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Countercyclical and time-varying reward to risk and the equity premium ☆

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ABSTRACT

We study whether the equity premium is related to volatility or variance, whether the reward to market risk is positive, and whether it behaves in a counter-cyclical fashion. Using APARCH models for the conditional market risk, we compare the traditional and the new testing approach of Antell and Vaihekoski (2019) on the monthly US equity premium from 1928 to 2018. The results from the new approach give stronger support for the pricing of volatility rather than variance and for positive reward to market risk. The support for timevarying and countercyclical reward to risk coefficient is smaller than previously thought.

1. Introduction

The equity premium is embedded in many central models in finance, influencing anything from asset management to firms' cost of capital and beyond. One of the most commonly used models for the equity premium is the Merton (1973, 1980) model which connects the equity market premium and the market risk via a positive reward to risk parameter. Although this relationship is theoretically intuitive and it has been studied in numerous papers, there are still a few open questions empirically.

First, many studies have had difficulties finding a significant and positive relationship between realized returns and variance (for a review of early studies, see Lundblad, 2007; for a more recent review, see Hong and Linton, 2020). More recently, Yu and Yuan (2011) find that the relationship is positive in low-sentiment periods but not in high-sentiment periods and that this result is strikingly robust to different methods. Müller et al. (2011) also find support for the risk-return relationship using a continuous-time GARCH analysis. Nyberg (2012), on the other hand, finds support for a positive relationship regardless of the state of the business cycle, as well as for a higher risk aversion in the recession regime. Guo et al. (2013) also report support for the positive risk-return trade-off when one controls for the illiquidity premium with the consumption–wealth ratio (CAY). All things considered, Feunou et al. (2014) review the literature and note that the empirical support for the relationship is "remarkably uneven".

Second, under certain assumptions (e.g., power utility), it can be shown that the reward-to-risk parameter equals to or is a function of the aggregate relative risk aversion (Merton, 1980; Taylor, 2005; Guo and Whitelaw, 2006). A number of empirical studies have argued that the risk aversion is time-varying (see, e.g., Smith and Whitelaw, 2009; Guo et al., 2013; Guiso et al., 2018;

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Cohn et al., 2015; Hong and Linton, 2020) although opposite results have also been presented (see, e.g., Andersen et al., 2008). In addition, the financial theory suggests that aggregate risk aversion should behave in a countercyclical way, that is, during recessions risk aversion should be higher (Guo et al., 2013). Cohn et al. (2015) note that the evidence for this is scarce. For example, Yu and Yuan (2011) note that it is very difficult for their empirical results to fit in with the existing hypotheses with either constant or time-varying risk aversion. Cohn et al. (2015) note that this is due to "the host of factors that simultaneously change during financial cycles". The risk aversion and the volatility both exhibit strong countercyclical patterns. Thus it is hard to separate the effects from each other in empirical tests.

Third, although Merton (1973) clearly advocates the use of variance as the measure of risk, Merton (1980) also considers volatility as a measure of risk, but ultimately he does not clearly favor one over the other. As most theoretical models connect risk premium particularly to variance, studies have mostly used variance as the measure of market risk and, as a result, risk aversion as the relevant reward to risk measure. On the other hand, Li (2007) develops a habit formation model where two reward to risk measures, risk aversion and price of risk, are indirectly linked. His model actually supports the notion that aggregate stock returns are more consistent with the model built on the idea of constant risk aversion and time-varying price of risk. As such, the choice of risk measure is open for debate and, at least for practical purposes, the choice can be made on the basis of the empirical results.

To find support for a positive risk-return relationship, numerous studies have focused on trying to improve the measure for variance (see, for example, Antell and Vaihekoski, 2019). However, this paper argues that some of the earlier mixed results for the pricing of market risk as well as for its time-variability are in fact due to the issues in the traditional empirical testing approach used to test conditional asset pricing models. Namely, as Antell and Vaihekoski (2019) argues, using realized returns or an auxiliary expectations model as a proxy for expected returns in tests of conditional asset pricing models is problematic. They show that under certain assumptions, one can derive a testable model for realized returns conditional on the tested asset pricing model being true – a reverse testing approach if you will.[1] Obviously, if the model tested is not the one used by the investors, the realized returns would reflect this and the model would be rejected. They test the approach on the Merton (1980) model and find strong support for a positive reward-to-risk relationship and pricing of the variance risk on the US market using a constant reward to market risk parametrization of the model.

However, the approach in Antell and Vaihekoski (2019) does not go without problems, either. In this paper, we take a step back and re-evaluate their results. We assess the effect of the underlying assumption of convergence in risk towards its long-term value and the ensuing term structure of required returns, in case the investors do not assume such a structure. In addition, we reconsider their method to extract dividends from the market index. As an extension to their model, we derive an empirically testable model in which the reward to market risk is allowed to be time-varying and test whether it behaves in a counter-cyclical fashion. Finally, we also test whether the equity premium is linked to volatility instead of variance.

We test the model using monthly equity premia for the US stock market from 1928 to 2018. Two measures of market risk are employed: volatility and variance, both of which are modeled using the asymmetric power ARCH (APARCH) model as it can embed both of them. The reward to risk parameter (here: lambda) is modeled linear on conditioning variables that are often found to be related to economic expectations. A number of ways to model economic expectations are applied in the empirical analysis. We also compare the results from the new estimation approach to those from the traditional estimation approach.

The results show strong support for a positive reward to market risk using the new testing approach. There is stronger support for the pricing of volatility rather than variance. The traditional testing approach cannot explain the risk-return relationship with either measure of market risk. We also find evidence for the term structure in the equity premium — investors take into account convergence in market risk towards the mean, but not as much as *a priori* expected. The reward to market risk behaves in a countercyclical way, especially when one uses variance as the measure of risk, but, in general, the main driver for the variation in the expected returns is the variation in the market risk.

The remainder of the paper is organized as follows. Section 2 presents the theoretical background, derives a model for the realized returns conditional on the Merton (1980) model for the return–variance relationship, and discusses the empirical research methodology as well as econometric issues. Section 3 introduces the data used in this paper. Section 4 shows the empirical results. Section 5 presents the conclusions and offers suggestions for further research.

2. Theoretical background

2.1. Risk-return models

Merton (1980) shows that the model for the excess return on the market portfolio, i.e. equity premium, can be reasonably approximated by

$$E\left[r^e_{m,t+1}\,|\Omega_t\right] = \lambda_{m,t+1}\sigma^{\delta}(r^e_{m,t+1}|\Omega_t), \tag{1}$$

[1] The idea of reverse testing is easy to demonstrate. Say, we have two competing asset pricing models for the expected returns. The first one argues that the conditional required return at time t is, say, 5% from that point onwards. The next period at time $t+1$, investors revise their views and increase the required rate to 10%. The competing model says the opposite. Now, given a dividend discount model, it is easy to show that if the first model is true, *ceteris paribus*, the realized return should be negative from time t to time $t+1$, whereas in the latter case, the return should be positive. In fact, all asset pricing models imply certain realized returns. Of course, even if we know the true model, the fit will not be perfect as the dividend growth expectations can also change — an issue often overlooked in all tests of asset pricing models. However, the model used here makes an effort to take them into account as well. To this end, the approach here is a further development of the Campbell and Hentschel (1992) model for realized returns.

where $E\left[r^e_{m,t+1}|\Omega_t\right]$ is the conditional expected excess return on the market portfolio and lambda, $\lambda_{m,t+1}$, is a time-varying measure referred to as the conditional reward to market risk. $\sigma^\delta(r^e_{m,t+1}|\Omega_t)$ is the conditional measure of risk for the excess market return. Following Merton (1980), two competing values are used for the delta parameter. Setting delta to one, Eq. (1) implies that the expected return is linearly related to the volatility (as in Theodossiou and Savva, 2016). When it is set to two, the market rewards variance instead. In the former case, the reward to risk parameter would equal the price of market risk, i.e., the slope of the capital market line (Sharpe ratio). In the latter case, the reward to market risk parameter can be seen as the aggregate relative risk aversion. All variables are conditional on the information set Ω_t available at time t, which implies that they vary over time. As such, Eq. (1) basically shows that investors must be compensated by a higher expected return if the conditional risk or the reward to market risk increases.

The theoretical model (1) has a number of empirical implications that can be tested. To do this, one must provide empirical proxies for expected returns, lambda, and conditional risk. Typically, the time-varying reward to market risk has been modeled as linear on information variables as

$$\lambda_{m,t+1} = \lambda_0 + \lambda_1' Z_t, \tag{2}$$

where λ_0 is a constant, λ_1 is a $K\times 1$ vector of parameters to be estimated, and Z_t is a $K\times 1$ vector of information variables available at time t. Some studies have also forced positivity for the reward to market risk using an exponential model (see, e.g., Bekaert and Harvey, 1995; De Santis and Gerard, 1997; Carrieri et al., 2007).[2]

Earlier empirical studies have often been conducted as if expected returns can be proxied with realized returns. An alternative is to use a linear expectations model for the realized returns; one of the first to use this approach were Gibson and Ferson (1985). Conditional risk, on the other hand, has been in most cases measured with the variance of returns as it is convenient to estimate. Now, given an estimate for the variance and a specification for the reward to market risk, one typically proceeds to estimate Eq. (1) using the following linear model:

$$r^e_{m,t+1} = \alpha + \lambda_{m,t+1}\sigma^2_{m,t+1} + \varepsilon_{m,t+1}, \tag{3}$$

where $r^e_{m,t+1}$ is the realized (or forecast from linear expectations model) excess market return from time t to t+1, α is a constant expected to be zero if excess returns are used and the asset pricing model is valid, $\sigma^2_{m,t+1}$ is the conditional variance for the period from time t to t+1, given the information available at time t, and $\varepsilon_{m,t+1}$ is the error term. Similar to Antell and Vaihekoski (2019), we refer to using this equation as the *traditional approach* to estimating lambda.

