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Suitability of the Fama & French Five-Factor Model in European Stock Markets (2014-2023)

A decade of statistical analysis across 32 European portfolios

Accounting and Finance
Bachelor's thesis

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The operating environment of financial markets has undergone significant changes in the recent decade, including the impact of events such as the COVID-19 pandemic and a substantial shift in interest rates. These changes have led to fundamental changes in the market environment. The five-factor model by Fama and French (2015) is one of the newest widely noticed asset pricing models which was developed to explain the cross-section of asset returns through factors. Leveraging insights of previous studies and refining the existing three-factor model Fama and French incorporate excess market returns, size, value, profitability, and investment factor to achieve improved explanatory power driving the excess returns of assets. The suitability of these factors can be precisely and comprehensively researched by analyzing portfolios including the elements of the five-factor model. This study aims to determine whether these factors are suitable for explaining the variances in portfolios' excess returns in the changed market environment in European stock markets.

The research delves into the performance of 32 European portfolios. By examining the individual returns of the portfolios and explanatory factors, it is possible to investigate the characteristics of the model's components and deepen our understanding of the selected factors' effectiveness. The model's suitability is first analysed using OLS regression, from which we obtain the degrees of explanation, beta coefficients and alphas. The regression results suggest a relatively high adjusted degree of explanation, but examining the significance of the factors individually it is revealed that that many of the factors have very limited statistical significance. To deepen our understanding, we use a test developed by Gibbons et al. (1989), also known as the GRS test, to assess the model's suitability as an explanatory variable of systematic risks. Combining statistical insights from the GRS test with comparisons of the regression results, we can provide objective analysis and multifaceted perspectives on the results.

The results can be interpreted in various ways. The adjusted degree of explanation of the five-factor model suggests that model has explanation power to the portfolios. However, the GRS test rejects the hypothesis that the five-factor model could capture all underlying systematic risks from the variance of the portfolios' excess returns. Also, the results for the statistical significance of the factors' beta-values, measured by p-values, were found weak. Nevertheless, the model's suitability empirically in the European stock markets have not decreased under the changed market environment and the results were in line with the previous research. Therefore, this research supports previous findings that portfolios' excess returns can be partly explained by the model's factors, but further research into the correct factors should be continued to make the results durable under empirical scrutiny. The model's weak statistical results can also be interpreted as that a factor-based approach may be inappropriate for empirically explaining the variance in excess returns on European stock markets.

Key words: Asset Pricing, Fama & French, five-factor model, European stock markets, international testing, GRS test, OLS regression, explaining portfolio excess returns.

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Rahoitusmarkkinoiden toimintaympäristö on kokenut huomattavia muutoksia viimeisen vuosikymmenen aikana. Vaikuttaviin tekijöihin ovat lukeutuneet COVID-19 pandemia sekä merkittävästi muuttunut korkoympäristö, jotka ovat saaneet aikaan fundamentaalisia muutoksia markkinaympäristössä. Faman ja Frenchin (2015) viisifaktorimalli on yksi uusimmista laajaa huomiota saaneista hinnoittelumalleista, joka kehitettiin selittämään kohde-etuksien poikkileikkaustuottoja faktorien avulla. Viisifaktorimalli sai inspiraatiota aiemmista tutkimuksista sekä sen avulla uudistettiin jo kehitettyä kolmefaktorimallia ottamalla huomioon markkinoiden ylituotot, sekä kohde-etuuden markkina-arvon, arvostuksen, tuottavuuden sekä investoinnit selittämään ylituottojen varianssia. Näiden faktorien sopivuutta ylituottojen selittäjänä voidaan tarkasti ja kokonaisvaltaisesti tutkia analysoimalla portfolioiden ylituottoja, joiden muuttujina käytetään selittäviä tekijöitä mallin faktoreista. Tämän tutkielman tavoitteena on selvittää, soveltuvatko nämä faktorit portfolioiden ylituottojen varianssien selittäjiksi muuttuneessa markkinaympäristössä eurooppalaisilla osakemarkkinoilla.

Tutkimuksessa tarkastellaan 32:n eurooppalaisen portfolion suorituskykyä. Portfolioiden ja selittävien tekijöiden yksittäisiä tuottoja tarkastelemalla voimme tutkia mallin komponenttien ominaisuuksia ja syventää ymmärrystämme valittujen tekijöiden suorituskyvystä. Mallin soveltuvuutta portfolioihin analysoidaan ensiksi OLS-regression avulla, jonka tuloksista saadaan selitysasteet, regressiokertoimet, eli beetat sekä alfat. Regressioanalyysin tulokset viittaavat suhteellisen suureen selitysasteeseen, mutta tarkasteltaessa faktorien tehokkuutta yksilöinä, voidaan havaita monen faktorin tilastollisen selityskyvyn olevan hyvin rajallinen. Syventääksemme ymmärrystä käytämme Gibbonsin ym. (1989) kehittämää testiä, eli GRS-testiä, arvioidaksemme tarkemmin mallin soveltuvuutta systemaattisten riskien selittäjänä. Yhdistämällä GRS-testistä saadut mallin systemaattisten riskien selitysvoiman tulokset OLS-regressiosta saatujen tuloksien vertailuun, kyetään muodostamaan monipuolinen ja objektiivinen analyysi mallin soveltuvuudesta portfolioiden ylituottojen varianssin selittäjänä.

Tuloksia voidaan tulkita monella tapaa. Viisifaktorimallia tutkittaessa korjatulla selitysasteella voidaan tulkita viisifaktorimallin omaavan kykyä selittää portfolioiden ylituottojen varianssia. Kuitenkin GRS testi hylkää oletuksen siitä, että viisifaktorimalli kykenisi selittämään kaikkia systemaattisia riskejä ylituottojen varianssista. Myös tulokset yksittäisten faktorien beeta-arvojen tilastollisesta selityskyvystä p-arvoilla mitattuina jäivät heikoiksi. Lisäksi korjatun selitysasteen tulokset ovat vaihtelevia. Mallin tilastollinen kyvykyys ei kuitenkaan ollut heikentynyt muuttuneessa markkinatilanteessa eurooppalaisilla osakemarkkinoilla ja tulokset vastasivat pitkälti aiempia tutkimustuloksia. Näin ollen tämä tutkimus tukee aiempia tuloksia siitä, että portfolioiden ylituotot voidaan osittain selittää mallin faktoreilla, mutta oikeiden faktorien tutkimusta olisi jatkettava edelleen, jotta tuloksista saataisiin myös empiirisen tarkastelun kestäviä. Myös mallin heikot tilastolliset tulokset voidaan tulkita siten, että faktoripohjainen lähestymistapa saattaa olla riittämätön empiirisesti selittämään ylituottojen varianssia eurooppalaisilla osakemarkkinoilla.

