ALGORITHMS FOR COALITIONAL GAMES

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PREFACE

A number of people have contributed to this thesis. First of all, I would like to thank my supervisor Professor Hannu Salonen for the guidance and comments, which helped to shape the content and presentation of this thesis. Furthermore, I am grateful to him for the opportunity to study at the Department of Economics of Turku School of Economics, University of Turku. Second, I would like to thank the pre-examiners of the thesis, Dr. Ulle Endriss and Dr. Mitri Kitti, for their valuable comments and suggestions for improvements. I acknowledge the financial support of Yrjö Jahnsson Foundation and Turku School of Economics Supporting Fund. Finally, I would like to thank my parents and friends for their constant support and encouragement along the years. My greatest dept is to my husband for his insight and consultation.

Abstract

Coalitional games are studied extensively in economics and computer science. They have recently shown to be a useful formalism and are applicable to a wide range of domains from electronic commerce to trade and negotiation systems. Key topics in coalitional games include both deciding the stability of given coalitions and calculating representative or all stable coalitions. However, typically the number of possible outcomes in coalitional games grows exponentially in the number of players involved. Thus, usually finding stable or optimal outcomes among all possible outcomes of a coalitional game is a challenging combinatorial task. The main objective of this research is to develop practically efficient algorithms for coalitional games with a large number of players. In particular, the research in the thesis focuses on hedonic coalitional games. As the main results, the thesis introduces new methods for finding a core stable coalition structure, calculating the set of all Nash stable coalition structures, and checking the core membership of a given hedonic game solution. In addition, the thesis presents a new technique to maximize social welfare in coalitional games represented as characteristic function games.

Keywords: game theory, algorithms, coalitional games, hedonic games, core, Nash stability

Contents

1 INTRODUCTION

Various social, political and economical situations require coordination and cooperation of multiple agents. Agents of real and virtual societies may need to make joint decisions and act cooperatively although they may have conflicting preferences over alternative moves or actions. For example, these kinds of agents may be human beings, companies, political parties or even robots. One of the main challenges in the cooperation, coordination and joint decision making of the agents is to reach a stable situation from which no agent has an incentive to deviate.

Game theory (von Neumann and Morgenstern 2004, Osborne and Rubinstein 1994, Myerson 1991) provides a well-defined, general and exact framework for modeling interaction of agents. Coalitional games, as a well established branch of game theory, have recently received a lot of research attention. Perhaps a reason for the attention is that coalitional games can be used to model several important real-world situations where multiple agents coordinate their actions in order to reach their private or joint goals. Among the application areas of coalitional games are cartels, environmental agreements, research and development agreements, political activities and voting (Kahan and Rapoport 1984, Ray 2007). Also, important recent application areas of coalitional games are automated negotiation and electronic commerce (Conitzer and Sandholm 2006).

In coalitional games, one essential question is to determine which coalitions will be formed. Game theory provides a number of solution concepts to single out the stable outcomes of a given game. However, typically finding stable outcomes among all possible outcomes of a coalitional game is a challenging combinatorial task. The main difficulty is usually that the total number of possible outcomes of a game grows exponentially in the number of players in the game. Furthermore, the search execution times and computational resources to find stable solutions in many application areas might be limited. A solution that cannot be provided within strict deadlines may be even useless. As Papadimitriou puts it:

> If an equilibrium concept is not efficiently computable, much of its credibility as a predictor of the behavior of rational agents is lost [...]. (Papadimitriou 2007, p. 29)

In practice, a limit in the size (i. e. the number of players) of a coalitional game that can be solved, is given by the available computational resources and algorithmic tools. In case of coalitional games, there is a need for algorithms that can tackle the involved combinatorics in practice. A discipline concerned with the study of computational aspects of these issues is called algorithmic game theory (Nisan et al. 2007). Algorithmic game theory integrates many techniques, concepts, and ideas from game theory, economics, computer science and artificial intelligence. This thesis essentially falls in the scope of algorithmic game theory.

In this thesis, we examine techniques to compute solutions in coalitional games that are optimal or stable with respect to the preferences of each player. Essentially, this thesis aims to show that practically efficient computational tools can be devised for many stability concepts in coalitional games. The aim is thus to develop algorithms that perform well in practice. In addition, the aim of this thesis is to provide us with new information concerning coalitional games via extensive computer simulations and numerical experimentation. To reach these objectives two lines of research have been carried out. First, we have studied coalitional games and related solution concepts to gain an insight to the structural and computational properties of coalitional games. Second, we have devised effective algorithms and studied their applicability to coalitional game settings.

In particular, the research focuses on hedonic coalitional games; the thesis introduces new methods for finding core stable coalition structures, finding the set of Nash stable coalition structures, and testing the core membership of given game solution. Furthermore, a new method to maximize social welfare in characteristic function games is devised.

A brief description of the contents of the publications in this thesis is given in Section 2. Section 3 contains a survey of basic concepts of coalitional games with an emphasis on characteristic function games and hedonic coalitional games. Section 4 surveys central techniques that can be used in order to find stable or optimal outcomes in coalitional games. Section 5 concludes.

2 STRUCTURE OF THE THESIS

This thesis consists of the following four publications and this summary. In the following Sections 3 and 4, the context and related work of each publication is given.

P1 Helena Keinänen, **Simulated annealing for multi-agent coalition formation.** In Anne Håkansson, Ngoc Thanh Nguyen, Ronald L. Hartung, Robert J. Howlett and Lakhmi C. Jain, editors, Agent and Multi-Agent Systems: Technologies and Applications, Third KES International Symposium, KES-AMSTA 2009, Uppsala, Sweden, Proceedings, volume 5559 of Lecture Notes in Computer Science, pp. 30–39, Springer, 2009.

P2 Helena Keinänen, **Stochastic local search for core membership checking in hedonic games.** Transactions on Computational Collective Intelligence, volume 6220 of Lecture Notes in Computer Science, pp. 56–70, Springer, 2010.

P3 Helena Keinänen, **An algorithm for generating Nash stable coalition structures in hedonic games.** In Sebastian Link and Henry Prade, editors, Foundations of Information and Knowledge Systems, 6th International Symposium, FoIKS 2010, Sofia, Bulgaria, Proceedings, volume 5956 of Lecture Notes in Computer Science, pp. 25–39, Springer, 2010.

P4 Helena Keinänen, **Core non-emptiness checking for hedonic games via difference logic.** In Piotr Jedrzejowicz, Ngoc Thanh Nguyen, Robert J. Howlett and Lakhmi C. Jain, editors, Agent and Multi-Agent Systems: Technologies and Applications, 4th KES International Symposium, KES-AMSTA 2010, Gdynia, Poland, Proceedings, volume 6070 of Lecture Notes in Computer Science, pp. 331-340, Springer, 2010.

2.1 Summary of the Publications in the Thesis

The contents of the publications included in the thesis are summarized below.