As stated in Antell and Vaihekoski (2019), realized returns are an inferior or even biased proxy for the expected returns for short return measurement intervals, and therefore the traditional approach is problematic for empirical tests of *conditional* asset pricing models. As a solution, they suggest a *reverse testing approach* which is based on the idea of analyzing realized returns conditional on the asset pricing model being true (c.f., Guo and Whitelaw, 2006). It is based on Campbell and Hentschel's (1992) result that continuously compounded realized returns, r_{t+1}, for a dividend-paying security can be written as

$$r_{t+1} \approx (1-\rho)\left[\sum_{i=0}^{\infty}\rho^i\left(d_{t+1,t+1+i} - d_{t,t+1+i}\right)\right] + \sum_{i=1}^{\infty}\rho^{i-1}\left(r_{t,t+i} - \rho r_{t+1,t+1+i}\right), \tag{4}$$

where $r_{t,t+i} = E_t[r_{t+i}]$ expresses the continuously compounded required rate of return for the period t+i ($i \geq 0$), conditional on information available at time t. $d_{t,t+1+i} = E_t[ln\left(D_{t+1+i}\right)]$ is the expected log dividend at time $t+1+i$ conditional on information available at time t. The terms $r_{t+1,t+1+i}$ and $d_{t+1,t+1+i}$ are defined in a similar fashion although both are conditional on information available at time $t+1$. The parameter ρ is positive and less than one by definition. Campbell et al. (1997) note that ρ should be 0.997 for monthly data.

Now taking our candidate asset pricing model (1), and using it to define conditional expected returns for the market portfolio, we can rewrite the last term of Eq. (4) for the market portfolio as

$$\begin{aligned}&\sum_{i=1}^{\infty}\rho^{i-1}\left(r_{t,t+i} - \rho r_{t+1,t+1+i}\right)\\ &= \sum_{i=1}^{\infty}\rho^{i-1}\left(r_{ft,t+i} - \rho r_{ft+1,t+1+i}\right) + \sum_{i=1}^{\infty}\rho^{i-1}\left(\lambda_{m,t+1}\sigma^\delta_{t,t+i} - \rho\lambda_{m,t+2}\sigma^\delta_{t+1,t+1+i}\right).\end{aligned} \tag{5}$$

Following earlier studies, we assume that the lambda is a linear function of information variables, and use Eq. (2) for $\lambda_{m,t+1}$ and $\lambda_{m,t+2}$. Utilizing these definitions, we can rewrite the second term in Eq. (5) as

$$\begin{aligned}&\sum_{i=1}^{\infty}\rho^{i-1}\left[\left(\lambda_0 + \lambda_1' Z_t\right)\sigma^\delta_{t,t+i} - \rho\left(\lambda_0 + \lambda_1' Z_{t+1}\right)\sigma^\delta_{t+1,t+1+i}\right]\\ &\quad= \lambda_0\left[\sum_{i=1}^{\infty}\rho^{i-1}(\sigma^\delta_{t,t+i} - \rho\sigma^\delta_{t+1,t+1+i})\right] + \lambda_1'\left[\sum_{i=1}^{\infty}\rho^{i-1}\left(\sigma^\delta_{t,t+i}Z_t - \rho\sigma^\delta_{t+1,t+1+i}Z_{t+1}\right)\right].\end{aligned} \tag{6}$$

[2] Merton (1980) actually recommends using the positivity constraint because short estimation samples can produce negative values.

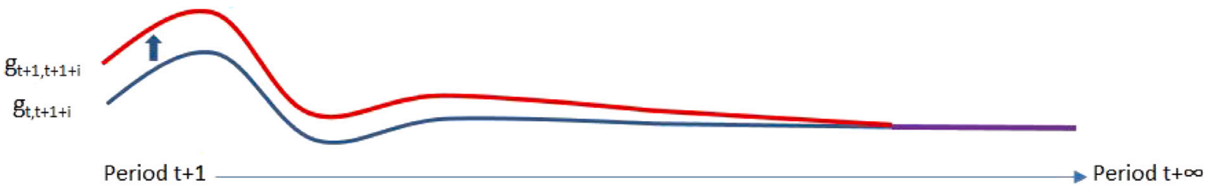

Fig. 1. Example of how investors' conditional expectations on dividend growth change from time t to $t+1$ for future period $t+i$ ($i \to \infty$). Here the investors have revised their views of future dividend growth upwards on the basis of new information received at time $t+1$.

Using the assumption that the conditional risk is a mean-reverting process, we express conditional risk for any future period as a function of the next period's forecast as $\sigma^{\delta}_{t,t+i} = \phi^{i-1}\sigma^{\delta}_{t,t+1} + \sigma^{\delta}(1-\phi^{i-1})$, where $|\phi\ |\le 1$ is a persistence parameter reflecting the speed of convergence of the conditional volatility (or variance) towards its long-term unconditional value σ^{δ}. The closer to zero ϕ is, the faster the convergence to the unconditional (long-term) value. At the other extreme, the closer to one ϕ is the slower the convergence. If ϕ equals to one, there is no convergence in risk. As such, there is no term structure in the risk premium, which also corresponds to the traditional approach. Using Eq. (2), Eq. (4) can be written as

$$\begin{aligned} r_{m,t+1} \approx\ & (1-\rho)[\sum_{i=0}^{\infty}\rho^{i}(d_{t+1,t+1+i}-d_{t,t+1+i})] + \sum_{i=1}^{\infty}\rho^{i-1}(r_{ft,t+1+i}-\rho r_{ft+1,t+1+i}) \\ & + \lambda_0\left[\left(\sigma^{\delta}_{t,t+1}-\rho\sigma^{\delta}_{t+1,t+2}\right)\cdot\varphi_{\Delta\sigma}+\sigma^{\delta}\cdot\varphi_{\sigma}\right] \\ & + \lambda_1'\left[\left(Z_t\sigma^{\delta}_{t,t+1}-\rho Z_{t+1}\sigma^{\delta}_{t+1,t+2}\right)\cdot\varphi_{\Delta\sigma}+\sigma^{\delta}(Z_t-\rho Z_{t+1})\cdot(1-\rho)^{-1}\cdot\varphi_{\sigma}\right], \end{aligned} \tag{7}$$

where $\varphi_{\Delta\sigma}$ is equal to $1/(1-\rho\phi)$ and φ_{σ} equals $\left(1-\varphi_{\Delta\sigma}\cdot(1-\rho)\right)$ derived using the sum of converging geometric series.

Now, for mathematical elegance, we apply certain simplifying assumptions on *changes* in expected dividend growth rates and risk-free rates from time t and $t+1$. In practice, we assume that, as investors revise their views on the future dividend growth rates from time t to $t+1$, the difference between the new and the old views converges geometrically to zero for period $t+i$ when $i\to\infty$ (c.f., the model (7) in Bansal and Lundblad, 2002). Fig. 1 demonstrates this. The same assumption is convenient also for the risk-free rates, i.e., the changes in expected risk-free rates are assumed to converge geometrically to zero the more distant future one considers. As a result, Eq. (7) can be expressed in a simplified form as

$$\begin{aligned} r_{m,t+1} \approx\ & k_2 + \left(g_{t+1,t+1}-g_{t,t+1}\right)\cdot\varphi_d + \left(r_{ft}-r_{ft+1}\right)\cdot\varphi_{rf} \\ & + \lambda_0\left((\sigma^{\delta}_{t,t+1}-\rho\sigma^{\delta}_{t+1,t+2})\cdot\varphi_{\Delta\sigma}+\sigma^{\delta}\cdot\varphi_{\sigma}\right) \\ & + \lambda_1'[(Z_t\sigma^{\delta}_{t,t+1}-\rho Z_{t+1}\sigma^{2}_{t+1,t+2})\cdot\varphi_{\Delta\sigma}+\sigma^{\delta}\cdot(Z_t-\rho Z_{t+1})\cdot(1-\rho)^{-1}\cdot\varphi_{\sigma}], \end{aligned} \tag{8}$$

where $g_{t+1,t+1}-g_{t,t+1}$ represents the change in the expectations of future dividend growth rates from time t to $t+1$. The parameters φ_d and φ_{rf} measure the impact of the changes in investors' expectations for the dividend growth rate and the risk-free rate, respectively. Both of them are by definition positive given the backward way of calculating changes in risk-free rates and risk. The constant k_2 corresponds to the risk-free rate.

Analyzing Eq. (8) reveals that realized returns are higher if investors' conditional expectations of the long-term dividend growth rate increase from period t to $t+1$, ceteris paribus. The same is true if the risk-free rate decreases. Assuming that the asset pricing model is correct, a decrease in conditional risk also leads to higher realized returns. If there is a positive relationship between the conditioning information variable and the reward to risk parameter (i.e., the corresponding element of λ_1 is positive), an increase in the value of the information variable should lead to a lower realized return. It is easy to see that this is the case. Thus all implications of the model are in line with the intuition. Eq. (8) also allows us to separate the effect of changes in the measure of market risk and in the reward to risk parameter.

2.2. Empirical estimation

The main objective of this paper is to test whether the reward to market risk, lambda, is time-varying and whether it behaves in a countercyclical fashion. In addition, we want to test whether the equity premium is a function of volatility or variance. Finally, we want to compare the estimate of lambda from the traditional approach with the estimate from the new estimation approach. To estimate the model, we need an empirical measure for the conditional volatility/variance. Antell and Vaihekoski (2019) compare GARCH, MIDAS, and the VIX-index as proxies for the variance, but they concluded that, overall, the choice of variance estimation method does not have a major effect on the empirical performance of the asset pricing model nor the conclusion of its validity. Hence, here we utilize GARCH type of models in the estimation.