Avainsanat: hinnoittelumalli, osakkeiden hinnoittelu, Fama ja French, viisifaktorimalli, eurooppalaiset osakemarkkinat, kansainvälinen empiirinen tutkimus, GRS-testi, OLS-regressio, portfolioiden ylituotot, 32 portfolioa

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1 INTRODUCTION

1.1 Background

Explaining the excess stock returns has been a captivating and widely researched topic in the field of investing and financial research. High returns of stocks that traditional economic models cannot sufficiently explain has been quoted as ‘equity premium puzzle’ (Mehra & Prescott 1985). Explaining stock returns with factors took a step forward when Sharpe (1964) developed the capital asset pricing model, commonly referred to as CAPM, which utilizes a single factor of β to explain systematic risk. This idea is consistent with the efficient market hypothesis as proposed by Fama (1970). CAPM is a foundational financial theory that measures risk and its relation to expected returns. CAPM requires many simplifying assumptions, and that can be one of the key reasons for CAPM having empirical issues.

Research on the topic indicates that using a single factor does not capture all risks, which influenced the further development of asset pricing models (Fama & French 1992). Many researchers have examined variance affected by factors in stock returns since then. The most known models to this day include the arbitrage pricing model (Ross 1976), the three-factor model by Fama and French (1992), Carhart’s (1997) four-factor model, which was based on the three-factor model, and the most recent five-factor model again developed by Fama and French (2015). There have been varying studies of explaining factors relating to stock returns, and the models keep developing over time. The five-factor model is based on five factors: market risk, size, book-to-market ratio, profitability, and investment factor (Fama & French 2015).

Fama and French (2015) were able to prove that the five-factor model clearly has an advantage compared to the CAPM and their previous three-factor model, since its explanatory power is increased. The results can be interpreted in a variety of ways. The five-factor model’s improved explanatory power over the three-factor model and CAPM shows that the five-factor model has made a difference by being more accurate and trustworthy as an asset pricing model, suggesting that a factor approach to explaining excess returns could be a viable method. However, the poor absolute performance of the five-factor model may imply that the approach is not suitable for the European stock markets.

1.2 Research questions

The five-factor model was developed by doing the research in the United States stock markets, but later they were able to show models explanatory power in Europe (Fama & French 2017). European stock markets differ from United States having smaller market capitalization. Also, some abnormal returns such as calendar anomalies have been proven to occur (Rossi & Gunardi 2018). The objective of this thesis is to examine the suitability of the Fama and French five-factor model for explaining the variance in excess returns in the 32 portfolios in European stock markets during the timespan of past decade. Additionally, to show if the model's empirical results in the last decade have changed compared with previous decades to determine if the model's effectiveness has changed. Macroeconomic environment has been changing rapidly due to changing interest rate environment and market shock caused by COVID-19 pandemic. These variables have created an optimal circumstance to critically evaluate and test the suitability of the model in the changed economic environment. The study will be conducted by using data from the European stock markets and portfolios constructed from European stocks.

The questions can be examined statistically with regression model by running regression model with the five-factor model and assess its ability to account for stock market returns of the 32 portfolios. Portfolios are constructed from the factor elements within European stock markets during the chosen decade of empirical results. Moreover, to evaluate the model's effectiveness and the significance of the alpha coefficients a Gibbons, Ross and Shanken, also known as the GRS test, will be used (Gibbons et al. 1989). This will address whether the alpha coefficients derived from the regressions are jointly zero. This statistical approach is critical for determining if the five-factor model adequately captures the excess returns of the portfolios or if there are unexplained systematic risk factors influencing the excess returns of the portfolios.

1.3 Limitations

This thesis contains numerous limitations. First limitation is the used time-series. The used time-series is 10-year period from the beginning of 2014 to the end of 2023. Second limitation of this thesis is that the study focuses on equities, leaving other asset classes out of the research. Third limitation is geographic limitation. The empirical

research only focuses on European stock markets leaving the rest of the world out of the scope. However, the results are interpreted with comparison with a broader geographic scope. Fourth limitation is the used datapoints. This study is done with 120 datapoint in the time series and it might leave out some interesting results from five-factor models behaviour during rapid market shocks.

1.4 Structure of thesis

This thesis is organized in the following manner: Section 2 is a literature review, offering an overview of various asset pricing models combined with a part of the history and chronological development of these asset pricing models. The literature review evolves from the basics and history of the asset pricing models towards the creation of five-factor model. Additionally, the literature review tries to give the reader a broad understanding how factors have been able to explain stock returns. Furthermore, it provides information about previous studies where researchers have evaluated the five-factor model in varying markets considering the strengths and weaknesses of the model and its implementation to different markets.

Section 3 introduces the construction methods of the examined 32 portfolios elucidating the methodologies adopted for the selection and categorization of assets. Furthermore, it provides a comprehensive overview of the methods used to calculate the factor data, which have been used to create this research. Finally, this section introduces the selected statistical methodologies implemented to derive research results, including models, tests and software used.

Section 4 begins by providing descriptive statistics through tables and figures, followed by an analysis of the empirical research results to determine their alignment with expectations and existing literature. The importance of results is discussed, and the more important statistics are highlighted to ensure the focus on the key metrics discussed in the previous studies and theoretical background.

Finally, Section 5 offers a succinct summary of the paper's findings and presents concluding thoughts. The section combines the key findings achieved by examining the topic with further discussion to propose possibilities for advance research of the topic. Moreover, it introduces critique to the model's capabilities and the factor-based approach to the European stock markets.

2 Theoretical background

2.1 Efficient market hypothesis

Efficient market hypothesis (EMH) is broadly recognized hypothesis explaining the structure of stock markets. The common belief in this hypothesis is that markets for securities operate efficiently. This theory assumes that new information spread rapidly and has immediate effect to the price of securities (Malkiel 2003). This hypothesis was firstly introduced by Fama (1970), but it is based on previous studies of Samuelson (1965), who was able to show that anticipated prices fluctuate randomly, and Markowitz (1952), who developed the idea of portfolio theory under the assumption that markets work under efficiency. Fama (1970) argued that markets can be efficient, and prices completely incorporate the accessible information under the condition that all the following criteria are met:

- No transaction fees and no market friction while exchanging securities.
- All market participants have all available information simultaneously and cost free.
- The comprehension of information at hand and its consequences for the present price and distribution regarding the future prices of each security are acknowledged and agreed by all market participants.

2.2 Factors explaining excess stock returns

2.2.1 Capital asset pricing model

Finance theory is largely based on belief that market returns are compensation on the risks taken as a stockholder. Sharpe (1964) developed a model to explain returns through a singular systematic risk factor in the capital asset pricing model (CAPM). The model is based on the idea of the efficient market hypothesis (EMH) as proposed by Fama (1970), which posits that stock prices fully incorporate all relevant information. This implies that stocks are always priced accurately and do not deviate from their true value. Therefore, in efficient markets, it is deemed unachievable to generate excess returns without taking on additional risk. The formula in equation 1 for this model as follows:

$$E(R_i) = \beta_i(E(r_m) - r_f),$$

where the beta is described as the ratio of the covariance of security's return to the market returns over the variance of market returns. $E(R_i)$ signifies the anticipated return of investment i , $E(r_m)$ is the market portfolio's expected return and lastly the r_f is the used risk-free rate.