P1 A simulated annealing algorithm is devised that finds within short runtimes nearly optimal coalition structures. It is demonstrated experimentally that a stochastic local search algorithm together with a specific neighbourhood function may find near optimal solutions to coalition formation problem in characteristic function games.

P2 Two algorithms to the check core membership in hedonic games are presented. Both of them are stochastic local search algorithms but are based on different heuristics. A motivation for this work is, e.g., that in many cases it is of interest to find out whether a given welfare maximizing coalition structure belongs to the core. Through experiments, it is shown that the proposed algorithms are practically efficient on large hedonic games. The experiments show that the core membership can be easily checked on multi-agent societies involving agents up to 5000. No other algorithms have been presented in the literature to tackle the problem.

P3 An algorithm is introduced which generates all Nash stable coalition structures of a hedonic game with additively separable preferences. In particular, the presented algorithm is useful in order to compare properties of different Nash stable coalition structures. The algorithm is based on a new theorem concerning hedonic games, which effectively allows to detect unstable coalitions, and thus the algorithm avoids unnecessary calculation on unstable coalition structures. It is shown through experiments that the algorithm can often generate all Nash stable coalition structures by checking only a very small portion (up to 0.0012%) of all possible solutions of a given hedonic game.

P4 Based on difference logic, a technique to solve the core non-emptiness checking problem in hedonic games, and to generate a representative core member (if such exists) is presented. It is shown through experiments that the problems can effectively be solved by combining compact problem encodings in difference logic and algorithmics for difference logic satisfiability. It turns out that hedonic games with a large number of players and a very large number of underlying coalition structures can be easily solved with the proposed technique.

2.2 Contributions of the Author

The author of the thesis is the sole author of all Publications P1, P2, P3, and P4 included in the thesis. Publication P1 extends results presented previously in (Keinänen and Keinänen 2008) where the author of the thesis is responsible for everything except implementation of the presented algorithm.

3 COALITIONAL GAMES

In this section, we survey some basic concepts concerning coalitional games. The aim is to present a formal framework for the algorithms in Publications P1, P2, P3 and P4. For a complete introduction to game theory the books of Osborne and Rubinstein (Osborne and Rubinstein 1994) and Myerson (Myerson 1991)) provide a good reference.

In general, coalitional games provide a theoretical foundation for the study of cooperation. Coalitional games are introduced by von Neumann and Morgenstern in (von Neumann and Morgenstern 2004, p. 583–584) where they use coalitional games to model markets. In the beginning, coalitional games gained interest slowly (Tucker and Luce 1959, p. 1) but the situation has changed drastically during the past two decades. Coalitional games have became an extensively studied area of game theory. Besides of economics, coalitional games and coalition formation have been popular research topics in political sciences and in artificial intelligence including multi-agent systems research.

In coalitional games, the players may act together and form coalitions in order to accomplish a joint task but they may also act together to achieve their private goals. In the context of coalitional games, a basic question is to determine which coalition structures are stable in the sense that no player or coalition has an incentive to deviate. The question is of great importance because, under certain assumptions, the stable coalition structures are those that will be formed in the course of the cooperation.

Characteristic function games (Section 3.2) and hedonic coalitional games (Section 3.3) together with the solution concepts for coalitional games (Section 3.4) and their computational complexity results (Section 3.5) provide a conceptual framework for the algorithmics presented in this thesis.

3.1 Players, Coalitions and Coalition Structures

The main elements in coalition formation are players (also referred to as agents). In the game theoretical literature (see e.g., Myerson 1991, pp. 2–5 and Luce and Raiffa 1957, p. 5) it is usually assumed that the players are rational in the sense that they aim to maximize their expected utility. A player might also aim to maximize the utility of her team instead of her own private utility. Furthermore, it is generally assumed in the literature that the players are intelligent meaning that they have some information and knowledge about the game, and that players can make correct inferences concerning the game.

In Publications P2, P3 and P4, where we concentrate on the stability of coalition structures, we assume that players are rational, intelligent and maximize their utility. However, we note that the emphasis of these publications is not on the processes of how optimal or stable coalition structures are formed. Instead, the emphasis is on the question how can we find effectively optimal or stable coalition structures among all possible coalition structures.

We now define the coalitional games formally. Let $N = \{1,2,\ldots,n\}$ be a set of players. A coalition S is a non-empty subset of the set players $S \subseteq N$. A coalition structure CS over N is a partition of N into mutually disjoint coalitions, that is, for all coalitions $S, S' \in CS$ we have $S \cap S' = \emptyset$ and $\bigcup_{S \in CS} = N$. The collection of all coalition structures over N is denoted by $\mathscr{C}(N)$. If a coalition S contains all players in N (i.e. $S = N$), then S is called the grand coalition.

Example 1 *Consider a set of players* $N = \{1,2,3\}$. *All possible coalitions are* $\{1\}$, $\{2\}$, $\{3\}$, $\{1,2\}$, $\{1,3\}$, $\{2,3\}$, $\{1,2,3\}$. *The coalition* $\{1,2,3\}$ *is the grand coalition. All possible coalition structures are contained in the set* $\mathscr{C}(N)$*, and are the following:* $\{\{1\},\{2\},\{3\}\}$, $\{\{1,2\},\{3\}\}$, $\{\{1,3\},\{2\}\}$, $\{\{1\},\{2,3\}\}$ *and* $\{\{1,2,3\}\}$.

Coalition structures can be represented as a coalition structure graph (Sandholm et al. 1999, Larson and Sandholm 2000). Following the usual graph-theoretic notations and definitions (see, e.g., Jungnickel 2008), we formally define the coalition structure graph as follows. A coalition structure graph Γ is a pair $\Gamma = (V,E)$, where V is the finite set of nodes and $E \subseteq V \times V$ is the set of edges. Nodes are coalition structures such that $V = \mathscr{C}(N)$. In (Sandholm et al. 1999, Larson and Sandholm 2000), the edges are defined as follows. For all $CS, CS' \in \mathscr{C}(N)$: if CS' can be obtained from CS by merging two coalitions of CS or by splitting some coalition of CS into two disjoint coalitions, then there is an edge $e \in E$ such that $e = (CS, CS')$. Figure 1 shows a coalition structure graph with four players.

The graph representation of the partitions of the set N is used in several algorithms for coalition formation problems (see, e.g., Dang and Jennings 2004, Larson and Sandholm 2000, Sandholm et al. 1999). Also, it provides us with the search space for the algorithms reported in P1 and P3. In addition, in P1 we study alternative coalition structure graphs based on other kinds of edge relations.