In practice, in the traditional estimation approach, we utilize the commonly used GJR-GARCH model by Glosten et al. (1993) to capture asymmetry in the variance process (cf., e.g., Bekaert and Wu, 2000; Cappiello et al., 2006). For the new estimation approach, we choose the APARCH model as it can be applied both to volatility and variance processes, and since it nests the GJR-GARCH. The APARCH(1,1) process for the excess market returns is given by

$$\sigma_{m,t+1}^{\delta} = \omega + \alpha_1 (|\varepsilon_{m,t}| - \gamma \varepsilon_{m,t})^{\delta} + \beta_1 \sigma_{m,t}^{\delta}, \tag{9}$$

where the parameters ω, α_1, γ, β_1, and δ are the parameters to be estimated. If we set $\delta = 1$, the model corresponds to the Threshold ARCH (TARCH) model for the volatility. If we set $\delta = 2$, we get the GJR-GARCH model (see, Ding et al., 1993).

To estimate lambda using the traditional approach, we combine Eqs. (2) and (3) to get the GJR-GARCH-in-mean model. To estimate the lambda using the reverse testing approach, we estimate model (8) in two steps. There are two reasons for this. First, the reverse testing specification uses the change in the conditional risk in the mean equation rather than the value of risk per se. Second, unless separated, the risk process specification affects the mean equation (through $\varphi_{\Delta\sigma}$ and φ_{σ}) which in turn affects the risk process estimation.

In the first phase, we estimate the conditional volatility/variance for the market using Eq. (9). Given the conditional risk series, we calculate the speed of convergence in the process (ϕ), unconditional volatility or variance (σ^{δ}), and the ensuing sigma multipliers. In the second step, we run a linear regression model where we have operationalized Eq. (8) in excess return form as follows:

$$\begin{aligned} r_{m,t+1}^{e} =& b_1 + b_2 \left(g_{t+1,t+1} - g_{t,t+1} \right) + b_3 \left(r_{ft} - r_{ft+1} \right) \\ &+ b_4 \left[(\sigma_{t,t+1}^{\delta} - \rho \sigma_{t+1,t+2}^{\delta}) \cdot \varphi_{\Delta\sigma} + \sigma^{\delta} \cdot \varphi_{\sigma} \right] \\ +\mathbf{b}_5' & [(\mathbf{Z}_t \sigma_{t,t+1}^{\delta} - \rho \mathbf{Z}_{t+1} \sigma_{t+1,t+2}^{\delta}) \cdot \varphi_{\Delta\sigma} + \sigma^{\delta} \cdot (\mathbf{Z}_t - \rho \mathbf{Z}_{t+1}) \cdot (1-\rho)^{-1} \cdot \varphi_{\sigma}] + u_{m,t+1}, \end{aligned} \tag{10}$$

where b_1 to b_4 and the elements of the vector $\mathbf{b}_5$ are the coefficients to be estimated. All coefficients except those in the vector $\mathbf{b}_5$ are expected to be positive. The coefficient b_4 is our estimate for the unconditional, long-term mean lambda given that the information variables have been demeaned. The parameters in $\mathbf{b}_5$ capture time-variation in lambda. The risk-free rate at time t is given by r_{ft}. The term $\sigma_{t,t+1}^{\delta}$ is the volatility or the variance of the continuously compounded excess market return from time t to $t+1$, conditional on information available at time t. Variables are defined similarly for time $t+1$. Other variables are as defined earlier.

To get an estimate for the reward to market risk and to assess whether it behaves in a counter-cyclical fashion, we begin our estimation with a baseline version of the model. For this purpose, Eq. (10) is simplified under the assumption that changes in both the interest rate level and the dividend growth rates are of lesser importance and, in the long run, the change in both averages to zero:

$$\begin{aligned} r_{m,t+1}^{e} =& b_1 + b_2 \left[(\sigma_{t,t+1}^{\delta} - \rho \sigma_{t+1,t+2}^{\delta}) \cdot \varphi_{\Delta\sigma} + \sigma^{\delta} \cdot \varphi_{\sigma} \right] \\ +\mathbf{b}_3' & [(\mathbf{Z}_t \sigma_{t,t+1}^{\delta} - \rho \mathbf{Z}_{t+1} \sigma_{t+1,t+2}^{\delta}) \cdot \varphi_{\Delta\sigma} + \sigma^{\delta} \cdot (\mathbf{Z}_t - \rho \mathbf{Z}_{t+1}) \cdot (1-\rho)^{-1} \cdot \varphi_{\sigma}] + u_{m,t+1}, \end{aligned} \tag{11}$$

where b_1 is expected to account for the mean effect from the components excluded from the model and b_2 is our estimate for the lambda. The parameter vector $\mathbf{b}_3$ accounts for the time-variation in lambda.

In practice, in the first step, we estimate the APARCH model for the excess market returns using only a constant in the mean equation. In the second step, we estimate Eq. (10) or (11) using estimates for the conditional risk as well as for $\varphi_{\Delta\sigma}$, φ_{σ}, and the unconditional measure of risk (volatility or variance) from the first step. To estimate $\varphi_{\Delta\sigma}$ and φ_{σ}, we use their definitions. This requires an estimate for the speed of convergence (persistence) for the conditional volatility or variance returning to their long-term means, i.e., the ϕ parameter and the dividend-to-price-related ρ parameter. The latter can be easily calculated from the data, but the former is specific to the chosen risk process. For the volatility process under conditional normality assumption, the speed of convergence parameter ϕ is the sum of $\sqrt{\frac{2}{\pi}}\alpha_1$ and β_1, and for the variance process it is $\alpha_1 \times (1-\gamma)^2 - \beta_1 - 2\alpha_1\gamma$.[3] The unconditional volatility and variance can be estimated with $\omega/(1-\phi)$ using the corresponding value for convergence.

3. Data

3.1. Main variables

We utilize monthly continuously compounded excess equity market returns for the US market to test the models. The sample period is from January 1928 to December 2018, i.e., 1,092 months of data. Month-end CRSP value-weighted total returns are used as a proxy for the market returns. The risk-free rate for any month is based on the one-month holding period return on US Treasury bills closest to one month at the end of the month. The excess return is obtained as the difference between the market return and the risk-free rate. Percentage excess returns are taken from Professor French's website (originally from the CRSP).

For the full model, we also need a measure for the change in the risk-free interest rate. Here, as a proxy we use the change in the long-term US government bond yield taken from the Ibbotson (2019). In addition, we need a measure for the change in the conditional dividend growth rates, i.e., investors' expectation for future dividend growth. For this, we note that modeling future dividend growth rates is not an easy task since there are a number of issues involved. First, one has to decide how aggregate

[3] Proofs are available upon request. For the variance process, we have first utilized Ding et al. (1993) to map the APARCH parameters to those of the GJR process, and then the fact that for the GJR-GARCH(1,1) model, the speed of convergence, ϕ, is the sum of alpha, beta, and half the gamma parameters for the GJR process.

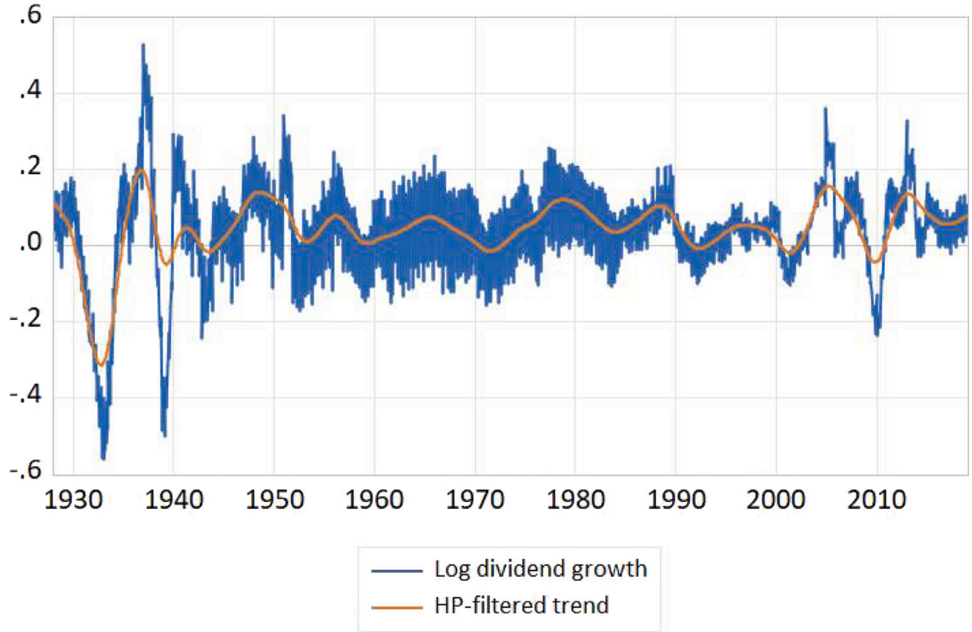

Fig. 2. Realized growth in annual dividends from month t to $t+1$. Original log dividend growth series and the Hodrick–Prescott trend series.

market-level dividends are measured. Second, one has to decide how to forecast dividend growth and how to take into account the cyclicality of the dividend payments.

To deal with the intra-year cyclicality of the dividends, Antell and Vaihekoski (2019) calculate annual dividends paid (USD) over twelve months, using two different methods. Their main method uses the fact that one can extract annual dividends by multiplying the CRSP price index a year ago with the difference between the total return and price index percentage returns in the twelve months.[4] The second method uses the same idea to extract monthly dividends and use their sum to derive the annual dividends. In the second step, they calculate the realized growth series g_t as the annual change in the log dividends and use it as a proxy for the expected future growth rate of dividends. Overall, their results give support for the effect on realized return through the change in the conditional dividend growth forecasts using the former measure, but not using the latter measure.

The sensitivity of their result to the dividend growth measure is due to the implicit feature built into the dividend extraction method. Namely, the extracted dividend series reflects the methodology used to build the stock market indices. With the CRSP indices, the dividends are always assumed to be re-invested in the market when paid. Thus, when the dividends are extracted less frequently (say, annually), market development has a bigger influence on the extracted dividend series. As it is more natural to analyze actual dividends and their growth, not their re-invested value, the latter approach (the sum of monthly dividends) seems to be a more appropriate measure for the annual dividends (c.f., Chen, 2009).

