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Long-term equity investing and withdrawal rules

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Abstract
This paper provides evidence on the outcomes of several different withdrawal policies for a long-term equity investor with a motive to preserve the real value of their assets and maximize withdrawals. Using data for the US and Finnish stock markets from 1913 to 2023, we find that historically, the maximum endowment-preserving withdrawal rates would have been 10.95% and 1.83% of the initial investment at the end of 1912 for the US and Finland, respectively. The maximum withdrawal rate varies with the timing of the initial investment. The average maximum withdrawal rate over the first 100 years is 7.79% for the USA and 6.63% for Finland. A rearview look shows that following a rule where the withdrawal is a given fixed rate of the nominal value of the portfolio, all the while either disallowing reductions in nominal withdrawals (Finland) or keeping them at least 90% level of the previous nominal withdrawal (USA), would have historically provided the highest average withdrawals in real terms. Looking forward, both countries offer withdrawal rates of four or even 5% with reasonable risk if one allows withdrawals to be adjusted for stock market development.

Keywords Long-term investing · Withdrawal · Endowment · Portfolio management

JEL Classification G11 · G23 · N3

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1 Introduction

Some say there is only one thing worse than dying too soon: to outlive your money. In this paper, we analyze a situation faced by an investor with a long investment horizon who aims to preserve the real value of her portfolio, all the while trying to maximize the cash withdrawals from the portfolio. The analyzed situation is common among many long-term investors, ranging from family trusts to different endowment funds, and even to pension funds, all of which aim to use withdrawals to finance their operations, yet at the same time, they wish to preserve the real value of the principal capital. The same situation is also faced by long-term retirement planners. The main research question is: *what kind of withdrawal policy is optimal?*

As stock market development in many Western countries has been remarkably high during the last 100 years (see, for example, Dimson et al. 2002), it has become popular to follow simple rules that typically allow for a fixed inflation-adjusted amount to be withdrawn. Probably the most well-known rule, the so-called *4% rule*, was introduced by Bengen (1994), building on Bierwirth (1994). It states that, for the US market, a safe withdrawal rate can be set at about 4% of the initial value of a portfolio containing 50% to 75% stocks and the remainder invested in intermediate-term treasuries, after which the withdrawal changes in accordance with inflation, regardless of the portfolio's evolution. However, the same withdrawal rate does not necessarily apply to other countries and times, as emphasized, for example, by Pfau (2010). Furthermore, due to volatility and crashes, long-term return and inflation averages do not necessarily guarantee a feasible long-run withdrawal scheme, nor do they prevent catastrophic downturns.

It is important to note that outcomes across different investment horizons can deviate substantially from the baseline scenario. For life-cycle investors, large idiosyncratic deviations from the average might occur, leading to adverse outcomes. Starting at a stock market peak makes it challenging to keep up with a chosen plan. On the other hand, the result could also be much better than expected, leading to unspent surpluses as stated by Scott et al. (2009). This is a built-in feature of inflexible static withdrawal rules.

In this study, we focus on an all-equity investment portfolio, as intuition and many theoretical models suggest that the share of equity in the optimal portfolio increases with the investment horizon (see, e.g., Warren 2019). Bengen (1994) notes that "holding too few stocks does more harm than holding too many stocks", where *stocks* refer to the overall stock market. We also assume that the investor does not have market timing ability. This could be due to perceived market efficiency or unwillingness to engage in such activity.

We compare the US and Finnish stock markets in our empirical analysis. Finland is selected for the empirical analysis for three reasons. First, Finland represents a smaller, inflation-prone country with a modest starting point. For example, Anarkulova et al. (2022) consider Finland a developed country as late as 1969, far later than other Nordic countries (except Iceland) and many other Western European countries. As Finland has shown convergence with other Western countries in terms of economic growth and prosperity, partly due to Nokia's success in the 1990s, it can offer the US market a run for its money. Second, Finland also represents the Nordic countries—an area often seen to combine Nordic capitalism and high quality of living.[1] As such, the Nordic model has often been brought up in the US political discourse either as something to admire or to avoid.[2] Finally, Finland's development can be seen to some degree as analogous to many present-day emerging countries, and, as such, the results can provide interesting insights for long-term investment in such an environment.

In the empirical analysis, we use annual stock returns from 1913 to 2023. The beginning of the sample period corresponds to the establishment of the Helsinki Stock Exchange in Finland in October 1912, as well as the introduction of an updated stock market index for Finland in Vaihekoski (2024). In practice, we compare several different withdrawal strategies, ranging from the classic 4% rule to more advanced ones. We begin by analyzing the historical outcome of these strategies. In the second step, we analyze a situation where the portfolio manager is faced with an uncertain future and is planning to follow a specific wealth withdrawal rule while aiming at preserving the real value of the portfolio. Assuming the same data-generating process (DGP) as in history, we use block bootstrap analysis to compare outcomes under different withdrawal rules.

We find that, historically, the geometric average nominal stock market return has been higher in Finland than in the USA (13.0% vs 11.5%). However, given Finland's higher inflation (7.3% vs 3.3%), the average real return has been lower than in the US market (5.3% vs 8.1%). Together with diverging early stock market development and lower volatility, one could have set the maximum (safe) fixed withdrawal rate of the initial savings clearly higher for the US (10.9%) than for Finland (1.8%) if the endowment had been set up in 1912. If the endowment had been established in any of the following 100 years, the average value-preserving rates would have been 6.6% and 7.8% in Finland and the US, respectively.

Looking ahead, we also analyze a situation in which the portfolio manager faces an uncertain future and plans to follow a specific wealth withdrawal rule while aiming to preserve the portfolio's real value. The results of the bootstrap analysis show that adhering to the classic 4% rule can lead to portfolio depletion in both countries, even though it offers the best chance to grow the portfolio's value. Therefore, it may be advisable to use a smoothing parameter that weights last year's outtake and the portfolio's current value. A smoothing parameter in the range of about 0.6 to 0.7, or even 0.8, for the USA seems feasible, smoothing the annual cash flows while preserving the portfolio's value at reasonable probabilities and keeping the risk of running out of cash acceptable.

Earlier results on different investment outcomes have been partly mixed. For example, Fama and French (2018) use a double bootstrap simulation and monthly nominal stock returns to evaluate the distribution of US buy-and-hold returns over 1963–2016 for horizons up to 30 years and conclude that the risk of landing in the

[1] For example, Finland has been ranked numerously as the happiest country in the world (Helliwell et al. 2024). See also https://www.forbes.com/sites/davidnikel/2025/03/20/5-life-lessons-from-finland-once-again-the-worlds-happiest-country/ (accessed May 26th, 2025).

[2] See, e.g., Anu Partanen and Trevor Corson, "Finland is a Capitalist Paradise," *New York Times*, December 7, 2019, https://www.nytimes.com/2019/12/07/opinion/sunday/finland-socialism-capitalism.html (accessed May 27th, 2025).

negative territory over a ten-year holding period is a mere 0.3%. The conclusion of Anarkulova et al. (2022) is the opposite. To alleviate the so-called easy data bias, which leads to the use of readily available data such as the US stock market, and the survivorship bias, they create an extensive sample covering 39 developed markets over the period 1841 to 2019. In their study, the markets are classified as developed using information available at the time of classification. Using monthly real returns and a block bootstrap procedure, they contend that, in the long run, stocks are not as safe as traditionally believed. More specifically, while the US data suggest a 1.2% probability of a loss of purchasing power of wealth over a 30-year period, the probability is 12.1% for the full sample of 39 markets. Hence, bad outcomes are not so rare that they can be ignored.

There are also several other studies on the topic.[3] Bengen (1994) uses a single realization of actual data, Guyton and Klinger (2006) simulate asset returns and inflation from a lognormal distribution with mean and volatility conformable to the underlying US assets over the period 1973 to 2004 to test different withdrawal rules. They find that initial withdrawal rates slightly exceeding 5% are feasible at a 99% confidence level for a 65% equity allocation. In certain situations, investors need to adjust and impose additional restrictions on their withdrawals so as not to exhaust the portfolio. Using a similar simulation approach, Klinger (2016) explores further the need for guardrails to avoid portfolio failures.

Instead of theoretical simulations, Spitzer et al. (2007) use bootstrap to resample actual US returns between 1926 and 2005. They find that a static rule, such as the 4% rule, may be an oversimplification in a complex world. Choosing between withdrawal schemes boils down to balancing the probability of success with the risk of financial ruin. Blume (2010) uses US annual data in combination with a simple bootstrap without replacement for different spending rules. For the *Ratchet rule*, i.e., taking the higher outcome of a nominal withdrawal equal to last year's withdrawal or a fixed percentage of the current value of the portfolio, the probability of failure of an equity-only investor after 50 years is about 40%. Blanchett and Frank (2009) use monthly data and bootstrapping to form a random annual return. For example, with an 80/20 stock/bond allocation, the failure rate is 4.7% over 30 years and 12.6% over 50 years.

In a recent study, Anarkulova et al. (2025) use a dataset covering 38 developed countries over the period 1890 to 2019, a block-bootstrap algorithm, different asset allocations, and different withdrawal schemes. Given that a 65-year-old couple retiring today would stick to a fixed 4% rule and a 60/40 stock/bond allocation, they would face a 16.8% probability of financial ruin within the lifetime of the longer-living member. In the US sample, the probability is 3.3%. Turning the point of view upside down, using a 100% stock allocation, and allowing for a 5% probability of ruin, the fixed withdrawal would be 1.9% for the entire sample, and 4.0% for the US sample. This result highlights the importance of broadening the perspective outside the USA.