The number of coalitions is exponential in the number of players. With n players there are $2^n - 1$ coalitions. The number of coalition structures that can be obtained by partitioning the set N into non-empty subsets corresponds to the Bell number $B(n)$ defined as

$$B(n) = \sum_{k=1}^{n} Z(n,k)$$

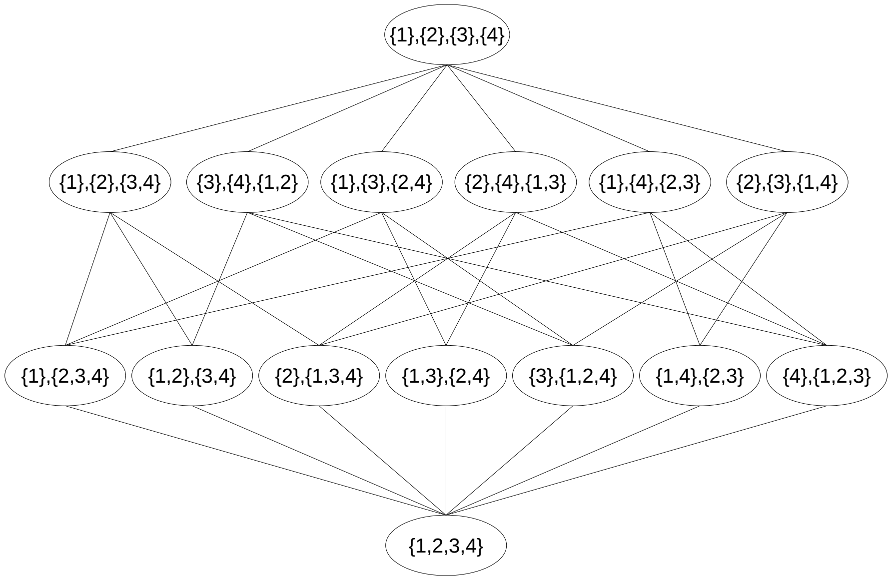

Figure 1: A coalition structure graph with four players.

where $Z(n,k) = kZ(n-1,k) + Z(n-1,k-1)$ is a Stirling number of the second kind. The Stirling number of the second kind provides us with the number of coalition structures where the players are partitioned into k, $k \leq n$, coalitions.

Therefore, the number of coalition structures grows quickly when the number of players is increased. For instance, already with nine players $|N| = 9$ we have 21 147 coalition structures and with $|N| = 15$ players the number of coalition structures is 1 382 952 545.

3.2 Characteristic Function Games

A characteristic function game (CFG) is a tuple $\langle N, v \rangle$ where N is a set of players and $v : 2^N \rightarrow \mathbb{R}$ is a characteristic function assigning a real value to each coalition S. The characteristic function can be interpreted as associating a worth to each coalition S. The worth may represent an amount of utility the players of the coalition S receive collectively from cooperating. Characteristic function can be seen as describing the possibilities of cooperation (Myerson 1991, p. 422). The sum, $V(CS) = \sum_{S \in CS} v(S)$, of the worths of the coalitions included in the coalition structure, i.e. the value of a coalition structure, is called social welfare. In P1 we study CFGs.

Example 2 *As an example of characteristic function games consider the following game, where $N = \{1,2,3\}$ and the characteristic function is $v(\{1,2,3\}) = 7$, $v(\{1,2\}) = v(\{1,3\}) = v(\{2,3\}) = 6$ and $v(\{1\}) = v(\{2\}) = v(\{3\}) = 2$. In this example the coalition structure which yields the maximal social welfare is any coalition structure which contains a coalition of two players and a singleton coalition.*

As usual in the literature (Sandholm et al. 1999, Larson and Sandholm 2000), we assume that that the values of the coalitions are non-negative, that is $v(S) \geq 0$.

If $v(S \cup S') \geq v(S) + v(S')$ for all $S \cap S' = \emptyset$ then the game is called super-additive. If $v(S \cup S') \leq v(S) + v(S')$ for all $S \cap S' = \emptyset$ then the game is called sub-additive. Finding a coalition structure yielding a maximal social welfare is trivial in super- or sub-additive CFGs. In the first case, the maximal social welfare is provided by the grand coalition and in the latter case by the coalition structure consisting of singletons. In this thesis, in particular in Publication P1 no assumptions concerning the super- or sub-additivity of the CFG are made.

3.3 Hedonic Coalitional Games

Another coalitional game studied in this thesis is called a hedonic coalitional game (or simply a hedonic game). Hedonic games are coalitional games where players express preferences only on the coalitions they may belong to. Thus, the players' preferences concerning coalition structures depend solely on the members of their coalitions. These games provide a particularly suitable framework to model collaboration among self-interested players.

Originally, hedonic games have been studied in the context of economics. The initial research on hedonic games (Drèze and Greenberg 1980, Banerjee et al. 2001, Bogomolnaia and Jackson 2002, Alcalde and Romero-Medina 2006) focuses mainly on defining the formal model for hedonic games as well as on defining stability concepts in hedonic settings. In addition to the initial work on hedonic games, there are several seminal examples in the literature which can be seen as special cases of hedonic games. These examples include stable matching problems such as the house allocation problem (Shapley and Scarf 1974), the kidney exchange problem (Roth et al. 2004, Saidman et al. 2006 and Abraham et al. 2007) and the stable marriage problem (Gale and Shapley 1962, Knuth 1976, Gusfield and Irving 1989). More recently, hedonic games are studied in the computer science, multi-agent systems and artificial intelligence research area (Sung and Dimitrov 2007, Sung and Dimitrov 2009). The research has concentrated on computational complexity issues in hedonic games. Also, questions concerning the representation of the hedonic preferences have received some attention (Cechlárová and Hajduková 2002, Cechlárová and Hajduková 2004, Elkind and Wooldridge 2009).

We now give a definition of the hedonic games. Let $\mathscr{A}^i$ be the set of coalitions containing player i. A hedonic game G is a pair $\langle N, (\succeq_1, \succeq_2, \dots \succeq_n)\rangle$ where N is finite set of players and tuple $(\succeq_1, \succeq_2, \dots \succeq_n)$ denotes, for each player $i \in N$, a complete, reflexive and transitive preference relation $\succeq_i$ of player i over $\mathscr{A}^i$. Note that a player's preferences concerning the coalitions structures are completely determined by the player's preferences concerning the coalitions she belongs to.

An important issue with hedonic games is the representation of the preferences. Roughly, the players' preferences can be represented such that the representation is either succinct or fully expressive. The issue is of great importance since the representation of the preferences affects to the computational complexity of the solution concepts for hedonic games (see Sections 3.4 and 3.5). The two main lines of representing hedonic preferences are as follows:

1. We can express the hedonic preferences in a way that the representation is fully expressive but not necessarily always succinct. As an example we have the individually rational coalition lists representation (IRCLs) (Ballester 2004). Hedonic nets (Elkind and Wooldridge 2009) provide an alternative, fully expressive representation scheme for preferences in hedonic games.

2. We can represent the hedonic preferences with a succinct representation scheme. This can be done by representing the preferences as additively separable preferences (Banerjee et al. 2001, Bogomolnaia and Jackson 2002, Sung and Dimitrov 2007). This representation requires only a space quadratic in the number of agents. Another concise way to represent hedonic preferences is based on B- and W-preferences introduced in (Cechlárová and Hajduková 2003, Cechlárová and Hajduková 2004).