Of course, one does not have to use growth in *annual* dividends as a proxy for investors' expectation for future dividend growth, we are free to choose the length of the period over which the dividends are accumulated as well as the lag used to calculate growth, although a typical choice has been to use two non-overlapping consecutive twelve-month periods to avoid the intra-year cyclical pattern in the dividend payments. We also follow this approach although we observe that it has a problem that can be clearly seen, e.g., around the development of the stock market crash that started in October 1929. Namely, due to the use of rolling twelve-month sums, the growth series indicated dividend growth several months past the crash. It is hard to believe that investors would not have understood the negative effect of the crash on future dividend growth. One could, in theory, shorten the cumulation period, say, even to a month but it would increase the volatility of growth, when, for our purposes, even the original growth series is too volatile — we need a measure for the changes in the investors' long-term growth expectations. Thus, ultimately we have decided to use the Hodrick–Prescott decomposition to the annual growth series to remove the short-term fluctuations from the series. This series can be seen in Fig. 2.

3.2. Conditioning variables for time-varying reward to risk

On an individual level, studies have linked several characteristics to risk aversion (for a review, see Guiso et al., 2018). Here, however, the main interest is on the aggregate level. The effect has to be unrelated to the market uncertainty, i.e., go over and

[4] This is based on the idea that we can calculate a proxy for the dividends by solving D_t from the equation $R_t - R_t^x = (\frac{P_t+D_t}{P_{t-12}} - 1) - (\frac{P_t}{P_{t-12}} - 1)$, where on the left-hand side we have the difference between realized percentage annual returns for the total return and the price indices.

above the effect caused by the time-varying market risk level. By the very nature of the relative risk aversion, we also argue that the potential time variation in the risk aversion is rather slow-moving compared to the variance especially. Hence, we expect the risk aversion as well as the price of market risk to behave fairly smoothly. A number of different variables have been suggested and their effect on the (aggregate) risk aversion has been studied in the literature. Yee (2006) studies the link between savings rate, earnings quality, and risk aversion. Others have studied the effect of e.g. war (Gilson et al., 2015), financial crisis (Guiso et al., 2018), liquidity (Gibson and Mougeot, 2004), catastrophe risk (Barro, 2006), uncertainty about government policy (Pastor and Veronesi, 2012), and the consumption surplus ratio (Li, 2007). De Santis and Gerard (1998) utilize financial variables – the dividend yield in excess of the short-term interest rate, the change in the U.S. term premium, the change in the one-month Eurodollar deposit rate, and the U.S. default premium – to model the reward to market risk. For some utility functions, there is also a relationship between relative risk aversion and the overall wealth level.

Here we concentrate on economic conditions and their effect on the reward to market risk coefficient. Using a priming method, Cohn et al. (2015) find that finance professionals are willing to take considerably less risk in a financial bust as compared to a boom, i.e., that risk aversion seems to be countercyclical. As a result, we test the hypothesis that bad economic conditions increase the reward to market risk, and vice versa. Moreover, it is natural to assume that the current economic conditions are not as important as the expectations regarding future development.

Now, it is an empirical and practical question of how to model economic expectations, which, in turn, is used to model the time-variation in the reward to market risk (lambda) parameter. We use three different approaches. The first approach allows lambda to vary freely with the selected conditioning variables that can be argued to be related to economic conditions. The second approach uses the actual knowledge of a recession to measure its effect on risk aversion. The third approach uses forecasting variables to generate a prediction for the probability of a recession. In the first approach, the estimated system is open and the focus is on the economic activity in general terms, whereas the latter approaches focus directly on modeling the economic situation or investors' expectations about it.

In the first approach, the reward to market risk is modeled linearly on variables that have been found to predict the economic activity and that have been readily available to investors throughout the sample. To keep the system tractable, only a few variables are chosen. The first variable is the continuously compounded monthly change in the US industrial production (dIP). The second variable is the U.S. default premium measured as the difference between Baa and Aaa-rated bond yields ($BaaAaa$).[5] The third variable is a measure of the long-end yield curve, namely the difference between 10-year government bond yield and 3-month T-bill rate (LTS). Our fourth variable is a measure of the short-end of the term structure, measured as the difference between 3- and 1-month T-bill rates (STS). All series are downloaded from FRED with the exception of the 10-year government bond yield which is taken from Ibbotson (2019).

We expect an increase in the dIP to lower the risk aversion since higher production signals improvement in the economy. Similarly, an increase in the LTS should be inversely related to risk aversion as a positive term structure has been found to be positively related to future economic activity (c.f., Estrella and Mishkin, 1998). The converse is true for the $BaaAaa$ variable as an increase in the corporate credit risk premium typically forecasts an economic downturn. The expected effect of the STS variables on risk aversion is more ambiguous as the values are partly affected by the decisions made by the Fed and partly by the short-term market anticipations.

Our second approach uses an ex post recession indicator (D_REC) for the US where a value of one indicates a recession period. The indicator is based on the announcements made by the Business Cycle Dating Committee of the National Bureau of Economic Research. Due to the way the indicator is calculated, the values are announced approximately twelve months after the fact.[6] As a result, the potential effect suffers from a look-ahead bias, but we do not consider it a major issue as one can expect investors as a group to be able to nowcast a recession even if it is officially announced afterward.

A recession indicator is only a crude measure of the economic cycles and it may not indicate the time-varying expectations of the investors. Hence, we also want to create a variable that captures investors' expectations. To construct a measure of economic expectations (economic cycles), we estimate the probability of a recession using a probit model with a set of selected information variables. Our approach is similar to Kauppi and Saikkonen (2008); in other words, the full sample is utilized in the estimation instead of a rolling estimation which produces a slight forward-looking bias (see their paper for the discussion). Dynamic and non-dynamic estimation approaches are used. The extracted recession probability estimates are used as one of the information variables.

In practice, we use the following dynamic prediction model for the recessions:

$$y_{t+1} = \delta y_t + x_t' \beta + \epsilon_{t+1}, \tag{12}$$

where y_{t+1} (y_t) is the (lagged) value of D_REC, x_t is a vector of prediction variables, and $\epsilon_{m,t+1}$ is the error term. A non-dynamic version excludes the first term from the right-hand side. For the probit model, we define $\Phi(\hat{y}_{t+1}) = \Phi(\hat{\delta} y_t + x_t' \hat{\beta})$, where $\Phi(.)$ is the normal cumulative density function, which means that $\Phi(\hat{y}_{t+1})$ is the cumulative probability of recession at time t+1.

[5] The NBER provides monthly values that are based on the average of daily data for the whole sample period as well as daily values from 1986 forward. We have augmented the former time series with the month-end values from the latter time series in the estimation.

[6] The NBER does not define a recession in terms of two consecutive quarters of decline in real GDP as it is often done. Rather, a recession is defined as a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale–retail sales. In practice, we have used the indicator series DREC from the FRED.

Table 1
Descriptive statistics.

Variable	Mean	Std. dev.	Skew-ness	Excess Kurtosis	J-B (p-val)	Autocorrelation ρ_1	ρ_2	ρ_3	ADF
Panel A: Main variables (%)									
Equity premium $R_m - R_f$	0.485	5.36	−0.52	6.80	<0.001	0.106*	−0.012	−0.081*	<0.001
Dividend growth rate p.a.	4.334	13.37	−0.99	3.32	<0.001	0.593*	0.574*	0.916*	<0.001
Long-term government bond yield p.a.	4.959	2.62	1.03	0.42	<0.001	0.996*	0.991*	0.987*	0.524
Panel B: Information variables (%)									
Baa-Aaa (BaaAaa)	1.067	0.66	2.42	8.24	<0.001	0.975*	0.939*	0.907*	0.019
Term structure, long (LTS)	1.603	1.28	−0.24	−0.10	0.005	0.961*	0.920*	0.884*	<0.001
Term structure, short (STS)	0.205	30.19	2.56	11.54	<0.001	0.518*	0.424*	0.331*	<0.001
Industrial production index (dIP)	0.259	1.77	0.37	15.44	<0.001	0.532*	0.244*	0.092*	<0.001
Economic Policy Uncertainty Index (dEPU)	0.020	4.82	0.29	0.64	<0.001	−0.309*	−0.073*	−0.095*	<0.001
U.S. Coincident Index (dUSCI)	0.236	0.92	−0.49	7.29	<0.001	0.282*	0.311*	0.212*	<0.001

Descriptive statistics for the main variables (Panel A), and the information variables (Panel B). The main variables are excess market returns, the dividend growth rate as well as the long-term government bond yield. Monthly data are used from January 1928 to December 2018 (1,092 observations) for the USA. All returns, yields, and growth rates are continuously compounded, in percentage form. The dividend growth rate per annum is the continuously compounded growth of the dividends paid during the past twelve months compared to dividends paid a year ago. Long-term government bond yield per annum is taken from the SBBI book. The information variables are the difference between the continuously compounded yield on Baa and Aaa-rated corporate bonds ($BaaAaa$), the 10-year government bond yield minus the 3-month T-Bill rate (LTS), a measure of the short-term yield curve (3-month minus 1-month TB rate, STS), the monthly change in industrial production (dIP), the monthly change in the Economic Policy Uncertainty Index ($dEPU$), and the monthly growth in the U.S. Coincident Index ($dUSCI$). Normality refers to the p-value for the Jarque and Bera (1987) test for normality. Autocorrelation coeffocients are for the first three lags. ADF is the MacKinnon (1996) one-sided p-value for the Augmented Dickey–Fuller test for stationarity. Autocorrelation coefficient estimates significantly (5%) different from zero are marked with an asterisk.

In practice, we utilize the same four variables as before as prediction variables in the empirical analysis. The probit analysis allows us to compute the fitted probabilities for recession. This variable *(PROB_REC)* is used to test whether risk aversion increases when the probability of recession increases. We do the analysis in sample similar to Kauppi and Saikkonen (2008). First, we use a dynamic version of the model in which investors know the prevailing state. Then we utilize a non-dynamic prediction model in which investors do not know the state of the economy. In both cases, similar to Kauppi and Saikkonen, we assume that even though data on recessions are announced with a delay, investors have some understanding of the prevailing economic conditions in real-time.