Nevertheless, the CAPM has found to have shortcomings in fully predicting stock returns, thereby inadequately reflecting risk exposures represented by singular risk factor β . For instance, studies have indicated that portfolios constructed on company size and price-to-earnings ratio consistently yield varying returns, indicating the existence of additional risk-influencing factors. (Reinganum 1981.)

2.2.2 Fama & French three-factor model

In the study of Fama and French (1993), it was observed that various factors were proposed to possess explanatory power. Previous studies indicated correlations among returns and market capitalization (Banz 1981), the book-to-market (B/M) ratio (Stattman 1980), the debt-to-equity ratio (Bhandari 1988) and earning-to-price ratio (Reinganum 1981). The commonality amongst these factors is their reference regarding the market capitalization of corporations.

To address multicollinearity and achieve simplicity, Fama and French (1992) tested with different factors explaining stock returns and developed model that can explain 90% of the cross-section of the excess returns. Outcome was drastically improved from the previous explanation power of CAPM. The model was tested with adding two additional factors to CAPM to elucidate stock returns more accurately. These factors were the size and the book-to-market factor which is more commonly referred to as value factor. The model's performance was tested in Fama and French (1996) research paper and they found out that the model possesses explanatory power to anomalies better than CAPM, but it still lacked possibility to explain momentum anomaly. The model can explain return of the portfolio with the sensitivity of three factors: market excess returns portfolio ($R_m - R_f$), size (SMB, small - big), and lastly the value factor (HML, high - low). The equation for this model can be presented as:

$$E(R_i) - R_f = \alpha_i + \beta_i(R_m - R_f) + s_iSMB + h_iHML,$$

where β_i , s_i and h_i are the factor loadings, also described as asset's sensitivity to the measured premium and α_i represents the excess returns that the model is incapable of explaining.

Fama and French (1992) discovered that portfolio with lower market value generates increased returns on average than big companies. It suggests a negative relationship between size and excess returns. This effect has been studied widely and has many possible explanations, but none are able to entirely demonstrate the existence of this effect (Van Dijk 2011).

In further research towards the effectiveness of HML and B/M factors, it was found that assets HML and B/M factors gain their significance from statement that companies with greater value can outperform companies labelled as growth stocks. This might be due to mispricing and non-optimal investor behaviour. (Lakonishok et al. 1994).

2.2.3 Carhart four-factor model

While Fama and French (1992) lacked the ability to elucidate abnormal returns that have the possibility to disobey efficient market hypothesis, De Bondt and Thaler (1985) introduced research indicating that stock prices tend to overreact to news, a phenomenon that ultimately leads to a market correction. De Bondt and Thaler created a portfolio using past losers and portfolio based on past winners. They showed that past losers had greater returns up to three to five years than portfolio with past winners. However, it has been argued that the higher earning compensates for elevated risk (Chan 1988).

Jegadeesh and Titman (1993) showed formerly poorly performed stocks generate significant return across three-month and twelve-month holding durations. They showed also that excess returns of these investing strategies cannot be a result of systematic risk or slow responses to factor influencing typical stock prices. This was the base idea for Carhart to start developing Fama and French three-factor model onwards to include factor that can take momentum anomaly in to account.

Carhart (1997) argued that the three-factor model cannot provide sufficient explanation to the momentum-sorted portfolio variation. Whereas the momentum factor combined with the three-factor model got improved results. Carhart's four-factor model as seen below:

$$E(R_i) - R_f = a_i + b_i(R_m - R_f) + s_iSMB + h_iHML + p_iPR1YR_t,$$

where in the end of equation, PR1YR describes the momentum factor and p_i denotes the sensitivity to the momentum factor. Other factors as previously mentioned in three-factor model.

Fama and French (2012) found issues capturing portfolios including size-momentum factors. They identified that the momentum issue is densely focused to the upper and lower quantiles. Fama and French three-factor model leaves momentum factor out of the model, but they still recognized that momentum factor can perform better than the three-factor model.

2.2.4 Fama & French five-factor model

Research of the topic suggest that additional factors are involved in increased explaining power. For instance, despite higher valuations, companies exhibiting higher profitability measured as profits-to-assets ratios tend to yield greater returns (Novy-Marx 2013). Additionally, Aharoni et al. (2013) were able to show that negative relation among anticipated investment and the average stock returns, whereas the three-factor model failed to account for a significant portion of the variability in average stock returns associated with profitability and investments factors. In response to this, Fama and French (2015) improved the previous model by including two more factors to cover for unexplained stock returns. In their research paper, they showed correlation of the profit matrices including improved performance with the stocks with more robust operating profitability, and stocks with more conservative investment strategy. They implemented these factors in to the previous three-factor model and gained improved results. The equation for the five-factor model is following:

$$E(R_i) - R_f = \alpha_i + \beta_i(R_m - R_f) + s_iSMB + h_iHML + r_iRMW + c_iCMA,$$

where RMW (robust-minus-weak) represents the profitability factor and CMA (conservative-minus-aggressive) implies that reduced investment results in increased yields.

The understanding of these factors relies on a crucial assumption proposed by Fama and French (1992; 2015). The objective is to define factors that reflects the risk associated with a particular excess return of portfolio. Fama and French formulated assumption

that returns act as a substitute measure for the risk. Therefore, these factors illustrate the variation in excess returns among companies with a substantial quantity and a small quantity of a specific factor.

2.2.5 Assessment of the five-factor model

In the research paper of Fama and French (2015) the capabilities of the five-factor model were tested in two parts. To begin with, they tested the model to portfolios created by size, B/M, profitability, and investment. They used different combinations of these factors to create matrices where they tried to show that the sensitivity of these factor would imply greater or lower return of the portfolios. Afterwards they continued in testing the model to anomalies and later advanced their testing to the international markets (Fama & French 2017).

Fama and French (2015) found out that the GRS test disproves the results of the model at capturing examined systematic risk. Nevertheless, the results implicated that the model's explanation capability had increased and varied between 71% and 94%. This meant that the model was able to explain that much of the variance in excess returns for the constructed portfolios of size, B/M, OP, and investment factor. However, the greatest issue with explanatory power of the model was regarding to RMW and CMA factors having negative exposures to small stock portfolios. The companies included in these portfolios tend to have greater amounts of investments, but the adverse reactions to RMW does not imply with decreased profitability, which differed from the results with bigger stock portfolios. Although Fama and French recognized the issue regarding to model's ability to explain small stocks, there has not been direct solution to the matter on their behalf.

2.2.6 International results and critique

To further investigate the suitability of the five-factor model outside the United States stock markets, Fama and French (2017) ran additional tests to the model in four regions: Japan, North America, Europe, and Asia Pacific. In the results for the last three regions, they found out that returns tend to improve in relation to SMB, RMW and CMA. These outcomes are consistent with the prior expectations. On the other hand, results were different in Japanese stock markets. In Japanese stock markets returns are not significantly impacted by profitability and investment (Kubota & Takehara 2018). Also

in the European stock markets, the investment factor does not significantly affect returns (Fama & French 2017).