Both of the approaches have their drawbacks which we consider in turn.

Hedonic nets represent hedonic preferences as a set of rules. The representation is fully expressive but in the worst case the number of rules is exponential in the number of players. IRCLs provide a fully expressive representation for the hedonic preferences. The main idea of an IRCL is that a player lists in the preference order all those coalitions she finds better or equally attractive as staying alone. Thus, the last coalition on the IRCL of player i is the singleton coalition $S = \{i\}$.

Example 3 *As an example consider an instance of the hedonic game G where* $N = \{1,2,3,4\}$ *and preferences are given in the form of IRCLs as follows:*

$$\begin{aligned}
1:&\ \{1,3,4\} \succ_1 \{1,2,3\} \succ_1 \{1,3\} \succeq_1 \{1\}\\
2:&\ \{1,2,3,4\} \succ_2 \{1,2,3\} \succeq_2 \{1,2,4\} \succ_2 \{2,3,4\} \succ_2 \{1,2\} \succ_2 \{2,3\} \succeq_2\\
&\ \{2,4\} \succ_2 \{2\}\\
3:&\ \{2,3\} \succ_3 \{3,4\} \succ_3 \{1,3\} \succ_3 \{1,2,3,4\} \succ_3 \{3\}\\
4:&\ \{1,2,4\} \succeq_4 \{1,3,4\} \succeq_4 \{2,3,4\} \succ_4 \{1,4\} \succeq_4 \{3,4\} \succeq_2 \{2,4\} \succ_2 \{4\}
\end{aligned}$$

The main difficulty with the approach is that in the worst case the lengths of the preference lists can be exponential in the number of players as it is the case with player 2 in Example 3.

Additively separable preferences provide a concise $|N| \times |N|$ representation of hedonic preferences. Additively separable preferences can be given as a matrix, where each player gives herself a value of 0 and a real value for all other players.

Example 4 *Consider a hedonic game G where* $N = \{1,2,3,4\}$ *and generic, additively separable preferences defined in the below matrix (for all* $i, j \in N$ *the value* $v_i(j)$ *is shown in line i and column j):*

0	3	0	2
1	0	1	1
4	5	0	-6
6	-1	-5	0

The first two rows of the matrix in Example 4 correspond to preferences of players 1 and 2 from the previous example (Example 3). As it can be seen, especially the preferences of player 2 can be expressed compactly using additively separable preferences. However, a drawback of the approach is that not all preference orders can be captured by such a succinct representation scheme. For example, the preferences of player 3 from Example 3 cannot be expressed with additively separable preferences. This difficulty is also with so-called B- and W-preferences, where the values the coalitions are determined by the best and worst members of the coalition. The intuitive idea of W-preferences is that the preferences of a coalition are derived from the worst member of the coalition. Similarly, B-preferences are in a sense dual notion such that the preferences of a coalition are derived from the best member of the coalition.

In this thesis, we use both additively separable preference representation (Publications P2 and P3) and IRCL representation (Publication P4) of the hedonic preferences.

3.4 Solution Concepts

Game theory provides a number of solution concepts to single out the outcomes of a coalitional game. For the definitions of the core, Nash stability and other solution concepts for coalitional as well as for the assumption of individual rationality, the reader is referred to (Osborne and Rubinstein 1994, Myerson 1991, Bogomolnaia and Jackson 2002, Banerjee et al. 2001, Kahan and Rapoport 1984). Here, these concepts are introduced informally in turn.

One of the most important solution concepts for coalitional games is the core. The concept of core relies essentially on the notion of player's preferences. Informally, players' may give their preferences by assigning values or giving orderings on other players or coalitions. Given players' preferences, intuitively the core of a coalitional game consists of all coalition structures where no set of players would prefer deviating from their coalitions. Thus, the core is a solution concept that considers coalitional deviation. A coalition where all its members prefer to deviate from their coalitions in a coalition structure is called a blocking coalition. If the game admits no blocking coalition for a coalition structure, then the coalition structure belongs to the core of the game. The concept of core is introduced by Gillies (Tucker and Luce 1959, pp. 47–85) and it has been extensively studied in the game-theoretic literature (Osborne and Rubinstein 1994, Myerson 1991). It is known (Banerjee et al. 2001, Bogomolnaia and Jackson 2002) that the core of a hedonic coalitional game may be empty or, in a special case, contain all possible coalition structures of the game. Actually, it is noted (Kahan and Rapoport 1984, p. 67) that a coalitional game with an empty core requires a social mechanism to resolve conflicts and sharing of the benefits. Otherwise, there may be an endless deviation from any proposed outcome. A technique for the core non-emptiness checking of a given hedonic game is presented in P4. Furthermore, P2 introduces two algorithms for the core membership checking of a given coalition structure in a given hedonic game.

There is another important solution concept for coalitional games and in particular for hedonic games, called Nash stability. It specifies the set of coalition structures, where no player finds it beneficial to deviate given that all other players stay in their coalitions. In contrast to the core, Nash stability deals only with individual deviations. An algorithm which generates the set of Nash stable coalition structures in hedonic games is presented in P3.

An important assumption behind the solution concepts of Nash stability and the core in hedonic games is the individual rationality. In an individually rational coalition structure each player prefers or finds equally good her coalition in the coalition structure to the alternative to stay alone in a singleton coalition. Thus, individual rationality is implied by both the core and Nash stability.

Nash and core stable coalition structures are also individually rational coalition structures (Bogomolnaia and Jackson 2002, p. 208). However, the existence of a core stable coalition structure or a Nash stable coalition structure does not depend on the existence of the other (Bogomolnaia and Jackson 2002, p. 208).

3.5 Computational Complexity Results for Coalitional Games

Computational complexity theory deals with time and space requirements of algorithms. The most important computational complexity classes for this thesis are NP and coNP. The complexity class NP is defined as the set of decision problems which can be solved using a non-deterministic polynomial time Turing machine. The complexity class coNP is the set of decision problems whose complements are in the complexity class NP. Furthermore, the complexity class P is the set of problems which can be decided using a deterministic polynomial time Turing machine. For formal definitions of these complexity classes and for an overview of computational complexity theory, the reader is referred to (Papadimitriou 1994).

A decision problem is called NP-hard, if it is known to be at least as computationally hard to solve as any other problem belonging to the class NP. The NP-hardness of a decision problem is usually proved by giving a polynomial time reduction from another decision problem which is already known to be NP-hard. The decision problems which are both known to be NP-hard and belong to the class NP are called NP-complete problems. No algorithm has been shown to exist which can solve an NP-complete problem by using only polynomial time in the size of the problem instance. The computational complexities of many problems related to coalitional games are known. Next, we will survey them briefly. The main complexity results for hedonic games are summarized in Table 1.

