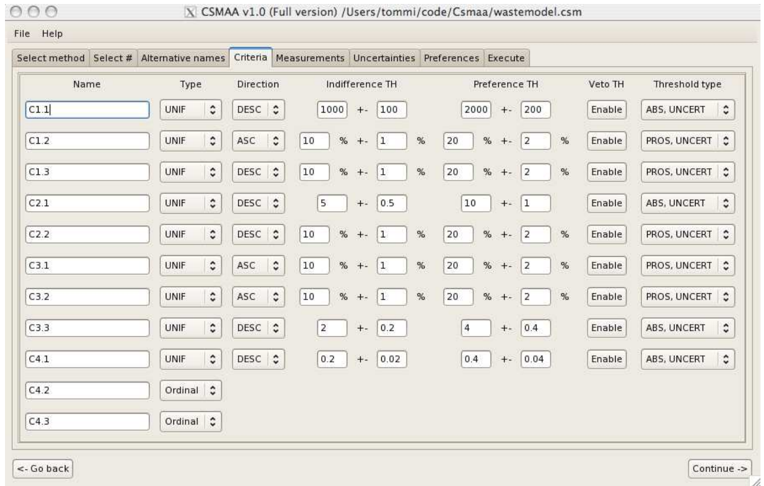

Figure 6.1: Criteria input screen in the software.

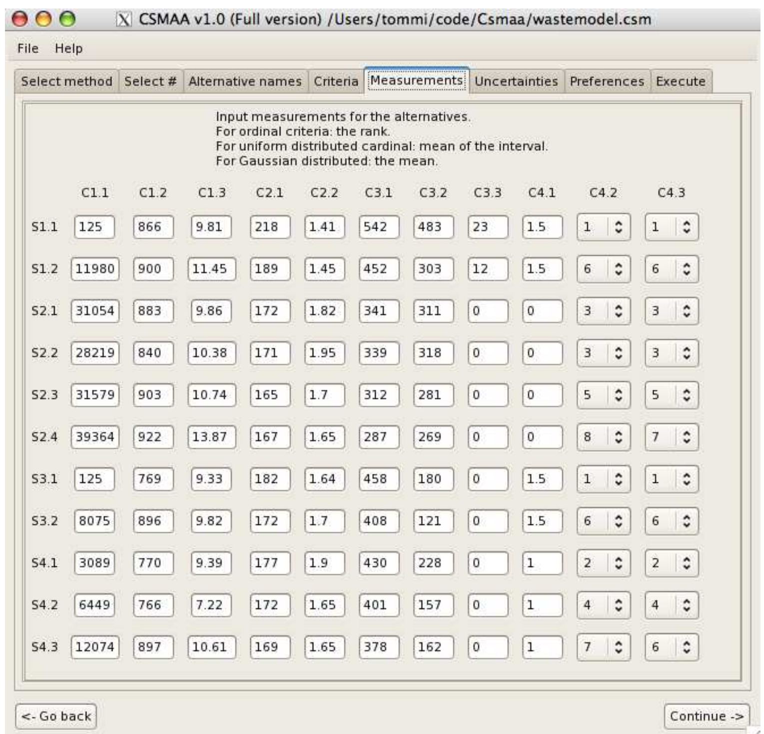

Figure 6.2: Criteria measurements input screen in the software.