Further international research done by Foye (2018) for five-factor model suitability to Asia, Eastern Europe, and Latin America it is shown that the five-factor model was unable to improve explanation power compared to the three-factor model in Asia. However, in Eastern Europe and Latin America the five-factor model do offer improved results explaining the average excess returns over the three-factor model. Additionally, the results of the study measured with GRS coefficient in Eastern Europe were 1,60 with p-value of 0,04 and in Asia GRS coefficient value was 2,28 with p-value of 0,00. These results implies that the model's alpha values jointly are not equal to zero with the 0,95-confidence level ($p < 0,05$) and the model's ability to explain all the underlying risk must be rejected.

3 Data and methodology

3.1 Description of the data

The time-series of this analysis covers from the beginning 2014 until the end of 2023, providing a sufficient period to examine if there will be impact of changed interest rate environment and COVID-19 market shock. It also provides enough data to evaluate the model's overall suitability to European stock markets. The data for the empirical analysis were gathered from Kenneth R. French online data library (2024). The data is originally from Bloomberg database.

The dataset comprises European countries such as Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. Using the data provided by the French data library, we can acquire more comparable results to the original study of international suitability of the five-factor model (Fama & French 2017). Taking the approach of a U.S. investor, calculating the returns in USD. The data spans from the beginning of 2014 to the end of 2023. The data is presented monthly creating 120 datapoints to each factor and portfolio.

3.2 Factors

The data of the factors is obtained from French's Data Library (2024). This dataset from previous years was formerly used by Fama and French (2017) in their testing of the five-factor model, which is the key reason for use of the same data library. The factor returns are measured in US dollars and consider both dividends and profits gained by asset sales, mergers, and acquisitions. The size breakpoints in the dataset are determined by the third, seventh, thirteenth, and twenty-fifth percentile rankings of the overall market value in Europe.

The factors of this model include the returns from: excess market returns ($R_m - R_f$), small size – big size (SMB), high value – low value (HML), robust profitability - weak profitability (RMW), and conservative investments - aggressive investments (CMA). Excess returns are calculated using risk-free rate, which is U.S. one month Treasury bill rate also known as T-bill rate. The calculations of the factors are shown more precisely

in the equations below. The SMB factor which is commonly referred as the size factor defines as:

$$SMB = \frac{1}{3}(SMB_{BM} + SMB_{OP} + SMB_{INV}),$$

$$SMB_{BM} = \frac{1}{3}(small\ value + small\ neutral + small\ growth) - \frac{1}{3}(big\ value + big\ neutral + big\ growth),$$

$$SMB_{OP} = \frac{1}{3}(small\ robust + small\ neutral + small\ weak) - \frac{1}{3}(big\ robust + big\ neutral + big\ weak),$$

$$SMB_{INV} = \frac{1}{3}(small\ conservative + small\ neutral + small\ aggressive) - \frac{1}{3}(big\ conservative + big\ neutral + big\ aggressive),$$

where the SMB results in the difference of average returns between nine small stock portfolios and the nine big stock portfolios measured by market capitalization. The HML factor is commonly referred to as value factor and the definition of the factor is:

$$HML = \frac{1}{2}(small\ value + big\ value) - \frac{1}{2}(small\ growth + big\ growth),$$

where HML factor measures the additional returns generated by value stocks over growth stocks. This classification is defined as those with high book-to-market stocks representing the value stocks and low book-to-market stocks accounted as growth stocks. The RMW factor, commonly referred to as profitability factor is constructed as:

$$RMW = \frac{1}{2}(small\ robust + big\ robust) - \frac{1}{2}(small\ weak + big\ weak),$$

where the RMW factor suggest that companies with higher profitability have higher expected returns. The factor represents the difference between companies with high operating profitability and low operating profitability. The profitability is calculated by earnings divided by book equity. The CMA which is commonly referred to as the investment factor and is defined as difference in returns between investment strategy portfolios. The equation for the factor is following:

$$CMA = \frac{1}{2}(small\ conservative + big\ conservative) - \frac{1}{2}(small\ aggressive + big\ aggressive).$$

3.3 Portfolios

The quantitative analysis is based on 32 portfolios constructed by French, and the data is gathered using the French Data Library (2024). These portfolios are used to precisely answer the research question of the model's suitability and to answer if the model's ability to explain excess returns have changed in the different market circumstances. This way the research results are more comparable with Fama and French (2017) international testing and different outcomes due to differences in data or different assumptions in the portfolio construction are avoided.

The portfolios are constructed from data obtained from the European stock markets. All returns are in U.S dollars. The stocks are sorted into portfolios using two market cap groups (ME1 and ME2), four book-to-market groups (BM 1-4) and four operating profitability (OP 1-4) groups. Recalculations of the portfolios stocks is done at the end of June each year.

More precise classification in French data library (2024) is done followingly. Big stocks are defined as those falling within the upper 90% of market capitalization in the region as of June, while small stocks make up the lower 10%. Within each size category, stocks are then divided into four groups based on their book-to-market ratios and operating profitability, using quartile breakpoints specific to the size group. This methodology results in the creation of 32 distinct portfolios in the European stock markets, effectively creating two 4x4 matrices of size, B/M and OP criteria.

3.4 Methodology

3.4.1 Ordinary Least Squares (OLS) regression

The empirical analysis in this study relies on regression techniques to investigate the relationships between various financial variables. To creation of this analysis, it was decided to utilise the R programming language to compute regression results. The regression analysis in R was conducted using the "lm" function, which takes advantage of the OLS method. OLS method is a statistical technique that can determine the optimal fit of a model and find the minimum sum of the squares of residuals. This method is widely used in regression analysis and estimations. (Miller 2006.)

3.4.2 Gibbons, Ross, & Shanken (GRS) test

To statistically assess the model's suitability, we selected to do tests of the joint α to strengthen the results acquired from the regression. To assess this hypothesis, a GRS test is used, which was invented by Gibbons et al. (1989). This methodology was chosen to gain similarities to the research of Fama and French (2015; 2017). In the case that the GRS results for the joint α equals zero, it can be interpreted that model captures all the systematic risk in variances of excess returns. The GRS statistic is computed according to the following equation:

$$GRS = \left(\frac{T - N - L}{N - T - L - 1} \right) \left[\frac{\hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha}}{1 + \bar{\mu}' \hat{\Omega}^{-1} \bar{\mu}} \right] \sim F(N, T - N - L)$$

where $\hat{\alpha}$ signifies Nx1 vector containing intercepts, while $\hat{\Sigma}$ denotes an unbiased estimate of covariance matrix of the residuals, $\bar{\mu}$ is an Lx1 vector denoting the average values or means of the factor portfolios' samples and $\hat{\Omega}$ represents an unbiased estimation of the covariance matrix associated with the factor portfolios. In the context of this study, the variant of the conventional methodology specifically tailored for smaller sample sizes is used. This is since we are working with smaller time-series compared to Fama and French (2015;2017). The formula for the GRS test used is determined as:

$$GRS = \left(\frac{T - N - K}{N} \right) \left[\frac{\hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha}}{1 + \bar{f}' \hat{\Omega}^{-1} \bar{f}} \right] \sim F(N, T - N - K)$$

This approach is designed specifically for smaller sample sizes, acknowledging that the covariance matrix, Σ can vary between instances. Consequently, every increase in the alpha values raise the GRS statistic, indicating increased deviation from the model's assumptions. (Cochrane 2014).