Deciding whether there exists a core stable coalition structure in a hedonic game is known to be NP-hard, if the preferences are given as additively separable (Sung and Dimitrov 2009). Furthermore, deciding whether there exists a core stable coalition structure for a hedonic game (with preferences given as IRCLs) is known to be NP-complete (Ballester 2004). The computational complexity of core non-emptiness checking problem for hedonic nets is studied in (Elkind and Wooldridge 2009), where it is shown that the problem is even harder than NP-complete. The problem of deciding whether a given coalition structure is core stable in a hedonic game with additively separable preferences is coNP-complete (Sung and Dimitrov 2007). However, the problem of deciding whether a given coalition structure is core stable with preferences represented as IRCLs is known to be in the complexity class P (Ballester 2004).

Deciding whether there exists a Nash stable coalition structure in a hedonic

Table 1: Computational complexities of some decision problems in hedonic games.

Decision problem	Preference representation	Complexity result
Is CS core stable?	IRCLs	In P (Ballester 2004)
Is CS core stable?	Additively separable	coNP-complete (Sung and Dimitrov 2007)
Is the core non-empty?	IRCLs	NP-complete (Ballester 2004)
Is the core non-empty?	Additively separable	NP-hard (Sung and Dimitrov 2009)
Is CS Nash stable?	IRCLs	Trivially in P
Is CS Nash stable?	Additively separable	Trivially in P
Is the Nash Stable set non-empty?	IRCLs	NP-complete (Ballester 2004)
Is the Nash Stable set non-empty?	Additively separable	NP-complete (Olsen 2009)

game with IRCLs is NP-complete (Ballester 2004). Also, deciding whether a Nash stable coalition structure exists in a hedonic game with additively separable preferences is NP-complete (Olsen 2009). The problem of deciding whether a given coalition structure is Nash stable in additively separable hedonic games is solvable with a polynomial algorithm.

Finally, the problem of calculating a coalition structure with an optimal social welfare in a CFGs is NP-complete (Sandholm et al. 1999). Since the complexity theory is defined for decision problems, this result actually is given for a corresponding decision variant of the problem (given the values of coalitions and a positive number k, does there exist a coalition structure with a value of at least k).

4 ALGORITHMS FOR COALITIONAL GAMES

Typically, combinatorial search problems have a large number of candidate solutions. These candidate solutions are evaluated with respect to an objective function. In case of decision problems those candidate solutions satisfying the objective function are considered as feasible solutions. In the case of optimization problems the aim is to find solutions with as optimal values of the objective function as possible. For the definitions of different types of decision and optimization problems, see (Kreher and Stinson 1998) and (Hoos and Stützle 2005).

The combinatorial search problems that arise in coalitional games also have a large number of candidate solutions. Mostly, the set of candidate solution of combinatorial problems in coalitional games consists of the set of all coalition structures. As noted earlier, the cardinality of this set grows exponentially with the number of players.

In this section we concentrate on techniques and algorithms presented in the literature to find feasible and optimal solutions for CFGs and hedonic games. Roughly, the existing algorithms can be divided into deterministic algorithms (Section 4.1) and into stochastic search algorithms (Section 4.2). The deterministic algorithms guarantee the optimal solution but may have large time and memory requirements. The stochastic search algorithms, in contrast, are usually more efficient in their time and space consumption but they are incomplete. Furthermore, this section concentrates on an approach which represents the coalitional games as a set of rules or formulas and thus makes possibles to employ existing NP-solvers in coalitional games (Section 4.3).

4.1 Deterministic Algorithms for Coalitional Games

Generating an optimal coalition structure in CFGs has been an extensively studied research problem in multi-agent systems. During the past ten years, in addition to the stochastic algorithms in (Shehory and Kraus 1998) and (Sen and Dutta 2000), a number of deterministic algorithms have been proposed in the literature. Roughly these algorithms can be divided into algorithms based on dynamic programming and on algorithms having an anytime feature. Let us explain both of these approaches.

The earliest algorithms to generate optimal coalition structures in CFGs were given as algorithms for complete set partitioning problem (Yeh 1986) and for solving the winner determination problem in combinatorial combinatorial auctions (Rothkopf et al. 1998). Both of these algorithms are based on dynamic programming and they are directly applicable to the optimal coalition structure generation problem, too.

In coalition formation settings, a dynamic programming algorithm that determines the optimal coalition structure by detecting the values of all coalitions was introduced in (Sandholm et al. 1999, p. 224). More recently, an improved version of the algorithm is introduced in (Rahwan and Jennings 2008a). The worst case complexity of the algorithms is $O(3^n)$. A drawback of the approach is large memory consumption which makes the approach practically useless for large games. Furthermore, the algorithms do not provide any solutions (not even suboptimal) before the whole search spaces are explored.

The coalition structure graph (see Fig. 1) serves as a starting point for several algorithms for coalitional games. Classical techniques that examine the graph systematically include a breath-first search (BFS) algorithm and a depth-first search (DFS) algorithm. The BFS algorithm is used as a basis in this thesis as well. The BFS algorithm traverses a directed graph and examines all its nodes in the order of their distance from the starting node. Since the BFS exhaustively searches through all the nodes of the graph, it cannot be used, as is, to solve combinatorial search problems in coalitional games involving several players. As noted in Sect. 3.1 the total number of coalition structures is often too large to allow an exhaustive search that checks through every coalition structure in order to find the optimal one. However, the basic BFS together with suitable search heuristics has proved to be an useful way to find optimal or stable coalition structures in coalitional games, as indicated in the results of (Sandholm et al. 1999, Dang and Jennings 2004) and Publication P3.

In order to be able to provide solutions that are within a bound from optimum, an any-time algorithm for CFGs is introduced in (Sandholm et al. 1999). A feature of any-time algorithms is that the search process can be stopped at any point of time and the algorithm provides us with the best value found so far. The advantage of the algorithm in (Sandholm et al. 1999, p. 218) is that it generates solutions that are guaranteed to be within a certain bound from optimum. The algorithm uses the BFS on the coalition structure graph (Fig. 1). The search is continued until either the search time ends or all the coalition structures of the graph has been visited. In order to guarantee that the optimal coalition structure of a given CFG has been seen, the algorithm in (Sandholm et al. 1999) needs to check through all coalition structures. Further variants of the algorithm in (Sandholm et al. 1999) are developed in (Larson and Sandholm 2000). A drawback of all these algorithms in (Sandholm et al. 1999, Larson and Sandholm 2000) is their memory consumption which is $O(n^n)$ with n players in a CFG. Thus, these algorithms can only be used for relatively small CFGs.

The algorithms in (Sandholm et al. 1999, Larson and Sandholm 2000) are revised and improved in (Dang and Jennings 2004). It is demonstrated in (Dang and Jennings 2004) that the revised approach improves the performance of the original anytime algorithm in (Sandholm et al. 1999, Larson and Sandholm 2000).