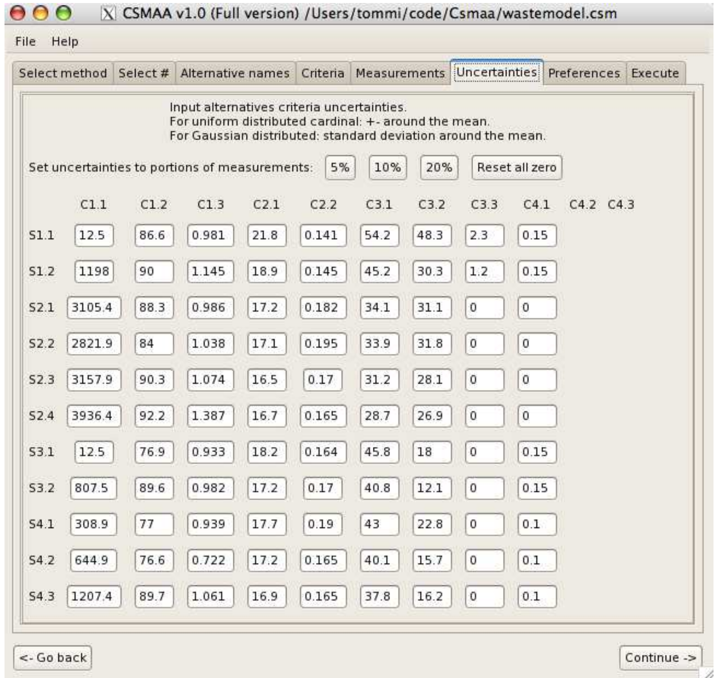

Figure 6.3: Criteria uncertainties input screen in the software.

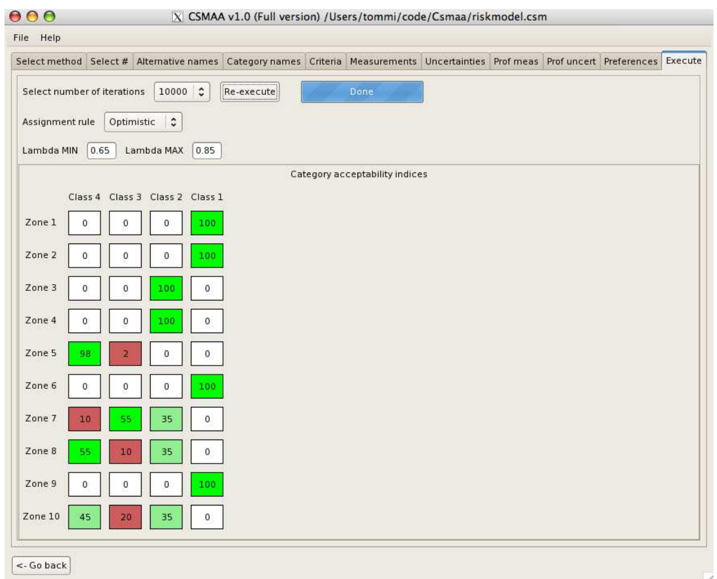

Figure 6.4: Results screen for SMAA-TRI in the software.

Chapter 7

Summary of publications

In publication [I], we present efficient methods for performing the SMAA computations. We analyze the complexity and assess the accuracy of the presented algorithms. We perform empirical efficiency tests as well. These tests show that our SMAA implementation is fast enough to analyze typical sized discrete problems interactively within seconds, if tight upper bounds for weights are not applied.

In publication [II], SMAA-2 is applied in elevator planning. This formulates a ranking problem, in which different elevator configurations are to be ranked with respect to both performance and non-performance criteria. We compare 10 feasible elevator group configurations for a 20-floor building. We evaluate the criteria related to the service level in different traffic situations using the KONE Building Traffic Simulator, and use analytical models and expert judgements for other criteria. The performance criteria are represented by a multivariate Gaussian distribution, others by deterministic values and ordinal information.

In publication [III], a new method, SMAA-TRI, is introduced. SMAA-TRI aims to analyze the stability of ELECTRE TRI results and to derive robust conclusions when SMAA-TRI is applied. SMAA-TRI allows ELECTRE TRI to be used with imprecise, arbitrarily distributed values for weights, profiles, and the lambda cutting level. The method computes for each alternative the share of parameter values that have it assigned to different categories. We illustrate application of SMAA-TRI by re-analyzing a case study in the field of risk assessment.

In publication [IV], we present a new method, SMAA-III. It allows ELECTRE III to be applied with imprecise parameter values. By allowing imprecise values, the method also allows an easily applicable robustness analysis. In SMAA-III, simulation is used and descriptive measures are computed to characterize stability of the results. We present a software implementing the methodology and illustrate its usage by re-analyzing an existing case study.