Our results contribute to the rather scarce academic literature on optimal selection of withdrawal policies for long-term investing. A wide range of fixed and dynamic

[3] For a survey of the early literature, see Salter and Evensky (2008). See also Kitches (2014).

withdrawal rules is analyzed using the block bootstrap, which better preserves the statistical properties of returns than the traditional bootstrap. We compare the US market with a small European country, Finland, which, by present-day standards, would initially have been considered an emerging market. Our sample covers the history from 1912 onwards, including, e.g., both world wars and periods of high volatility. The results can be used to answer the question of how large an endowment is needed if the aim is to use the proceeds to finance certain operations (say, a hospital), and to analyze the risk of running out of money associated with the selected withdrawal policy. The results have major importance to long-term investors, some of whom may have very large portfolios. For example, the market value of endowment funds at universities and colleges alone accounted for $927 billion in the USA in 2021 (National Center for Education Statistics 2023).[4]

The remainder of the paper is organized as follows. Section 2 presents the theoretical background. Section 3 introduces the data used in this paper. Section 4 shows the empirical results. Section 5 presents the conclusions and offers suggestions for further research.

2 Theoretical background

One of the main questions in the classic optimal asset allocation literature is the choice between the market portfolio and the risk-free asset, and how the investment horizon affects the optimal choice. It is also one of the main issues in everyday investment decision-making. The question is how much risk one is willing to take, or, more specifically, what percentage of the portfolio should be allocated to the market portfolio and how much to the nominally risk-free asset. To answer the question, one needs to consider, for example, expected returns and their distribution, investors' risk aversion, investment horizon, potential cash flows and their constraints, other externalities, and bequest motive.

It is well known, since Samuelson (1969), that in a simple classic setup, an investor's investment horizon does not affect the optimal asset allocation. Here, however, the situation is slightly different. In our empirical analysis, we focus on an all-equity strategy without any market timing, akin to market efficiency. The investor wants to preserve the real value of the investment portfolio (endowment) for perpetuity, and, as a result, the investment horizon is infinite. As such, from the practical point of view, there is no risk-free asset available in real, or even in nominal, terms, for this kind of investment horizon (or even for much shorter ones), and the optimal choice has to be made between the market portfolio (here taken to be the stock market) and a low-risk asset.[5] Nevertheless, an all-equity strategy asks for an explanation of whether it is consistent with the theoretical recommendations for long-term investors.

[4] Note that endowment funds are also commonly used by hospitals, churches, nonprofit charities, and foundations.

[5] We also note that for most of our sample period, there were no functioning money markets in one of our countries (Finland), and as such investors' closest alternative to the nominally risk-free asset was bank deposits which did not offer positive real returns.

The question of whether investors should hold more equities over the long term is a debated topic. The workhorse for much of the work in the portfolio optimization literature, the power utility function, suggests that this is not the case under iid returns. However, there are several ways to adjust the standard model to allow the investment horizon to be considered in the optimization, which typically leads to a higher weight on equity (see, e.g., Cuthbertson and Nitzsche 2004; for a review of the literature). One may, for example, assume that stocks become less risky over the long run, which is based on the idea that (continuously compounded) returns are cumulated linearly with time while volatilities cumulate with the square root of time. Another approach is to modify the utility function or to consider the uncertainty of the parameters, or additional constraints and externalities (e.g., labor income). However, despite the adjustment, in many cases, the optimal investment decision does not depend on the investment horizon, even though the optimal decision, depending on the investor's risk aversion, may suggest an all-equity investment.

If we, on the other hand, take the reference-dependent utility functions as the starting point, the investment horizon most typically influences the portfolio decision. Reference-dependent utility functions usually give increasing weights to higher-returning assets, such as equities, as the horizon lengthens despite their higher volatility (Warren 2019). Using a standard setup, Warren shows how the reference-dependent utility function suggests an all-equity strategy if the investment horizon increases beyond 10 years. In addition, the result indicates that the portions of the equity distribution that provide a worse outcome than bonds decrease as the horizon lengthens.

Thus, overall, taking into account empirical evidence on historical stock market development (cf., Bessembinder 2018), an all-equity strategy can be considered a feasible starting point at least in certain situations for an investor with an infinite investment horizon, and as such, we can proceed with the analysis of the all-equity investment strategy. Furthermore, we avoid the issues in defining the risk-free rate, which is often quite problematic in a historical setup for many countries.

3 Research method and data

3.1 Withdrawal rules

We will evaluate a set of withdrawal rules, some of which are identical or close to rules in the previous literature, and some of which are developed for this article. The classic 4% rule, advocated for example by Bengen (1994), suggests withdrawing 4% of the initial value of the portfolio during the first year, keeping later withdrawals at this level in real terms. Our first rule generalizes this to a *fixed withdrawal* rule, where a fixed/static percentage of the initial wealth in real terms, i.e., a fixed real monetary amount, is withdrawn from the portfolio each year. In nominal terms, the withdrawal varies year-to-year with inflation, but not with the changes in the value of the portfolio. Note that under the fixed withdrawal rule, somewhat counterintuitively, the withdrawal as a percentage of wealth will change year-to-year.

In times of low inflation and stock market volatility, the first rule is likely to be feasible. However, in times of high inflation, especially when coupled with weak stock market performance, the portfolio might soon be depleted. Therefore, our second rule introduces a cap and a floor to the inflation adjustment. For as long as the inflation remains within the limits, the real value of the withdrawal stays unchanged. As a modification of the second rule, our third rule allows the cap to be lifted for as long as the value of the portfolio is above a certain threshold, for example, the initial endowment, in real terms. Correspondingly, the floor is removed if the value of the portfolio is above the threshold.[6] The second rule decreases the risk of exhausting the portfolio while it brings the risk of decreasing the real value of the withdrawal. The third rule tries to alleviate this risk by allowing one to benefit from good stock market development.

The advantage of classic fixed withdrawal rules is cash flow predictability. This might be crucial if the withdrawals are supposed to fund commitments already made, such as professorships, hospital operations, or retirement spending. However, the drawback is the inflexibility to adjust the withdrawals to reflect the stock market development. To circumvent this, we introduce a smoothing parameter γ (gamma) in the following way, representing our fourth rule:

$$CF_t = \gamma \cdot CF_{t-1} + (1-\gamma) \cdot FR \cdot W_t, \tag{1}$$

where CF_t is the withdrawal at the end of the period t $(t > 0)$, FR is a fixed withdrawal rate, W_t is the value of the portfolio at the end of period t just before the withdrawal, and γ is a smoothing parameter. For practical purposes, we assume gamma to be in the interval from zero to one. All monetary values are in real terms. As the portfolio is created at $t=0$, the imaginary initial (target) cash flow used in Eq. (1) is defined to be $CF_0 = FR \cdot W_0$. Note that the equation embeds the first rule if we set $\gamma=1$. On the other end of the spectrum, for $\gamma=0$, the withdrawal varies in tandem with the real stock market development. In the former case, the cash flow is steady but might deplete the portfolio. On the other hand, if the stock market develops favorably, the portfolio's value can increase manifold as the withdrawals are low relative to the increased value of the portfolio. In the latter case, the portfolio will never be completely depleted, but the withdrawals may be too volatile. For values of gamma between zero and one, a value closer to one leads to smoother withdrawals, whereas a value closer to zero leads to smoother withdrawal percentages. Rule four allows one to balance between predictable withdrawals and a chance to benefit from a good stock market development.

To introduce some flexibility and to increase the odds of getting back to the original portfolio value after a negative stock market development, we further develop the fourth rule such that γ is not a prespecified constant, but can vary according to market conditions. In our fifth rule, we set $\gamma_t = W_t / W_0$, and limit gamma to be between zero

[6] The second rule resembles the *inflation rule* of Guyton and Klinger (2006) who set the maximum increase due to inflation at 6%. The third rule again is similar to their *modified withdrawal rule*, however with the difference that they restrict the withdrawal if the portfolio return is negative, while we restrict it if the real value of the portfolio has decreased below the starting portfolio.

and one. When the value of the portfolio is above its initial value, gamma has a value of one, making the withdrawals constant, similar to rule one. When the value of the portfolio is low, then gamma becomes less than one, and the value of the portfolio has a higher impact on the withdrawals, protecting from the risk of running out of money. The downside of this rule is that the withdrawals can get really low if the stock market develops unfavorably. In addition, if the value of the portfolio starts to increase after low levels of wealth, the withdrawals typically level off below the original cash flows.

If it is crucial to keep the withdrawals at least at a certain level relative to previous withdrawals in nominal terms, and yet increase the cash flows if the stock market development allows for it, one can follow our sixth rule as follows:

$$CF_t^n = max\left(\phi \cdot CF_{t-1}^n, FR \cdot W_t^n\right), \tag{2}$$

where superscript n indicates nominal values and ϕ (phi) is the minimum requirement parameter. For example, if $\phi = 1$, the withdrawals will never be smaller than the previous withdrawal in nominal terms, but they can be higher given good stock market development. In times of poor stock market performance, this rule can also act as a softened saving mechanism, as it effectively reduces the real value of the withdrawal by inflation without the need to decide on the reduction of the withdrawal. One disadvantage of rule six, if compared to rule four, is that the risk of depleting the portfolio increases with stock market crashes.