A recent algorithm for optimal coalition structure generation in CFGs is proposed in (Rahwan et al. 2009). The algorithm is based on a widely used technique of integer linear programming (see e.g., Jungnickel 2008) where the aim is to maximize a linear objective function with respect to linear constraints together with a condition that the variables are integers. The main idea of the algorithm (Rahwan et al. 2009) is to use a search space of partitions that are based on sub-spaces determined by the cardinalities of the coalitions in the coalition structures. The sub-spaces are detected independently and for every sub-space an integer linear optimization problem (maximize the social welfare value of coalition structure) is solved. Solving the maximization problem of a sub-space provides upper and lower bounds which may be used to prune other sub-spaces. As with anytime-algorithms, the worst-case complexity of the algorithm in (Rahwan et al. 2009) is $O(n^n)$. Also, in (Rahwan and Jennings 2008b), the linear integer programming and dynamic programming approaches have been combined by extending the algorithm in (Rahwan et al. 2009) with techniques from (Rahwan and Jennings 2008a).

4.2 Stochastic Local Search Techniques for Coalitional Games

Stochastic local search (SLS) algorithms (Hoos and Stützle 2005) are widely used techniques for finding high quality solutions to various combinatorial problems. They have been successfully used to solve many decision and optimization problems. However, previously the SLS algorithms have been applied only in (Shehory and Kraus 1998, Sen and Dutta 2000) to solve problems related to coalitional games.

A drawback of the SLS techniques is that they do not guarantee optimal solutions. Indeed, the SLS algorithms are incomplete in the sense that they search usually through only a part of the search space. However, the experimental results in this thesis indicate that, in practice, with the SLS methods one can find nearly optimal (or sometimes even optimal) solutions within relatively short runtimes.

The SLS methods seek to find the solutions from a search space which consists of a finite set of candidate solutions. When applied to coalitional games, the search space of the SLS may consist of the set of all coalition structures (as in Publication P1) or of the set of all coalitions (as in Publication P2). Selecting a proper neighbourhood relation is crucial for the performance of the SLS algorithms. A neighbourhood relation used in this thesis is based on 1-player exchange. In such a neighbourhood two candidate solutions are neighbours if and only if they differ in at most 1 player. This neighbourhood relation is used in both Publications P1 and P2. In addition to 1-exchange neighbourhood, we use in P1 a split/merge neighbourhood relation. In a split/merge neighbourhood two candidate solutions are neighbours if and only if one solution can be obtained from the other by splitting (merging) a coalition into two coalitions (merging two coalitions into one) in the original coalition structure. We notice that other appropriate neighbourhoods could be used as well, for example a k-exchange neighbourhood where two candidate solutions are neighbours if and only if they differ in exactly k players.

An iterative improvement algorithm (also known as a hill-climbing algorithm) is a basic SLS algorithm. For the details of the algorithm see e.g., (Kreher and Stinson 1998, pp. 157–158) and (Hoos and Stützle 2005, pp. 47, 61–75). The iterative improvement algorithm starts from either a random candidate solution or from a given candidate solution and aims to improve the value of an objective function. The improving candidate solution is searched among the neighbours of the current candidate solution. There are several ways to choose the improving neighbour solution. Firstly, we may choose the neighbour yielding the largest improvement in the value of the objective function. Such a greedy algorithm for coalition formation and task allocation problems is proposed in (Shehory and Kraus 1998). Secondly, we may choose the first neighbour which improves the value of the objective function. In Publication P2 we propose an algorithm based on the iterative improvement using both of these neighbour selection techniques.

A drawback of the iterative improvement algorithm is that it easily gets stuck in local optima. In other words, for a candidate solution there might not be an improving neighbour. Then, the algorithm is not anymore able to provide improving values of the objective function and search terminates although the value of the objective function might be far from the global optimal value.

To escape the local optimal solutions a number of stochastic heuristics have been developed, among them a widely used heuristic of the Metropolis search (Metropolis et al. 1953) and simulated annealing (Černý 1985, Kirkpatrick et al. 1983). The main idea of of these heuristics is to sometimes allow choices of neighbouring solutions which yield worsening values. The algorithms start with random initial solutions and iteratively improve the value of the objective function. In Metropolis algorithm the probability to accept worsening neighbours is controlled by a constant stochastic parameter. In simulated annealing algorithm we have, in addition to the stochastic parameter, an annealing schedule which reduces the probability of acceptance of worsening steps as the search proceeds. The performances of the simulated annealing algorithm and the Metropolis algorithm in coalitional game settings are studied in Publications P1 and P2.

Although the simulated annealing algorithm is an incomplete algorithm, there exists a theoretical result that the algorithm may reach the optimum with an extremely slow annealing schedule (Geman and Geman 1984). Unfortunately, the schedule is too slow to be used in practice. However, in practice with a carefully chosen annealing schedule and with an appropriate neighbourhood relation, the algorithm often reaches near optimal or even optimal solutions.

An alternative stochastic search algorithm based on a genetic algorithm is proposes in (Sen and Dutta 2000) for searching the optimal coalition structures. Such algorithms typically start with a set of candidate solutions and create a new set of candidate solutions by means of selection, mutation and recombination. A difficulty with the approach is that mutation and recombination methods may create a large number of invalid candidate solutions. The experimental results from the performance of the genetic algorithm in (Sen and Dutta 2000) cannot be directly compared with the experimental results in P1, since it is assumed in the experiments of (Sen and Dutta 2000) that there is certain regularity in the search space.

A most recent SLS-algorithm which searches for the coalition structure with optimal social welfare is presented in (Mauro et al. 2010). The algorithm is based on greedy randomized adaptive search procedure (GRASP) and its performance is compared with the simulated annealing algorithm in P1 and algorithms in (Sandholm et al. 1999, Yeh 1986, Rahwan and Jennings 2008a). According to the experimental results in (Mauro et al. 2010), as already indicated in P1, the simulated annealing algorithm of P1 outperforms (Sandholm et al. 1999).The proposed GRASP (Mauro et al. 2010) outperforms both (Sandholm et al. 1999) and the algorithm in P1. Furthermore, the authors report that GRASP is orders of magnitude faster than the algorithm based on dynamic programming in (Rahwan and Jennings 2008a) and it is indicated in (Mauro et al. 2010) that GRASP also outperforms the integer programming algorithm in (Rahwan et al. 2009).

4.3 Using NP-solvers in Coalitional Games

A promising approach to tackle the combinatorics related to coalitional games is to employ solvers directed to tackle NP-hard combinatorial problems which implement various effective algorithmics. An example of this approach is to represent the characteristic function of CFGs as a set of rules given as marginal contribution nets (Ieong and Shoham 2005) or as synergy coalition groups (Conitzer and Sandholm 2006). In this way, one can apply existing constraint programming techniques to solve the coalition structure generation problem (Ohta et al. 2009).