In publication [V], a complete survey of SMAA methodology is presented. Methods of this family allow solving MCDA problems of various types. Even though the methods have been applied in the past in various real-life decision-making situations, the structure of a unified SMAA framework has not been studied. This publication describes the methods of the family, and defines a unified SMAA framework. We also point out the key points in the methodology for future research.

Chapter 8

Concluding remarks

Decision support with Multiple Criteria Decision Making (MCDM) methods has become increasingly important for organizations of various sizes, because modern decision making situations often oblige Decision Makers (DMs) to consider several aspects of the problem and the trade-offs between them. Sometimes there are also multiple DMs whose opinions have to be taken into account. The problem settings often contain various types of uncertainties. Therefore methods that allow modelling of uncertainty in the parameters and possibly clashing preferences are needed.

Stochastic Multicriteria Acceptability Analysis (SMAA) family of methods include ways to handle various types of uncertainties and imprecision. Uncertain weights or other preference parameters can be used to model clashing or missing preferences. In this thesis, we showed how the SMAA approach can extend third-party MCDM methods to use imprecise parameters. This allows to perform an automated parameter stability analysis in addition to solving the two above-mentioned problems.

Although the possibility of defining uncertain parameters facilitates the elicitation process, the weight information should be consistent with the underlying preference model. For example, utility-theory based SMAA models should not use intervals for weights except for stability analysis. Imprecise trade-off ratios should be used instead for the weights to be consistent with the preference model. However, there does not yet exist an efficient weight generation technique for them. Future research should address this subject, and new efficient algorithms for generating weights with various types of constraints should be developed. In addition to being of importance for the SMAA methodology, they can be used with other Monte Carlo simulation applications within MCDM as well as in other disciplines.

We presented the basic SMAA method and its most important extensions. We also analyzed complexity of the algorithms and presented an application in the field of elevator planning. This application shows how the methodology can be used to solve problems traditionally beyond the scope of MCDM. Following this, we presented the two new SMAA methods: SMAA-TRI and SMAA-III. These extend ELECTRE TRI and ELECTRE III, respectively, to allow imprecise parameter values. For an MCDM method to enjoy widespread accentance, a user-friendly software is needed. Part of the work leading to this thesis composed of programming a software implementing the SMAA-III and SMAA-TRI methods. This software was briefly presented in this thesis. Free demo versions of the software can be obtained from the author.

The comprehensive decision making process as supported by SMAA or the more traditional decision making methods differs in many aspects. In the traditional methods, the model has to be defined with exact values straight in the beginning, and elicitation of the preference parameters from DMs is usually slow. In many cases these parameters do not change dramatically with time. On the contrary, SMAA models can be defined with no preference information, and the model iterated until sufficiently precise results are obtained. This can help for a more dynamic decision making process with more space for discussion. For example, in the context of multiple DMs, usually most of the preferred alternatives of different DMs obtain some first rank acceptability. This can stimulate further discussion for redefining the parameters more precisely and for finding good compromise alternatives.

Even though SMAA methods allow flexible decision making process, they should not be used in automated decision making. The results are always somewhat vague and need to be interpreted as such. This is an important difference between the SMAA model and many other MCDM models allowing imprecise values. Although the results are more imprecise than of other methods, they explicitly show the uncertainties present in the parameters. This can lower the possibility of accepting an "incorrect" model. This is somehow the main idea of SMAA philosophy – Monte Carlo simulation is used to bring visible the consequences implicated by the uncertain data, but inside SMAA are still the traditional MCDM methods.

The current state of research in SMAA methodology is quite young and the proposed new directions in this thesis are the initial steps in diversing the methodology. This thesis has tried to bring together the somewhat heterogenous parts of the SMAA methodology. Although being applied in various real-life cases, the theoretical basis needs to be defined firmer and the different SMAA methods bound together in a consistent way. Future research should concentrate in this direction instead of developing new methods to the family.

This thesis is comprised of smaller works in various areas, mainly because of having been completed in different universities under supervision of professors from different fields. Somehow this characterizes the whole field of MCDM: it is a synthesis of various disciplines, and for succesful research, we need economists to tell what is needed, mathematicians to provide the theoretical basis for it, as well as computer scientists for providing tools and methods to achieve the goals.

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