3.5 Hypothesis

To answer the research question of this thesis if the five-factor model can explain excess return in the European stock markets, we will examine the adjusted R^2 , beta-values and p-values of the factors as individuals and the interception terms of the model. First, we will formulate a question regarding to the explanation power of the model and its

statistical significance. This question can be answered with the regression results and the hypothesis is following:

$H_0 = \text{Model does not explain the variation in excess returns,}$

$H_1 = \text{Model is significantly explaining the variation in excess returns.}$

To formulate second hypothesis, we apply the previously mentioned GRS test to the regression results. By using this approach, we can determine whether there is systematic risk that the model is inadequately able to explain. The second hypothesis is determined followingly.

$H_0 = \text{The alpha coefficients will not significantly differ from zero,}$

$H_1 = \text{The alpha coefficients will significantly differ from zero.}$

4 Empirical results & discussion

4.1 Descriptive statistics

In this section, we embark on a detailed examination of the descriptive statistics pertaining to the five-factor model and the 32 examined portfolios. The aim is to provide a comprehensive overview of its fundamental characteristics and performance metrics to give a robust background for the further analysis.

Table 1. Descriptive statistics of the five factors 2014-2023

	<i>Mkt-RF</i>	<i>SMB</i>	<i>HML</i>	<i>RMW</i>	<i>CMA</i>
Mean	0,004	0,001	-0,001	0,003	-0,001
Standard Error	0,004	0,002	0,003	0,001	0,001
Median	0,006	0,000	-0,003	0,004	-0,001
Mode	0,018	0,007	0,004	0,007	0,008
Std. Deviation	0,048	0,017	0,030	0,016	0,016
Sample Variance	0,002	0,000	0,001	0,000	0,000
Kurtosis	0,958	0,459	4,079	0,926	1,173
Skewness	-0,056	0,152	0,556	-0,486	0,123
Range	0,321	0,098	0,234	0,092	0,096
Minimum	-0,154	-0,051	-0,113	-0,054	-0,044
Maximum	0,166	0,047	0,121	0,038	0,052
Sum	0,496	0,071	-0,084	0,354	-0,162
Count	120	120	120	120	120

These descriptive statistics shows time series consisting of data from 10-year period as a monthly data adding the amount of data points to 120 per factor. The statistics shows positive returns accumulated from Mkt-Rf, SMB and RMW factors during the examined period whereas the HML and CMA factors indicate negative returns. These values can also be described as the factor premiums.

Mkt-Rf factor yields an equity premium of 0,4% monthly, RMW factor contributes 0,3% monthly equity premium, and the SMB factor delivers 0,1% monthly equity premium. On the other hand, the HML and CMA factors exhibit negative monthly equity premiums of -0,1%. The descriptive statistics also show that the market factor has the greatest standard deviation, while the RMW and CMA factors have significantly lower standard deviations.

These statistics highlight certain disparities from the Fama and French (2017) test of the model to the international markets, where they recorded the monthly equity premiums

to be: 0,5% for the Market-Rf factor, 0,1% for the SMB factor, 0,4% for the RMW factor, 0,3% for the HML factor and 0,2% to the CMA factor.

To enhance the understanding of the factors and their behaviour it is important to inspect the data also using correlation matrix to see whether the factor data behave similar way over each other. The correlation matrix is presented in the table 2.

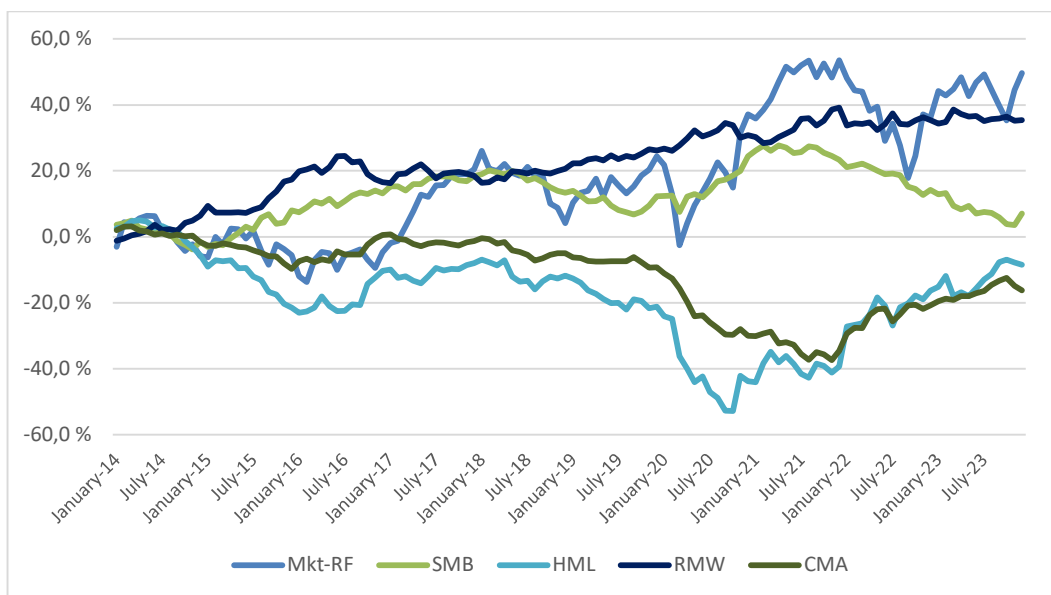
Table 2. Correlation matrix of the factors

	<i>Mkt-RF</i>	<i>SMB</i>	<i>HML</i>	<i>RMW</i>	<i>CMA</i>
Mkt-RF	1				
SMB	0,194	1			
HML	0,140	-0,011	1		
RMW	-0,034	-0,054	-0,782	1	
CMA	-0,227	-0,302	0,745	-0,572	1

These results implicate that the excess return of CMA and HML factor have high positive correlations, whereas the HML factor and the RMW factor have high negative correlation. All the other correlations fall in categories of moderate, low, or negligible correlation.

Finally, to provide graphic perspective to the behaviour and results of the factors we counted the cumulative returns to factors from the examined decade. Spanning from the beginning of 2014 to the end of 2023, these visualizations serve to demonstrate the trajectory of each factor's cumulative returns over the examined period.

Figure 1. Cumulative returns of the five factors



These cumulative returns support the previous descriptive statistics highlighting the positive equity premiums of the Mkt-Rf, RMW and SMB factors. In addition, they clearly depict the negative equity premiums of the HML and the CMA factor.

Furthermore, this visualization clearly confirms and accentuates the higher standard deviation number of the Mkt-Rf factor.

To show correlation between explaining factors and increasing profits, we decided to include the framework they used for presenting the portfolio returns in matrices, a method that was already used in Fama and French (1992) study. This approach helps us stay consistent with existing research while capturing the core goals of our research by focusing on explaining the portfolio excess returns related to variable. Table 3 displays the average returns of the 32 portfolios, organized into (2x4x4) portfolios with size, B/M and OP criteria.