In a similar way, one can use various other existing solvers in coalitional games. In this thesis, we have represented coalitional games as a set of rules given as difference logic formulas. Difference logic (for a reference, see Nieuwenhuis and Oliveras 2005) is a propositional logic extended with the theory of integer differences. Fortunately, there exist practically efficient recent implementations of difference logic satisfiability solvers. The state-of-the-art solvers for difference logic satisfiability include Yices (Dutertre and de Moura 2009, Dutertre and de Moura 2006) and Barcelogic for SMT (Satisfiability Modulo Theories) (Nieuwenhuis and Oliveras 2005, Bofill et al. 2008).

In Publication P4, we present a difference logic encoding for the core non-emptiness checking problem in hedonic games. The basic idea of the encoding is to represent the preference lists of the players as difference logic formulas. The formulas contain variables for the players so that the integer values of the variables encode the coalitions where the players reside. In addition, the encoding has integer constants to represent the positions of coalitions in the preference lists. In this representation, the most preferred coalition has position 1 and the values increase as we move to less preferred positions. Based on this kind of encoding, the two solvers Yices and Barcelogic can effectively be used to decide the core non-emptiness of hedonic games, and to generate coalition structures residing in the game core. In our difference logic problem encoding, we use only integrity constraints. To solve such formulas, the Yices implements a simplex algorithm (Dutertre and de Moura 2006) and the Barcelogic implements a combination of a branch-and-bound and cutting planes algorithms (Bofill et al. 2008). In brief, what we have shown is that the players' hedonic preferences can be represented as a set of rules (i.e. difference logic formulas), and we can effectively solve the core non-emptiness problem of hedonic games using difference logic satisfiability solvers.

5 CONCLUSIONS

In this thesis we have presented techniques for finding stable and optimal outcomes in coalitional games. On the one hand, the proposed methods are based on our new research results concerning properties of coalitional games and related solution concepts. On the other hand, the proposed methods apply in new ways already existing algorithmical tools to coalitional game settings.

5.1 Results and Summary

In brief, in Publication P1 we have devised a new stochastic local search algorithm to find nearly optimal social welfare values in characteristic function games. We demonstrate via numerical experiments and comparisons to previous state-of-the-art algorithms that the proposed stochastic local search performs quite well in practice, if a suitable search neighbourhood is selected. Furthermore, in Publication P2 we have developed two stochastic local search algorithms to check the core membership of a given coalition structure in a hedonic game. We show experimentally that core membership can be checked quickly even for considerably large hedonic games by choosing an appropriate initial coalition for the algorithms. In Publication P3, a complete algorithm is given to generate the set of all Nash stable coalition structures in hedonic games. The algorithm is based on a new theorem concerning Nash stable coalition structures. Finally, in Publication P4 we introduced a difference logic encoding for the core non-emptiness problem in hedonic games. We show that using a compact problem encoding in difference logic together with a practically efficient implementation of difference logic satisfiability solvers, the problem of core non-emptiness can be easily solved even for hedonic games with quite large number of players.

5.2 Discussion and Further Work

Publication P1 indicates that the proposed simulated annealing algorithm performs well on the random benchmarks used in the reported experiments. However, in evaluating the performance of the simulated annealing algorithm in P1 the following issues should be noticed.

In P1 we have used a method to generate problem instances, which is originally introduced and used in (Larson and Sandholm 2000). Essentially, using this method to generate the problem instances involves drawing, for every possible coalition, a value from the uniform distribution over the interval $[0,1]$. As a result, the expected social welfare of a coalition structure with k coalitions is $k/2$, and it's maximal social welfare is k. This fact seems to explain the poor performance of the anytime algorithm from (Sandholm et al. 1999, Larson and Sandholm 2000), since it first checks the grand coalition, and then the $2n - 1 - 1$ coalition structures with exactly 2 coalitions which all can be expected to have a very low social welfare. The anytime algorithm from (Sandholm et al. 1999, Larson and Sandholm 2000) seems to perform relatively weakly on these particular benchmarks but the weak performance of the algorithm in (Sandholm et al. 1999, Larson and Sandholm 2000) seems to be merely a feature of the particular problem instances. Therefore, in order to draw conclusions concerning the performances of the algorithms in general, one needs to conduct more extensive experiments. In future work, it would be also important to investigate whether it is possible to derive some theoretical guarantees for the simulated annealing algorithm in P1.

In P1, Theorem 1 states that short paths always exist from any initial coalition structure to the optimal coalition structure. However, it must be noted that in these kinds of coalition structure graphs which are used in P1, the likelihood of the proposed algorithm actually finding an optimal coalition structure by following such a path is very small. This is due to the high number of edges in the underlying coalition structure graphs.

In Publication P2, we propose two new algorithms to check core membership. It must be noted that, although these algorithms seem to perform well on large problem instances, the strengths and weaknesses of the algorithms will only become clear when alternative approaches have been proposed, and extensive comparative studies have been carried out. In (Elkind and Wooldridge 2009), an algorithm is sketched in the proof of Theorem 6 to check core membership in hedonic nets. Since additively separable hedonic games can be trivially represented as hedonic nets (see Section 3.1 in Elkind and Wooldridge 2009), one could use such an algorithm to solve the problem in P2 as well. An important topic for further work would be to compare the performances of these two distinct approaches.

In Publication P3, the experimental results suggest that the proposed algorithm performs well on randomly generated hedonic game instances with aversion to enemies preferences. However, it is observed that for hedonic games with generic preferences the results are not as promising because the median run-times vary significantly and are reasonably high. Also, concerning the experimental results presented in P3 one must notice that the method of generating random problem instances in P3 makes it unlikely that a Nash-stable coalition structure will include a coalition of cardinality 3 or more (as pointed out also on page 36 in P3). This leaves open whether the proposed algorithm will also perform well in the context of problem instances that do not have this particular feature. These kinds of important further experiments are left as future work.

It must be noted that Publication P4 only reports on experiments in which the non-emptiness problem has a positive answer and thus all the corresponding formulas are satisfiable. Interestingly, we were not able to generate a single problem instance with an empty core using any of the random benchmarks. It would be an important topic for further research to study the performance of the difference logic solvers on encodings of negative problem instances.

In conclusion, the experiments in this thesis are done in order to test initially the applicability of the methods in practice. Some of these initial tests are based on examples taken from the previous literature, and all the tests are based on randomly generated coalitional games. It would be an important topic for further research to apply the proposed methods to some real-world applications, because problem instances from real-world applications have often different structure than random problem instances, which may affect the performances of the algorithms.

