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Spontaneous Focusing on Numerical Order and Numerical Ordering Skills in Early Mathematical Development

Heidi Harju

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To Mummi and Ukki,
who taught me the joy of exploration and were always proud of me.

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ABSTRACT

Early mathematical skills are highly predictive of later mathematical achievement. Importantly, children have large individual differences in these mathematical skills developing before school entry, that not only tend to persist but often widen as children progress through school. Therefore, it is crucial to identify the factors that lead to the emergence of these individual differences in early mathematical skills. Previous research has shown that varying numerical ordering skills explain individual differences in later mathematical skills. However, the development of numerical ordering skills has remained poorly understood. Next to differences in early numerical skills, some of the variation in mathematical skills can also be explained by children's tendencies to spontaneously focus on mathematical aspects in everyday situations. Some children more frequently recognize and use mathematical aspects in various situations without any adult-guidance, which may lead to increased practice in their mathematical skills. It is possible, that some children also focus on the aspect of numerical order spontaneously more than others, giving them an advantage in learning numerical ordering skills. Therefore, this dissertation aims to provide novel insights into (a) the development of numerical ordering skills (i.e., the skills to order sets of items based on their numerical value) within mathematical development before school entry, and (b) the associations between Spontaneous Focusing On Numerical Order (SFONO) and early mathematical skills, especially numerical ordering skills. In the current dissertation, numerical ordering skills refer to skills required to order sets of items according to their exact number (in numerical order). The specific goals of the four studies of this dissertation were to identify to what extent children exhibit a spontaneous tendency to recognize and use numerical order in non-mathematical situations, and to assess how this tendency, alongside early numerical skills, is related to individual differences in numerical ordering skills.

Study I presented a longitudinal study following 36 children between the ages of 3 and 12 years, with detailed assessments of focusing on numerical order (FONO) and early numerical skills conducted between the ages of 3 to 6 years, and a final follow-up at the age of 12 assessing their mathematical achievement. The results revealed that there were substantial individual and developmental differences in children's FONO. Furthermore, focusing on numerical order was predictive of

mathematical achievement at the age of 12 years, even when other early numerical predictors such as subitizing-based enumeration and spontaneous focusing on numerosity (SFON) were controlled. Focusing on numerical order in this study was measured with a task that required applying numerical ordering skills in a novel play-based situation. Thus, the results showed that the children who noticed the numerical order aspect and had better numerical ordering skills in this novel situation, also had better mathematical skills later in their development. Importantly, the results suggested the importance of numerical ordering skills for later mathematical achievement, and indicated that children may exhibit individual differences in their spontaneous recognition and use of numerical order.

Study II continued the work of Study I, by explicitly measuring individual differences in children's spontaneous recognition and use of numerical order in 150 three- to four-year-old children. The study used three SFONO tasks that were developed based on findings from Study I in addition to earlier investigations of spontaneous mathematical focusing tendencies, and is the first to specifically report on the relations between SFONO and early numerical skills. The cross-sectional data revealed that children had substantial individual differences in their tendency to spontaneously focus on numerical order, that were related to their early numerical skills. These individual differences in SFONO tendency explained unique variance in children's numerical ordering skills. In other words, those children who tended to recognize and use numerical order more frequently than others in situations that were not explicitly mathematical, had better numerical ordering skills. These results are the first to suggest that SFONO tendency may be a relevant factor contributing to individual differences in numerical ordering skills.

After Study II, it was still not known whether the individual differences observed in SFONO tendency were due to individual differences in the requisite skills needed to spontaneously focus on numerical order in the SFONO tasks or varying task demands. Therefore, Study III investigated individual differences in SFONO tendency in 51 five- to six-year-old children from Belgium, aiming to address the previous gaps. The results showed individual differences in SFONO tendency across four SFONO tasks with similar task demands. Moreover, these individual differences could not be fully accounted for by the requisite skills involved in the SFONO tasks—namely, the ability to order sets numerically, recognize cardinal values, or recite the number sequence. This study was therefore successful to demonstrate a partial dissociation between SFONO tendency and these underlying skills, suggesting that SFONO represents a distinct construct.

Study IV was a longitudinal study of the participants from Study II, examining the development of numerical ordering skills over a one-year time period between the ages of 3 and 5 years. During the follow-up, children completed measures of numerical ordering skills, cardinality recognition, and number sequence production at three time points. The results indicated that children developed at varying rates in their numerical ordering skills during the follow-up. While some children developed steeply from having very little knowledge on numerical ordering to fully mastering the skill, some children showed very low numerical ordering skills across the follow-up. Furthermore, the findings showed not only that numerical ordering skills started to develop only some time after learning to recognize the smallest cardinal values and learning to recite the number sequence, but also that the development may

include a conceptual shift where children integrate their existing knowledge about cardinal values and the number sequence.

Overall, the current dissertation sheds light on the role of numerical ordering skills and SFONO tendency in early mathematical development. The findings suggest that the development of numerical ordering skills is a complex process, that may require the integration of pre-existing numerical knowledge, which is important for later mathematical development. Furthermore, those children who are more likely to notice and use numerical order in situations that are not explicitly mathematical seem to be ahead in their numerical ordering skills. It may be, that these children gain more self-initiated practice with numerical ordering skills in different situations due to increased SFONO tendency. By examining numerical ordering skills and SFONO tendency in preschool-aged children, the dissertation contributes to a more detailed understanding of early mathematical development. These findings may help inform the design of assessment tools aimed at identifying individual differences in early numerical skills, which could be valuable for supporting mathematics learning before the start of formal schooling.

KEYWORDS: Spontaneous focusing on numerical order, numerical ordering skills, early numeracy, development, children

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TIIVISTELMÄ

Varhaiset matemaattiset taidot ennustavat vahvasti myöhempää matemaattista osaamista. Lapsilla on suuria yksilöllisiä eroja näissä jo ennen koulun alkua kehittyvissä matemaattisissa taidoissa, jotka eivät ole ainoastaan pysyviä, vaan usein kasvavat koulun edetessä. Tämän vuoksi onkin erittäin tärkeää tunnistaa varhaisten matemaattisten taitojen kehitykseen vaikuttavia yksilöllisiä eroja aiheuttavia tekijöitä. Aiempien tutkimusten mukaan erot numeerisen järjestämisen taidoissa selittävät myöhemmin esiintyvää vaihtelua matemaattisissa taidoissa. Tästä huolimatta, numeerisen järjestämisen taitojen kehityksestä ei ole vielä juurikaan tutkimustietoa. Edellä mainittujen tekijöiden lisäksi matemaattisten taitojen yksilöllisten erojen muodostumista voidaan selittää lasten taipumuksella kiinnittää oma-aloitteisesti, eli spontaanisti, huomiota matemaattisiin piirteisiin arkielämän tilanteissa. Tämä tarkoittaa sitä, että jotkut lapset tunnistavat ja hyödyntävät matemaattisia piirteitä erilaisissa tilanteissa ilman aikuisen ohjausta enemmän kuin toiset ja saattavat näin ollen tulla harjoitelleeksi enemmän matemaattisia taitojaan. Vastaavasti on mahdollista, että jotkut lapset kiinnittävät oma-aloitteisesti enemmän huomiota numeeriseen järjestykseen, mikä voi antaa heille etulyöntiaseman numeerisen järjestämisen taitojen oppimisessa. Tämän väitöskirjan tarkoituksena on tarjota uusia näkökulmia (a) numeerisen järjestämisen taitojen kehityksestä ennen kouluikää ja (b) spontaanin numeeriseen järjestykseen huomion kiinnittämisen (SFONO) ja varhaisten matemaattisten taitojen, erityisesti numeerisen järjestämisen taitojen, välisistä yhteyksistä. Tässä väitöskirjassa numeerisen järjestämisen taidoilla tarkoitetaan taitoja järjestää esineryhmiä niiden lukumäärän perusteella. Väitöskirjan neljän osatutkimuksen erityisenä tavoitteena oli selvittää, missä määrin lapset oma-aloitteisesti tunnistavat ja käyttävät hyödykseen numeerista järjestystä ei-matemaattisissa tilanteissa ja miten erot tässä SFONO-tendenssissä ovat yhteydessä yksilöllisiin eroihin numeerisen järjestämisen taidoissa.

Ensimmäinen tutkimus oli pitkittäistutkimus, jossa seurattiin 36 lapsen kehitystä kolmen ja kahdentoista ikävuoden välillä. Lasten huomion kiinnittämistä numeeriseen järjestykseen sekä varhaisia matemaattisia taitoja arvioitiin vuosittain 3–6 vuoden iässä. Lisäksi tutkimuksen viimeisessä mittapisteessä, lasten ollessa 12-vuotiaita, mitattiin heidän matemaattista osaamistaan. Tulokset osoittivat huomattavia

yksilöllisiä ja kehityksellisiä eroja lasten huomion kiinnittämisessä numeeriseen järjestykseen. Huomion kiinnittäminen numeeriseen järjestykseen viisivuotiaana ennusti lasten matemaattista osaamista 12 vuoden iässä, vaikka muut varhaiset matemaattiset ennustajat, kuten lukumäärän tunnistaminen subitisoimalla sekä spontaani huomion kiinnittäminen lukumääriin, kontrolloitiin analyysissä. Tässä tutkimuksessa huomion kiinnittämistä numeeriseen järjestykseen mitattiin tehtävällä, joka edellytti numeerisen järjestämisen taitojen soveltamista uudessa, leikinomaisessa tilanteessa. Tulokset osoittivat, että ne lapset, jotka tunnistivat numeerisen järjestyksen tehtävässä ja osoittivat parempia numeerisen järjestämisen taitoja, olivat myöhemmin parempia matematiikassa. Tulosten perusteella pääteltiin, että numeerisen järjestämisen taidot voivat olla keskeisiä myöhemmän matemaattisen osaamisen kannalta, ja että lasten välillä saattaisi esiintyä yksilöllisiä eroja myös siinä, kuinka usein he spontaanisti kiinnittävät huomiota numeeriseen järjestykseen.

Toinen tutkimus jatkoi ensimmäisessä tutkimuksessa aloitettua tutkimuslinjaa. Tutkimuksessa tarkasteltiin yksilöllisiä eroja 150 kolme–neljävuotiaan lapsen SFONO-tendenssissä, eli taipumuksessa spontaanisti tunnistaa ja käyttää numeerista järjestystä hyödykseen. Arviointiin käytettiin kolmea SFONO-tehtävää, jotka kehitettiin ensimmäisen tutkimuksen sekä aiempien tutkimusten perusteella. Aiemmat tutkimukset ovat keskittyneet taipumuksiin kiinnittää spontaanisti huomiota erilaisiin matemaattisiin piirteisiin. Kyseessä on ensimmäinen tutkimus, jossa raportoidaan erikseen SFONO-tendenssin ja varhaisten matemaattisten taitojen välisiä yhteyksiä. Poikkileikkausaineisto osoitti, että lasten välillä oli merkittäviä yksilöllisiä eroja SFONO-tendenssissä. Nämä erot olivat yhteydessä lasten varhaisiin matemaattisiin taitoihin ja ne selittivät tilastollisesti merkitsevästi lasten välisiä eroja numeerisen järjestämisen taidoissa. Toisin sanoen, ne lapset, joilla oli vahvempi taipumus tunnistaa ja käyttää numeerista järjestystä tilanteissa, jotka eivät olleet varsinaisesti matemaattisia, omasivat paremmat numeerisen järjestämisen taidot. Tulokset osoittivat ensimmäistä kertaa, että taipumus kiinnittää spontaanisti huomiota numeeriseen järjestykseen voi olla keskeinen tekijä numeerisen järjestämisen taitojen kehityksessä.

Toisen tutkimuksen jälkeen oli vielä epäselvää, johtuivatko havaitut yksilölliset erot SFONO-tendenssissä eroista niissä taidoissa, joita tarvittiin spontaaniin huomion kiinnittämiseen numeeriseen järjestykseen SFONO-tehtävissä, tai eroista tehtävävaatimuksissa. Kolmannessa tutkimuksessa tutkittiin täten yksilöllisiä eroja SFONO-tendenssissä 51 belgialaisella 5–6-vuotiaalla lapsella tavoitteena vastata edellä mainittuihin avoimiin kysymyksiin. Tulosten mukaan lasten välillä oli yksilöllisiä eroja SFONO-tendenssissä neljässä tehtävässä, joilla oli keskenään samankaltaiset tehtävävaatimukset. Lisäksi nämä yksilölliset erot eivät selittyneet pelkästään taidoilla, joita tehtävien suorittaminen edellytti—eli taidoilla järjestää esineryhmiä niiden lukumäärän perusteella, tunnistaa lukumääriä tai lukujonon luettelutaidolla. Tämä tutkimus oli ensimmäinen, joka osoitti SFONO-tendenssin ja SFONO-tehtävissä vaadittavien taitojen osittaisen erillisyyden, mikä viittaa siihen, että spontaani huomion kiinnittäminen numeeriseen järjestykseen on itsenäinen erillinen kehitystekijä.

Neljäs tutkimus oli pitkittäistutkimus, jossa seurattiin toisen tutkimuksen osallistujia ja tarkasteltiin numeerisen järjestämisen taitojen kehitystä yhden vuoden ajan lasten ollessa 3–5-vuotiaita. Seurantajakson aikana arvioitiin lasten numeerisen

järjestämisen taitoja, lukumäärän tunnistamisen taitoja sekä lukujonon luettelemisen taitoja kolmessa mittapisteessä. Tulokset osoittivat, että lasten numeerisen järjestämisen taidot kehittyivät yksilöllisesti eri tahtia seurantajakson aikana. Osa lapsista eteni nopeasti siirtyen lähes olemattomasta osaamisesta taidon täyteen hallintaan, kun taas toisten kohdalla kehitystä ei tapahtunut juuri lainkaan. Lisäksi tulokset osoittivat, että numeerisen järjestämisen taitojen kehitys alkoi sen jälkeen, kun lapset olivat oppineet tunnistamaan pieniä lukumääriä ja luettelemaan lukujonoa. Kehitykseen saattoi sisältyä myös muutos käsitteellisessä ymmärryksessä, jossa lapset alkoivat rakentaa yhteyksiä aiemmin oppimiensa lukumäärien ja lukujonon välille.