Table 3. Average monthly excess returns of portfolios formed on Size, B/M and OP

B/M ->	Small				Big			
	low	2	3	high	low	2	3	high
Low OP	-0,54 %	-0,49 %	0,07 %	0,14 %	0,46 %	-0,26 %	0,58 %	0,28 %
2	-0,02 %	0,16 %	0,34 %	0,61 %	0,75 %	0,47 %	0,37 %	0,59 %
3	0,65 %	0,53 %	0,53 %	0,86 %	0,47 %	0,47 %	0,29 %	0,09 %
High OP	0,62 %	0,56 %	0,52 %	0,27 %	0,64 %	0,40 %	0,50 %	0,25 %

These results implicate that the high book-to-market value and high operating profitability could explain some of the excess returns in small-stock portfolios, giving ground to these factors suitability to European stock markets. However, the results shown in the big stocks portfolios differ from the ones seen in smaller stocks, giving more non-linear outcomes of excess returns of the portfolios. In the case of big stocks, both book-to-market and operating profitability factor gives more redundant results as an explanator.

4.2 Regression and GRS-test results

Table 4 focuses on evaluating the five-factor model through a comprehensive assessment that involves regressions results and the application of the GRS test. The columns of the table show statistics for each of the 32 portfolio alphas followed by p-values of alphas, beta-values of factors followed by their p-values and lastly are the adjusted R^2 values in the last column on the right. Additionally, in the bottom of the

table is calculated averages of the previous results and the GRS test with its corresponding p-value.

Table 4. Regression results

Model's regressions result of the 32 portfolios (size, B/M and OP) combined with averages and GRS-test result.

Portfolio	α	Mkt-Rf	SMB	HML	RMW	CMA	Adjusted R ²						
SMALL LoBM LoOP	-0,006	0,011	1,109	0,000	1,171	0,000	-1,141	0,000	-1,793	0,000	-0,109	0,703	0,853
ME1 BM1 OP2	-0,002	0,228	0,891	0,000	0,919	0,000	-0,607	0,000	-1,034	0,000	-0,446	0,052	0,862
ME1 BM1 OP3	0,002	0,094	1,017	0,000	0,861	0,000	-0,491	0,000	-0,219	0,087	0,069	0,646	0,938
ME1 BM1 HiOP	0,001	0,464	1,079	0,000	0,965	0,000	-0,220	0,000	0,081	0,237	-0,237	0,004	0,985
ME1 BM2 LoOP	-0,006	0,056	1,131	0,000	1,264	0,000	-0,767	0,002	-1,585	0,000	-0,250	0,523	0,766
ME1 BM2 OP2	-0,002	0,157	0,975	0,000	0,918	0,000	-0,208	0,050	-0,390	0,006	-0,047	0,778	0,923
ME1 BM2 OP3	0,001	0,248	0,991	0,000	0,869	0,000	-0,111	0,032	-0,045	0,513	0,014	0,864	0,981
ME1 BM2 HiOP	-0,001	0,491	1,026	0,000	0,903	0,000	0,285	0,000	0,546	0,000	0,050	0,564	0,980
ME1 BM3 LoOP	-0,001	0,528	1,031	0,000	0,987	0,000	-0,132	0,442	-0,997	0,000	-0,156	0,567	0,845
ME1 BM3 OP2	0,000	0,753	0,925	0,000	0,662	0,000	-0,047	0,601	-0,136	0,261	0,063	0,661	0,931
ME1 BM3 OP3	0,000	0,731	0,966	0,000	0,866	0,000	0,348	0,000	0,260	0,000	-0,050	0,498	0,984
ME1 BM3 HiOP	-0,001	0,592	1,015	0,000	0,874	0,000	0,636	0,000	0,492	0,000	-0,078	0,574	0,955
ME1 BM4 LoOP	0,001	0,813	1,108	0,000	1,549	0,000	-0,227	0,372	-1,381	0,000	0,744	0,067	0,762
ME1 BM4 OP2	0,003	0,031	0,937	0,000	0,678	0,000	0,524	0,000	-0,099	0,391	0,061	0,654	0,950
ME1 BM4 OP3	0,004	0,001	0,966	0,000	0,771	0,000	0,490	0,000	0,145	0,211	0,085	0,537	0,950
SMALL HiBM HiOP	-0,002	0,474	1,112	0,000	1,079	0,000	0,701	0,005	0,167	0,610	0,098	0,800	0,773
BIG LoBM LoOP	0,000	0,962	0,884	0,000	0,254	0,441	-0,323	0,396	-0,486	0,340	-1,302	0,032	0,478
ME2 BM1 OP2	0,004	0,143	0,961	0,000	0,015	0,924	-0,765	0,000	-0,131	0,582	0,105	0,708	0,772
ME2 BM1 OP3	0,001	0,559	0,981	0,000	-0,181	0,009	-0,709	0,000	-0,127	0,229	-0,032	0,796	0,948
ME2 BM1 HiOP	0,002	0,048	0,950	0,000	-0,229	0,000	-0,385	0,000	0,282	0,001	0,083	0,379	0,966
ME2 BM2 LoOP	-0,004	0,137	1,012	0,000	0,157	0,378	-0,729	0,001	-0,960	0,001	0,298	0,361	0,716
ME2 BM2 OP2	0,002	0,267	1,018	0,000	-0,194	0,028	-0,707	0,000	-0,295	0,031	0,399	0,014	0,911
ME2 BM2 OP3	0,000	0,787	1,016	0,000	-0,163	0,019	-0,091	0,251	0,363	0,001	0,148	0,240	0,947
ME2 BM2 HiOP	-0,002	0,122	1,038	0,000	-0,072	0,448	0,372	0,001	0,771	0,000	-0,038	0,829	0,915
ME2 BM3 LoOP	0,003	0,075	1,006	0,000	0,072	0,491	-0,244	0,046	-0,422	0,010	0,180	0,349	0,882
ME2 BM3 OP2	-0,001	0,422	1,008	0,000	0,155	0,038	0,172	0,046	0,261	0,024	0,182	0,179	0,942
ME2 BM3 OP3	-0,002	0,181	1,037	0,000	-0,184	0,072	0,398	0,001	0,463	0,004	0,155	0,404	0,903
ME2 BM3 HiOP	0,000	0,936	0,951	0,000	0,068	0,591	0,673	0,000	0,434	0,028	-0,270	0,245	0,856
ME2 BM4 LoOP	0,001	0,512	1,047	0,000	0,000	0,998	0,635	0,000	-0,852	0,000	-0,416	0,006	0,953
ME2 BM4 OP2	0,002	0,089	1,069	0,000	-0,084	0,289	0,773	0,000	-0,089	0,465	-0,141	0,331	0,954
ME2 BM4 OP3	-0,005	0,021	1,065	0,000	-0,132	0,333	1,097	0,000	0,631	0,003	-0,350	0,162	0,877
BIG HiBM HiOP	-0,004	0,343	1,099	0,000	0,176	0,540	0,878	0,009	1,196	0,008	0,596	0,256	0,601
Averages	-0,000	0,352	1,013	0,000	0,468	0,175	0,002	0,071	-0,155	0,126	-0,019	0,431	0,877
GRS coefficient	1,623	0,041											

The table reveals a nuanced performance of the five-factor model across the 32 portfolios when analysed the performance of the five-factor model. Portfolio is based on low to high categories, with names for size (ME1 – ME2), book-to-market categories (BM1, BM2, BM3, BM4) and operating profitability (LoOP, OP2, OP3, HiOP). The first and last portfolios of size-categories is marked with low/high categories for all the variables to highlight portfolio size category change.