To summarize, we consider the following rules:

Rule 1. Fixed withdrawal.

Rule 2. Fixed withdrawal with inflation cap and floor.

Rule 3. Fixed withdrawal with adjusted inflation cap and floor.

Rule 4. Cash flow smoothing I: $CF_t = \gamma \cdot CF_{t-1} + (1-\gamma) \cdot FR \cdot W_t$, with a prespecified value for γ.

Rule 5. Cash flow smoothing I as in rule 4, with $\gamma_t = W_t/W_0$, where $0 \leq \gamma_t \leq 1$.

Rule 6. Nominal cash flow smoothing II: $CF_t^n = max\left(\phi \cdot CF_{t-1}^n, FR \cdot W_t^n\right)$, with a prespecified value for ϕ.

As investors are interested in the purchasing power of money, we use real returns and real values throughout the calculations unless otherwise stated. The conclusions would be identical using nominal values, although the numbers would differ. In fixed rules, the first withdrawal is set as a specific percentage of the initial wealth, i.e., $CF_1 = FR \cdot W_0$. Depending on further adjustments, the actual withdrawal might be different, for example, as a result of restrictions on inflation or due to the stock market development. Initial wealth is set either at 1 monetary unit or alternatively at 100,000 monetary units. The withdrawals are made at the end of each year.[7] We also make some further refinements in the subsections. We start by looking at the history as it was and comparing these rules with the aim of preserving the real value of the initial

[7] For example, Guyton and Klinger (2006) and Klinger (2016) make the withdrawals in the beginning of the year.

portfolio.[8] After that, we continue by applying a bootstrap technique, i.e., resampling the history.

3.2 Sample data

We use Finland and the USA as our sample countries. To measure the stock market returns for Finland, we use the recently updated stock market index for Finland from the end of 1912 to 1981 in Vaihekoski (2024) and augment it with the WI index of Berglund et al. (1983) from 1982 to 1990, and the official OMXH general stock market index from the end of 1990 to the end of 2023. All indices are value-weighted, all-share total return indices aimed to measure the development of a stock market portfolio in Finland in nominal terms from the end of 1912 to the end of 2023. Year-end values are used to calculate percentage returns (111 observations). To calculate real returns, we use the annual inflation rate, calculated using the percentage change in the Finnish Consumer Price Index (CPI) values in December, and the Fisher equation. As the CPI index is only available from 1921, the inflation values for the early part of the sample are taken from Nyberg and Vaihekoski (2014).[9]

For the US market, we utilize the stock market returns from July 1926 onwards from Professor French's webpage.[10] For the period before, we use data from Professor Shiller's webpage.[11] The US inflation rate is calculated using the official CPI for all urban consumers from the Bureau of Labor Statistics, again augmented with the data from Shiller's webpage. All variables have been calculated similarly to those for Finland.

Table 1 provides descriptive statistics. Panel A shows that the arithmetic mean stock market return, i.e., the mean of all observations, is high in nominal terms for Finland, 17.0%. However, to get a better understanding of how the value of the investment portfolio develops, we need to take a look at the geometric mean, which also corresponds to the constant annual growth rate (CAGR). In nominal terms, the CAGR is 13.0%, which is surprisingly high compared to many other countries. However, nominal returns are highly influenced by inflation. During the sample period, Finnish inflation was also rather high. The geometric average equals 7.3% (the arithmetic average is 9.2%), which gives us an annual geometric average real stock market return of 5.3% (the arithmetic average is 9.6%), which, unlike the nominal return, is not internationally exceptionally high (cf., Dimson et al. 2023).

Looking at the other descriptive statistics for the three series, we can see that the standard deviation of Finnish inflation is as high as 25.5%. This reflects the high inflation periods during (and after) both World Wars and the oil crisis in the 1970s.

[8] The analysis can be easily adjusted for other goals as well. If the goal is to keep the value of the withdrawals, for example, in line with the salary level, the goal could be to increase the value of the portfolio in line with the growth of the GDP instead of inflation. Historically the GDP growth has been higher than the inflation.

[9] Heikkinen (2017) also creates an annual inflation index for 1910–1925. Overall, the difference is not substantial but on an annual level, there are some differences which may be due to the timing of the index as well as slightly different handling of black-market prices.

[10] https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[11] http://www.econ.yale.edu/~shiller/data.htm.

Table 1 Descriptive statistics

Variable	Arithmetic mean	Geometric mean	Standard deviation	Min	Max	Autocorrelation		
						ρ_1	ρ_2	ρ_3
Panel A: Finland								
Nominal stock market return	16.96	13.01	32.41	−51.31	167.04	0.111*	−0.143*	−0.174*
Inflation rate	9.15	7.32	25.48	−40.68	198.58	0.298*	0.231*	0.097*
Real stock market return	9.56	5.30	31.47	−55.95	161.76	0.156*	−0.140	−0.039
Panel B: USA								
Nominal stock market return	13.52	11.51	20.47	−44.03	57.36	0.012	−0.164*	0.099*
Inflation rate	3.27	3.16	4.77	−10.82	20.44	0.645*	0.261*	0.036*
Real stock market return	10.14	8.10	20.56	−38.28	56.17	0.021	−0.164	0.090*

Descriptive statistics for annual nominal and real stock market returns and the annual inflation rate. Sample periods for monthly data are from January 1913 to December 2023 (111 annual observations). All returns and rates are calculated using year-end values and in simple percentage form. Real returns have been calculated using the fisher equation. Autocorrelation coefficients significantly (5%) different from zero are marked with an asterisk (*)

The minimum and maximum values can also raise some questions. The highest inflation rate, +198.6%, took place in 1918, reflecting the Finnish Civil War in the early part of 1918 as well as a deficit in state finances (Heikkinen 2017). In general, there are a few years with small negative inflation, i.e., deflation, but the minimum value, −40.7% in 1919, also asks for an explanation. This value is mostly driven by a decrease in black market prices that took place after the economic situation stabilized following the Civil War.

Panel B in Table 1 shows the descriptive statistics for the US. As we can see, the nominal stock market returns have been, on average, lower than in Finland (arithmetic and geometric means are 13.5% and 11.5%), but so has the inflation rate. The real stock market returns have been clearly higher. The geometric mean is 8.1% which is almost 2.8% points higher than in Finland, all the while the volatility is approximately one-third lower.

4 Empirical results

4.1 Rearview look

As a starting point, we consider an investor who does not withdraw any money from her portfolio. For an institution, this implies an infinite investment horizon, and for an individual, the goal is to maximize the bequest ("die as a millionaire"). The data provides an opportunity to analyze what would have happened to an all-equity investor who invested 100 at the beginning of the period (the end of 1912) in the value-weighted stock market portfolio as presented by the market index. In Finland, given the monetary reform of 1963 and the transition to the euro in 1999, an investment of

FIM100 at the end of 1912 corresponds to 467.88 euros at the end of 2023.[12] Assuming a starting value of 100 and indexing the development, the portfolio would be valued at 78,836,450 (corresponding to 788,364.50 new FIM or 132,593 euros) by the end of 2023, reflecting the constant annual growth rate (CAGR) of 13.1%. Considering the inflation with a geometric average of 7.3%, the CAGR in the portfolio's value in real terms would be 5.3% and as a result, in real terms, the value of the portfolio would be 31,030. At the same time, the value of the portfolio for an investor in the US would be much lower, USD 17,923,770 in nominal terms in 2023, but in real terms, it would have been 567,073, reflecting the much higher real return of 8.10% due to lower inflation in the US. Note that the initial values of FIM100 and USD100 are not directly comparable, as their purchasing power at the time was different.

Next, we study the outcome of the six withdrawal strategies selected for closer analysis. We analyze them with the goal of preserving the real value of the portfolio at the end of the sample period. The solution can be calculated analytically for the first and the second rule, but for the other rules, one minimizes the difference between the initial and end values of the portfolio by changing the value of the parameter *FR*. First, we consider an investor who follows the classic 4% fixed withdrawal rule. Using an initial saving of 100,000 as an example, it implies that at the end of the year 1913, the investor would withdraw 4% of the initial value of the portfolio, i.e., 4000. In all subsequent years, the withdrawal is adjusted by inflation.[13] Following this rule would not have been feasible for Finland, as the last year allowing a full 4% withdrawal would have been 1924. Hence, only 12 full withdrawals would have been possible. Now, solving for the highest value-preserving *FR*, we get a surprisingly low value, 1.83%, which is by a wide margin less than that for the USA, 10.95%. As the difference in the average real returns was much smaller, the large difference may come as a surprise. However, it can be explained by the underdeveloped state of the Finnish stock market at the beginning of the sample, as well as by higher volatility in the Finnish stock market.

Note that in the long run, the margins between portfolio exhaustion and a decent result are minuscule. For example, withdrawing 1836.90, i.e., 1.84%, would have exhausted the portfolio exactly in 2023, while withdrawing only slightly less, 1830.98, would preserve the portfolio's real value. Note also that following rule one would have required a lot of guts from the investor in Finland. The wealth would have exceeded the initial wealth only in 11 years. In fact, after falling to only 13.4% of its initial value in 1916, the portfolio did not exceed its initial value again until 1999. The lowest value is as recent as 1992, when it was 12.5% of the initial value. The withdrawal during that year would have been as large as 14.7% of the portfolio.