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A ADDITIONS AND CORRECTIONS TO THE PUBLICATIONS

P1

- On page 31 the sentence ”A coalition structure CS is a partition of A into mutually disjoint coalitions in which, for all S, $S' \subseteq A$, we have $S \cap S' = \emptyset$, $S \neq S'$ [...].” should be ”A coalition structure CS is a partition of A into mutually disjoint coalitions; that is, for all coalitions $S, S' \in CS$ with $S \neq S'$ we have $S \cap S' = \emptyset$”.
- On page 31, the following incorrect statement should be removed: ”Note that the characteristic function can easily be represented compactly such that the size of its representation is only polynomial in the number n of agents”.
- On page 32, in the line following Algorithm 1, 2^N should be 2^A.
- On page 33, in the proof of Lemma 1, within item 1, $j - i$ times should be $j - 1$ times.
- On page 34, we notice that the bounds established by Lemma 1 and Lemma 2 are tight.
- Figure 1 on page 35, the title of the figure is ”Minimum runtime to find optimal social welfare”, the x-axis label is ”split/merge (number of seen coalition structures)” and the y-axis label is ”shift (number of seen coalition structures)”.
- On page 36 the sentence ”[...] it is of interest to investigate the behaviour of a rather simpler algorithm [...].” should be ”[...] it is of interest to investigate the behaviour of a rather simple algorithm [...].”
- Figure 2 (a) on page 36, the title of the figure is ”Minimum social welfare”, the x-axis label is ”split/merge (social welfare)” and the y-axis label is ”shift (social welfare)”.
- Figure 2 (b) on page 36, the title of the figure is ”Median social welfare”, the x-axis label of is ”split/merge (social welfare)” and the y-axis label is ”shift (social welfare)”.
- Figure 2 (c) on page 36, the title of the figure is ”Maximum social welfare”, the x-axis label is ”split/merge (social welfare)” and the y-axis label is ”shift (social welfare)”.

P2

- On page 56, ”residing in core” should be ”residing in the core”.
- On page 57, ”exists valuation function” should be ”exists a valuation function”.

- On page 57, in the last paragraph ”characteristic” should be ”characterising the relation”.
- On page 58, ”A core of game” should be ”The core of game”.
- On page 56–70, the terms ”generic hedonic games” and ”generic hedonic preferences” refer to hedonic games with no restrictions to players’ preferences.
- On page 58, in Definition 2, notice that the coNP-completeness result of the core membership checking problem concerns only the problem of deciding whether CS is in the core of G, and does not concern the problem of computing a blocking coalition.
- On page 59, ”based on iterative” should be ”based on the iterative”.
- On page 60, ”added to current” should be ”added to the current”.
- On page 63, ”each agent integer” should be ”each agent an integer”.
- On page 69, ”operation research” should be ”operations research”.

P3

- On page 25, the sentence ”Despite hedonic games have received much research attention recently ” should be replaced by the sentence ”Despite hedonic games having received much research attention recently”.
- On page 27, the sentence ”for all players $i \in N$, $v_i(\cdot) \in [-n, 1]$ and $v_i(i) = 0$” should be replace by the sentence ”for all players $i \in N$, $v_i(\cdot) \in \{-n, 1\}$ and $v_i(i) = 0$”.
- On page 27, ”the Problem 1” should be replaced by ”Problem 1”.
- On page 28, the proof of Theorem 1 could be simplified by omitting the induction proof and pointing out that preferences can trivially be set up so as to ensure that all agents consider all possible coalitions equally desirable.
- On page 29, in Theorem 2 the sentence ”where player i and players in S” should be replaced by the sentence ”where player i and the players in S”.
- On page 29, in the proof of Theorem 2 the sentence ”Also, we know by the assumption of Nash stable coalition structures that” should be replaced by the sentence ”Also, we know by the assumptions of Nash stable coalition structures and additively separable preferences that”.
- On page pages 32–37, the terms ”generic hedonic games” and ”generic hedonic preferences” refer to hedonic games with no restrictions to players’ preferences.

- On page 34, the following sentence should be added to the end of the first paragraph of Section 5.3: "Notice that all coalition structures are checked by the second variant of the algorithm without the stability test."

P4
- On page 331, in the first paragraph of Section 1 "in coalitional games is core" should be "in coalitional games is the core".
- On page 333, in Definition 2 "oven N" should be "over N".
- On page 333, the following incorrect statement should be removed: "The representation of the preferences by IRCL is often compact, but in the worst case it reduces to the naive representation of the preferences".
- On page 334, in Definition 4 should be added the following sentence: "We use $(x_i \neq c)$ as a shorthand for $\neg(x_i = c)$".
- On page 335, "let $p_i(S)$ be integer constant" should be "let $p_i(S)$ be an integer constant".
- On page 336, "Φ_G evaluates" should be "Φ_G evaluate".
- On page 336, the encoding Φ_G may be simplified by removing the sub-formula Φ_l and extending the sub-formula Φ_p to handle singleton coalitions.
- On page 336, in the proof of Theorem 1 the sentence "satisfy the sub-formula Φ_S" should be replaced by "satisfy the sub-formula Φ_S in conjunction with the other sub-formulas of Φ_G".
- On page 336, the proof of Theorem 2 should be added as follows: "The number of conjuncts in Φ_c is $2 \times n$, where all conjuncts are atomic formulas. For both Φ_p and Φ_l, the number of conjuncts is $O(k \times (n \times n))$ and each conjunct consists of an implication of atomic formulas. The number of conjuncts in Φ_s is k and each conjunct consists of a disjunction of at most n atomic formulas. Thus, the size of the formula Φ_G is $O(n) + O(k \times n^2) + O(k \times n^2) + O(k \times n) \leq O(n^2 \times k)$."
- On page 337, "number players" should be "number of players".
- On page 338, Table 1, the runtimes are measured in seconds.

PUBLICATION 1

Keinänen Helena (2009) Simulated annealing for multi-agent coalition formation. In Agent and Multi-Agent Systems: Technologies and Applications, Third KES International Symposium, KES-AMSTA 2009, Uppsala, Sweden, Proceedings, ed. by Anne Håkansson, Ngoc Thanh Nguyen, Ronald L. Hartung, Robert J. Howlett and Lakhmi C. Jain, 30–39. Lecture Notes in Computer Science, Vol. 5559, Springer.

PUBLICATION 2

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PUBLICATION 3

Keinänen Helena (2010) An algorithm for generating Nash stable coalition structures in hedonic games. In Foundations of Information and Knowledge Systems, 6th International Symposium, FoIKS 2010, Sofia, Bulgaria, Proceedings, ed. by Sebastian Link and Henry Prade, 25–39. Lecture Notes in Computer Science, Vol. 5956, Springer.

PUBLICATION 4

Keinänen Helena (2010) Core non-emptiness checking for hedonic games via difference logic. In Agent and Multi-Agent Systems: Technologies and Applications, 4th KES International Symposium, KES-AMSTA 2010, Gdynia, Poland, Proceedings, ed. by Piotr Jedrzejowicz, Ngoc Thanh Nguyen, Robert J. Howlett and Lakhmi C. Jain, 331–340. Lecture Notes in Computer Science, Vol. 5070, Springer.

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