As a final test, we utilize two variables that also measure economic conditions and market sentiment. The first variable, $dEPU_t$, is the monthly change in the Economic Policy Uncertainty (EPU) Index. The second variable, $dUSCI_t$, is the monthly growth in the U.S. Coincident Index (USCI). The EPU index measures policy-related economic uncertainty as reflected in newspaper articles (for details, see Baker et al., 2016). USCI is a broad measure of the current economic activity. It is closely related to the NBER economic recession indicator, but it is a more refined measure of the US economy's development. It is calculated by the Economic Cycle Research Institute (ECRI).[7] Of course, neither series was available in the past, but as they both reflect matters that were observable to investors, they should be reflected in the reward to market risk.

3.3. Descriptive analysis

Table 1 provides descriptive statistics for the asset returns (Panel A) and the information variables (Panel B). The sample period is from January 1928 to December 2018. The arithmetic mean risk premium is 0.49 per cent per month (or 5.8% per annum), with a volatility of 5.4 per cent per month (18.6% p.a.). The average dividend growth rate is 4.3%. The government bond yields have been, on average, 5.1%.

All time series are non-normally distributed according to the Jarque and Bera (1987) test for normality. The monthly risk premia are negatively skewed and show excess kurtosis. As expected, the monthly risk premium shows fairly low, albeit significant, positive first-order autocorrelation. The dividend growth rate shows fairly high autocorrelation (0.594) as expected, as does the government bond yield series (0.996).

Before using the information variables in the estimation, we test whether they are stationary using the augmented Dickey–Fuller test with a constant and four lags. MacKinnon (1996) one-sided p-values are reported in Table 1. The results show all but one of the tests reject the null hypothesis of unit root (non-stationarity). As such, the long-term government bond yield series shows nonstationarity (p is 0.524), but since we using the first difference in the series, the nonstationarity disappears.

Finally, to study whether the information variables for the recession predictability analysis show multicollinearity and whether some of them might be redundant, we calculate their cross-correlations (not reported). They are all quite low – the highest cross-correlation is between $BaaAaa$ and LTS (0.32).

[7] The series is available on their website at https://www.businesscycle.com/.

Table 2
Recession model results.

	Constant	y_{t-1}	$BaaAaa_{t-1}$	LTS_{t-1}	STS_{t-1}	dIP_{t-2}	McFadden R^2
Non-dynamic model	−1.478***		69.075***	−18.041***	0.063	−35.157***	0.258
	(−12.807)		(7.859)	(−4.087)	(0.397)	(−9.850)	
Dynamic model	−1.909***	3.594***	19.417	−29.994***	−0.247	−5.326	0.769
	(−10.952)	(16.031)	(1.441)	(−3.801)	(−0.980)	(−0.939)	

Quasi-maximum likelihood estimates for the probit model under different model specifications are reported. In-sample estimation is conducted using the full sample from January 1928 to December 2018 (1,092 observations). In the estimation, the recession indicator (D_REC) is regressed against four lagged information variables. The information variables are the difference between the continuously compounded yield on Baa and Aaa-rated corporate bonds ($BaaAaa$), a measure of the short end of the yield curve (3-month minus 1-month TB rate, STS) as well as the long-end of the yield curve (10-year government bond yield minus 3-month T-Bill rate, LTS), and the monthly change in industrial production (dIP). Non-dynamic and dynamic versions of the model are estimated. In the latter model, the lagged realized value for the dependent variable appears as an additional explanatory variable. McFadden R^2 is also reported. t-values are provided below the parameter estimates in parentheses. Coefficient estimates significantly (10%, 5%, or 1%) different from zero are marked with one, two, or three asterisks, respectively.

3.4. Probit estimation

Table 2 presents results for the probit model based on Eq. (12). All prediction variables are lagged by one month with the exception of the industrial production which is lagged by two months to allow for publication lag. This means that when we model the recession indicator for month $t+1$, we utilize the information that is available at the end of month t. We report the parameter estimates first for the non-dynamic model which does not utilize information on the concurrent recession, and then for the dynamic model. In addition to the parameter estimates, an adjusted McFadden R^2 is reported.

The results show, as expected, that the dynamic version is much better at capturing the recession periods, as the knowledge of the past state of the economy is a natural predictor for the next period. Higher credit yield spread and decreasing industrial production are positively related to the recession probability with the non-dynamic version of the model. A steep long-end yield curve is negatively related to future recessions in both versions of the model. Fig. 3 shows the development of the fitted probabilities from both estimations.

Although we utilize the probabilities from both dynamic and non-dynamic versions of the probit model, our main interest is with the latter version. The dynamic model uses information available after the fact and thus it is unlikely to give false signals. In reality, investors may anticipate recessions that do not materialize after all, yet they affect investors' behavior.

4. Empirical results

4.1. Preliminary analysis

We begin our empirical analysis by testing whether the equity premium is linearly related to volatility or variance. To keep the estimation as simple as possible, we test Eq. (1) with a constant lambda and δ set to one or two for volatility and variance, respectively. APARCH(1,1) is used for the volatility and variance processes. In the traditional approach, either the variance or its square root is used in the mean equation to find out whether the market rewards variance or volatility. In all cases, we assume the disturbances to be normally distributed. The results are reported in Table 3.

The volatility and variance process parameter estimates are significant in all four cases. There is clear evidence of asymmetry in the risk process as a response to negative shocks in returns. However, in line with earlier studies, the lambda parameter estimates from the traditional approach are clearly lower than suggested by the theory, and they are statistically insignificant in both cases, suggesting that neither volatility nor variance is priced in the market. We cannot make conclusions about which one of the models is more appropriate for the returns. The new reverse estimation approach, on the other hand, shows significant reward to market risk parameter estimates using both measures of the risk. The data provides stronger support for the pricing of volatility risk, though, as the explanatory power of the model is clearly higher (47.8% vs 25.4%) than of variance risk. To run a horse race between volatility and variance, we re-estimate the model with both measures of risk (c.f., Glosten et al., 1993). The result (not reported) is clear, only volatility is statistically significant which supports the notion that for practical purposes market prices volatility rather than variance.

Obviously, one can raise the issue that the comparison between the traditional and the new approaches is not fair as it stands at least for two reasons. First, the new approach is based on a two-step estimation utilizing linear regression which could make it easier to gain significant lambda estimates. To study this, we re-estimate the traditional model (Eq. (3)) in two steps with linear regression, where we use volatility/variance forecasts from the APARCH as the explanatory variable. The results are similar to those reported in Table 3 – the lambda estimate is not even close to being significant. Thus, the failure of the traditional approach cannot be attributed to the estimation approach.

Second, the new approach is a joint test of the asset pricing model and convergence in the risk process. To study this, we set the risk persistence parameter to one instead of the estimated value of 0.957. It implies that every time the risk level changes, investors use the new required return from there on *ad infinitum* – the same assumption implicitly built into the traditional testing approach.