[12] Obviously, the adjustment for inflation does not account for the development in the standard of living. Nowadays, an investment of 500 euros is within everybody's reach, but in 1912, monthly salaries for blue-collar workers in Helsinki were less than FIM150 (cf., Statistical Yearbook of Helsinki 1912 [Helsingin Kaupungin Tilastollinen Wuosikirja 1912], 1915).

[13] Note that end-of-year withdrawals can also be interpreted as taking place at the beginning of the next year. Note also that if an investor plans to use the savings to support herself even during the first year, the analysis that follows implicitly assumes that she has 4% of the initial savings set aside for the first year. If we assume that she withdraws 4% immediately at the beginning of the first year, the savings would in effect be only 96% of the initial wealth. This assumption can be easily adjusted.

Analyzing the results closer reveals that the starting year plays a significant role in determining the allowable withdrawal rate. This is especially the case if the sample begins with a negative (real) stock market development, as fixed withdrawals and asymmetric behavior of percentages lead to long periods to recover the lost value of the portfolio. To study this, we solve the maximum withdrawal rates under rule 1 if the portfolio had been set up in 1913, 1914, and so on until the year 2012. The results are shown in Fig. 1.

For Finland, it is obvious that if the analyzed period had started a decade later, the allowable withdrawal rates would have been much higher. The opposite had been true for the USA. In fact, if the starting year had been 1925, the withdrawal rate would have been higher for Finland than for the USA. The highest withdrawal rate (22.74%) for Finland would have been achieved if the initial year had been 1993. For the USA, the maximum rate (18.71%) would have been in the year 1923. The average maximum withdrawal rates are also marked in the Figure. They are 6.63% for Finland and 7.79% for the US.

For our second withdrawal rule, we adjust the first rule by setting a lower and upper bound for inflation adjustment for the withdrawals. For example, the Finnish

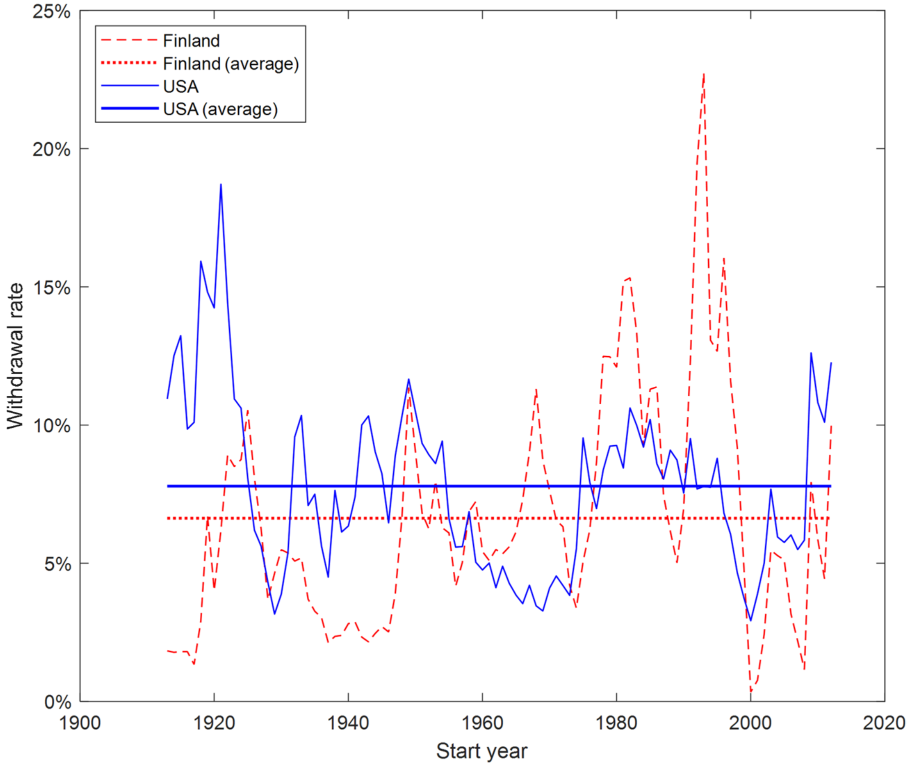


Fig. 1 Value-preserving withdrawal rates. The figure shows the maximum withdrawal rates that preserve the real value of the endowment until the end of 2023 if one sets up the endowment and starts to withdraw money from it a year later, as shown on the x-axis, and adjusts the withdrawal by the inflation rate in the subsequent years. Values are shown for the first 100 years

inflation of 130% and 199% in 1917 and 1918 would have been detrimental to the portfolio, erasing the portfolio in 1924 if rule one with a fixed withdrawal of 4% of the initial portfolio value had been followed. To use rule two in different countries with different inflation levels, we set the upper bound (cap) to the average of the inflation rates plus one standard deviation in the country in question. As a result, the upper cap is set to 34.6 and 8.0% for Finland and the USA, respectively. The inflation exceeded the cap six times in Finland during the sample period and fourteen times in the USA. If the inflation has exceeded the cap, this restriction in effect decreases the real value of the withdrawal. To set the lower bound, we note that negative inflation rates (deflation) will decrease the nominal withdrawals, which is likely to be behaviorally difficult. As a result, we set the floor to zero. In addition, this serves as a counterbalance to the cap, as if the deflation exceeds the floor (i.e., inflation is less than the floor), in effect, the purchasing power of the withdrawal increases. The floor is used for 12 years in Finland and 11 years in the USA.

Targeting again to preserve the real value of the portfolio at the end of the sample allows for an initial withdrawal of 4352.60, corresponding to a fixed withdrawal rate of 4.3% for Finland (USA: 11.7%). This is clearly more than under rule one. The downside is that when the inflation exceeds the cap, the real value of the withdrawal decreases. On the other hand, due to the selected floor, the purchasing power of the withdrawal increases with deflation. The effect of both on the real value of the withdrawal in Finland can be seen in Fig. 2 for different withdrawal rates and three strategies either to deplete the portfolio ("Target 0") at the end of the sample, keeping its real value unchanged ("Target 100"), or increasing its value five-fold ("Target 500"). The baseline strategy, Target 100, shows that the real value of the withdrawal would decrease to 1146.32 in 2023, i.e., its real value would decrease by 80% from the beginning of the sample.

Overall, the second rule is good in the sense that the withdrawals are not influenced by the (negative) stock market development, and they are quite stable and foreseeable to some degree, even in nominal terms. The risk of depleting the portfolio is also smaller than for a fixed withdrawal rule. The downside of this rule is that the real value of the withdrawal can decrease dramatically if the inflation exceeds the upper bound. Second, the withdrawals do not take advantage of the (good) stock market development unless one adjusts the rule along the way.

Rule three allows removing the cap and keeping the floor of rule 2 if the accumulated wealth is above a certain threshold. In practice, we make the inflation adjust-

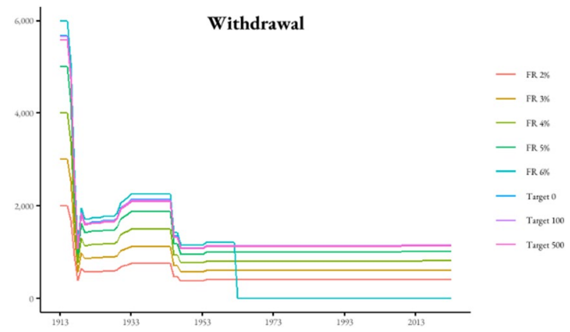


Fig. 2 The real value of cash withdrawal under rule two for Finland. The withdrawals for different fixed withdrawal rates, and for different end-of-period wealth levels (Target 100 equals preserving the portfolio's real value, Target 500 equals a five-fold increase in value, and Target 0 equals full consumption of the portfolio)

ment to the withdrawal and forfeit the deflation adjustment if the value of the portfolio exceeds the sum of the initial value and the previous withdrawal. As a result, rule three allows us to set the initial withdrawal rate slightly higher at 6.50% for Finland and 11.74% for the USA. Interestingly, rule three does not seem to work in Finland. Although the initial withdrawals are slightly higher than under rule two, given the high inflation from 1917 onwards, combined with poor stock market performance, the withdrawals are lower than under rule two after that. However, the situation in the USA is different. After the 18% inflation in 1946, rule three has provided higher withdrawals than rule two.