Tämä väitöskirja tarjoaa uutta tietoa numeerisen järjestämisen taitojen ja SFONO-tendenssin merkityksestä varhaisessa matemaattisessa kehityksessä. Tulokset viittaavat siihen, että numeerisen järjestämisen taitojen kehitys on monivaiheinen prosessi, joka saattaa edellyttää aiemmin omaksutun matemaattisen tiedon yhdistelemistä, ja on keskeistä myöhemmän matemaattisen osaamisen kannalta. Lisäksi lapset, jotka todennäköisemmin huomaavat ja käyttävät numeerista järjestystä tilanteissa, jotka eivät ole varsinaisesti matemaattisia, näyttävät olevan pidemmällä numeerisen järjestämisen taidoissaan. On mahdollista, että nämä lapset saavat enemmän omaehtoista harjoitusta numeerisen järjestämisen taidoissa erilaisissa tilanteissa vahvemman SFONO-tendenssinsä ansiosta. Tarkastelemalla numeerisen järjestämisen taitoja sekä SFONO-tendenssiä varhaiskasvatusikäisillä lapsilla, väitöskirja syventää ymmärrystämme varhaisten matemaattisten taitojen kehityksestä. Tulokset voivat myös tukea arviointimenetelmien kehittämistä, joiden avulla voidaan tunnistaa yksilöllisiä eroja varhaisissa matemaattisissa taidoissa ja siten edistää oppimisen tukemista jo ennen peruskoulun alkua.

ASIASANAT: Spontaani huomion kiinnittäminen numeeriseen järjestykseen, numeerisen järjestämisen taidot, varhaiset matemaattiset taidot, kehitys, lapset

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List of Original Publications

This dissertation is based on the following original publications, which are referred to in the text by their Roman numerals:

I Harju, H., Lehtinen, E., & Hannula-Sormunen, M. (2022). Focusing on numerical order in preschool predicts mathematical achievement nine years later. In Fernández, C., Llinares, S., Gutiérrez, A., & Planas, N. (Eds.), *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (vol. 2, pp. 347–354).

II Harju, H., Van Hoof, J., Nanu, C. E., McMullen, J., & Hannula-Sormunen, M. (2024). Spontaneous focusing on numerical order and numerical skills of 3- to 4-year-old children. *Educational Studies in Mathematics*, *117*(1), 43–65. https://doi.org/10.1007/s10649-024-10327-3

III Harju, H., Van Belle, L., Van Dooren, W., McMullen, J., & Van Hoof, J. (Under review). “I need to follow the numbers” - developing and validating a more comprehensive measure of Spontaneous Focusing On Numerical Order.

IV Harju, H., Van Hoof, J., McMullen, J., Nanu, C. E., Laakkonen, E., & Hannula-Sormunen, M. (Under review). When "one, two, three" gains ordinal meaning: Investigating the development of numerical ordering skills in 3- to 5-year-old children.

The original publications have been reproduced with the permission of the copyright holders. In all publications, Harju contributed to the conceptualization, data analysis and interpretation, and was responsible for writing the first drafts of the manuscripts. All co-authors participated in the conceptualization, provided critical feedback, and contributed to all manuscript revisions. Studies III and IV are under review in *Educational Studies in Mathematics* and *Developmental Psychology*, respectively.

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1 Introduction

A child states while putting on clothes to go out "I need two gloves, one for each hand.”. While going down the stairs, the child counts each step and declares “I counted 10 stairs, there were so many!”. During block play, a child explains: “I have one red, two blue, and three green blocks. Together I have six blocks!”, while another proudly announces, "I’m four now” and shows four fingers and continues, “But next birthday I’ll be five!", while putting up one finger more. The blocks become a tower with the red block first, followed by the blue and the green blocks: “First one, then two, and then three!”. It is lower than the other child’s tower which has seven blocks. “Can I build a tower that is taller than me?”, a child asks. These everyday moments reveal something remarkable. As stated by Clements and Sarama (2007): “Young children possess an informal knowledge of mathematics that is surprisingly broad, complex, and sophisticated” (p. 462). Indeed, this is something that I have learnt during the course of this dissertation process. It seems that for young children, there are endless possibilities to use mathematics in their everyday life.

Importantly, these informal experiences with mathematics are more than just playful moments — they are foundational building blocks for later mathematics learning (Clements & Sarama, 2007; LeFevre et al., 2009). Just as a house requires a solid foundation to support its structure, mathematical competence is built on the early development of core numerical skills. A growing body of evidence shows that children’s numerical skills prior to school entry are crucial predictors of later mathematical achievement (Aunola et al., 2004; Duncan et al., 2007; Foster et al., 2015; Jordan et al., 2009; Liu et al., 2025; Nguyen et al., 2016). Notably, children show substantial individual differences in their early numerical skills, which tend to be highly stable, often widening as mathematical development progresses (Aunola et al., 2004; Jordan et al., 2009). This highlights the importance of understanding how early numerical skills develop, and what factors contribute to these developmental differences.

Among the foundational numerical skills, children begin to grasp core concepts that lay the foundation for later mathematics learning (Clements & Sarama, 2007). Among these early milestones are understanding two fundamental properties of numbers: cardinality and ordinality (i.e., numerical order). Cardinality refers to the property of numbers that denotes how many items there are in a set—that is, the underlying quantity a number represents (Fuson, 1988; Wynn, 1992). In contrast, ordinality refers to the order relations among numbers in a sequence, such as knowing that six comes after five and before seven (Lyons et al., 2016). Because numerical order reflects systematic changes in cardinality (e.g., five comes after four because it is one more), these two aspects of number may be closely intertwined, particularly in early development (C. Xu & LeFevre, 2021).

While the development of cardinality has been the focus of extensive research (e.g., Bermejo et al., 2004; Fuson, 1988; Mix et al., 2012; Wynn, 1992), ordinality understanding has only recently begun to receive research attention. This growing interest is motivated by findings showing that ordinality skills are strong predictors of later mathematical achievement (Liang et al., 2023; Lyons et al., 2014; Malone et al., 2022). Moreover, ordinality is suggested to play a central role in understanding how numbers relate to each other and form a coherent system (for a review, see Lyons et al., 2016). Despite its clear relevance, relatively little is known about how ordinality skills develop in early childhood, or the factors that contribute to the emergence of individual differences in ordinality skills.

Critically, early numerical development is shaped not only by formal instruction or adult-guided learning, but also by children's spontaneous engagement with mathematical aspects. Research has shown that some children are more inclined than others to spontaneously (i.e., in a self-initiated way) recognize and use mathematical information in non-explicitly mathematical situations, such as everyday environments (Hannula & Lehtinen, 2005; McMullen et al., 2019; Verschaffel et al., 2020). While the opening examples illustrate how some children spontaneously engage with mathematical ideas in everyday life, it is important to recognize that this is not the case for all children. For example, some children may spontaneously focus on the number of plates on a table, or even that there are twice as many cutleries as there are plates, while others may focus on the colour of the plates or what food is for dinner. Importantly, children who tend to spontaneously focus on mathematical aspects more than their peers, have been shown to have better concurrent and later mathematical skills (e.g., Li et al., 2025; McMullen et al., 2019; Verschaffel et al., 2020). These children may notice more opportunities to practice their mathematical skills in everyday lives, leading to enhanced mathematical skills (e.g., Hannula & Lehtinen, 2005). However, while studies have shown that children spontaneously focus on mathematical aspects such as exact number, quantitative relations, Arabic number symbols, and patterns (for reviews, see McMullen et al., 2019; Verschaffel et al., 2020), much less is known about whether children differ in their tendency to spontaneously focus on the aspect of numerical order. This is of great relevance in the investigation of individual differences in ordinality skills. A stronger tendency to spontaneously focus on numerical order may be one explaining factor for individual differences in numerical ordering skills.

To address the identified gaps in existing research, the present dissertation aims to investigate the development of numerical ordering skills as part of early mathematical learning before school entry in 3- to 6-year-old children, and how children's spontaneous focusing on numerical order (SFONO) is associated with these skills. Specifically, the four studies conducted in this dissertation examined whether, and to what extent, children exhibit a tendency to spontaneously recognize and use numerical order in situations that are not explicitly mathematical, and how this tendency is associated with early numerical ordering skills. In addition, the studies investigated the development of numerical ordering skills, and how early numerical skills contribute to the individual differences in this development. By exploring these issues, this dissertation aims to broaden our understanding of early mathematical development and shed light on the factors that shape individual differences in children's numerical skills before school entry.

The current doctoral dissertation consists of four main sections that together provide the theoretical foundation, methodological considerations, and a synthesis of four original empirical studies. The first section outlines the theoretical background by reviewing (a) the definition, operationalization, and association of ordinality with mathematical skills, (b) the numerical development central to the development of numerical ordering skills, and (c) the role of spontaneous mathematical focusing tendencies in mathematical development. The second section details the methodology used across the studies included in this dissertation. The third section summarizes the four empirical articles in relation to the overarching research aims. The final section discusses the main findings, their theoretical and practical implications, and directions for future research.

1.1 Unpacking ordinality: Definitions, measures, and the focus of the current dissertation

Numerical order, also known as the ordinality of numbers, is an important characteristic of the number system. It reflects the structured nature of numbers as inherently ordered entities, which includes crucial information about the natural number system (Anderson & Cordes, 2013). Specifically, the logic behind the natural number system is such, that each number has an immediate predecessor (n-1), and successor (n+1) (P. Cheung et al., 2017). Although ordinality as a construct is not uniquely associated with numbers as there are also other types of ordered sequences like letters of the alphabet or months in the calendar year, in the context of the present dissertation, ordinality refers specifically to the property of numbers, in other words, to numerical ordinality.

Ordinality is defined in many different ways in the literature. For example, Lyons et al. (2016) define ordinality in their review as a property of numbers, that denotes the relative position of an item in a sequence that answers to a question "What position?". Furthermore, ordinality knowledge is often defined as understanding the ordinal relations among numbers, such as knowing that 5 comes before 6 and after 4 in a sequence of numbers (e.g., Attout et al., 2014; Devlin, Moeller, Reynvoet, & Sella, 2022; Finke et al., 2022 Malone et al., 2021; Sasanguie & Vos, 2018). These definitions often consider ordinality mainly as a property of symbolic numbers, distinct from the magnitudes they represent. On the other hand, some studies consider that ordinality is tied to the magnitudes of numbers in a sense that larger numbers follow smaller numbers (e.g., Bryant & Nuñes, 2010; Dunn et al., 2019; Le Corre, 2014; Spaepen et al., 2018). From this perspective, ordinality involves understanding that numbers appearing later in the number sequence correspond to larger quantities (the later-greater principle; Le Corre, 2014), that within this ordered series three is larger than two and one (Bryant & Nuñes, 2010), or more specifically (in the case of natural numbers), that each number is exactly one more than the previous one (Spaepen et al., 2018). Furthermore, C.-N. Cheung & Lourenco (2019) consider ordinality constructing from two perspectives: position-based ordinality which refers to the relative positions between items in a sequence, and magnitude-based ordinality which describes the magnitude relations between the items. For example, in a sequence of 1-2-3, number 2 is the second number and comes after 1 but before 3 (position-based ordinality), and 2 is larger than 1 but smaller than 3 (magnitude-based ordinality). In fact, some definitions consider ordinality knowledge constructing from both of these perspectives, requiring joint understanding of magnitude and position (i.e., 3 comes after 2 because 3 is larger than 2) (e.g., Bryant & Nuñes, 2010; Clements & Sarama, 2007; C. Xu & LeFevre, 2021).

In addition to the different definitions on ordinality, also different aspects are associated with it. In the literature, mainly three distinct but interrelated aspects of ordinality are recognized: numerical ordering, use of numerical relations (e.g., seven is more than five / seven comes after five), and use of ordinal number words (Clements & Sarama, 2007; Fuson, 1988). Numerical ordering includes ordering sets of items or numbers in numerical order. Numerical relations may be used in a situation where two or more nonequivalent items are compared. Furthermore, the repeated use of a given numerical relation results in numerical ordering. For example, sets of items can be ordered based on the magnitude relations between the sets (e.g., one is less than two, and two is less than three), or thinking of the quantities the sets represent, and their order in the number sequence (e.g., two comes after one and before three) (C.-N. Cheung & Lourenco, 2019; Sasanguie and Vos, 2018; C. Xu & LeFevre, 2021). In addition, in any situation where an order is established, such as in a line of people or in the number sequence, the relative position of one item with respect to other items can be described by using ordinal number words (e.g., the third person in the line, two is the second number in the number sequence).

The different definitions and aspects related to ordinality cause for differences in how ordinality knowledge is measured in the existing studies. Over the past decade, measures of ordinality have particularly focused on using number symbols as stimuli. Common measures in these studies involve tasks where participants quickly determine if a sequence of numbers is in numerical order or not (order processing) (e.g., Goffin & Ansari, 2016; Hutchison et al., 2022; Lyons et al., 2014; Malone et al., 2021), or arrange numbers from smallest to largest (number ordering) (e.g., Di Lonardo Burr et al., 2022; Liang et al., 2023; O'Connor et al., 2018; C. Xu et al., 2023). These studies have demonstrated that good performance in these measures is strongly predictive of more complex mathematical abilities, such as arithmetic, and becomes increasingly important as children develop, eventually surpassing cardinality skills in predicting arithmetic proficiency (Liang et al., 2023; Lyons et al., 2014; Malone et al., 2021; Sasanguie & Vos, 2018; C. Xu & LeFevre, 2021). For instance, Lyons et al. (2014) found that while number comparison was the best predictor of early arithmetic skills in 7- to 8-year-old children, the importance of order processing grew from the age of 8 years onwards, eventually becoming the most significant predictor over a variety of other important domain-specific and domain-general skills such as number line estimation, reading skills, and non-verbal intelligence. Similarly, Liang et al. (2023) discovered that from non-symbolic comparison, cardinality skills, number ordering, and executive functions (i.e., working memory, inhibitory control, shifting) assessed at the age of 4 years, only number ordering significantly predicted math performance growth between the ages of 4 and 5 years. Additionally, Morsanyi et al. (2024) showed in their intervention study that training in ordinality using the measure of order processing over a 3-week period (6 sessions with 2-3 days in between) improved mathematical performance in 4- to 5-year-old children, while training of order working memory or listening comprehension did not. However, recent studies suggest that the order processing task may primarily reflect memory-retrieval strategies of familiar sequences rather than true ordinality skills (Devlin et al., 2024; Dubinkina et al., 2021, Sella et al., 2020). Furthermore, Gilmore and Batchelor (2021) found no evidence that order processing measures a distinct aspect from verbal count sequence knowledge, and some researchers argue that the task may be too challenging for young children (Di Lonardo Burr et al., 2022; Hutchison et al., 2022; O'Connor et al., 2018; C. Xu et al., 2023). Taken together, these considerations indicate a need for alternative ways to measure children's ordinality knowledge.