Results of the model's alpha values, or intercept terms, indicate the average excess returns that the model does not explain. These values are relatively low, suggesting that there may not be significantly important additional systematic risk factors. Certain portfolios exhibit positive small positive alphas, indicating relatively small level of excess returns beyond the model's explanations power. However, negative alpha values

in some portfolios indicate that the model underperforms in explaining their returns. This suggests that certain aspects of these portfolios' performance are not captured by the model. It also should be noted that the p-value of alphas is high implying statistical insignificance for the alphas.

The Mkt-Rf factor shows a positive relation with portfolio return, which aligns with the concept of the market risk premium. The SMB and RMW factors also display positive contributions, although results vary more between the portfolios. HML and CMA factors predominantly show negative coefficients implying that value stocks and conservative investment strategies may have underperformed during the examined period, which supports our previous Figure 1 depicting the cumulative results of factors.

The adjusted R^2 values are mostly robust with average value of 87,7%, indicating that the model takes in to account a significant part of the variance in portfolios' excess returns and has statistical significance explaining variance in excess returns. However, the p-values of beta coefficients observed in the analysis mostly tend to be over the conventional threshold of 0,05, thereby making their relevance too high to support that the many of the factors would be statistically proven to work in European stock markets in the academic standards. All the factors: SMB, HML, RMW and CMA on average go over the significance level of 0,05, but still can provide statistical significance to certain portfolios. The average p-value of CMA factor is the highest implying redundancy to explaining excess returns. Despite the individual factors tending to have poor performance explaining the variance of excess returns, the model still can uphold explanation power to excess returns with relatively robust adjusted R^2 , and with the significant Mkt-Rf factor and low alpha values. This is why we reject the null hypothesis of the model not having any statistical significance explaining the variances in stock returns.

Evaluating the model, the GRS test coefficient, with a value of 1,623 and p-value of 0,041 rejects the second null hypothesis of model's alpha coefficients being jointly 0. The GRS F-value is compared to critical value of F-distribution, which is 1,606 with 5% significance rate for this research. This reject suggests that that the Fama and French five-factor model cannot adequately explain all the systematic variation in excess returns across the 32 portfolios with 5% significance level ($p < 0,05$) and there are still statistically significant abnormal returns unexplained by the model. However, the

relatively low result of GRS value implies that while there is evidence against model's adequacy, the extent of the model's inadequacy may not be substantial.

The result aligns with Fama and French (2017) results where they measured GRS value of 1,85 with corresponding p-value of 0,01 for the similarly constructed portfolios. They captured relatively high mean R^2 , but the poor performance of alpha coefficients fails to prove the model's suitability internationally. This finding also aligns with the previously addressed study of model's suitability to Eastern Europe. Foye (2018) reported a GRS test coefficient of 1,60, accompanied by p-value of 0,04, thereby showing similar conclusions regarding to the model's explanatory capacity within this geographic context. It is important to mention that Foye's test was created with a larger dataset having lower critical value of F.

These results also suggest that research is needed for investigation and potentially alternative factor approaches to better understand excess return drivers. Given the complexities observed and the limitations of certain factors in explaining portfolio returns, it is crucial to further develop the analysis of the European market behaviour and potential underlying variables for excess returns. This could involve looking into additional variables or refining existing factors to better capture the nuances of European market behaviour. The concepts of behavioural finance might be the future of explaining anomalies and uncovering new points of view to the factor approach. Furthermore, comparing different models or methodologies may offer insights into more effective frameworks for explaining and predicting excess returns in European stock markets.

5 Conclusions

The search for better asset pricing models has long been an important subject of interest in the asset pricing literacy. Asset pricing models have been improving and better empirical results have been recorded to more developed models. This investigation into the Fama and French five-factor model's application in European stock markets highlights both its strengths and limitations. While the model demonstrates a degree of success in capturing stock returns through its factors measured with adjusted R^2 , the analysis from a decade marked by unparalleled market shifts reveals its inadequacy explaining all the underlying systematic risks. Notably, the model's performance varies significantly across different market conditions and portfolio characteristics. These findings prompt a critical reflection on the model's universality and effectiveness, suggesting further research and development of asset pricing models.

As an individual factors the Mkt-Rf, SMB and RMW factors were able to yield positive returns in the examined time-series. HML and CMA factor yielded negative returns. CMA factors beta coefficient had the greatest p-value in the factors implying that the CMA is redundant in the European stock markets, which was in line with Fama and French (2017) results with similar portfolios. Previous studies have been able to explain increasing explaining power to excess returns with the use of growing number of factors measured with adjusted R^2 values. However, this thesis among the Fama and French (2017) research shows that even though the model has relatively high average explanatory rate, the model still cannot explain all the underlying variance of excess returns in the portfolios. The GRS statistics show that the model leaves a possibility for systematic excess returns to be captured.

Future work should prioritize on refining existing risk factors, where the individual risk factors should all, or at least most of them have empirically proven statistical significance. It could also be beneficial to do research for other explaining factors to enhance the model's robustness and adaptability to diverse market environments or research possibilities of geographically specialised risk factors. These models might require continuous development, because market dynamics keeps changing and anomalies have proven to occur. However, by challenging the boundaries of current asset pricing theories, it might be possible to achieve a more accurate and comprehensive model that reflects the complex dynamics of global stock markets.