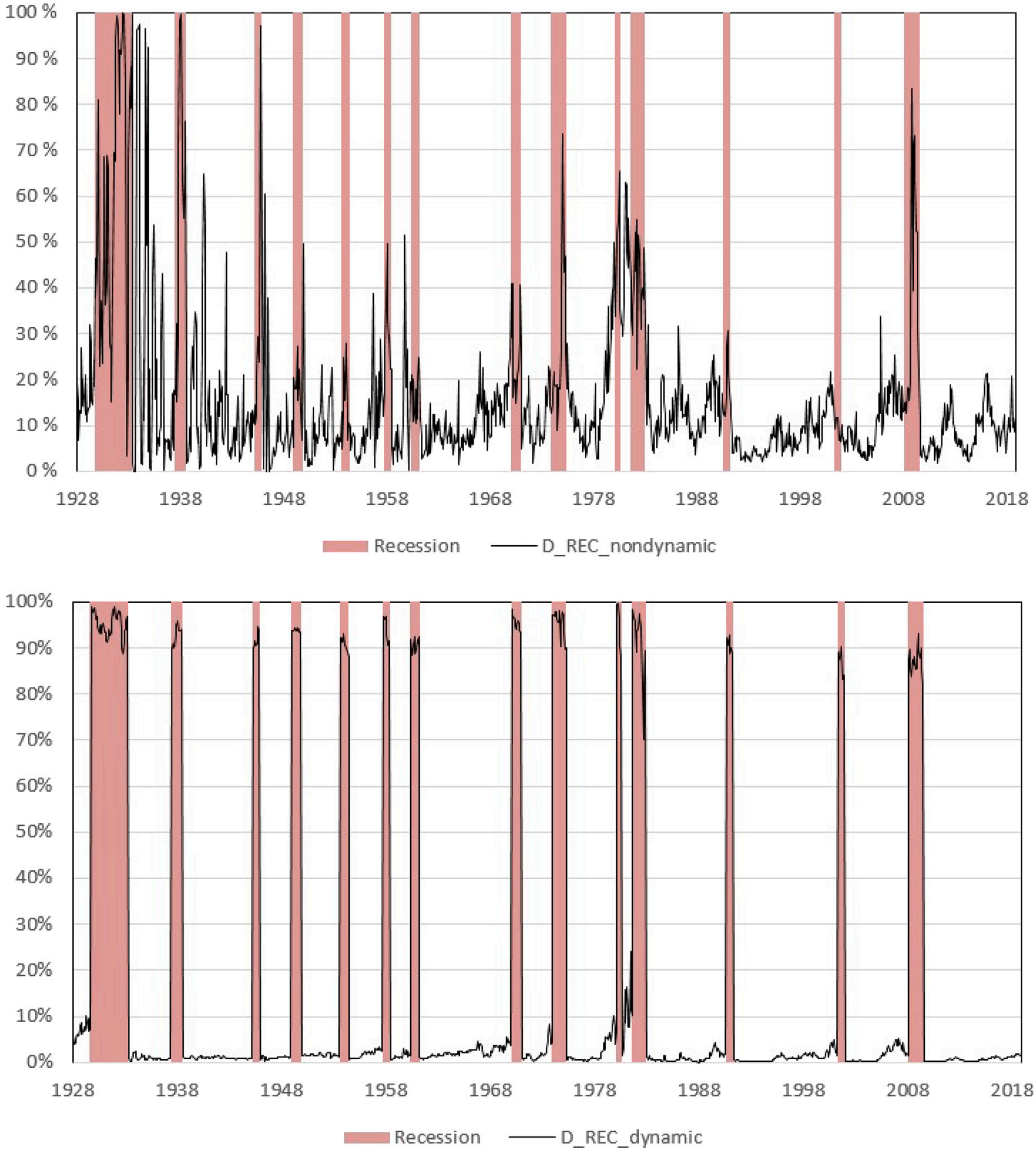

Fig. 3. Fitted probabilities for a recession from non-dynamic (3a, above) and dynamic (3b, below) probit models estimated with four conditioning variables. The dynamic model uses previous realizations in the model. Monthly US data from January 1928 to December 2018 are used in the estimation. Recession periods as indicated by NBER are marked with colored background. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The results (not reported) show that this assumption does not reduce the statistical significance of the lambda estimates. However, the results show interestingly that the assumption of convergence in risk has a meaningful impact on lambda. For example, when one uses volatility as a measure of the risk, the lambda estimate becomes 0.020 which is clearly lower than the 0.293 reported in Table 3. If one takes the average Sharpe ratio for the sample period (0.090) as a proxy for the mean value for the price of risk, the result suggests that investors do take convergence in risk into account, i.e. persistence has to be less than one, but at the same

Table 3
Volatility vs. variance and the equity premium.

	Traditional approach		New approach	
	Volatility σ	Variance σ^2	Volatility σ	Variance σ^2
Panel A: Mean equation				
Constant α	0.004	0.006***	−0.010**	0.003*
	(1.182)	(2.890)	(−7.605)	(1.734)
λ_0	0.018	0.161	0.293***	0.894***
	(0.179)	(0.197)	(36.254)	(8.630)
Adj. R^2	−0.001	−0.001	0.478	0.255
Panel B: Volatility/variance equation				
ω	0.002***	0.000***	0.002***	0.000***
	(5.190)	(4.968)	(5.730)	(5.091)
α_1	0.124***	0.109***	0.124***	0.109***
	(8.561)	(5.775)	(8.716)	(5.789)
γ_1	0.465***	0.236***	0.469***	0.239***
	(4.320)	(2.774)	(4.450)	(2.822)
β_1	0.857***	0.850***	0.858***	0.850***
	(65.089)	(56.551)	(75.047)	(57.652)
Log-likelihood	1819.100	1810.509	1819.077	1810.491

Estimates for the constant price of market risk and relative risk aversion models are reported from the traditional and the new estimation approaches. The price of market risk model assumes investors trade off return and volatility, whereas the reward to market risk model assumes that investors trade off return and variance. Panel A reports parameter estimates for the traditional and new testing approaches. For the traditional approach, volatility or variance in the mean equation has been used. For the new testing approach, the parameter estimates are from the second stage OLS with Newey–West (1987) robust standard errors. Panel B reports parameter estimates for the volatility (variance) process in the second and fourth (third and fifth) columns. The APARCH power parameter is set to one or two for the conditional volatility and variance processes, respectively. Normality is assumed. Excess US continuously compounded returns (CRSP total return index) from January 1928 to December 2018 (1,092 observations) are used. Adjusted R^2 is calculated for the mean equation. Log-likelihood is for the risk process. Coefficient estimates significantly (10%, 5%, or 1%) different from zero are marked with one, two, or three asterisks, respectively.

time persistence used by the market could be higher than that implied by the APARCH process for volatility to match the observed Sharpe ratio.[8]

Since one of our main interests is to study the argument that the reward to risk is time-varying, we conduct a rolling window estimation as our final preliminary analysis. We do this with a sixty-year estimation window (720 observations) utilizing both the traditional and the new estimation (Eq. (11)) approaches while using variance as our measure of risk (with volatility the results are quite similar). To keep the estimation as simple as possible and to minimize the risk of convergence issues, we assume here that the reward to risk is constant in each estimation and use the GARCH(1,1) model with normally distributed errors for the variance. The time-series evolution of the lambda estimates can be seen in Fig. 4.

Analyzing the results, one can make several conclusions. First, most of the lambda estimates with the traditional and some with the new approach are not significantly different from zero. This suggests that the traditional approach – with the ‘in-mean’ specification – is sensitive to the sample period and requires careful attention to the estimation setup, whereas the new estimation approach is more robust to the sample period. Second, it is clear from Fig. 4 that lambda is time-varying regardless of the estimation approach when one uses variance as a measure of risk. Finally, during the Second World War, the reward to risk has been higher, potentially reflecting investors’ higher risk aversion during wartime (c.f., Gilson et al., 2015). The rolling estimation approach, however, can only be applied after the fact and as such it does not utilize the most recent information to the fullest. Hence, in order to get a more detailed view of the behavior of the reward to risk and how the economic cycles affect it, we proceed to estimate the conditional model.

4.2. Time-varying reward to market risk and the traditional estimation approach

We first study the time-variation of lambda using the traditional estimation approach. As the results in Table 3 did not favor either of the risk measures in the traditional testing approach, we use, for simplicity and comparison with earlier studies, variance as the relevant measure of risk. To induce the time-variability into lambda, we combine Eqs. (2) and (3) and utilize variance estimates from the variance process. In effect, we estimate a GJR-GARCH in mean model with a time-varying coefficient for the explanatory variable (variance) in the mean equation. Researchers have either used multiple-step estimation or estimated the model using maximum likelihood, which requires a joint likelihood function. Here we choose the latter approach.[9] The parameter estimates

[8] For the variance process, the estimated persistence is 0.966.

[9] Since no statistical program, as far as we know, offers this as an option out of the box, we use the ARMAX-GARCH-K-SK Toolbox 1.5 for Matlab by Alexandros Gabrielsen in the estimation. The toolbox is available on the Mathworks website. We have modified the routines to allow for an ‘in-mean’ specification. All other estimations are done with Eviews.

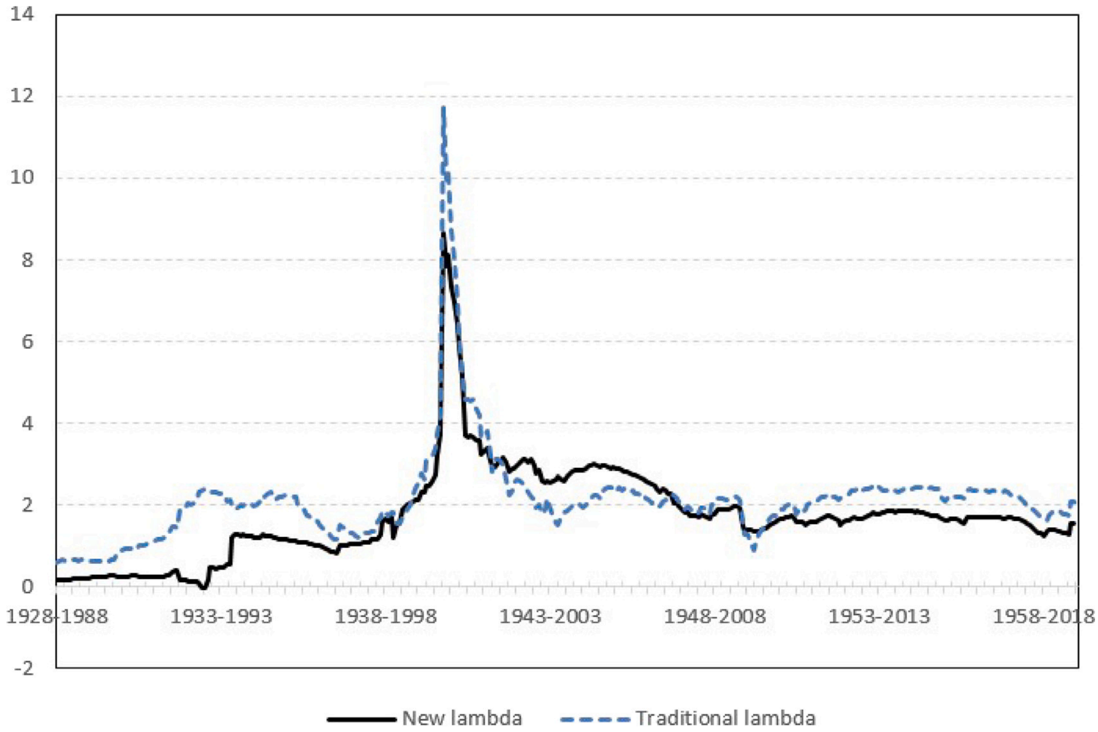

Fig. 4. Reward to risk (risk aversion) estimates from the traditional and the new estimation approaches using a sixty-year rolling window. The variance is modeled as a GARCH(1,1) process with normal distribution. Years on the x-axis indicate the sample period.

from the unconditional lambda model are used as initial values for the estimation of the conditional version of the model while the initial values for the conditioning variables' coefficients are set to zero. Conditioning variables are demeaned with the exception of the recession indicator variable.

Our initial finding is that it is very challenging to estimate the model jointly with more than one conditioning variable for the reward to market risk even with our rather long, 90-year sample. And even with one conditioning variable, there are a number of issues and caveats that have to be taken into account.[10] As a result, to test the main hypothesis, we let the reward to market risk (relative risk aversion) vary with only one variable that reflects the overall expectations regarding economic development. This makes it possible to estimate the model in one step. The results are reported in Panel A of Table 4.