Ordinality has also been measured using non-symbolic stimuli (e.g., Finke et al., 2021; Rubinsten & Sury, 2011; Vogel et al., 2017). The non-symbolic order processing measure is similar to the symbolic order processing measure, but instead of using three number symbols, three collections of dots are used. Comparisons between non-symbolic and symbolic order processing have showed that participants show different response patterns in these tasks, and only symbolic order processing explains unique variance over arithmetical skills with the both tasks included in the analyses (Rubinsten & Sury, 2011; Vogel et al., 2017). This may explain the recent favoring for symbolic stimuli. However, more research on non-symbolic ordinality is needed for the following reasons. First, with the current information, it can be hypothesized that the different response pattern and unique relation to arithmetic performance in the symbolic order processing task may be due to the symbolic order processing measuring fact retrieval rather than ordinality (for a review, see Devlin, Moeller, Reynvoet, & Sella, 2022). Second, the studies comparing these symbolic and non-symbolic ordinality measures included only adult participants. Non-symbolic measures could be especially important in young children, as it has been suggested that children may initially depend on their cardinal knowledge of numbers (i.e., magnitude comparison) to reason about ordinality (C. Xu & LeFevre, 2021).

Next, there are also studies that investigated ordinality from the aspect of numerical ordering, by asking participants to order sets of items from least to most. These studies have associated performance in numerical ordering with other numerical abilities (Berteletti et al., 2010; Desoete et al., 2009; Purpura & Lonigan, 2013; Spaepen et al., 2018; Stock et al., 2009). For example, Purpura and Lonigan (2013) found that numerical ordering skills correlated with symbolic number ordering, number comparison, addition, subtraction, and other numerical skills in a cross-sectional study of 3- to 6-year-old children. Furthermore, Stock et al. (2009) observed in their follow-up study, that numerical ordering at the age of 5 years explained unique variance in arithmetical reasoning and arithmetic fluency measured one year later at Grade 1 when counting knowledge, classification, and general intelligence were included in the analyses. However, for Grade 2 the results were inconsistent, as numerical ordering at the age of 5 years significantly predicted only arithmetic fluency in one of two cohorts. In addition, it is suggested that children's ability to order relative magnitudes is one of the first indications of understanding the logical structure of the number system (Geary, 2013). Given the observed importance of numerical ordering skills in children between 3 and 6 years of age, it is surprising that relatively few studies have investigated ordinality knowledge from this perspective.

In this dissertation, ordinality knowledge is defined to construct from a joint understanding of magnitude (three is larger than two) and position (three comes after two), and it was investigated from the less explored aspect of numerical ordering. Accordingly, the term numerical ordering skills is used from this point onward to refer to children's ability to arrange sets of items in numerical order, from least to most (or vice versa). This approach was taken, as it may be more relevant in young children for several reasons. First, a few studies have observed a shift from magnitude comparison as the best predictor of arithmetical skills around the age of 7 years, to symbolic order processing one year later at the age of 8 years (Lyons et al., 2014; Sasanguie & Vos, 2018), indicating that symbolic order processing becomes more relevant later in the development. Second, Sury and Rubinsten (2012) argue that ordinality tasks likely require the manipulation of quantity, magnitude, or semantic information before ordinal relationships can be identified and a solution reached. This suggests that number symbols and the magnitudes they represent may be difficult to distinguish from each other. Third, since young children first encounter numerical concepts through non-symbolic quantities, examining ordinality in a non-symbolic context can offer valuable insights into the less explored area of the early development of ordinal understanding. Additionally, numerical ordering may require children to think relationally between the cardinal values of non-symbolic sets, and to understand that these values follow a structured sequence where larger numbers come after smaller ones (Spaepen et al., 2018). Thus, numerical ordering may be closely related with the conceptual understanding of numerical order.

1.2 Early numerical development

Extensive research has shown that early numerical skills are strong predictors not only for later mathematical achievement (Aunola et al., 2004; Duncan et al., 2007; Foster et al., 2015; Jordan et al., 2009; Liu et al., 2025; Nguyen et al., 2016; Watts et al., 2014) but also for overall academic success, future employment prospects, and well-being (Claessens et al., 2009; Davis-Kean et al., 2022). Thus, early numerical skills are highly important, and form the foundations for later mathematics learning (Liu et al., 2025). For example, a recent meta-analysis with 54 longitudinal studies and over 58 000 children by Liu et al. (2025) examining the predictive nature of early numerical skills found that early mathematical skills measured between the ages of 3 and 7 years, significantly predicted mathematics achievement measured between the ages of 6.5-13.5 years (a minimum of 6 months later). In addition, the prediction of early numerical skills for later mathematical achievement was stronger with longer prediction periods, and the earlier they were measured (Liu et al., 2025). This suggested, that learning early numerical skills may trigger a snowballing effect, cumulatively affecting mathematics development over time (Liu et al., 2025). This suggestion is also supported by studies observing that children who enter school with lower numerical skills often experience slower growth in mathematical skills during elementary school (Aunola et al., 2004; Jordan et al., 2009). Thus, the individual differences in numerical skills observed already before school-entry are highly stable, putting children with initially lower skills at a disadvantage in mathematics learning compared to their peers. Therefore, it is crucial to understand how these skills develop and how these large individual differences emerge.

Early numerical skills are the initial set of mathematical skills that are learnt before the start of formal schooling (Liu et al., 2025; Purpura & Lonigan, 2013). However, the specific skills encompassed by this term can vary across different studies. In a narrative review of recent research on the structure of early numerical skills in children aged 4 to 8 years, Devlin, Moeller, and Sella (2022) identified five commonly recognized domains: (1) procedural counting, (2) counting knowledge, (3) understanding numerical relations, (4) symbolic and nonsymbolic comparison and estimation, and (5) basic arithmetic. However, it should be noted that the included specific skills can be categorized into different domains in different studies (Devlin, Moeller, & Sella, 2022). Additionally, some studies also incorporate curriculum-aligned skills such as geometry, patterning, and measurement (Devlin, Moeller, & Sella, 2022). For most children, the acquisition and mastery of early numerical skills occurs spontaneously through activities at home and other experiences in the child's everyday environment (LeFevre et al., 2009).

From these five domains of numerical skills, *procedural counting* and *counting knowledge* are sometimes seen as one domain (e.g., Jordan et al., 2006; Purpura & Lonigan, 2013), while some make a distinction between procedural and conceptual counting skills (e.g., Cirino, 2011; Ribner et al., 2018). In general, these domains include skills relating to children's knowledge of the rules and processes related to counting and determining the quantity of a set (Purpura & Lonigan, 2013), such as number sequence production (i.e., reciting the number sequence from one), counting forward and backward from numbers, subitizing, object counting, and understanding the cardinality principle (i.e., the last number counted indicates the total number of objects in a set) (Clements & Sarama, 2007; Cirino, 2011; Jordan et al., 2006; Purpura & Lonigan, 2013). *Understanding numerical relations* involves knowledge of how two or more items (non-symbolic or symbolic numbers) relate to each other (Fuson, 1988; Purpura & Lonigan, 2013), including specific skills such as comparing, matching, and ordering sets of quantities or Arabic numerals (Clements & Sarama, 2007; Fuson, 1988; Jordan et al., 2006; Purpura & Lonigan, 2013). In addition, understanding numerical relations may include understanding the relation between sets of quantities and Arabic numerals, and knowledge of the names of Arabic numerals (Clements & Sarama, 2007; Purpura & Lonigan, 2013). Thus, numerical ordering skills (i.e., ordering sets of dots from least to most) is included in this domain of numerical skills. Furthermore, numerical skills related to *symbolic and non-symbolic comparison and estimation* are sometimes separated from understanding numerical relations, although these domains overlap. In addition to comparing sets of quantities or numbers (i.e., "which is bigger", "which is more"), estimating the total set size without counting is included in this aspect of numerical skills (Aunio & Räsänen, 2016; Jordan et al., 2006; Purpura & Lonigan, 2013). Lastly, *basic arithmetic* includes numerical skills such as addition and subtraction with or without objects, and composing or decomposing sets (Clements & Sarama, 2007; Purpura & Lonigan, 2013).

Skills in these domains may develop in a hierarchical manner. For example, Liu et al. (2025) suggested counting knowledge to lay the foundations for understanding numerical relations, which is important for learning basic arithmetic. Regarding the development of ordinality skills, a few studies have similarly suggested the need to develop adequate cardinality skills before beginning to understand ordinality (Anderson & Cordes, 2013; Knudsen et al., 2015; Spaepen et al., 2018). For example, Spaepen et al. (2018) found that among 3- to 4-year-old children, only those who had mastered the cardinal principle showed improvement in numerical ordering and in understanding that each successive number in the counting sequence represents a quantity one greater than the previous. Concurrently, studies have suggested cardinality to be a prerequisite for understanding numerical relations (Geary et al., 2018; Geary & vanMarle, 2018; Krajewski & Schneider, 2009), which is essential for numerical ordering of non-symbolic sets (Spaepen et al., 2018). Furthermore, Knudsen et al. (2015) observed in a cross-sectional sample of 4- to 7-year-old children, that the groups of 4- and 5 -year-old children had the highest performance in cardinality skills, followed by naming Arabic numerals, knowing the cardinal value of an Arabic numeral, and lastly ordering Arabic numerals, indicating a developmental pattern for these skills.

Next to cardinality skills, Hutchison et al. (2022) suggested that young children's initial understanding of numerical order may be closely tied to the verbal count list, as their study found that 4- to 5-year-old children struggled to recognize that sequences of numbers can be in numerical order even if they are not consecutive (e.g., 2-4-6). Thus, the current dissertation focused on cardinality skills and number sequence skills in relation to numerical ordering skills, as there is limited knowledge on how these skills develop in relation with numerical ordering skills. In the next chapters, the development of cardinality and number sequence skills is reviewed, as they may be prerequisites for developing numerical ordering skills. In addition, other theoretically proposed contributors, such as infant preverbal abilities, comparison and mathematical language, which may also play a role in the development of ordinality, will be discussed.

1.2.1 Prenumerical abilities in infancy

A body of research has shown that even human infants have some innate capacities to process numerical information regarding quantity and ordinality (for a review, see

Cantrell & Smith, 2013). Although these infant abilities were not in the scope of the current dissertation, it is important to review these skills as they may form the foundations for developing more complex numerical abilities. First, studies have shown that 6-month-old infants can discriminate between two non-symbolic quantities (e.g., 8 and 16 dots) (e.g., Cordes & Brannon, 2009; Lipton & Spelke, 2003; F. Xu & Spelke, 2000). However, these quantity discriminations are imprecise, and limited by their numerical ratio (e.g., while 6-month-old infants successfully discriminate 8 vs. 16 dots, they fail with 8 vs. 12 dots) (Feigenson et al., 2004). The mechanism supporting these discriminations is called the approximate number system (ANS), which represents numerical quantities as approximate mental magnitudes centered on the target number (Feigenson et al., 2004; Libertus et al., 2013).

Next to discriminating between sets with large enough numerical ratio, 10-14-month-old infants can also discriminate between two small quantities (Cantrell & Smith, 2013; Feigenson et al., 2004). For example, when infants followed an experimenter sequentially hiding one cracker in a bucket on the left, and two crackers in the bucket on the right, the infants crawled towards the bucket with more crackers (Feigenson et al., 2002). However, studies investigating infant discrimination of small quantities have observed that infants successfully discriminated between sets of one, two, and three objects, but failed when the number of objects reached four (Feigenson et al., 2002). Thus, it is suggested that infants are able to accurately represent small sets of objects by creating working memory models in which each item is encoded as a distinct mental symbol, with a system called parallel individuation (Feigenson et al., 2002; Le Corre & Carey, 2007).

In addition, studies have suggested ordinality processing as an innate ability present at infancy (Brannon, 2002; Suanda et al., 2008; Sury & Rubinsten, 2012). For example, Suanda et al. (2008) habituated 9- and 11-month-old infants to a given ordinal direction (i.e., repeating sequences of increasing or decreasing numerical quantities, such as sets of 2-4-8 squares), and then presented the infants with new sequences that increased or decreased in numerosity. The results showed, that 11-month-old infants looked longer at sequences that did not correspond to the ordinal direction of the habituation sequence, thus indicating that they detected the reversed order of ordinal direction (Suanda et al., 2008). Also 9-month-old infants were able to detect changes in ordinal directions, when element size, and cumulative surface area increased with number (Suanda et al., 2008). The ANS and parallel individuation are suggested to serve as the foundation to learn more sophisticated, human created numerical concepts (Feigenson et al., 2004). Concurrently, the innate sense of ordinality may form the basis for understanding of ordinality. However, this relation is hypothetical, as there are yet no studies investigating the relation between innate sense of ordinality and later ordinality skills.

1.2.2 Number sequence skills

Fluent number sequence skills are highly important, as they work as a mechanism for solving arithmetic problems such as addition and subtraction (Butterworth, 2005; Fuson, 1991), and strongly predict later mathematical achievement (e.g., Aunola et al., 2004; Koponen et al., 2016, 2019; Krajewski & Schneider, 2009). Fuson (1988) suggests there are five levels that can be distinguished in children's number sequence skill development, which include the increasing integration of counting and cardinal meanings into the number sequence. Notably, children may be on different levels of this development depending on the part of the number sequence (Fuson, 1988). For example, children may be competent counters in the numerical string of one to 10, while still learning to recite the numbers above 10 in the correct order.

Learning the number word sequence starts around the age of 2 years, when children are starting to learn to recite the list of number words from the beginning (Fuson, 1988; Krajewski & Schneider, 2009; Wynn, 1990; 1992). The number sequence is first learned as a memorized poem, where many of the number words have initially no meaning, and are rather considered as a whole instead of separable entities (e.g., onetwothreefour…) (i.e., *string level*, Fuson, 1988). At this level, the number sequence is not yet related to other mathematical situations like cardinality recognition. Learning of the number sequence continues long after children are able to recite the first number words correctly (Fuson, 1991).