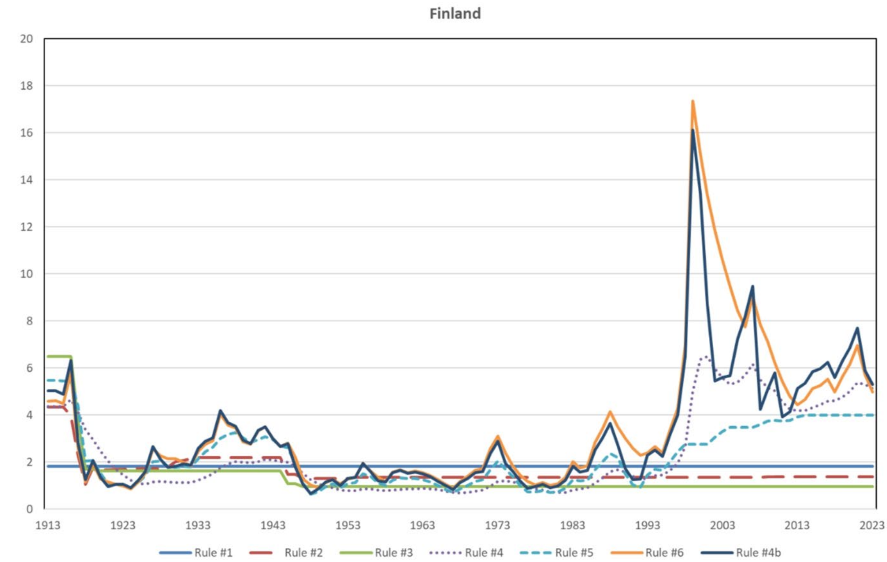
The downside of the first two rules is that they do not utilize positive stock market development. For the most part, this is also the case for the third rule. As a result, we analyze rule four, where the withdrawal is a weighted average of the previous withdrawal and a fixed percentage of the value of the portfolio before the withdrawal at the end of each year.

Now, rule four asks to select the value for the smoothing parameter, gamma. If gamma is set to zero and the stock market develops favorably, rule four will allow one to increase the value of the portfolio and, as such, it will lead to higher withdrawals. On the other hand, this rule has the inconvenience that the withdrawals are influenced directly by the yearly stock market movements, which may not be acceptable in practice if the withdrawals are used to finance certain committed activities (living expenses, hospital operations, pensions, etc.) that cannot be adjusted yearly. However, for academic curiosity, we can calculate the result. Solving for the maximum withdrawal rates that preserve the real value of the portfolio, we get 5.04% for Finland and 7.49% for the USA.

For a more realistic case, we test three different values for the smoothing parameter. In practice, we set it to 0.25, 0.50, and 0.75. We again solve for the maximum withdrawal rates using optimization that preserves the value of the portfolio at the end of the sample period. We can see that the rates are higher than before, 4.96% (gamma = 0.25), 4.79% (gamma = 0.50), and 4.37% (gamma = 0.75) for Finland. For the US, the rates are 7.44%, 7.34%, and 7.02%, respectively. In both cases, we can see that the closer the value of gamma is to zero (leading to less smoothing), the higher the fixed withdrawal rate one can use.

As a modification of the rule, we allow gamma to vary with the current-to-initial wealth ratio, which we label rule five. Limiting gamma to vary between zero and one, and solving again for the maximum withdrawal rates that preserve the real value of the portfolio, we get 5.04% for Finland and 12.46% for the USA. Under rule five, withdrawals react more quickly to the stock market development if gamma gets higher values than those used in rule four. A feature of rule five is that after good stock market development, gamma becomes one, and the withdrawal stays put despite further good development. This can be seen especially in the US market (cf., Fig. 3).

The downside of rules four and five is that they allow the withdrawal to decrease in nominal terms, which might be undesirable or behaviorally difficult. As a result, we analyze a situation where the investor follows rule six. Now, the nominal cash withdrawal is always a fixed percentage of the nominal value of the wealth ($FR \cdot W_t^n$), but no less than the previous nominal withdrawal times the smoothing parameter ($\phi \cdot CF_{t-1}^n$). Setting the smoothing factor ϕ to one, i.e., the nominal value never



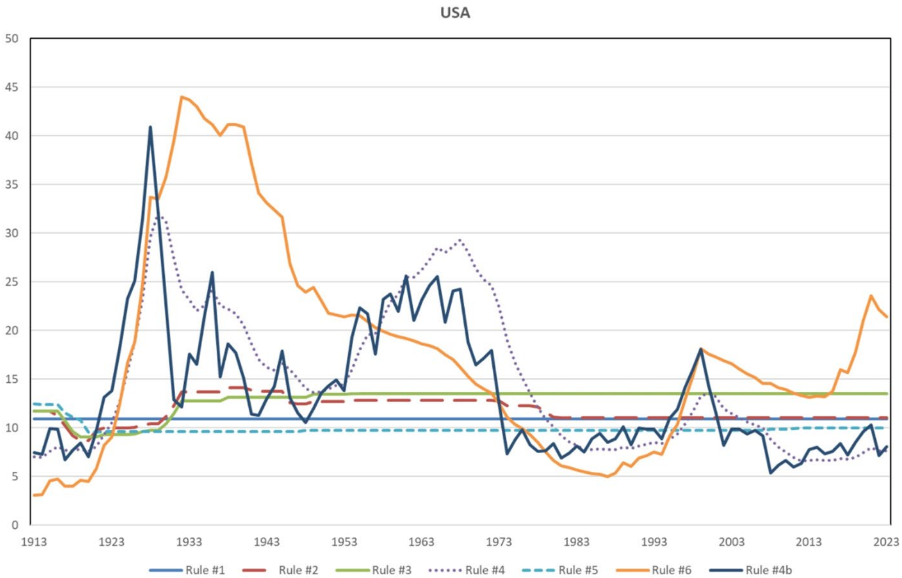


Fig. 3 Real withdrawals under different rules. The upper figure is for Finland, and below is for the USA. Rules have been defined in the text. For rule #4, gamma is set to 0.75. #4b refers to the special case where gamma is set to zero, and the withdrawals vary with the development of the stock market. For rule #6, phi has been set to 0.90

decreases. In this case, solving for the maximum nominal withdrawal rate during the sample period, we get *FR* to be 3.02% for Finland and 3.10% for the US. Now, setting the term ϕ to 0.90, the withdrawals are always at least 90% of the previous year's withdrawal in nominal terms, but given a good year on the stock market, the withdrawal can be higher. Solving for the value-preserving rates shows that this time the withdrawal rate can be higher: 4.58% in Finland and 5.12% in the US. If the smoothing factor is set lower, the withdrawal rate can be set even higher, but then one loses part of the original motive for this rule.

Now, taking the results together, Fig. 3 shows the real values of the withdrawals in Finland and the US under different withdrawal rules for an investment of 100 in 1913. It is obvious that the rules differ in their payouts and how they have developed over time. For example, rule 1 provides constant withdrawals, whereas rule four shows widely varying withdrawals, especially when gamma is set to zero (we label it as rule 4b, i.e., a fixed percentage of the market value rule). When gamma is set to less than one (here 0.75), the withdrawals are smoothed, as one can see from the Figure. Interestingly, rules two and three allow initially higher withdrawals than rule one, but at the cost of later development. On the other hand, rule six provides initially lower withdrawals than rule one, but later one can increase them above the level of rule one.

It is also easy to see that none of the rules is superior to all other rules. To compare the rules statistically, we report descriptive statistics for the withdrawals in Table 2. If the purpose of the portfolio is to pay out as much as it can while preserving the value

Table 2 Historical realizations

Withdrawal rule	FR (%)	Mean	Std. dev.	iCV	iCV(*n*)	Min
Panel A: Finland						
Rule 1, fixed withdrawal	1.83	1.83	0	n/a	0.90	1.83
Rule 2, fixed withdrawal with inflation floor/cap	4.35	1.62	0.59	2.74	0.90	1.05
Rule 3, fixed withdrawal with adjusted inflation f/c	6.50	1.35	1.07	1.26	0.90	0.96
Rule 4, cash flow smoothing (gamma = 0.00)	5.04	3.16	2.58	1.22	0.63	0.66
Rule 4, cash flow smoothing (gamma = 0.75)	4.37	2.30	1.76	1.31	0.64	0.69
Rule 5, cash flow smoothing (time-v. gamma)	5.47	2.22	1.26	1.76	0.69	0.64
Rule 6, nominal cash flow smoothing (phi = 1.0)	3.02	8.86	11.22	0.79	0.61	0.83
Rule 6, nominal cash flow smoothing (phi = 0.9)	4.58	3.46	3.02	1.15	0.64	0.87
Panel B: USA						
Rule 1, fixed withdrawal	10.95	10.95	0	n/a	1.19	10.95
Rule 2, fixed withdrawal with inflation floor/cap	11.70	11.77	1.23	9.58	1.16	8.68
Rule 3, fixed withdrawal with adjusted inflation f/c	11.74	12.86	1.34	9.61	1.23	9.08
Rule 4, cash flow smoothing (gamma = 0.00)	7.49	13.65	6.75	2.02	1.00	5.36
Rule 4, cash flow smoothing (gamma = 0.75)	7.03	14.80	7.41	2.00	1.01	4.21
Rule 5, cash flow smoothing (time-v. gamma)	12.46	9.87	0.56	17.77	1.11	9.55
Rule 6, nominal cash flow smoothing (phi = 1.0)	3.10	17.71	10.89	1.63	0.76	3.10
Rule 6, nominal cash flow smoothing (phi = 0.9)	5.12	29.04	14.29	2.03	1.17	5.12

The table shows descriptive statistics for annual cash withdrawals in real terms under different withdrawal rules in Finland and the USA. In all cases, the withdrawal rate (FR) has been maximized, all the while the real value of the portfolio at the end of 2023 has to equal that at the beginning of the sample. iCV stands for the inverse of the coefficient of variation, i.e., the average withdrawal divided by the standard deviation. iCV(n) is calculated using the nominal cash flows of the principal, the best-performing strategy for Finland, based on average cash flow, is rule 6 with phi set to one. The average (median) withdrawal is 8.86 (3.02), which is clearly more than for any other rule. The downside is that withdrawals under rule 6 also have the highest standard deviation (11.22). The classic fixed withdrawal and its inflation-adjusted variants (rules 1–3) have performed the worst for Finland.