Using a recession indicator, D_REC, as a proxy for the economic expectations, our estimate for the lambda parameter is 0.769 (t-value 0.891) and it is still not significant. The coefficient estimate for the recession indicator is unexpectedly negative, −0.067 (t-value −1.602), indicating a relationship that is opposite to economic theory, i.e., that recessions induce a lower, not higher, risk aversion. Of course, the estimate is not significant, and the unexpected relationship could be due to the fact that the recession variable is only known after the fact and it may not capture investors' expectations of economic conditions. Thus, we estimate the model again with the forecasted probability for a recession, Pr_REC.

The model is estimated separately using probabilities from dynamic and non-dynamic probit models. With the dynamic and non-dynamic recession forecasts, the results are similar to the actual recession indicator (the λ_0 estimates are 0.771 and 0.771 and neither is significant). Similarly, lambda is estimated to be lower when the probability of a recession increases (both λ_1 estimates are −0.009) but neither of them is significant. Finally, we estimate the model using $dEPU$ and $dUSCI$. Again the results prove challenging for the traditional approach; the coefficient estimates are not significant.

Overall, the explanatory power of the asset pricing model is low when tested with the traditional estimation approach, regardless of the model chosen, with an adjusted R^2 very close to zero. As summary, the results from the traditional estimation approach do not provide evidence in favor of time-varying reward to market risk, nor for its counter-cyclical behavior.

4.3. New approach and the baseline model

Next, we turn to the reverse testing approach. Although the results suggested that investors price volatility instead of variance, we proceed in estimating our baseline model (11) using both measures of risk. The estimation proceeds in two steps. In the first

[10] For example, the estimation results are sensitive to the starting values. In addition, achieving convergence in the estimation seems to be sensitive to the optimization method and, more so, the more complicated models one estimates.

Table 4
Time-varying reward to risk.

	D_REC_{t+1}	Pr_REC_{t+1}	DPr_REC_{t+1}	$dEPU_t$	$dUSCI_t$
Panel A: Traditional approach (GJR-GARCH variance)					
Constant α	0.005	0.004	0.004**	0.005**	0.005**
	(0.517)	(0.919)	(2.454)	(2.762)	(2.796)
λ_0	0.769	0.771	0.771	0.430	0.550
	(0.891)	(0.358)	(1.308)	(1.308)	(0.708)
λ_1	−0.067	−0.009	−0.009	−0.004	0.228
	(−1.602)	(−0.534)	(−0.841)	(−0.342)	(1.359)
Adj. R^2	0.004	−0.007	−0.006	−0.006	−0.004
Panel B: New approach (APARCH variance)					
Constant α	0.003*	0.003*	0.003*	0.003*	0.003*
	(1.811)	(1.776)	(1.852)	(1.744)	(1.785)
λ_0	0.868***	0.875***	0.865***	0.890***	0.897***
	(8.209)	(8.207)	(7.863)	(8.675)	(8.550)
λ_1	0.028**	0.030*	0.053*	0.040*	−0.902**
	(2.107)	(1.765)	(1.936)	(1.666)	(−2.250)
Adj. R^2	0.259	0.258	0.269	0.256	0.284
Panel C: New approach (APARCH volatility)					
Constant α	−0.010***	−0.010***	−0.010***	−0.010***	−0.010***
	(−7.205)	(−7.333)	(−7.238)	(−7.171)	(−7.220)
λ_0	0.291***	0.291***	0.290***	0.292***	0.291***
	(35.122)	(36.386)	(32.168)	(35.218)	(35.172)
λ_1	0.001	0.002***	0.001	0.001*	−0.028
	(1.019)	(2.681)	(1.395)	(1.683)	(−1.611)
Adj. R^2	0.478	0.482	0.484	0.479	0.487

Estimates for the conditional reward to risk parameters are reported using the traditional and the new estimation approaches under different model specifications. The conditional reward to risk is modeled linear on the variable given in the first row. Conditioning variables have been demeaned (except for the recession indicator, D_REC). The variable Pr_REC and DPr_REC are the in-sample probability of a recession from non-dynamic and dynamic probit models, respectively. The variable $dEPU_t$ is the monthly change in the Economic Policy Uncertainty Index. The variable $dUSCI_t$ is the monthly growth in the U.S. Coincident Index. GJR-GARCH/APARCH assuming normality is used for the variance/volatility as indicated in the panel headings. Excess US continuously compounded returns (CRSP total return index) from January 1928 to December 2018 (1,092 observations) are used. Adjusted R^2 is for the mean equation. Coefficient estimates significantly (10%, 5%, or 1%) different from zero are marked with one, two, or three asterisks, respectively.

step, we estimate the parameters for the APARCH process with delta set to one (volatility) or two (variance) using only a constant in the mean equation. We then use the results to calculate the convergence parameter (ϕ) and the long-term unconditional value for the risk process, as well as the sigma multipliers as defined earlier. In the second step, we estimate a linear regression model according to Eq. (11) and again with the information variables demeaned. The results are reported in Panels B and C of Table 4. As there are indications of first-order autocorrelation in the residuals, we use the Newey–West (1987) adjustment for autocorrelation and heteroskedasticity with automatically selected fixed bandwidth and Bartlett kernel for the standard errors of the OLS parameter estimates.

In line with the traditional approach, the estimated risk process parameters (not reported) are always significant. However, in contrast to the results in Panel A, the lambda estimates are significant in all cases regardless of the risk measure. The explanatory power is also considerably higher than for the corresponding traditional model, although again more so when one uses volatility as the measure of risk.

The results in Panel B provide support for time-varying reward to market risk (relative risk aversion), using variance as the risk measure. Our first candidate variable for the time-variability is the explicit measure for the recession, D_REC. We can clearly see that risk aversion is significantly and positively related to a recession. The impact 0.028 (t-value 2.107) is not large in economic terms, but it confirms the theoretical expectation. When we use other variables to model expectations on economic conditions, the results show a positive relationship (although at the 10 percent level) between the probability of a recession and risk aversion with dynamic and non-dynamic estimates for the recession, respectively. Our last two variables, the monthly change in the Economic Policy Uncertainty (EPU) Index and in the US Coincident Index (USCI), also show results that are in line with the expectations. Higher uncertainty of the economic policy increases lambda (coefficient estimate 0.040) although this value is significant again only at the ten percent level (t-value 1.666). If the economic activity increases, the risk aversion decreases (coefficient estimate −0.902 with t-value −2.250). However, the economic significance of the time-variation in lambda is smaller than expected. For example, with the conditioning variable $dUSCI$, the conditional lambda varies from 0.850 to 0.945 (a range of 11.2%) depending on the value for $dUSCI_t$. Moreover, analyzing the variance of the expected premium, the proportion of variance due to the time-varying lambda (variance) is less than one percent (98%), and the rest is due to their cross-effect.[11]

[11] Here we have used the fact that for two variables, x and y, we can write $Var(xy) = \bar{x}^2 Var(x) + \bar{y}^2 Var(y)$+cross-terms, where x and y correspond to conditional lambda and variance, respectively.

The results in Panel C differ interestingly regarding time-variation in lambda from those in Panel B. Using volatility as the measure of risk, the evidence in favor of time-varying reward to market risk (price of market risk) is much weaker. With the same conditioning variables as in Panel B, we find most of them with matching signs but insignificant. The only variable found significant is the forecasted probability of recession from the non-dynamic probit model implying a higher price for the market risk ahead of the perceived notion of a forthcoming recession. Its coefficient (0.002 with t-value 2.681) is small compared to that of the lambda (0.291) indicating again that the economic significance of the time-variation in lambda is likely to be smaller than previously thought.

Now, the difference in conditioning variables' impact on the reward to risk parameter under volatility and variance asks for an explanation. We argue that there is a natural reason for it. Namely, as shown earlier, the risk premium is more likely to be linear on volatility rather than variance. Thus, using a nonlinear model (volatility squared) instead of a linear one, one can find statistically significant results for the conditioning variables even when there are none.[12] This result implies that the price of risk is quite stable, or that perhaps it is not driven by the variables related to economic seasonality. For this reason, we test the model using a totally different exogenous variable as the conditional variable. As Fig. 4 implies, a war would be a natural cause for investors to increase their price of risk. As a result, we create an indicator for the Second World War (i.e. December 1939–August 1945) and re-estimate our model using the war indicator as the conditioning variable. The results (not reported) show that the war indicator is significant with an estimated coefficient of 0.003 (with t-value 7.219). The increase in the price of risk is, however, economically quite small, as the increase in lambda is only 1.04% resulting in a 0.194% higher risk premium per annum *ceteris paribus* if measured using the average conditional volatility during the same period, 5.40% per month, surprisingly not much higher than the sample average.

4.4. Full model

Finally, we test our full model as given by Eq. (10). As before, we estimate the model using linear regression with conditional volatility and variance estimates from the first-pass estimation. The results using the volatility are reported in Table 5. Results with the variance are available upon request.

The results show that the long-term (unconditional) lambda estimate is statistically significant and basically at the same level as before. As far as the time-variability in lambda concerns, again only the variable measuring the probability of a recession from the non-dynamic probit model is significant. The second main explanatory variable, the change in the risk-free rate, is significant with a positive coefficient estimate, as the model implies. The third main explanatory variable, the change in the expected dividend growth rate, also loads up with a positive coefficient estimate as expected, although it is insignificant. It could be that the chosen variable does not capture, for example, the turning tide in investors' growth expectations fast enough. To study this, we estimate our model with realized dividend growth calculated using dividends for one month (instead of twelve months) and comparing it with the dividend a year ago. From there on we proceed as before. In addition, we estimate ARMAX(1,1) based forecasts for the dividend growth using our original dividend growth series and log dividend-to-price ratio as the forecasting variable. This last approach is motivated by Chen (2009) and Binsbergen and Koijen (2010). The results (not reported) from both cases are in line with the reported results. This goes on to show the difficulties in predicting dividend growth.