After reaching the *string level*, children start to learn that the number words are separated, but they can start the recitation only from the beginning and go only forwards (i.e., *unbreakable list level*, Fuson, 1988). Reaching this level enables children to use the number sequence in counting, and eventually pair the last counted item with the last counted number word to describe the manyness of all items in the counted set (i.e., cardinality principle) (Fuson, 1988). Next, children advance to being able to recite the number sequence starting from a number other than one (i.e., *breakable list level*), after which the number words themselves become units in a numerical sense, meaning that the number words can represent numerical situations, and can be matched, added, and subtracted (i.e., *numerable chain level*) (Fuson, 1988).

Lastly, at the *bidirectional chain level*, children can recite the number sequence fluently and flexibly in both directions (Fuson, 1988). At this stage, "the sequence is both seriated and embedded" (Fuson, 1988, p. 413), meaning that each number word represents both, a word within the sequence and a cardinal quantity. Each successive number word represents a quantity that is one more than the previous, reflecting an understanding that numbers build upon one another in a structured, additive way. Concurrently, the children understand that there are exactly n-1 entities preceding the nth entity. Thus, according to Fuson (1988), although children may learn the cardinality principle as early as the *unbreakable list level*, the deeper understanding that each successive number represents a quantity that increases by one typically emerges later, specifically at the *bidirectional chain level*, where the cardinal meanings become integrated with the number sequence.

1.2.3 Cardinality skills

An important milestone in children's numerical development is learning the cardinality principle (i.e., the last number word in counting represents the total number of items in the whole set) (e.g., Bermejo et al., 2004; Fuson, 1988; Gelman & Gallistel, 1978, Spaepen et al., 2018; Wynn, 1992). Acquiring this understanding requires many subskills that are learnt in parallel, without first knowing how these skills are related (Fuson, 1988; Mix et al., 2012; Wynn, 1990, 1992). These subskills include being able to recite the number sequence in the correct order, counting skills, and likely recognizing numerosities of a few small sets without counting them (Mix et al., 2012). Thus, around the same time as children are learning to recite the number sequence, they are learning the exact meanings of small cardinal number words, and how to count objects (Fuson, 1988; Le Corre & Carey, 2007; Mix et al., 2012; Wynn, 1990, 1992).

Children learn the meanings of small number words (i.e., one, two, three, and sometimes four) sequentially and one by one (e.g., Le Corre & Carey, 2007; Sarnecka & Lee, 2009; Wynn, 1992). For example, children learn first that "one" corresponds to a set of one object, and only after that the number word "two" corresponds to a set of two objects. The number of items in these small sets is recognized rapidly without counting – a process called subitizing-based enumeration (e.g., Clements & Sarama, 2007). It is suggested that subitizing-based enumeration is supported by infant parallel individuation, which would explain the small number range in subitizable sets (e.g., Feigenson et al., 2004).

In addition to enumerating sets of items by subitizing, children are beginning to learn how to use counting to tell how many items there are in a set. The process of learning how to count is a complex process requiring the mastery of many subskills. According to Gelman & Gallistel (1978), counting requires the understanding and coordination of several principles: 1) understanding that only one number word is assigned to each one of the items to be counted (i.e., *the one-one principle*), 2) knowing that the number sequence is always recited in the same order (i.e., *the stable-order principle*), 3) the number allocated to the last counted object represents the total number of objects in the set (i.e., *the cardinal principle*), 4) understanding that the preceding principles can be applied to any set of items (i.e., *the abstraction principle*), and 5) knowing that the order in which the items are counted is irrelevant (i.e., *the order-irrelevance principle*). In addition, Hannula (2005) suggested that to recognize the exact number of items by subitizing or counting, the child's attention first needs to be focused on the aspect of exact number.

The process of learning all the necessary subskills of counting is a lengthy process, taking time somewhere between one and two years for a child to learn the cardinal principle (Mix et al., 2012; Wynn, 1992). After acquiring this understanding of cardinality (i.e., each number word represents the number of items in a set), children become able to produce any set size within their counting range (e.g., Le Corre & Carey, 2007; Wynn, 1992). Understanding cardinality is considered as a gatekeeper in numerical development, as children's numerical knowledge accelerates after acquiring this knowledge (Geary & van Marle, 2018). In addition, Geary et al. (2018) observed that children who acquired understanding of cardinality earlier than their peers by the age of 4 years, had better number system knowledge (i.e., arithmetic, number line estimation) at the start of primary school, suggesting that these children had more time to develop their understanding of numerical relations.

According to Krajewski's model (Krajewski, 2008, as cited in Krajewski & Schneider, 2009) the previously reviewed numerical skills are acquired in three levels that lead towards an understanding of relations between quantities and number words:

1) *Level 1: Basic numerical skills*: Children are able to differentiate between discrete quantities (cf. infant quantity discrimination) perceptually and verbally, and approximately compare them with words like "more", "less". At this level, children acquire the precise number words by reciting the number sequence. However, the number words are yet not linked to the quantities they represent, as this requires perceiving the number words as discrete entities (Fuson, 1988; *unbreakable list level*).

2) *Level 2: Linking number words with quantity*: Children acquire the knowledge of how number words and quantities are connected to each other (i.e., understanding of cardinality) in two steps. First, the number words are linked to imprecise quantifiers (i.e., a bit, much, very much) possibly based on the time it takes to recite the number sequence until the target number. Thus, they become able to compare numbers in different categories (e.g., three = "a bit", and 20 = "much"), but this does not yet allow for precise comparisons between close numbers (e.g., three and four). In the second step, the children link the number words to exact quantities. This understanding develops through frequent object counting, during which children gradually realize that the final number word in the count represents the exact quantity of items (e.g., "three" = •••), rather than simply being a procedural cue to repeat the last number said. In addition, children link the duration of recitation to the number word – quantity -representations, allowing them to make an induction that the later number words in the number sequence represent larger quantities. With this

knowledge acquisition children are able to compare two consecutive numbers, and order them based on their quantities. In addition, at this level the children learn about the relations between quantities, such that quantities change only if an item is added or taken away from a set.

3) *Level 3: Linking quantity relations with number words*: At the third level, children start to understand composition and decomposition of numbers (e.g., a set of five objects can be divided into sets of two and three objects). In addition, they discover that a difference between two numerical quantities can be presented with a third numerical quantity (e.g., a set of five objects has two objects more than a set of three objects), thus gaining the understanding that the difference between two numbers is another number.

Similar to Krajewski's model (Krajewski, 2008, as cited in Krajewski & Schneider, 2009), other studies have suggested that children acquire the knowledge of the cardinal principle by connecting the ordinal structure of the count list with the cardinalities they represent (Carey, 2009; Marchand & Barner, 2018; Sarnecka & Carey, 2008). This means that children notice that successive number words "one", "two", "three", and "four" refer to quantities that always increase by one (e.g., •, ••, •••, ••••), and make the general induction that this is how all the numbers in the number sequence are related (Carey, 2009).

For instance, Sarnecka & Carey (2008) observed in their cross-sectional study of 2- to 4-year-old children, that only those children who understood the cardinality principle, performed above chance in two tasks assessing their understanding of how adding or removing items from a set represents moving forward or backward in the number sequence. In the Direction task (Sarnecka & Carey, 2008), children were first shown two plates with five (or seven) bears on both and told that each plate has five bears. After this, one bear was moved from one plate to another, the child was told that now there was a plate with four and a plate with six bears, and asked to indicate the plate with six bears (Sarnecka & Carey, 2008). In the Unit task (Sarnecka & Carey, 2008), the child was shown a wooden box, and the experimenter placing N (e.g., four) items in it. After making sure that the child remembered how many items there were in the box, one or two items were added to the box, and the child was asked whether the box now included N +1 or N + 2 (e.g., 5 or 6) items (Sarnecka & Carey, 2008). The successful performance of only cardinal principle knowers in these two tasks was taken as an indication that the acquisition of understanding how adding one item to a set implements counting forward in the number sequence, is the key factor that contributes to learning the cardinal principle and separates cardinal principle knowers from those children who can successfully produce sets under five items (i.e., subset-knowers) (Sarnecka & Carey, 2008).

However, there are a few limitations to the conclusion that learning the cardinal principle entails understanding how changes in cardinal value correspond to moving along the number sequence. While in neither task, counting of items was possible (Sarnecka & Carey, 2008), it could be argued that the Direction task could be solved with comparison (e.g., knowing that adding an item to a set leads to more items, and six is more than four). This may explain why Sarnecka & Carey (2008) observed that children who could produce a set of four but were not considered as cardinality principle knowers succeeded in the Direction task as well as the cardinal principle knowers. In addition, the Unit task only assessed children's knowledge of adding items and going forward in the number sequence. Thus, it is not entirely clear whether success in these tasks truly reflects a generalized understanding of the systematic relationship between the number sequence and cardinal values. However, if learning the cardinal principle includes the knowledge of how the number sequence and the cardinal values they represent are related, numerical ordering skills should automatically follow learning this important milestone, contradictory to Fuson's (1988) suggestion. On the other hand, it may be that only after children acquire the cardinality principle, they notice that cardinal values can be ordered in correspondence to the number sequence, which may allow children to discover that the sets always differ by exactly one and to make the generalized induction that this is true for all numbers in their counting range (Fuson, 1988; P. Cheung et al., 2017).

1.2.4 The co-development of cardinality and number sequence skills: A knowledge integration perspective

From the knowledge integration perspective, learning is not merely about the accumulation of knowledge, but rather requires the integration of different pieces of knowledge towards an interconnected network (Linn, 2006; M. Schneider & Stern, 2009). This theoretical perspective is supported by a range of work across disciplines, which suggests that children often acquire related pieces of knowledge in isolation, without initially understanding the connections between them (Baroody, 2003; Linn, 2006; Linn & Eylon, 2011; M. Schneider et al., 2011; M. Schneider & Hardy, 2013; Smith III et al., 1994). Such knowledge fragmentation, where abstract relations between different pieces of knowledge are not yet recognized, is a frequently occurring phenomenon in learning (M. Schneider & Stern, 2009).

M. Schneider and Stern (2009), outline three core assumptions for the knowledge integration perspective. First, children's knowledge is constructed from different elements, which may be fragmented or connected to varying degrees. Second, children often fail to recognize the underlying structure linking these elements. Third, these separate elements can be integrated into a well-connected, coherent knowledge. This integration may require restructuring the existing knowledge, and reflection on the underlying conceptual connections (Clark & Linn, 2013; M. Schneider & Stern, 2009; M. Schneider & Hardy, 2013). The interconnectedness of knowledge is also seen as a factor that separates experts from novices in different fields, as expert knowledge is organized into a coherent and well-structured network, that continues to expand and becomes better organized (Rittle-Johnson & Schneider, 2015; M. Schneider & Stern, 2009; Smith III et al., 1994).

Knowledge fragmentation may be influenced by children's prior experiences, particularly when concepts are introduced or encountered in separate contexts without explicit connections (Baroody, 2003; Linn & Eylon, 2011; M. Schneider & Stern, 2009). Concurrently, children's early knowledge is often loosely connected, even when developed with the support of adults (Baroody, 2003). This is consistent with Karmiloff-Smith's (1992) account of cognitive development, in which children may first acquire procedural or context-bound knowledge, and only later begin to understand the underlying concepts through a process of representational redescription. In this process, earlier knowledge is internally reorganized and re-encoded in increasingly explicit formats, allowing the child to reflect on, manipulate, and apply that knowledge more flexibly across contexts (Karmiloff-Smith, 1992). For example, early experiences with number often involve a procedural use of the number sequence for counting, which may initially develop separately from an understanding of cardinality (Fuson, 1988; Gelman & Gallistel, 1978). These early experiences may lay the groundwork for integration, but until children actively reflect on the relationships between the two concepts, their knowledge may remain fragmented.

Contradictory to the previous studies that suggested understanding of ordinality to follow the learning of cardinality principle (Carey, 2009; Krajewski & Schneider, 2009; Marchand & Barner, 2018; Sarnecka & Carey, 2008), and in line with Fuson (1988) and the reviewed knowledge integration perspective, some studies have observed that even though children have learned to use counting to tell the numerosity in a set, it does not mean that they understand how the number sequence and cardinal values are related (P. Cheung et al., 2017; Davidson et al., 2012; Le Corre, 2014; R. M. Schneider et al., 2021; Sella & Lucangeli, 2020). For example, Davidson and colleagues (2012) observed in their cross-sectional study of 3- to 5-year-olds, that many children who understood the cardinal principle, failed to indicate that adding one item to a set of N items results to the next number in the number sequence N + 1 even among small numbers (e.g., 4 and 5) using the Unit task (Sarnecka & Carey, 2008), even though majority of them they could label the successive number word in the number sequence (i.e., "What comes After N?"). However, the children who could recite the number sequence up until somewhere between 30 and 100 and were considered as "High counters" in addition to knowing the cardinality principle, performed above chance in the Unit task, while cardinal principle knowers with less counting proficiency did not. However, even the "High counters" were observed of failing in the Unit task for numbers beyond 10. Thus, Davidson et al. (2012) concluded that the knowledge of how adding one item to a set corresponds to moving forward in the number sequence emerges somewhere later acquiring the cardinality principle knowledge. In addition, Davidson et al. (2012) observed that around 20 % of the children who knew the cardinality principle, did not know which of two small number words (between five and nine) is larger.