Table 2 reports the results also for the USA. Typically, US investors would have been able to make, on average, two to three times higher withdrawals. The ranking of the analyzed withdrawal rules is also different from that of Finland, although again, the best-performing rule would have been rule 6, but now with phi set to 0.9. The average withdrawal would have been 29.04 (with a standard deviation of 14.29). For the USA, rule five would have performed the worst in terms of the total payout.

Overall, looking back, choosing the rule that maximizes the average payoff, which also maximizes the total sum of payoffs, often comes at the cost of higher variation in the payoffs. As the smooth series of withdrawals is often desired, one may consider the ratio of average withdrawal to standard deviation (akin to the Sharpe ratio), i.e., the inverse of the coefficient of variation iCV, as another ranking measure for selecting the optimal withdrawal rule. Obviously, we cannot calculate it for rule 1 in real terms, as the withdrawals are constant. Table 2 reports the ratios for all other cases. Now, the ranking of the withdrawal rules is different. The best rule by far with this ratio is rule two (ratio is 2.74) in Finland, followed by rule five (ratio 1.76). For the US, using this criterion, rule five is found to be the best as it offers fairly high withdrawals with minimal volatility. The inflation-adjusted variants of the classic fixed rate rule, i.e., rules two and three, are also surprisingly good with this criterion.

Finally, we also calculate the iCV ratio using nominal cash flows, as the decision makers in practice may prefer steady cash flows in nominal terms. Now, the withdrawals even under rule one show volatility due to inflation. The ranking is somewhat different. For Finland, rule three shows the highest ratio (0.904), but the first and second rules are not far behind (0.900 and 0.899, respectively). For the USA, rule three is also found to show the highest ratio (1.234), followed by rule one (1.190).

Looking at the results, we can see that there is no universal best rule for both countries. Similarly, using different criteria to compare the rules gives different rankings for them. This suggests that the decision-maker has to take a stand on what the goal of portfolio management is: to maximize withdrawals or to balance them with their volatility, possible in nominal terms.

4.2 View forward

The analysis thus far has been subject to a forward-looking bias, i.e., assuming that we *know* the returns, and the inflation rates and the order in which they take place. Now we turn to the withdrawal strategy one should follow given an uncertain future. We assume that the historical data-generating process remains intact and can be used as a starting point for predicting the future. We focus on rule four as it is versatile in the sense that it allows for different withdrawal strategies from smoothed series to fully fixed (rule 1) and a fully market-dependent one (rule 4b). Now, we ask two main questions. First, what is the maximum (safe) withdrawal rate to keep the value of the portfolio intact, and how is the rate distributed? Second, if one chooses a certain withdrawal rate and gamma, what would happen to the value of the portfolio, and what kind of risk would be involved?

We use a stationary circular block bootstrap method as suggested by Politis and Romano (1992 and 1994), following the approach of Anarkulova et al. (2022 and 2025). The circular block bootstrap will give each observation in the original sample the same probability. Block lengths are random following a geometric distribution with a minimum and maximum values of two and 10 years. Given the block size, we pick a random year from a uniform distribution as the starting point for the block. Using these parameters, we resample realized annual real returns with replacement to represent the returns for different investment horizons and repeat this procedure one million times. Using these returns, we can analyze how a given withdrawal rule has performed during the investment period in question, focusing especially on how feasible and risky it would be. For convenience, we assume an initial wealth of 100,000.

To answer the first question, we use Eq. (1) and the same investment horizon as in the original data to solve for the value-preserving withdrawal rates under four different values for gamma. At one end, we assume that the gamma parameter is one (no smoothing), which is the classic fixed withdrawal rule (rule 1). As a result, the fixed withdrawals stay the same in real terms, but their nominal value and the value as a percentage of the portfolio will vary over time. On the other hand, we also set gamma to zero (rule 4b), allowing annual withdrawals to vary with the portfolio's market value. In addition, we use two intermittent values (0.50 and 0.75) for gamma.

From the simulations, we obtain the distribution of the safe rates. The mean values for the *FR* are 5.00% (gamma = 0), 4.76% (gamma = 0.50), 5.63% (gamma = 0.75), and 5.41% (gamma = 1). For the US market, the values for the *FR* go from 7.48% (gamma = 0), 7.32% (gamma = 0.50), 9.70% (gamma = 0.75), and 8.17% (gamma = 1).[14] As expected, the rates are often 2% points higher for the US market. It is evident that the solution is not a linear function of the gamma parameter. Note that the averages for *FR* differ from those derived earlier in the historical analysis (e.g., for gamma = 1, the historical *FR* for Finland was estimated to be 1.83%). This can be explained by the asymmetry of percentage returns.

Figure 4 shows the distribution of solutions for *FR* when gamma is set to zero or one for the Finnish and US stock markets. Note that it is also possible to observe negative values. They are examples of poor stock market development, which has led to negative withdrawals, i.e., one has to add more money to the portfolio to preserve its real value at the end of the period. For example, if we set gamma to one, 3.14% of the solutions are negative in Finland. In fact, using all four values above for gamma, the number of negative solutions is quite stable, between 2.8 and 2.9% in all cases, indicating that, regardless of the rule followed, there is the possibility that positive withdrawals are not always possible if history repeats itself. The likelihood of observing a negative solution is much lower in the USA, occurring in only three out of million bootstrap samples. It is also evident from the Figure that the optimum rate does not follow a normal distribution for gamma equal to one. For rule 4b (gamma = 0), the distribution is much closer to the normal distribution.

[14] For gamma equal to 0 and 0.50, the medians are almost the same as the averages. For gamma equal to 0.75 and 1.00, the medians are 4.37% and 4.06% for Finland, and 7.39% and 7.54% for the USA.

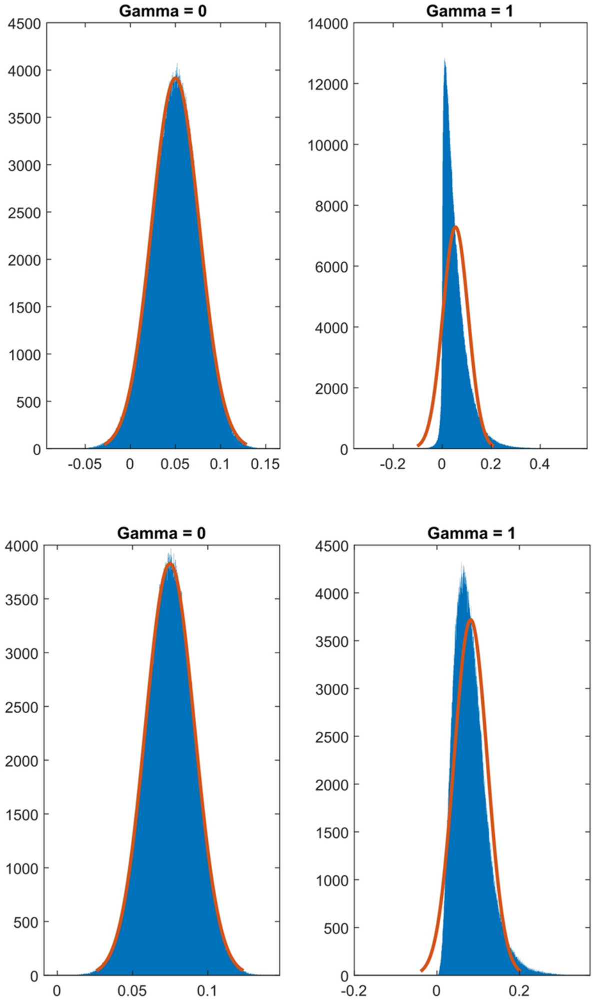


Fig. 4 Distribution of maximum allowable withdrawal rates under rule 4 using bootstrapped Finnish (above) and US (below) real returns. The left figure is simulated with gamma set to zero (rule 4b, fixed percentage of the value rule) and the right figure with gamma set to zero (rule 1, fixed sum rule). The normal distribution curve has been fitted to both figures

Analyzing the simulation results for Finland in Table 3, we can draw several interesting conclusions. If we set gamma to one (rule 1), the median withdrawal across all bootstrap samples is 4000, but since none of the withdrawals can be more than that, the mean value is less than 4000, and more so the longer the investment period, as there is always the risk that one runs out of money. For example, the risk of running out of money at the end of the investment horizon increases from 11.5% to 47.9% when the horizon increases from 15 years to 100 years. On the other hand, given favorable stock market development, rule one can lead to high growth of the investment portfolio, and more so, the longer the investment horizon. For example, over 100 years, on average, the size of the portfolio has multiplied by the factor of 5012, far more than under any other rule. This is caused by the fact that if the withdrawal rate is lower than the geometric average of the stock market returns, the investment portfolio benefits from the compounding effect. However, the standard deviation of the outcomes is also the largest, and the median result is only 2.4. All in all, it seems