Another explanation for the insignificant coefficient estimate for the Δg variable could be our decision to use the normal distribution with the APARCH. It is well known that a t-distribution could be a more realistic one. Hence, we re-estimate the models in Table 5 with t-distributed errors.[13] The results (available upon request) are in line with the reported results with two main differences. First, the estimates for the unconditional lambda are somewhat higher, above 0.4, whereas the time-variability parameter estimates remain pretty stable. For example, for the first model in Table 5, the lambda estimate is 0.332 (t-value 43.587). Second, the explanatory power of the model increases approximately by five to ten percentage points. For the first model in Table 5, the adjusted R^2 is 55.3% – an increase of 12.2 percentage points. The coefficient estimate for the change in the dividend growth remains insignificant, but the t-value is slightly higher (1.334).

Overall, the explanatory power of the full model is always slightly higher than that of the corresponding baseline model. As the lambda estimates are basically the same as for the baseline model, the results indicate that including or excluding the risk-free and dividend growth components does not influence the main conclusion regarding the asset pricing model. The variation in realized returns is mostly driven by the changes in the required risk premia.

4.5. Additional analysis

Although the goal here is not trying to predict realized returns per se – there are other types of models for that purpose[14] – it is of interest to see how the asset pricing model performs, especially out of sample. To test this, we utilize data from January 1928 to December 1947 to estimate the model using both the traditional and new approaches with an APARCH process for the market risk. Using the estimated parameters, we calculate the expected risk premium for January 1948 and compare it against the

[12] To demonstrate this, we make the following trial statement for random time series variable: $R_t = R_t\sigma_t/\sigma_t = \lambda_t\sigma_t$. On the other hand, we can also state $R_t = R_t\sigma_t^2/\sigma_t^2 = \gamma_t\sigma_t^2$. Now, to make the statements economically meaningful, one needs a theory to provide a reduction in dimensionality for the lambda or the gamma parameter. Say, a theory suggests that λ is constant, then the implied gamma parameter is time-varying by construction (i.e., constant lambda divided by time-varying volatility).

[13] As a result, the speed of convergence parameter changes. Its new value is 0.949.

[14] Two recent examples with references to literature in the area include Batten et al. (2022) and Kuok and Ausloos (2022).

Table 5
Full model with time-varying price of market risk.

	Constant	D_REC_{t+1}	Pr_REC_{t+1}	DPr_REC_{t+1}	$dEPU_t$	$dUSCI_t$
Constant α	−0.010***	−0.010***	−0.010***	−0.010***	−0.010***	−0.010***
	(−7.522)	(−7.500)	(−7.472)	(−7.286)	(−7.495)	(7.493)
λ_0	0.289***	0.288***	0.287***	0.286***	0.289***	0.287***
	(35.272)	(34.639)	(35.152)	(31.079)	(34.747)	(34.252)
λ_1		0.001	0.002***	0.001	0.001	−0.029
		(0.933)	(2.773)	(1.393)	(1.630)	(−1.621)
Δg	0.540	0.539	0.525	0.532	0.542	0.543
	(1.040)	(1.041)	(1.025)	(1.039)	(1.044)	(1.056)
Δr_f	1.120**	1.095**	1.173**	1.105**	1.092**	1.163**
	(2.159)	(2.107)	(2.220)	(2.138)	(2.098)	(2.230)
Adj. R^2	0.481	0.481	0.485	0.484	0.482	0.491

OLS estimates for the unconditional (column 1) and conditional price of market risk are reported using the new estimation approach. The conditional price of market risk is modeled linear on the variable indicated. The variables are explained in Table 4. APARCH(1,1) assuming normality is used for the volatility process. Excess US continuously compounded returns (CRSP total return index) from January 1928 to December 2018 (1,092 observations) are used. Δg is the first difference in the expected dividend growth rate per annum proxied by the Hodrick–Prescott filtered trend in realized growth rates. Δr_f is the first difference in the long-term US government bond yield. The Newey and West (1987) adjustment is used to calculate the standard errors. t-values are provided in parentheses. Coefficient estimates significantly (10%, 5%, or 1%) different from zero are marked with one, two, or three asterisks, respectively.

Table 6
Risk premium prediction.

Approach	Mean (%)	SD (%)	SSE	R_2 (%)	Smaller SE (%)
Realized average	0.540	4.248			
Historical average	0.489	0.070	1.550		
Traditional approach (σ^2_{t+1}, λ)	0.009	0.103	1.573	−1.47	41.2
Traditional approach (σ_{t+1}, λ)	−0.007	0.020	1.573	−1.48	41.4
New approach (σ^2_{t+1}, λ, baseline)	0.141	0.110	1.558	−0.51	42.5
New approach (σ^2_{t+1}, λ, full)	0.130	0.102	1.559	−0.57	42.5
New approach ($\sigma^2_{t+1}, \lambda_{t+1}$, full)	0.142	0.111	1.558	−0.50	42.6
New approach (σ_{t+1}, λ, baseline)	0.970	0.386	1.563	−0.84	51.6
New approach (σ_{t+1}, λ, full)	0.954	0.380	1.562	−0.74	51.2
New approach ($\sigma_{t+1}, \lambda_{t+1}$, full)	0.946	0.377	1.561	−0.69	51.6

This table presents forecast statistics for the realized risk premium using different models for expected market premium. In-sample estimations are done initially using a sample of 20 years from January 1928 to December 1947 after which the in-sample is extended by a month in each estimation. Out-of-sample (OOS) forecasts are always for the first OOS month and they based on the in-sample estimations. Forecasts are done from January 1948 to December 2018. Realized average is for the OOS period. The historical average is calculated using the OOS forecasts based on the in-sample average. All other models are based on APARCH(1,1) for variance σ^2 or volatility σ with normal distribution with either constant or time-varying lambda. The full model adds Δg and Δr_f in the estimation. Models with time-varying lambda λ_{t+1} use $dUSCI_t$ as a conditioning variable. The forecasted premium is based on (conditional) lambda times the volatility or variance forecast. Mean and SD are the average and standard deviation (both % per month). SSE is the out-of-sample sum of squared errors for the forecast. Out-of-sample R-squared and percentage of months with smaller error are calculated against the historical average approach.

realized value. Then we extend the sample by one month and repeat the analysis one month at a time until the end of the sample.[15] Estimations are done using both volatility and variance as measures of market risk. For the new approach, we utilize the baseline model (Eq. (11)) and the full model (Eq. (10)) with and without time-varying lambda. If time-varying lambda is used, we utilize $dUSCI_t$ in the estimation. Note that in all cases, the forecasted risk premium is estimated using Eq. (1), i.e. (conditional) lambda times the market risk forecast.

The results are reported for a select few combinations in Table 6. Following Campbell and Thompson (2008) we also estimate the historical average and compare our results for the pricing model with it using three metrics: sum of squared errors (SSE), out-of-sample R-squared, and the percentage of months with smaller squared error against the historical average. Positive values for the R-squared indicate that the selected model performs better than the historical average. Values above 50% for the months with smaller squared errors indicate the same. Finally, we report the average monthly risk premium (forecast) and its standard deviation.

We can clearly see that the historical average does expectedly a good job of predicting the risk premium as shown in Campbell and Thompson (2008). The sum of squared errors is the best of all models. On average, its forecast of 0.489% is slightly below the realized out-of-sample average of 0.540% per month. The traditional approach, on the other hand, does not do well in any of the metrics reported. Its forecasts are, on average, way too low (even negative −0.007% for the volatility model). Its out of sample R-squares are also the worst of all the tested models and it can beat the historical average in accuracy only in 41.2% (41.4% for

[15] Extending sample estimation was selected instead of the rolling estimation to improve risk process estimation.

the volatility model) of the months. If we utilize the variance risk process and estimate lambda using the new approach, we can see that the results improve slightly, the expected premium is more realistic, and the model is slightly better in terms of accuracy on a monthly basis. However, if we utilize the volatility risk process, we can beat the historical average in accuracy on a monthly basis in terms of squared errors, although the historical average still shows the best overall forecasting accuracy.[16] But, as said, the role of asset pricing models is not to forecast accurately realized returns — on efficient markets, it is not even possible beyond the return justified by the risk.

5. Summary and conclusions

In this paper, we utilize the reverse testing approach developed in Antell and Vaihekoski (2019) to study the relationship between the equity market risk premium and volatility and variance using the conditional asset pricing model of Merton (1980). The reverse testing approach is based on the idea of deriving an equation for the realized returns conditional on the tested asset pricing model being true. Its main advantage is that one does not need to utilize a proxy for the expected returns. Here our main interest is to study whether the equity premium is related to volatility or variance, whether the reward to risk is time-varying, and whether it behaves in a counter-cyclical fashion. We compare our results with the traditional estimation approach which uses realized returns as a proxy for the expected returns.

Tests are conducted using monthly data for the equity premium on the US stock market from 1928 to 2018. In the empirical tests, we utilize different ways to estimate economic expectations that are related to cyclicality in the economy. For robustness, we also utilize different specifications for the volatility and variance processes. The traditional estimation approach cannot find a positive risk-return relationship regardless of the risk process chosen. Similarly, it cannot provide support for the hypothesis of time-varying reward to risk or its counter-cyclical behavior.

The results from the new estimation approach, on the other hand, give strong support for a positive risk-return relationship. Investors can expect to receive higher returns if they are willing to take on more market risk. A horse race between volatility and variance shows clear evidence in favor of volatility. As such, the results suggest that there is a linear relationship between expected return and volatility, not variance. Thus, earlier evidence of time-variation in the relative risk aversion could be due to using variance instead of volatility as the measure of risk. The results also support the notion that investors take into account convergence in risk over time. In addition, the variation in the price of market risk is smaller than perhaps thought *a priori*. As a result, the data are consistent with the idea that the main driver for the variation of the expected risk premium is the conditional volatility, not the time-variation in the reward to risk.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Appendix A. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.ribaf.2023.102017.

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[16] An Internet Appendix to this article shows a figure with realized premia against forecasts from the historical average approach as well from the last model in Table 6.

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