Concurrently, instead of only assessing children's knowledge of how adding an item to a set leads to the successive number in the number sequence, Sella and Lucangeli (2020) investigated also how children perform when an item is removed from a set. The results of their cross-sectional study with 5- to 6-year-old children showed, that children who understood the cardinality principle, performed well in trials where an item was added to a set (corresponding to moving forward on the number sequence), while displaying limited knowledge in trials where an item was removed from a set (corresponding to moving backward on the number sequence) (Sella & Lucangeli, 2020). In addition, Sella and Lucangeli (2020) observed that some children answered the successive number in the number sequence irrespective of the transformation, namely whether an item was added or removed from the set. A similar pattern of responses was found in a task where children were asked to indicate a number that comes before or after two given numbers, filling in the missing position in the sequence (e.g., [ ] [2] [3]) (Sella & Lucangeli, 2020). These results may indicate that the children were reciting the number sequence forward as a rote behavior, and did not yet have the conceptual understanding of how the changes in cardinal values are related with going forward and backward in the number sequence (Sella & Lucangeli, 2020). Thus, it seems that understanding the cardinality principle may not be enough to understand how cardinal values and number sequence are connected, which may be essential for understanding numerical ordering. In conclusion, knowledge of cardinality and the number sequence may initially exist as disconnected elements, and recognizing the abstract structure that links these concepts, such as understanding that each successive number represents a set with one more item, may help children integrate them into a more coherent conceptual framework.

1.2.5 Additional factors to numerical ordering: The role of comparison and mathematical language

Understanding the development of numerical ordering skills may benefit from a broader view of how children's mathematical reasoning evolves. Resnick's (1991) theoretical framework describes this development as progressing through layers of mathematical knowledge from intuitive reasoning about physical quantities to abstract numerical concepts. At the earliest level, known as mathematics of protoquantities, children can compare sets perceptually (e.g., one glass has more milk than another), using terms like more, less, big, or small. At this level, children may be able to order sets based on salient size differences from least to most, but they do not yet associate these quantities with exact numbers.

In the next level, the mathematics of quantities, children begin to use number words to describe physical sets (e.g., "eight apples"). Numbers still function as adjectives tied to physical quantities, but now allow for more precise comparisons (e.g., recognizing that eight is more than three). Here, children may be able to order sets labeled with different numbers, but the reasoning is still grounded in the physical size of the objects. At this level, children may gain early experiences about the more and less relations between different numbers of items. However, it is unclear whether children are able to order quantities based on their cardinal value, if the differences between the sets of items are not large enough to discriminate physically.

Finally, in the level of mathematics of numbers, numbers become conceptual entities. Children understand the abstract numerical relations (e.g., that six is less than eight) without a need to imagine the physical materials. Resnick (1991) describes this as the level where "the properties of numbers are defined in terms of other numbers." (p. 41). If numerical ordering requires the conceptual understanding that numbers themselves can be ordered based on their cardinal value (Spaepen et al., 2018), numerical ordering may occur only at the level when numbers become conceptual entities themselves that follow a certain order. From this perspective, early experiences with comparison and relational language may play an important role in supporting the development of numerical ordering.

Empirical research supports the idea that comparison may contribute to early numerical ordering skills. While studies have repeatedly suggested comparison and ordering to be distinct processes (e.g., Finke et al., 2022; Goffin & Ansari, 2016; Vogel et al., 2015, 2017), studies with especially young children using symbolic ordinality tasks have showed positive relations between number comparison and ordering. For example, C. Xu and LeFevre (2021) investigated the relations between number ordering, number comparison, and sequential relations (i.e., missing number task; which number is missing from the sequence of 1 _ 3 4) in two samples of children, in the beginning of school in Grade 1 at the age of 6, and after a year of formal schooling in Grade 2 at the age of 7. They observed that number ordering was strongly predicted only by number comparison in children in Grade 1, while for children in Grade 2, number ordering was predicted by number comparison, missing number, and inhibitory control. The results suggested that at the early phases of development, number comparison and ordering are highly intertwined, as children may rely on cardinal information (i.e., Which is more?) to reason about order (Lyons et al., 2016; Sasanguie & Vos, 2018; C. Xu & LeFevre, 2021). Moreover, later in the development, number ordering may reflect the integration of cardinal and sequential relations, as children become able to use both of these associations flexibly to reason about order (C. Xu and LeFevre, 2021). Similarly, Finke et al. (2022) observed cross-lagged relations between number comparison and number ordering in a longitudinal study involving two samples from different countries, followed from Grade 1 to Grade 3 (mean ages approximately 6.3 and 7.2 years at baseline). Furthermore, Sella, Lucangeli, et al. (2020) observed in a cross-sectional study of 3- to 6-year-old children, that after controlling for cardinality skills (measured with a Give-N task) and memory, performance in ordering numbers on a number line explained unique variance in comparison tasks with number words and Arabic number symbols (see also Sella et al., 2019). They suggested that mastering the cardinality principle does not imply a full understanding of the magnitudes of symbolic numbers. Instead, experience with ordering numbers on a number line may contribute to this understanding by providing a conceptual structure to access the magnitude relations between number symbols (Sella, Lucangeli, et al., 2020). Together, these results indicate that the development of comparison and ordering skills is highly intertwined in early mathematical development.

In addition to comparison, mathematical language may play a supporting role in children's ability to reason about numerical order. For example, Hornburg et al. (2018) proposed that complex numerical skills, like number ordering, may build on basic procedural competencies (e.g., verbal counting and number symbol identification), and children’s understanding of mathematical language such as terms “more”, “less”, “before”, and “after”. Similarly, Chan et al. (2022) suggested that children with difficulties in number relation skills, including number ordering, may benefit from instruction that explicitly draws connections between counting, number symbol identification, and number ordering. Indeed, stronger proficiency in mathematical language has been shown to predict better performance in number ordering (Chan et al., 2022; Hornburg et al., 2024), likely because such language may support children's understanding of numerical relations (Chan et al., 2022). Overall, empirical studies suggest that acquiring number ordering skills likely requires the coordination and integration of multiple numerical skills, including number sequence knowledge, number symbol identification, comparison, and mathematical language (Chan et al., 2022; Hornburg et al., 2018; C. Xu & LeFevre, 2021). Similarly, learning to order non-symbolic quantities may depend on understanding the connection between the number sequence and cardinal values, with comparison skills and mathematical language likely contributing to this process. The integration of cardinal and ordinal knowledge has been proposed as essential for a well-developed concept of number (Anderson & Cordes, 2013; Fuson, 1988). Since one of the earliest signs of this understanding is learning that magnitudes can be ordered based on their cardinal values (Geary, 2013), developing numerical ordering skills may be highly important.

1.3 Spontaneous mathematical focusing tendencies and mathematical development

Early numerical skills are acquired not only through formal, adult-led instruction but also through numerous opportunities in informal, unguided situations (Lehtinen et al., 2017). For decades, studies have reported children to engage with mathematics spontaneously – that is, in a self-initiated way – in their everyday environments even before reaching school age (Ginsburg, 1977; Hannula et al., 2005; Ramani et al., 2015; Saxe et al., 1987). Thus, examining the sub-processes that trigger the use of mathematical skills in these informal situations is crucial to better understand the development of mathematical skills (Hannula, 2005; Hannula & Lehtinen, 2005).

Since the early 2000's first study about spontaneous focusing on numerosity (Hannula & Lehtinen, 2001, 2005), a line of research on *spontaneous mathematical focusing tendencies* has shown that some children are more inclined to spontaneously recognize and use mathematical aspects in non-explicitly mathematical situations, which has been associated with their concurrent and later mathematical development (for reviews, see Li et al., 2025; McMullen et al., 2019; Verschaffel et al., 2020). These tendencies include spontaneous focusing to specific mathematical features, such as numerosity (SFON; Hannula & Lehtinen, 2001, 2005), quantitative relations (SFOR; McMullen et al., 2014), Arabic number symbols (SFONS; Rathé et al., 2019), patterns (SFOP; Wijns et al., 2020), space (Perez & McCrink, 2019), and orientation towards different dimensions of magnitude (SOMAG; Viarouge et al., 2018).

To encompass these distinct tendencies, the broader term *spontaneous mathematical focusing tendencies* has been introduced (McMullen et al., 2019; Verschaffel et al., 2020). Importantly, this term does not imply that a child spontaneously focuses on all mathematical aspects in general, but instead captures these domain-specific tendencies grouped under a broader umbrella term. For example, some children may spontaneously focus on the number of flowers in a vase and start to count them (i.e., SFON), while others will focus on different aspects, such as the colors of the flowers. Furthermore, the term *spontaneous* refers to the "unguided, self-initiated nature of recognition and use of numerical features within a specific moment or situation (i.e., without external prompting)" (McMullen et al., 2019, p. 72; see also Hannula & Lehtinen, 2005). The different spontaneous mathematical focusing tendencies are described as attentional processes that trigger the recognition of the specific mathematical feature and use of the relevant mathematical skills across different situations (Hannula-Sormunen, 2015). They are considered as distinct and domain-specific components of mathematical development, while also interacting with the required skills (e.g., mathematical and cognitive skills) and contextual factors (e.g., perceptual salience, social goals) present in a given situation (McMullen et al., 2019).

1.3.1 Spontaneous focusing on numerosity (SFON)

The first discovered, and most investigated spontaneous mathematical focusing tendency is children's spontaneous focusing on numerosity (SFON) (for a recent meta-analysis, see Li et al., 2025). SFON was first introduced by Hannula and Lehtinen (2001, 2005), and it refers to a process of spontaneously (i.e., in a self-initiated way, not prompted by others) focusing attention on the aspect of exact number of items or incidents, and using that information in action within situations that are not explicitly mathematical (Hannula et al., 2010). Many studies have observed substantial individual differences in children's tendency to spontaneously focus on numerosity, which are not entirely attributable to their existing mathematical or cognitive skills (Batchelor et al., 2015; Bojorque et al., 2017; Gray & Reeve, 2016; Hannula et al., 2010; Hannula & Lehtinen, 2005; Liang et al., 2021; McMullen et al., 2015; Rathé et al., 2019; Torbyens et al., 2018). Furthermore, Hannula & Lehtinen (2005) showed that out of a sample of children who did not spontaneously focus on numerosity in SFON tasks and got a score of 0, almost all children succeeded in the same tasks at least once when their attention was explicitly guided towards the aspect of exact number (i.e., when they were told to focus on the number of items in the task). In addition, the children (n = 4) who could not carry out the SFON tasks even with explicit guidance towards the aspect of exact number, had been earlier diagnosed with either linguistic or attentional deficiency. Therefore, the individual differences in SFON were not due to a lack of the mathematical or cognitive skills required by the tasks, but rather reflected a lack SFON itself—supporting the view that SFON tasks measure a construct distinct from the skills they rely on (Hannula & Lehtinen, 2005).

Prior research has consistently shown that individual differences in SFON tendency are positively associated with early numerical skills (Gray & Reeve, 2016; Hannula, Räsänen, & Lehtinen, 2007; Hannula & Lehtinen, 2005; Liang et al., 2021; Poltz et al., 2022; Silver et al., 2020; Torbyens et al., 2018) and serve as predictors of later mathematical achievement (Batchelor et al., 2015; Gloor et al., 2021; Hannula et al., 2010; Hannula-Sormunen et al., 2015; Lepola & Hannula-Sormunen, 2019; McMullen et al., 2015; Nanu et al., 2018). Many of these associations were domain-specific, and remained significant after accounting for domain-general factors (Gloor et al., 2021; Hannula et al., 2010; Li et al., 2025; Nanu et al., 2018; Poltz et al., 2022). For example, Poltz et al. (2022) observed that SFON tendency measured at the age of 5 years was a significant predictor of magnitude estimation and calculation (i.e., sum score of visual and verbal arithmetic, subitizing, magnitude comparison, and quality estimation of quantities) measured 9 months later, in addition to counting skills, number knowledge, and domain-general factors such as nonverbal intelligence, visual attention, and visuospatial working memory. In addition, Hannula et al. (2010) found that SFON tendency measured at the age of 5 years significantly predicted arithmetical skills at the age of 8 years even after controlling for nonverbal IQ, rapid naming, instruction comprehension, and spontaneous focusing on spatial locations. Notably, a similar predictive relation was not observed for reading skills (Hannula et al., 2010). These results provided support for the domain-specificity of SFON.

SFON has been measured with a variety of different tasks (Li et al., 2025). The most used tasks include behavioral tasks, such as Imitation, Model, and Finding tasks (Hannula & Lehtinen, 2005), and a verbal Picture description task (Batchelor et al., 2015) (Li et al., 2025; Rathé et al., 2016). In Imitation tasks, children reproduce actions demonstrated by the experimenter, for example, giving a toy parrot a specific number of berries or inserting a certain number of letters into a postbox (Hannula & Lehtinen, 2005). Furthermore, in the Model task, children observe the experimenter constructing a model, such as stamping a specific number of spikes on a dinosaur's back, and are then asked to replicate the model (Hannula & Lehtinen, 2005). In the Finding task, children observe the experimenter hide a treasure under a series of hats placed next to each other in a semicircle, and then indicate where the treasure was hidden (Hannula & Lehtinen, 2005). While these behavioral tasks require children to respond through actions, the Picture description task involves a verbal response. In this task, children are shown pictures that include both numerical (e.g., different numbers of objects) and non-numerical (e.g., shapes, colors) aspects, and are asked to describe the pictures as accurately as possible (Batchelor et al., 2015). Importantly, children are not advised to focus on number in any of these tasks, but are rather left to notice it themselves (Batchelor et al., 2015; Hannula & Lehtinen, 2005).

In the behavioral SFON tasks, the children score SFON if they notice or use the exact number to produce the answer (e.g., feed the same number of berries to the parrot as the experimenter, or stamp the same number of spikes onto a dinosaur), or if they show any quantifying acts while solving the task, such as a) use of number words referring to the number of items in the task, b) use of fingers to show numbers, c) counting acts (e.g., whisper a number word sequence, show pointing acts with fingers), d) other comments referring to counting or number of items (e.g. "I miscounted them."), or e) interpreting the goal of task as quantitative (e.g., "I gave exactly the same number.") (Hannula & Lehtinen, 2005). In the verbal Picture description task, the children score SFON if they mention at least one exact number in their description of the picture (e.g., "There are three chicks.") (Batchelor et al., 2015). Importantly, in any of these SFON tasks the children get a SFON score even though there would be mistakes in the enumeration process, as in the behavioral SFON tasks a SFON score is given if the children show any quantifying acts but make a mistake along the process, and in the Picture description task even if the exact number of objects in the description would not be the same as in the picture

(Batchelor et al., 2015; Hannula & Lehtinen, 2005). The total score on the SFON tasks serves as a measure of children's *SFON tendency*, which is considered as an indicator of how likely children spontaneously focus on the aspect of exact number in explicitly non-mathematical situations, such as everyday life (Hannula & Lehtinen, 2005; McMullen et al., 2019).