Table 3 Block bootstrap results with rule 4 for Finland

	15 years				30 years				100 years			
	γ=0	γ=0.5	γ=0.75	γ=1.0	γ=0	γ=0.5	γ=0.75	γ=1.0	γ=0	γ=0.5	γ=0.75	γ=1.0
Panel A: End-of-period cash flow												
Mean	9 078	8 875	8 357	3 539	19 060	19 119	18 945	2 770	592 839	666 503	827 556	2 083
Standard deviation	15 927	14 843	12 686	1 277	67 850	69 121	68 161	1 845	12 470 883	15 450 252	22 659 102	1 998
Minimum	16	18	0	0	4	3	0	0	0	0	0	0
Median	5 063	5 087	5 100	4 000	5 866	5 763	5 594	4 000	12 505	10 859	6 910	4 000
Maximum	2 033 577	1 488 966	1 048 219	4 000	12 773 318	15 490 004	16 095 178	4 000	>3 533 M	>5 656 M	>10 685 M	4 000
Prob(≥3 000)	68.3%	68.7%	69.9%	88.5%	66.8%	66.1%	64.7%	69.3%	69.6%	67.0%	59.6%	52.1%
Prob(≥4 000)	58.6%	59.0%	59.7%	88.5%	59.9%	59.2%	58.1%	69.3%	65.9%	63.4%	56.3%	52.1%
Prob(≥5 000)	50.5%	50.6%	50.8%	0.0%	54.2%	53.6%	52.8%	0.0%	62.9%	60.5%	53.8%	0.0%
Panel B: End-of-period wealth / the initial wealth												
Mean	2.2	2.2	2.3	2.8	4.6	4.8	5.3	9.7	142.3	167.8	229.3	5 012.4
Standard deviation	3.8	4.2	4.6	6.0	16.3	18.8	23.1	44.7	2 993.0	3 966.8	6 435.0	127 768.0
Minimum	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Median	1.2	1.2	1.2	1.3	1.4	1.4	1.3	1.4	3.0	2.5	1.6	2.4
Maximum	488.1	559.9	641.7	824.3	3 065.6	3 750.3	4 857.8	9 296.9	847 896.8	1 180 114.4	2 001 846.1	3 632 563.3
Prob(≥0.75)	66.9%	65.8%	64.4%	62.3%	63.8%	64.2%	61.1%	57.2%	68.0%	66.1%	57.9%	51.3%
Prob(≥1.00)	57.2%	56.4%	55.6%	55.9%	58.8%	57.5%	55.1%	54.3%	65.3%	62.5%	54.7%	51.1%
Prob(≥1.25)	48.9%	48.5%	48.3%	50.1%	53.1%	52.0%	50.7%	51.7%	62.4%	56.6%	52.2%	50.9%
Prob(Nothing)	0.0%	0.0%	0.1%	11.5%	0.0%	0.0%	0.7%	30.7%	0.0%	0.0%	4.5%	47.9%

This table reports results for end-of-period cash flows (Panel A) and wealth (Panel B) after conducting one million block bootstraps using percentage realized real annual stock market returns for Finland from 1913 to 2023. Wealth is assumed to be 100,000 in the beginning. The selected initial withdrawal rate is 4%. The investment period is assumed to be either 15, 30, or 100 years. The gamma parameter measures the weight between the realized return and the target payout (4% equaling, 4000). A value of zero always pays out 4% of the current value of the portfolio, whereas a value of one always pays 4% of the initial wealth. Values in between weigh these alternatives linearly. In panel A, Prob(≥3,000) gives the probability that the cash flow at the end is higher than or equal to 3000 in real terms. Other values are calculated similarly. In panel B, the end-of-period wealth has been divided by the initial wealth that the risk of losing the portfolios is likely to be too high for most endowments, and as such, one may want to consider either a lower value for *FR* or setting gamma to less than one.

Now, if we set gamma to zero (rule 4b), we can see that the outcome in terms of withdrawals is extremely favorable in the long run. Both mean and median end-of-period cash flows and wealth grow the longer the investment horizon. For example, the median cash flow after 100 years is 12,505, and the end-to-initial wealth ratio is 3.0. For any horizon, the median end-of-period cash flow is almost always higher than for any other value for gamma. Interestingly, this is not the case for the means. This is due to the fact that if gamma is set to zero, the withdrawals take full benefit from a positive stock market development, whereas positive values for gamma utilize it only partially. Nonetheless, rule 4b seems to offer the potential to increase the withdrawals and wealth in real terms, all the while having a low risk of depleting the investment portfolio. Note, however, that this rule is not risk-free in the sense that there is a 32% probability of losing 25% of the investment portfolio over 100 years. Thus, one may look for a smoother withdrawal rule by adjusting the gamma parameter.

If we analyze the results for the US in Table 4, they are very much in line with those for Finland. However, given the historically better stock market performance on average, setting gamma to less than one leads to higher median end-of-period cash flows. Interestingly, the same is not the case with the means, which reflect the higher skewness of the outcome for Finland. This can be seen, for example, by the maximum end-of-period wealth. It is always greater for Finland, reflecting the outcome of bootstrapping the best years from the sample. Nonetheless, the main conclusion remains. Rule one comes with a risk of depleting the portfolio in the long run, but it can lead to a major increase in the value of the portfolio, which may or may not be desirable in practice. Rule 4b leads to higher withdrawals and almost negligible risk of depleting the portfolio.

Now, it would be tempting to state that the optimal strategy for endowments is to pay 4% of the value of the portfolio (under rule 4b). However, for many reasons (e.g., from the recipients' perspective), this is not the case, as it would make the cash flows vary with the real stock market returns. Thus, a strategy with annual withdrawals that are more stable than the stock market might be considered more feasible and thus more optimal across all issues considered. Looking at the results with different smoothing parameters, it becomes evident that the gamma should be less than one, yet fairly high. For example, for $\gamma = 0.75$, the average and median payouts increase compared to $\gamma = 1$, at the same time decreasing the probability of running out of wealth altogether from 11.5% to 0.1% for a 15-year horizon in Finland (0.6 and 0.0% for the US). The corresponding numbers are 47.9% and 4.5% for a 100-year horizon (12.2 and 0.0% for the US).

Now, the analysis thus far has been done with the classic value, 4%. An obvious question is to ask what happens if we select a different value. To analyze the risk of running out of cash, we repeat the bootstrap analysis with *FR* set to three, four, and 5%. The results are reported in Table 5. Four different values for gamma are used. Panel A shows the results for Finland, and Panel B for the USA. Note that the risk of depleting the portfolio increases with the withdrawal rate, gamma, and investment horizon, especially if the investment period is longer than 10 years. This is as

Table 4 Block bootstrap results with rule 4 for the USA

	15 years				30 years				100 years			
	$\gamma=0$	$\gamma=0.5$	$\gamma=0.75$	$\gamma=1.0$	$\gamma=0$	$\gamma=0.5$	$\gamma=0.75$	$\gamma=1.0$	$\gamma=0$	$\gamma=0.5$	$\gamma=0.75$	$\gamma=1.0$
Panel A: End-of-period cash flow												
Mean	9 431	9 209	8 647	3 976	21 133	21 290	21 195	3 749	917 159	1 069 537	1 405 212	3 511
Standard deviation	8 128	7 742	6 801	310	30 613	31 292	31 208	969	5 434 860	6 974 943	10 246 863	1 309
Minimum	163	230	270	0	50	62	17	0	23	17	0	0
Median	7 320	7 232	6 972	4 000	12 623	12 599	12 516	4 000	168 436	176 382	194 564	4 000
Maximum	206 857	184 099	147 755	4 000	2 636 574	2 555 997	2 719 783	4 000	> 1 860 M	> 2 604 M	> 3 811 M	4 000
Prob(≥3 000)	89.5%	89.6%	89.7%	99.4%	92.8%	92.5%	91.8%	94.2%	98.7%	98.5%	98.0%	87.8%
Prob(≥4 000)	80.4%	80.3%	80.2%	99.4%	87.9%	87.5%	86.8%	94.2%	98.1%	97.9%	97.3%	87.8%
Prob(≥5 000)	70.9%	70.7%	69.9%	0.0%	82.7%	82.3%	81.6%	0.0%	97.5%	97.2%	96.5%	0.0%
Panel B: End-of-period wealth / the initial wealth												
Mean	2.3	2.3	2.5	3.9	5.1	5.4	6.0	10.7	220.1	271.3	398.5	7799.9
Standard deviation	2.0	2.1	2.3	3.0	7.3	8.3	10.0	19.4	1 304.4	1 824.3	3 156.9	56 016.5
Minimum	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Median	1.8	1.8	1.8	2.1	3.0	3.1	3.3	5.3	40.4	43.4	50.7	935.8
Maximum	49.6	55.5	62.7	80.8	632.8	769.0	982.0	1 878.1	446 461.7	649 599.4	1 213 134.8	> 21.8 M
Prob(≥0.75)	88.4%	87.8%	86.9%	84.9%	92.2%	91.6%	90.6%	86.5%	98.7%	98.4%	97.9%	87.7%
Prob(≥1.00)	78.8%	78.4%	78.0%	78.1%	87.0%	86.5%	85.7%	84.0%	98.0%	97.7%	97.1%	87.7%
Prob(≥1.25)	68.9%	68.8%	69.1%	71.4%	81.7%	81.3%	80.8%	81.5%	97.3%	97.0%	96.4%	87.7%
Prob(Nothing)	0.0%	0.0%	0.0%	0.6%	0.0%	0.0%	0.0%	6.3%	0.0%	0.0%	0.0%	12.2%

this table reports results for end-of-period cash flows (Panel A) and wealth (Panel B) after conducting one million block bootstraps using percentage realized real annual stock market returns for the USA from 1913 to 2023. Wealth is assumed to be 100,000 in the beginning. The selected initial withdrawal rate is 4%. The investment period is assumed to be either 15, 30, or 100 years. The gamma parameter measures the weight between the realized return and the target payout (4% equaling 4,000). A value of zero always pays out 4% of the current value of the portfolio, whereas a value of one always pays 4% of the initial wealth. Values in between weigh these alternatives linearly. In panel A, Prob(≥3000) gives the probability that the cash flow at the end is higher than or equal to 3,000 in real terms. Other values are calculated similarly. In panel B, the end-of-period wealth has been divided by the initial wealth expected. Higher values for FR and gamma reduce flexibility in the withdrawals, and a longer horizon makes it more likely to run out of cash.