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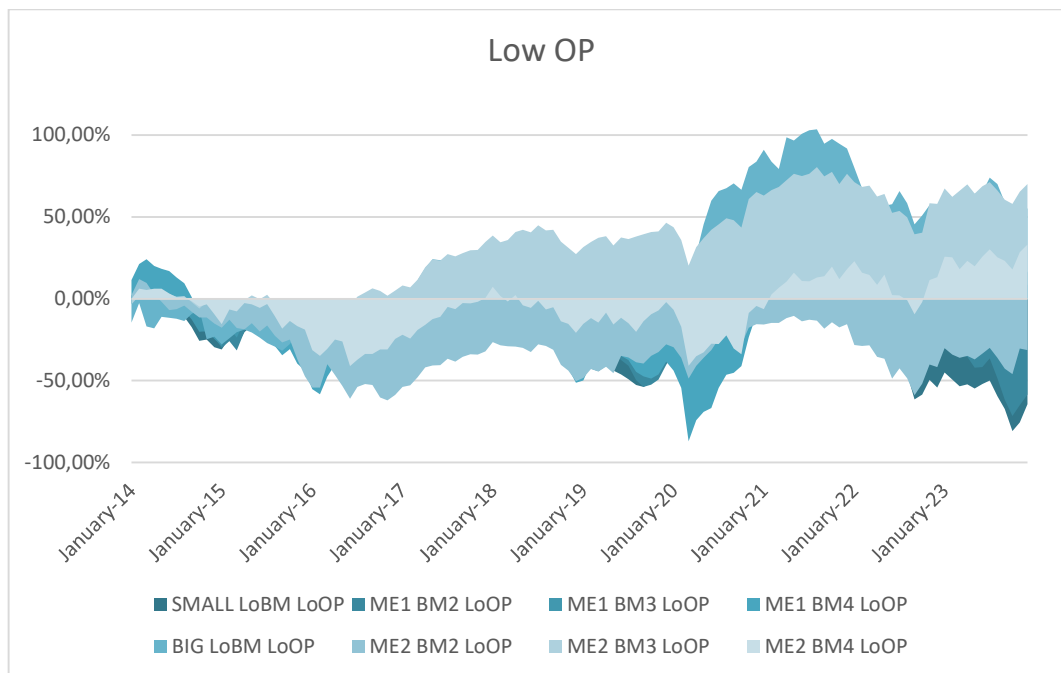
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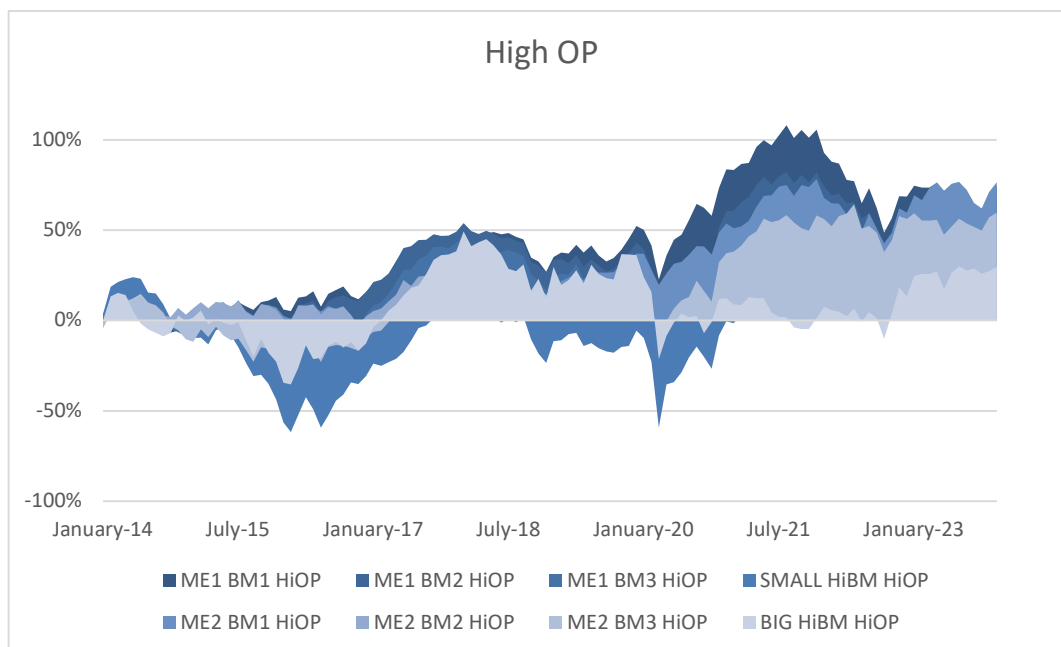
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Appendices

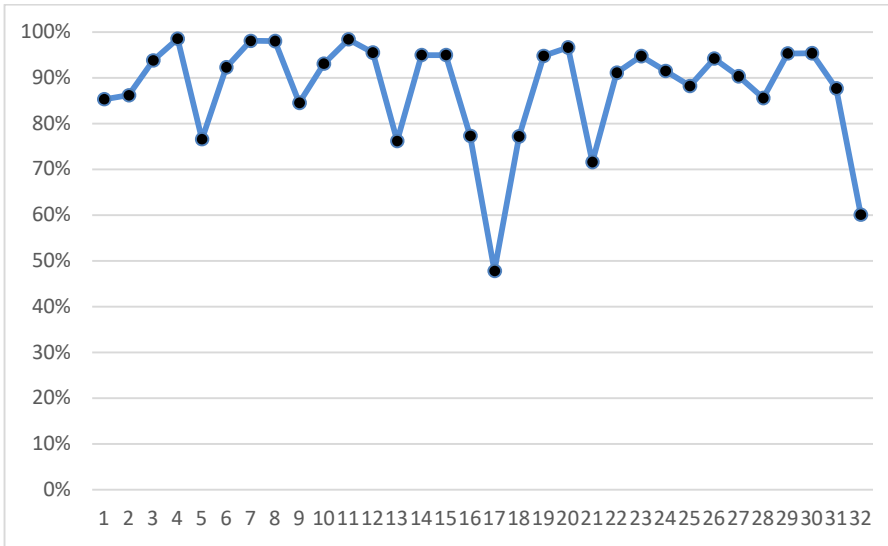
Appendix 1. Excess returns for low operating profitability portfolios



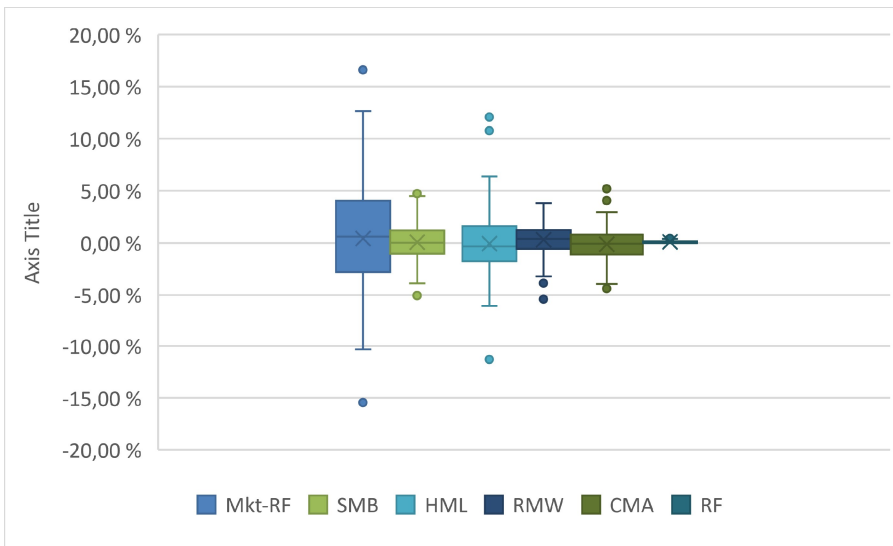
Appendix 2. Excess returns for high operating profitability portfolios



Appendix 3. Adjusted R² across the 32 portfolios.



Appendix 4. Variation in factor returns (Mkt-Rf, size, value, profitability, investment)



Appendix 5. 3D model of volatility during COVID-19 pandemic (1/2020-12/2022)