The behavioral and verbal SFON tasks have their limitations. Batchelor (2014) investigated the psychometric properties of the previously described three behavioral SFON tasks, and raised important methodological considerations for these tasks. First, Batchelor (2014) suggested the tasks to have little to focus on besides than exact number of objects, and have high working memory demands, as they require children to remember the number of objects (e.g., stamped spikes on a dinosaur) to correctly solve the task. Second, as most of the children received a SFON score on the behavioral tasks based on the correctly reproduced number of items and not the quantification acts, the SFON tasks may be driven by children's counting accuracy (Batchelor, 2014; Elliot et al., 2022). However, Nanu et al. (2018) investigated the construct validity of SFON Imitation tasks and found that children typically responded either with the correct number of items or with a grossly inaccurate number (e.g., using all available items). This bimodal response pattern, rather than a typical, normally distributed curve associated with counting accuracy, led the authors to conclude that individual differences in the SFON tasks reflected differences in children's SFON tendency rather than counting ability. Third, studies have observed low to moderate correlations between the different behavioral SFON tasks (Elliot et al., 2022; Liang et al., 2021). However, the low correlations may be explained by observations suggesting that spontaneous focusing may be influenced by task context, such as features, demands, and the salience of numerical information (Batchelor et al., 2015; Chan & Mazzocco, 2017; Mazzocco et al., 2020). Notably, these aspects differ also in everyday situations.

The verbal Picture description task has its own limitations too, as it requires sufficient verbal skills to describe the content of the picture, and may thus hinder children with limited verbal skills, such as young children (Savelkouls et al., 2020). In addition, similarly to the behavioral SFON tasks, using pictures with different contexts may result in low correlations between the items and reduced internal consistency, possibly due to varying salience of the mathematical aspects in the different pictures (Määttä et al., 2024). In the studies included in this dissertation, behavioral tasks were used to assess children's SFONO. A recent meta-analysis of Li et al. (2025) found that behavioral SFON tasks and scoring methods that included both the accuracy of reproduced number of objects and quantifying acts were more strongly correlated with mathematics performance. Furthermore, these limitations highlight the complexity of measuring spontaneous mathematical focusing tendencies. Thus, there are several guidelines that need to be considered in designing new measures, that were followed in designing the new SFONO measures for the studies in this dissertation. These criteria are described in detail in the Methods section.

1.3.2 Spontaneous mathematical focusing tendencies in mathematical development

In addition to the aspect of exact number, studies have identified individual differences in spontaneous focusing on other mathematical aspects, such as quantitative relations (SFOR; McMullen et al., 2014), Arabic number symbols (SFONS; Rathé et al., 2019), and patterns (SFOP; Wijns et al., 2020). These spontaneous mathematical focusing tendencies have also been associated with better mathematical skills (e.g., McMullen et al., 2014, 2016, 2017; Rathé et al., 2019; 2022; Van Hoof et al., 2016; Wijns et al., 2020). Furthermore, similarly as higher SFON tendency has been associated with better cardinality and counting skills (e.g., Hannula & Lehtinen, 2005), higher SFOR scores are associated with better rational number knowledge (McMullen et al., 2014, 2016, 2017; Van Hoof et al., 2016), higher SFONS scores with better numerical mapping abilities (Rathé et al., 2022a), and higher SFOP scores with better patterning ability (Wijns et al., 2020). Thus, it seems that the different spontaneous mathematical focusing tendencies may be especially relevant for the development of different mathematical skills (McMullen et al., 2019). This suggestion is supported by results from a 7-year follow-up study of Nanu et al. (2018), that found no predictive relation between SFON tendency measured at the age of 5 years and rational number knowledge measured at the age of 11 years. However, there are studies with contradictory results. For example, McMullen et al. (2015) found that SFON tendency at the age of 6 years predicted rational number knowledge measured 6 years later. Notably, McMullen (2014) proposed that although early individual differences in SFON tendency are predictive of later mathematical achievement and seem to contribute to mathematical development, the impact of self-initiated practice with solely counting and enumeration skills (indicated by SFON tendency) may diminish as mathematical content becomes more complex.

The different spontaneous mathematical focusing tendencies (e.g., SFON, SFOR, SFONS, SFOP) are treated as distinct (but related) constructs (McMullen et al., 2019). While most studies include only one or two spontaneous mathematical focusing tendencies (Verschaffel et al., 2020), existing evidence points to partial empirical separability (Rathé et al., 2019, 2022a, 2022b). For example, Rathé et al. (2022a) investigated longitudinal relations and the factor structure of SFON and SFONS constructs from the age of 4 years to the age of 7 years in three time points, and found low correlations ($0.21 < r < 0.28$) between SFON and SFONS across the follow-up. In addition, the two spontaneous mathematical focusing tendencies were best represented by two separate (but related) latent factors, SFON and SFONS, in all three time points (Rathé et al., 2022a).

In general, it is hypothesized that children who show a higher tendency to spontaneously focus on mathematical aspects are more inclined to recognize and use those mathematical aspects in situations that are not explicitly mathematical (e.g., everyday lives), and may thus gain more self-initiated practice in the relevant mathematical skills (e.g., Hannula & Lehtinen, 2005; Lehtinen et al., 2017; McMullen et al., 2019; Rathé et al., 2019). For instance, in a longitudinal study with children aged 3.5 to 6 years, Hannula and Lehtinen (2005) found a reciprocal relationship between SFON and early numerical skills (i.e., a composite score of number sequence production and cardinality or counting skills), with each contributing to the development of the other over time. Similarly, McMullen et al. (2017) followed 9- to 11-year-old children over a four-year period measuring their SFOR tendency and rational number conceptual knowledge in four timepoints, and found that SFOR tendency and rational number conceptual knowledge were related reciprocally. These findings suggest that the development of (the relevant) mathematical skills (e.g., counting and cardinality) and spontaneous mathematical focusing tendency (e.g., SFON) may follow an iterative, mutually reinforcing process (McMullen et al., 2019). In this cycle, more advanced mathematical skills may enhance a child’s tendency to spontaneously attend to mathematical aspects in their environment, which in turn increases opportunities for self-initiated practice, further supporting the development of mathematical skills.

The findings of the reciprocal development between spontaneous mathematical focusing tendencies and mathematical skills (Hannula & Lehtinen, 2005; McMullen et al., 2017) are similar to the developmental associations between deliberate practice and mental representations, where deliberate practice results in developed mental representations, improving the effectiveness of later deliberate practice (Ericsson, 2016; Lehtinen et al., 2017). Furthermore, in their study of expert and exceptional performance, Ericsson and Lehman (1996) found that future experts were capable of recognizing possibilities in everyday situations to practice their skills, without any external guidance. Similarly, it may be that children exhibiting a higher tendency to spontaneously focus on a mathematical aspect, may recognize more situations to practice their relevant mathematical skills in everyday lives, and thus acquire more self-initiated practice. For example, children with higher SFON tendency may recognize more opportunities to practice their cardinality and counting skills, thus leading to enhanced skills in these domains.

According to Ericsson (2016), skill acquisition is a process of building and refining different abilities. As the level of acquired performance increases, the nature of the concrete practice activities will differ. Furthermore, one key characteristic of deliberate practice is that it involves attempting things that exceed a person's current skill level (Ericsson, 2016; Lehtinen, 2017). Thinking of the development of mathematical skills, a similar progression may be relevant. First, when learning the basic numerical skills such as cardinality recognition, recognizing small numbers of items and practicing object counting skills is highly relevant. However, after children become competent counters, the relevance of practicing these skills will diminish as more advanced skills start to develop. In a similar way, spontaneous mathematical focusing tendencies may evolve over time, supporting the development of increasingly complex mathematical skills throughout the learning trajectory (McMullen, 2014).

Considering the reviewed evidence from studies on spontaneous mathematical focusing tendencies, it may be that for the development of numerical ordering skills, a spontaneous mathematical focusing tendency that triggers the self-initiated practice with numerical ordering becomes increasingly important. Thus, the current dissertation investigated whether such a spontaneous mathematical focusing tendency, namely Spontaneous Focusing On Numerical Order (SFONO) can be identified in children. More specifically, it examined whether individual differences in children's SFONO tendency exist, and how these differences are related, in particular, to children's numerical ordering skills.

1.3.3 Spontaneous focusing on numerical order and numerical ordering skills

Studies have shown, that already 3- to 5-year-old children have developed some knowledge regarding numerical ordering of non-symbolic sets (e.g., Purpura & Lonigan, 2013; Spaepen et al., 2018; Stock et al., 2009) and symbolic numbers (e.g., Chan et al., 2022, Liang et al., 2023; Morsanyi et al., 2024). This suggests, that children start to learn numerical ordering skills informally, before the start of formal schooling. In addition, there is evidence that already 3- to 5- year-old children and their caregivers spontaneously engage in math talk regarding the order of numbers (e.g., *"What comes after nine*", *"Two comes next."*) in activities like book reading, puzzle play, and playing a board game (Fuson, 1988; Ramani et al., 2015; Trickett et al., 2022). Thus, it seems that children are starting to develop and practice their numerical ordering skills already from the early age.

One study in the domain of spontaneous mathematical focusing tendencies is in particular interest to the current dissertation. Sharir & Mevarech (2022) showed that 4- to 6-year-old children had individual differences in spontaneous focusing on mathematical structures. They measured spontaneous focusing on mathematical structures with Picture description tasks and Imitation tasks, where one possible aspect to focus on was one out of three mathematical structures. In the Imitation task, a child was asked to imitate the researcher inserting different colored discs into a box, while in the Picture description task a child was shown pictures with squares, and pictures presenting items in their natural environments. The mathematical structures included were random order (e.g., three discs put into the box one at a time), multiplication patterns (e.g., two pairs of two discs put into the box), and arithmetic series (e.g., sets of one, two, and three discs put into the box) (Sharir & Mevarech, 2022). Thus, spontaneous focusing on mathematical structures included spontaneous focusing on exact number (i.e., random pattern), and numerical order (i.e., arithmetic series), in addition to multiplication patterns. The results of the cross-sectional study showed, that children who spontaneously focused on mathematical structures more frequently, had higher concurrent mathematical skills (Sharir & Mevarech, 2022). However, the study did not specifically investigate the relation between spontaneous focusing on arithmetic series (i.e., numerical order), the Imitation task regarding arithmetic series was highly demanding for working memory, and there were not many aspects (besides the mathematical structures) to focus on in the measures. Thus, although the study of Sharir & Mevarech (2022) provided some evidence that children may spontaneously focus on the aspect of numerical order, it did not specifically measure SFONO or investigate its relation with mathematical skills.

In light of the reviewed evidence, it was hypothesized that a separate construct of spontaneously focusing on numerical order exists, which can be distinguished from children's existing mathematical skills. In line with previous research, SFONO refers to a process of spontaneously focusing on numerical order in situations that are not explicitly mathematical. In the current dissertation, SFONO was measured using non-symbolic stimuli (e.g., sets of items), and thus the recognition and use of numerical order in the SFONO tasks included numerical ordering of sets of items based on their cardinal value. This was done, as it was decided that numerical ordering would be investigated from the non-symbolic aspect in the current dissertation. In line with the existing research on spontaneous mathematical focusing tendencies, it was hypothesized that SFONO tendency would be especially related to numerical ordering, as SFONO may reflect children's amount of self-initiated practice in the relevant skills. Thus, children with a higher SFONO tendency may also have better numerical ordering skills, as they may be more inclined to notice more opportunities in explicitly non-mathematical situations to practice their numerical ordering skills.

1.4 A proposed model of the subskills involved in numerical ordering

This chapter presents a proposed model of the subskills involved in numerical ordering, based on the theoretical and empirical literature reviewed in the preceding chapters, and the empirical studies conducted in the present dissertation. The purpose of the model is to clarify how core numerical competencies may interact and contribute to the development of numerical ordering skills, particularly in a non-symbolic context. By integrating these elements, the model offers a conceptual framework for understanding the underlying factors of numerical ordering skills in early childhood. Figure 1 illustrates the key subskills of numerical ordering addressed in the current dissertation.

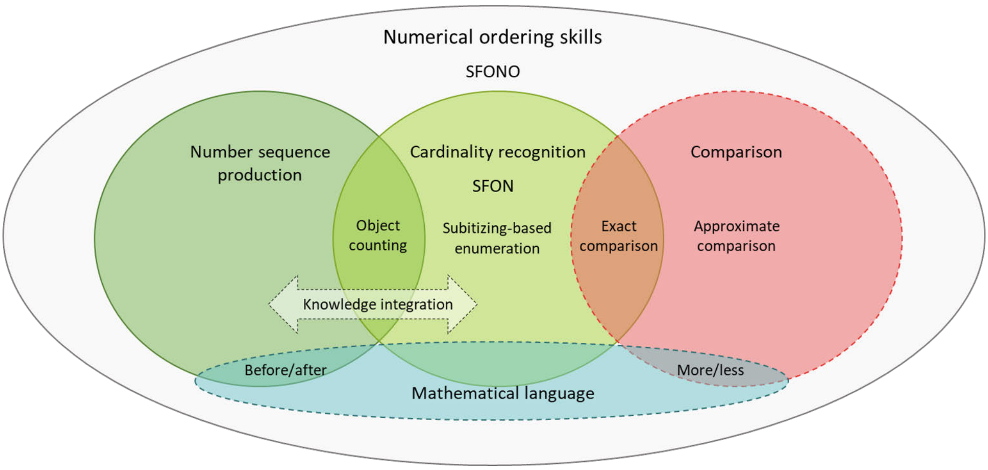

Figure 1. The proposed subskills of numerical ordering.

Developing numerical ordering skills appears to be a complex process that involves many subskills. As numerical ordering skills were investigated from the non-symbolic aspect, cardinality recognition sits at the core of numerical ordering skills. To order sets of items in exact numerical order, children must be able to recognize the precise number of items in each set. This can be achieved either by rapidly recognizing the number of items in a set by subitizing-based enumeration, or by counting how many items the set has (e.g., Bermejo, 1996; Mix et al., 2012; Wynn, 1990). If the child's attention is not guided towards the aspect of exact number of items in the sets, SFON may be needed to trigger the process of exact number recognition (Hannula, 2005; Hannula & Lehtinen, 2005).