For Finland, the risk is quite minimal, even for $\gamma = 0.67$, zero to one percent, depending on the investment horizon and the value for *FR*. However, if we set gamma to 0.84, the risk starts to increase rapidly, especially if the FR has been set to four or five percent. Setting gamma to one (akin to the classic withdrawal rule 1), the likelihood of running out of cash over the long term is more likely than the opposite in some cases. For the USA, the results are basically quite similar, but due to better stock market development, the probabilities are typically clearly lower. For example, by setting FR to 5% and gamma to 0.84, the likelihood of running out of cash in the long run is a reasonable 2.8%, whereas for Finland, the risk is 59.8%.

Looking at the results, an obvious question arises. What kind of recommendations can be given to decision-makers, either in Finland or in the US? Of course, it is always difficult to make this kind of recommendation because the analysis makes strong assumptions and because we have analyzed only one of the withdrawal rules, although with some flexibility due to various values for gamma. If one acknowledges the analysis as a starting point, the next step for the decision-maker is to choose the appropriate investment horizon. For a retiree, the horizon can be anything up to 50 years, whereas for an institution, an infinite horizon (proxied here by 100 years) is more appropriate. Next, one needs to consider what kind of volatility one is willing to tolerate in the withdrawals and choose gamma accordingly. Finally, one needs to consider the risk level attached to the choice—is it something one can live with?

4.3 Additional considerations

One can question whether the comparison between the countries is fair, as Finland can be considered a developing country during the early part of the long sample period.[15] Now, following Anarkulova et al. (2022 and 2025), we define Finland as a developing country before 1969 and a developed country afterwards, and repeat the forward-looking bootstrap analysis for both periods separately. In nominal terms, the stock market did almost equally well in both periods (16.2% vs 17.7%), but in real terms, the latter period did much better, with an average return of 13.0% as compared 6.19% for the first period. However, higher returns were accompanied by higher volatility, 34.6% (27.9% for the first period).

Looking at the bootstrap results for both periods (available upon request for both countries), it is clear that the portfolio would have done much better in the latter period. For example, the probability of running out of wealth in 15 years with rule 1 would have been 18.0% before 1969 yields and only 4.7% for the latter period. This is still above 0.6% for the US market (0.5% and 0.7% for the corresponding first and second periods). The same result is observed consistently across all other cases as well. Overall, even if we limit the analysis to the period during which Finland can be considered a developed country, i.e., 1969 and onwards, the main conclusion remains—the US stock market has been able to provide surprisingly robust real

[15] We thank the reviewer for suggesting this.

Table 5 Cash runout probabilities in Finland and the USA

Smoothing (γ)	$\gamma=0.5$			$\gamma=0.67$			$\gamma=0.84$			$\gamma=1.0$		
Horizon (below) Target FR (right)	3%	4%	5%	3%	4%	5%	3%	4%	5%	3%	4%	5%
Panel A: Finland												
10 years	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	1.1	1.2	3.0	6.3
20 years	0.0	0.0	0.0	0.0	0.0	0.0	0.8	3.1	7.9	11.4	19.5	27.7
30 years	0.0	0.0	0.0	0.0	0.0	0.2	2.3	7.4	16.4	20.6	30.6	40.0
40 years	0.0	0.0	0.0	0.0	0.0	0.3	3.8	11.8	24.5	26.5	37.1	46.8
50 years	0.0	0.0	0.0	0.0	0.0	0.4	5.5	16.1	32.1	30.4	41.2	50.9
100 years	0.0	0.0	0.0	0.0	0.1	1.1	13.3	34.7	59.8	37.1	47.9	57.1
Panel B: USA												
10 years	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2
20 years	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.4	2.2	6.5
30 years	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	1.9	6.2	13.4
40 years	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.7	3.2	10.5	17.4
50 years	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	1.0	4.0	10.7	19.5
100 years	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.3	2.8	5.0	12.2	21.7

This table reports the probabilities of running out of cash over various time horizons for three withdrawal rates and four cash flow smoothing strategies. Gamma measures the smoothing between the previous withdrawal and the given percentage of the assets. Gamma equal to one implies that the same cash flow in real terms is withdrawn every year, i.e., there is no smoothing. panel A reports the results for Finland, and panel B reports the results for the USA. The analysis employs one million block bootstrap replications based on realized real returns for the Finnish and US stock markets from 1913 to 2023 returns to investors, and as such, it has been able to provide higher withdrawal rates with lower risk.

The analysis has excluded all taxes for comparability across markets. Obviously, they are not a trivial issue for any investor, although for many long-term investors, especially endowment funds, there may be reasons to expect them to be lower than for someone investing in the short term. For example, in Finland, there are at least two ways in which taxation could influence the outcome. First, for most of the sample period before 1966, investors had to pay a stamp duty every time they traded stocks. On the other hand, and more importantly, investors had to pay tax on dividend payments and capital gains whenever the stocks were sold. The taxation, however, differed across different investors. For most associations and foundations, which in many ways correspond to endowment funds in the US, dividends and realized capital gains were tax-free.

Similarly, trading costs were not taken into account in the analysis. This is because the costs vary considerably during the sample period. In general, before the 1980s, the costs of buying and selling stocks were subject to rather monopolized trading fees. The costs were typically high fractions of a percentage. On the other hand, for long-term investors who follow the buy-and-hold strategy for the most part, the role of costs is again lower than for short-term investors.

A wide range of rules used in the literature and by practitioners has been analyzed. We have assumed that the investor adheres to a single rule throughout the investment period. In practice, however, one may encounter different rules, at times even highly complex ones, and investors may change strategy in different circumstances. In addition, in situations where there is an intention to grow the portfolio's real value (a build-up phase), the optimal strategy might differ. Similarly, if one can expect to receive additional funds into the portfolio over time (e.g., a university endowment receiving donations), the optimal withdrawal strategy may differ from those analyzed.

5 Summary and conclusions

In this paper, we study several well-known withdrawal rules for an investor with the goal of preserving the value of the investment portfolio. As a starting point, we assume an infinite investment horizon and an all-equity investment strategy. We also assume that the investor does not have timing ability or that the market is efficient in the sense that there are no risk-adjusted returns available after costs. We utilize one of the Nordic countries, Finland, and the USA as our sample countries. The US stock market has been selected as the benchmark. Finland has been selected for two reasons. First, Nordic countries, like the US, have been successful in their economic development during the past century, although at the same time, the stronger economic development in Finland began later than in the US, and to some degree, even other Nordic countries, and as such, it makes an interesting market to study. Second, the Finnish economy has suffered from high inflation from time to time, and it offers a great chance to study the role of inflation in withdrawals.

Analyzing the sample period, the average real return for Finland has been slightly lower than for the US (5.3% vs 8.1%). However, due to good early development and lower volatility in the US stock market, one could have set the maximum value-preserving fixed withdrawal rate much higher for the US (10.9%) than for Finland (1.8%) if the endowment had been set up in 1912. On the other hand, if the endowment was set on any one of the following 100 years, the average value-preserving rates are 6.6% and 7.8% for Finland and the US, respectively. Of the analyzed policies, rule six would have provided the highest withdrawals in total for both countries, with slight differences in the smoothing parameter, but if one compares the rules with the average nominal withdrawal dividend by the volatility of the withdrawals, it is rule two that becomes the most favorable for Finland and rule three for the US.

Looking forward, we also analyzed a situation where the portfolio manager is faced with an uncertain future, and she is planning to follow a specific wealth withdrawal rule while aiming to preserve the real value of the portfolio. This is a common goal for many investors after retirement who wish to leave a bequest to their heirs. Similarly, many associations and scholarship-granting organizations are faced with the same goal—withdrawal is used to finance (part of) their operations or grants, yet at the same time, they wish not to endanger their future.

Using rule 4 and a block bootstrap method with real equity returns, the results show that sticking strictly to the classic 4% of the value of the initial portfolio might result in portfolio depletion in both countries, even though it offers the best chance to grow the value of the portfolio. Therefore, it may be advisable to use a smoothing parameter that weights last year's outtake and the current value of the portfolio. A smoothing parameter in the range of about 0.6 to 0.7, or even 0.8, for the USA seems feasible, smoothing the annual cash flows, yet preserving the value of the portfolio at reasonable probabilities and keeping the risk of running out of cash acceptable.

As an extension of the analysis, one could consider other withdrawal rules. One could also analyze the withdrawal rules using a portfolio of equities and bonds, or more dynamic withdrawal rules that take behavioral factors into account, but these questions are left for future study.

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Declarations

Conflict of interest The authors declare no competing interests.

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