Furthermore, the ability to recognize cardinal values alone may not be sufficient for understanding ordinality. Instead, children may need to additionally understand that cardinal values themselves can be ordered so that each one is more than the previous one (Spaepen et al., 2018). This understanding may require integrating one's knowledge of the number sequence and cardinal values towards a coherent understanding that each number in the number sequence corresponds to a cardinal value. While object counting inherently involves the use of the number sequence to determine the quantity of a set, it is not self-evident that children understand the conceptual relation between the two (P. Cheung et al., 2017; Davidson et al., 2012; R. M. Schneider et al., 2021; Sella & Lucangeli, 2020).

Additionally, empirical evidence shows that comparison and mathematical language are positively related to numerical ordering, at least with symbolic numbers (Chan et al., 2022; Finke et al., 2022; Hornburg et al., 2024; C. Xu & LeFevre, 2021). In numerical ordering with non-symbolic stimuli, comparison may allow children to recognize that the sets of items differ in quantity, and to understand the more and less -relations between the numbers of items (e.g., four is more than three). Therefore, comparison and understanding of mathematical language (e.g., terms like more, less, before, after), may be needed to order sets of items from least to most. In addition, previous research has suggested that alternate strategies may be used to reason about ordinality (e.g., C.-N. Cheung & Lourenco, 2019; Sasanguie and Vos, 2018; C. Xu & LeFevre, 2021). For instance, children may rely on making repeated pairwise comparisons (e.g., four is larger than three, and therefore comes later), or scanning their memory of the number sequence (e.g., recite the number sequence) to arrive at the correct numerical order. In addition, ordinality tasks with small consecutive numbers (e.g., 1, 2, 3) may be easier to order due to the high familiarity of the sequence and stronger associative links between the numbers (e.g., Devlin et al., 2024; Sella, Sasanguie, et al., 2020; C. Xu & LeFevre, 2021). Importantly, the whole numerical ordering process may require attentional focusing on the aspect of numerical order (cf. Hannula, 2005). If the child's attention is not purposefully directed towards numerical order (e.g., in everyday situations), SFONO may serve a critical role in triggering the recognition and use of numerical order in the situation.

2 Research Aims

As discussed, early numerical skills are highly predictive of later mathematical achievement. As large individual differences exist in numerical skills already before formal schooling (e.g., Jordan et al., 2009), it is important to understand the factors behind these individual differences. One milestone in early mathematical development is understanding the ordinality (i.e., numerical order) of numbers. The predictive role of ordinality skills for later mathematical performance has only recently been emphasized (e.g., Lyons et al., 2016). However, there is very little research on how these skills develop. Furthermore, studies of ordinality have used mainly number symbols as stimuli, and focused on children's performance on the ordering of numbers when this was given as their explicit goal. Nonetheless, ordering sets of items (e.g., collections of six and seven dots) numerically may be more closely related to the conceptual understanding of ordinality, as in understanding that the next number is always more than the previous one, instead of merely reciting the counting sequence from rote memory. In addition, previous studies have shown that it is not only the learning of numerical skills in adult-guided situations, but also children's self-initiated recognition and use of mathematical features in non-explicitly mathematical situations that contributes to numerical development, as stronger tendencies to spontaneously focus on mathematical features have been associated with better mathematical skills (e.g., Hannula & Lehtinen, 2005; McMullen et al., 2019; Verschaffel et al., 2020). If some children spontaneously focus on numerical order more than others, it may be a relevant factor contributing to individual differences in numerical ordering skills. Given this context, the overarching aim of the current dissertation was to investigate the role of numerical ordering skills and SFONO tendency in mathematical development. This aim was addressed through three specific research objectives:

1) Investigate whether there is a separate attentional process of spontaneous focusing on numerical order, that is distinct from children's numerical ordering skills (Studies I, II, III)

Spontaneous focusing on different mathematical aspects in non-explicitly mathematical situations has been associated with better concurrent and later mathematical skills (McMullen et al., 2019; Verschaffel et al., 2020). One less researched mathematical aspect that can be spontaneously focused on is numerical order. To investigate whether, and to what extent, children spontaneously recognize and use numerical order in non-explicitly mathematical situations, valid measures that are distinct from tasks that explicitly ask children to use numerical order need to be developed. Study I introduced a novel task to measure children's recognition use of numerical order in a situation where children were not explicitly asked to use numerical order. The results indicated that children had individual differences in focusing on numerical order, but the task instruction included some numerical hints and did not fully measure children's spontaneous focusing. Study II expanded on the previous methodology of Study I, by introducing three novel tasks that measured spontaneous focusing on numerical order, and was the first to specifically measure children's SFONO tendency. Study III aimed to measure SFONO with a more comprehensive set of tasks and numerical sequences than Study II, and with a different sample of children from another education system. In addition, Study III aimed to provide evidence that the individual differences in SFONO tendency were not due to individual differences in the skills needed in order to spontaneously focus on numerical order in the SFONO tasks.

2) Examine the development of numerical ordering skills in 3–6 -year-old children in relation to early numerical skills and later mathematical performance (Studies I and IV)

Better numerical ordering skills predict later mathematical achievement (e.g., Liang et al., 2023). Nevertheless, there is limited knowledge on how these skills develop. Thus, Study IV investigated this development in detail between the ages of 3 and 5 years, when it was expected that numerical ordering skills are starting to develop. In particular, this study aimed to identify different growth trajectories in numerical ordering skills over the course of a one-year follow-up, and to examine not only how children with different growth trajectories attempted to solve the task but also how these attempts evolved over time. In addition, Study IV examined how cardinality skills and number sequence production were related with differences in numerical ordering skill development, aiming to uncover developmental relations between the three distinct but intertwined numerical skills. Study I examined children's development between the ages of 3 and 6 years in a task where, in order to succeed, numerical ordering skills needed to be applied, thus broadening the knowledge on the development of numerical ordering skills in relation to early mathematical development. In addition, it was not known whether numerical ordering skills measured with non-symbolic stimuli predicted later mathematical achievement similarly as to the results of those studies using number symbols. Thus, Study I longitudinally examined whether the application of numerical ordering skills in a play-based task at the age of 5 years predicted individual differences in general mathematics achievement at the age of 12 years. This focus was important, as the ability to apply numerical skills beyond specifically designed mathematical tasks represents a valuable and desired goal for education.

3) Explore the associations between SFONO tendency and early numerical skills, especially numerical ordering skills (Study II)

It is suggested, that children who more frequently spontaneously focus on mathematical features, may acquire more self-initiated practice, leading to better numerical skills (e.g., Hannula & Lehtinen, 2005). Study II was the first that aimed to investigate the relations between SFONO tendency, numerical ordering skills, and other aspects of early numeracy cross-sectionally in 3–4 -year-old children. In addition, this study investigated whether higher SFONO tendency was particularly associated with better numerical ordering skills, in line with the hypothesis of self-initiated practice.

3 Methods

For the studies presented within this work, a mixture of cross-sectional and longitudinal methods was used. This allowed to examine the development of numerical ordering skills within early numeracy, their importance for later mathematical achievement, and individual differences in SFONO tendency and its relation to numerical ordering skills. Study I used an existing longitudinal dataset to investigate children's development in the recognition and use of numerical order in a novel task, as well as its predictive role for mathematical achievement. This study acted as a starting point for designing a longitudinal data gathering investigating the development of numerical ordering skills, and especially for developing tasks measuring spontaneous focusing on numerical order. Therefore, this study was followed by developing and piloting multiple versions of SFONO tasks. After piloting several novel SFONO tasks, three of them were used to investigate individual differences in 3- to 4-year-old children's SFONO tendency, and its relation to numerical ordering skills in the first timepoint of the planned longitudinal data gathering (Study II). Study III developed a more comprehensive measure of SFONO tendency covering more aspects of ordinality, and examined factors that may influence the process of spontaneously focusing on numerical order. Furthermore, it examined whether SFONO tendency could be distinguished from the underlying skills needed to spontaneously focus on numerical order in the SFONO tasks. Study IV included a longitudinal dataset over three timepoints investigating the early development of numerical ordering skills, and the precursors for this development with a mixture of quantitative and qualitative methods. The next section outlines the various methods, samples, and procedures employed across the studies. A summary of methods is provided in Table 1.

Table 1. Overview of the methods in the studies.

	AIMS	**PARTICIPANTS**	**PROCEDURE AND MEASURES**	**ANALYSES**
STUDY I	Examine how focusing on numerical order develops, and its relation to later mathematical achievement	N = 36, 18 girls and 18 boys, $M_{age, T1}$ = 3.0 years	Children were followed longitudinally from 3- to 6-years completing the following measures: • Focusing on numerical order • SFON tendency • Early mathematical skills: Cardinality recognition, number sequence production, subitizing-based enumeration A follow-up was conducted at the age of 12 measuring: • Math achievement (RMAT)	• Descriptive comparison of children's development in focusing on numerical order, cardinality recognition, and number sequence production • Correlations and regression for the relations between preschool numerical skills and math achievement at 12 years
STUDY II	Investigate individual differences in SFONO tendency and its relation to early numerical skills	N = 150, 73 girls and 77 boys, M_{age} = 4.1 years	Children were individually tested on the following measures: • SFONO tendency (Duck Family, Tower Building, and Find the Mother -tasks) • SFON tendency (Bird and Postbox Imitation tasks) • Early mathematical skills: numerical ordering skills, cardinality recognition, number sequence production	• Frequencies of children's SFONO scores, and behavioral manifestations of SFONO • Correlations for the relations between the measures • Hierarchical regression for SFONO tendency's unique relation to numerical ordering skills (controlling for age, and other early mathematical skills)
STUDY III	Explore SFONO with a more comprehensive measure, and delineate it from the requisite skills needed to spontaneously focus on numerical order	N = 51, 22 girls and 29 boys, M_{age} = 5.75 years	Children were individually tested on the following measures: • SFONO tendency (Tower building, Ladybug, Laundry, and Boxes -tasks) • Guided focusing on numerical order • Cardinality recognition • Number sequence production	• Frequencies of children's SFONO scores, and behavioral manifestations of SFONO • Frequencies of children's SFONO scores among a group of children indicating the necessary skills to spontaneously focus on numerical order in the SFONO tasks

				• Comparisons for the mean SFONO sum scores between a) task contexts, and b) trial types (i.e., different numerical sequences)
STUDY IV	Examine differences in the development of numerical ordering skills in relation to cardinality skills and number sequence production	*N* = 150, same participants as in Study II	Same procedure as Study II, continued to a one-year longitudinal study (3 measurement points, 6-month intervals) on the following measures: • Numerical ordering skills • Cardinality recognition • Number sequence production	• Growth mixture modelling analysis on the development of numerical ordering skills to identify different developmental trajectories • Group comparisons for differences in the development in cardinality recognition and number sequence production • Video-analysis on children's numerical behavior during the numerical ordering task

3.1 Research context

Studies I, II, and IV were conducted at the Centre for Research on Learning and Instruction (Department of Teacher Education, University of Turku, Finland). Study I used a pre-existing dataset from a previous research project Pythagoras (PI: Professor Erno Lehtinen). The Pythagoras research project included a longitudinal study following children's mathematical development from the age of 3 years to the age of 6 years. In addition, a follow-up of children's general mathematical achievement at the age of 12 years was conducted.[1] Studies II and IV were conducted within the framework of a research project Focusing On Numerical Order (FONO; PI: professor Minna Hannula-Sormunen). The FONO project examined children's numerical development and the role of spontaneous mathematical focusing tendencies in this development. Both of these research projects were funded by the Research Council of Finland. The data used in Study III were originally collected for a master's thesis at KU Leuven, Belgium. The present doctoral work was funded through the Research Council of Finland's FONO project (June 2021 to August 2024), the Doctoral Programme on Learning, Teaching and Learning Environments (OPPI) doctoral programme in the University of Turku Graduate School (September 2024 to September 2025), and Turun Opiskelijain Tukisäätiö.

3.2 Ethical considerations

Study I was conducted according to the ethical guidelines of University of Turku. All required research permissions were gathered within the Pythagoras research project as part of their longitudinal data gathering. Approval was granted by the Municipality of Turku and participating Early Childhood Education and Care (ECEC) centers at the outset of the study. At each measurement point, permissions were reaffirmed by the respective ECEC centers and the children's groups. Written informed consent was obtained from parents at the beginning of the follow-up and again when the children reached 12 years of age. When the children were 6 years old, parents were asked for permission to be contacted later regarding potential future follow-up assessments. Verbal assent was obtained from the children at each measurement point. The personnel of the ECEC centers were informed that the study was about children's general quantitative development, without further specifications. The participation for the ECEC centers, parents, and children was voluntary, and they were allowed to withdraw their participation at any time. The

[1] Other data from the follow-up have been previously reported in Hannula, 2005; Hannula, Eskola, & Lehtinen, 2007; Hannula & Lehtinen, 2001, 2005; Hannula, Räsänen, & Lehtinen, 2007, and Hannula-Sormunen et al., 2015.

data used in Study I had been previously pseudonymized by replacing children's names with ID numbers in the quantitative data file, ensuring that personal data could no longer be linked to any specific child.

Studies II and IV were conducted as part of the same data collection and adhered to the ethical principles of research with human participants and ethical review in the human sciences in Finland (Finnish National Board on Research Integrity, 2019). The Ethics Committee for Human Sciences of University of Turku reviewed the reseach plan of the FONO research project, and assessed the planned research ethically acceptable. All required research permissions were gathered from participating cities, ECEC centers, and parents of the participating children. All recruited ECEC centers were called and asked about their willingness to participate, and teachers provided a list with all the children fulfilling the inclusion criteria in their group. Parents received a parent letter and a consent form about the study from the ECEC teachers. The parent letter outlined the purpose of the study, the practical requirements for participation by both the child and the parents, details about the research team and obtained ethical approvals, the target group of the study, and the confidentiality measures regarding the collected data.

The purpose of the study was communicated to the ECEC centers and parents as an investigation of children’s thinking and observation skills. The specific focus on numerical development was not disclosed, in order to avoid influencing the children’s behaviour and to allow for a more reliable assessment of their spontaneous focusing on mathematical aspects. This was considered ethical, as nothing in the study was outside children’s everyday activities, and the procedure was approved by the ethics committee. Written parental consent was a prerequisite for children’s participation. Participation was voluntary for all ECEC centers, parents, and children, and they were allowed to withdraw their participation at any time. All collected data (except for the original video recordings of the children’s testing) was pseudonymized, by replacing children’s names with ID-numbers before analysis. In addition, information about children’s ECEC centers was replaced with numbers from which the specific ECEC center could not be identified. Only researchers of the FONO research project have the access to the original material that has personal data on individual children. All original data were securely stored either in a locked cabinet within a locked room and on protected servers compliant with EU data protection regulations (GDPR). These data will be stored for 10 years after the last publication.

Data for Study III was originally collected for a master’s thesis project at KU Leuven, Belgium. The study got an ethical permission from the Sociaal-Maatschappelijke Ethische Commissie (SMEC) of KU Leuven (G-2022-6019-R2(MIN)). The required research permissions were collected from the participating schools, and parents of the participating children. The schools were approached with an information letter and a written consent form about the study. After the schools had given a written consent, the parents were approached with an information letter and a consent form, which were distributed by the teacher, inviting their children to participate in the study. The information letters explained the focus of the study, how it is conducted (i.e., what type of tasks are presented, the length of the testing sessions), what type of data will be collected and the confidentiality, voluntarity of the participation and a right to withdrawal at any time, and the obtained ethical approval. The mathematical nature of the study was explained in the information letter, and the teachers and parents were told that the research was about children's attention for numerical order. However, these letters explicitly explained that for the spontaneous nature of the study, it was important that the children were not told that they were participating in a study about counting, numbers, numerical order, etc. Written parental consent was a prerequisite for children's participation. All collected data (except for the original video recordings of the children's testing) was pseudonymized, by replacing children's names with ID-numbers before analysis. The information letter provided to schools and parents stated that the collected data may be used for scientific research and may be published. It also emphasized that the anonymity and confidentiality of the data will be strictly maintained throughout the entire research process. The collected data is stored securely, and processed in accordance with the GDPR.

Due to the protocols of both of the research projects, the data used in the studies was not opened to the research community. However, the data that support the findings of Studies II, III and IV were made available on request from the corresponding author to review the analyses. In general, the dissertation abided by the guidelines of Finnish National Board on Research Integrity TENK, and the University of Turku policies.

3.3 Participants

Participants in Studies I, II, and IV were 3-6-year-old children attending municipal ECEC in Finland. In addition, at the last measurement point of Study I, the children were 12-year-old children attending their last year of primary school. In Study III, the participants were 5-6-year-old children attending their final year of kindergarten in Flanders, Belgium. A detailed overview of participants is provided in Table 1. In Finland, 90 percent of 3-5-year-old children attend voluntary ECEC (Statistics Finland, 2024), that aims to promote children's holistic growth, development, and learning. In the August of the year when children turn 6 years, they attend government funded pre-primary education for one year, after which primary school starts in the year children turn 7 years. In Belgium, school attendance becomes compulsory in September of the year the child turns 5 years, and education is provided free of charge.

Study I was based on a longitudinal dataset collected in 1999 - 2008, which included 36 children (18 girls and 18 boys) from seven ECEC centers at the beginning of the study. All children were Finnish-speaking and had no diagnosed needs for special support. At the beginning of the study, the children were between the ages of 2 years 10 months and 3 years 2 months (M_{age} = 3.0 years). The children were followed longitudinally, with assessments of early mathematical skills conducted in one-year intervals between ages 3 and 6 years. Their mathematical achievement was evaluated again when they were in their final year of primary school at the age of 12 years. On average, the educational level of the parents was representative of the corresponding Finnish men and women, and is reported in the dissertation of Hannula (2005).

Participants in Studies II and IV were 150 children (77 boys, 73 girls) at 13 daycare centers in southwest Finland. At the start of the study the children were from 3 years 5 months to 4 years 9 months old (M_{age} = 4 years 1 month, SD = 4 months). All children had Finnish as their first language and they did not have any diagnosed needs for special support. All children completed SFONO and SFON tasks, and measures of numerical ordering skills, cardinality recognition, and number sequence production. For Study II, data from the first measurement point was used, consisting of a cross-sectional sample of 3- to 4-year-old children. For Study IV, longitudinal data from the same children over the first three time points was used. For this longitudinal study, 4 children (2.7 % of the sample) relocated during the data gathering. On average, the educational level of the parents exceeded the educational level of the corresponding Finnish men and women (Statistics Finland, n.d.). The reported educational levels of parents varied from 0.8% of mothers and 3.8% of fathers not having completed upper secondary education, 16.7% of mothers and 29.0% of fathers having completed upper secondary education, and 82.6% of mothers and 66.9% of fathers having completed tertiary education.

Study III was based on data collected for a Master's Thesis, including 51 children (22 girls, 29 boys) at two schools in the region of Flanders in Belgium. The children were between the ages of 5 years 3 months and 6 years 10 months (M_{age} = 5 years 9 months, SD = 4 months), and did not have any diagnosed needs for special support. The children first completed SFONO tasks, followed by measures of guided focusing on numerical order, cardinality recognition, and number sequence production.

3.4 Measures

All the measures of the studies are presented in Table 2. Detailed descriptions of the measures can be found in Studies I–IV. Studies II and IV were part of the same data gathering. In all studies, the children were assessed individually in a quiet room in their ECEC centers, and the sessions were video-recorded.

Table 2. The ages (in years) at which the measures included in the four studies were used.

Measure	Study I	Study II	Study III	Study IV
Focusing on numerical order	3, 4, 5, 6	-	-	-
SFONO tendency	-	3.5-4.5	5-6	-
Guided focusing on numerical order	-	-	5-6	-
SFON tendency	6	3.5-4.5	-	-
Numerical ordering skills	-	3.5-4.5	-	4, 4.5, 5
Cardinality recognition	3, 4, 5, 6	3.5-4.5	5-6	4, 4.5, 5
Number sequence production	3, 4, 5, 6	3.5-4.5	5-6	4, 4.5, 5
Subitizing-based enumeration	5	-	-	-
Mathematical achievement	12	-	-	-

Note. In studies II and IV, the children's age range was 3.5–4.5 years at the beginning of the study. For clarity, the mean age of participants is presented in Study IV.

3.4.1 Assessments of SFONO

As one aim of the current dissertation was to distinguish a separate attentional process of SFONO, novel measures were created. The development of the SFONO tasks was highly informed by previous development of tasks assessing SFON (Hannula, 2005; Hannula & Lehtinen, 2005), other spontaneous mathematical focusing tasks (Wijns et al., 2020), and the previously developed Focusing On Numerical Order (FONO) task that was published in Study I. SFONO tasks are intended to measure a general tendency to spontaneously (i.e., in a self-initiated way without adult-guidance) focus on numerical order in situations that are not explicitly mathematical, such as everyday situations. Thus, it is of high importance that the children's attention is not guided towards mathematical aspects before or during the tasks (e.g., Hannula, 2005; Hannula & Lehtinen, 2005; Hannula-Sormunen, 2015). This also includes, that the tester should not provide any hints towards the mathematical aspects in the tasks nor feedback on their actions. In addition, the children should not be aware that the study is about mathematics. Thus, if the children notice and use the mathematical aspects in the SFONO tasks, it can be said that it was done spontaneously. This has to be considered also in planning the order of the tasks in the sessions. As children's attention should not be guided towards mathematics, the spontaneous focusing tasks should be presented first, before any mathematical measures (e.g., Hannula, 2005; Hannula & Lehtinen, 2005; Hannula-Sormunen, 2015).

There are also important criteria to be considered in designing the SFONO tasks, to ensure capturing spontaneous focusing on the mathematical aspect of interest, in this case numerical order (e.g., Hannula-Sormunen, 2015; McMullen, 2019). First, the task instruction should be open for multiple interpretations, and the tasks should include several (mathematical and non-mathematical) aspects to focus on. This means, that there should be several options for actions for children in the tasks. Second, the tasks should not be associated with typical mathematical activities (e.g., counting money). Third, the tasks should be within children's numerical (i.e., include only such small numbers that are possible for all children to enumerate and order), cognitive, and motorical competencies, as well as sustain the attention and situational interest of the children throughout the task. This approach aims to minimize the likelihood that the individual differences in SFONO reflect variations in children's skills needed in the tasks, attention, or interest towards the task. Third, to get a reliable indicator of children's general tendency to spontaneously focus on numerical order, multiple tasks with different task contexts should be used. This is important, as for some children the mathematical aspects may be more salient in different task contexts compared to others (Chan & Mazzocco, 2017; Mazzocco et al., 2020). Using various task contexts also mirrors everyday situations, where the contexts in which mathematical aspects are found and used can vary.

Focusing on numerical order. Study I included a Caterpillar Ordering task (see Fig. 2), which was the first attempt to measure SFONO. The task's objective was to measure children's recognition and use of numerical order in a novel play-like situation. The task material included boxes of caterpillars with a unique number of legs (i.e., 7 boxes with caterpillars from 1 to 7 legs), and a corresponding number of socks, one for each leg. Children were presented with the boxes and caterpillars one by one. After this, the caterpillars were collected away from the boxes, which were arranged in numerical order based on the number of socks (from 1 to 7) in front of the child by the researcher. The child was then asked to remember where each box was. Importantly, the child was not explicitly told that the boxes were in numerical order. Instead, this was left for the child to notice. After a few seconds, the boxes were closed so that the socks were not visible to the child anymore, and the child was asked to help the caterpillars to find their own boxes. The idea in the task was, that if the child had (spontaneously) focused on the numerical order of the socks, it would be easier to remember the order of boxes and thus place the caterpillars in numerical order according to the number of their legs. In addition, there were that many boxes (i.e., 5 or 7), that it would be difficult to remember each boxes' place individually without noticing the numerical order aspect. Thus, if the child arranged

the caterpillars in numerical order based on the number of legs, they were regarded to have focused on the numerical order aspect in the task.
However, the Caterpillar Ordering task had a few limitations regarding the previously discussed guidelines of measuring spontaneous focusing. First, the task had some numerical hints towards the number of the socks and legs, as in presenting the boxes individually to the child, the researcher pointed out that the box had *as many socks as the caterpillar had legs*, which guides the child's focus towards the number of legs and socks. Second, the score of the task was based only on the child's performance in the task, as the score corresponded to the highest number of caterpillars in the correct order. However, it is possible that the child could have focused on numerical order but instead decided to do something else, or made a mistake along the way in ordering the caterpillars. As spontaneous focusing measures should not measure the child's level of mathematical skills needed in the task, but instead whether the child focuses on the mathematical aspect of interest, the scoring was a limitation. Thus, it was decided that the task was not fully indicative of SFONO, but rather of children's *focusing on numerical order*.

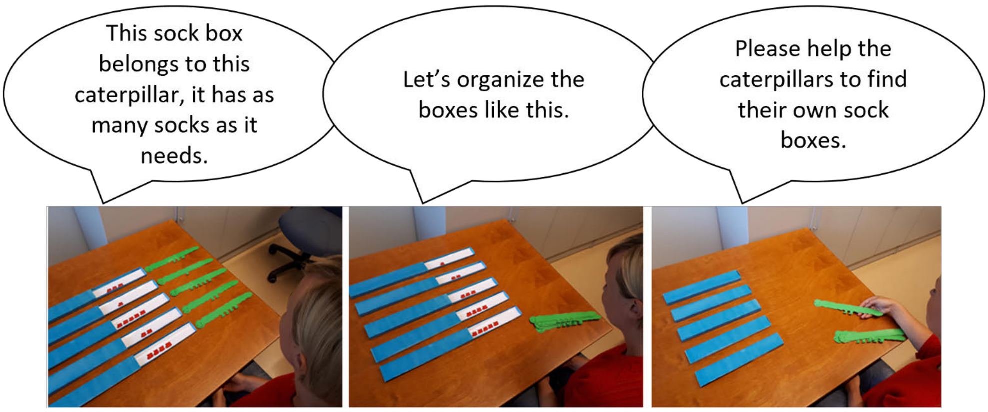

Figure 2. The Caterpillar Ordering task measuring children's focusing on numerical order. Modified from Harju, H., Lehtinen, E., & Hannula-Sormunen, M. (2022). Focusing on numerical order in preschool predicts mathematical achievement six years later. In C. Fernández, S. Llinares, A. Gutiérrez, & N. Planas (Eds.), *Proceedings of the 45th conference of the International Group for the Psychology of Mathematics Education* (*Vol. 2*, pp. 347–354).

Spontaneous focusing on numerical order. To capture children's SFONO tendency, three novel SFONO tasks were developed and used in Study II. In these tasks, the prior limitations were addressed, and multiple SFONO tasks were designed to capture an indicator of children's general tendency to spontaneously focus on numerical order in different situations. As the Caterpillar Ordering task had potential to measure SFONO, it was used as the starting point to develop the SFONO tasks.

First, it was decided that all tasks would include a possibility to numerically order sets of items. Additionally, we decided that there should be at least three sets of items to order numerically, so that it would be possible to distinguish comparison and ordering, as ordering should include the repeated use of numerical relations (more/less, before/after) (Fuson, 1988, p. 14). In addition, including the minimum of three sets of items to order reduced the probability of producing a numerical order in the task by luck. The final set of SFONO tasks consisted of two tasks where one option in the task was to arrange sets of items in numerical order, namely 1) to help duck families with different number of ducklings to swim on a river (Duck Family task, Fig. 3a), and 2) to build a tower in numerical order based on numbers of stickers in the blocks (Tower Building task, Fig. 3b) (modified from the Construction task of Wijns et al., 2020), and from one task where one option in the task was to reproduce a numerical order that was visible at the start of the task (similarly to the Caterpillar Ordering task), namely to help puppies to find their own mothers (Find the Mother task, Fig. 3c). These tasks were designed so, that they also included other possible mathematical and non-mathematical aspects to focus on, and did not include any numerical hints in the instructions. To capture all possible indications of SFONO, the tasks were video-recorded.

Figure 3. The initial setup of a) the Duck Family task, b) the Tower Building task, and c) the Find the Mother task. Modified from Harju, H., Van Hoof, J., Nanu, C. E., McMullen, J., & Hannula-Sormunen, M. (2024). Spontaneous focusing on numerical order and numerical skills of 3- to 4-year-old children. *Educational Studies in Mathematics*, *117*(1), 43–65.

Results from Study II showed that the SFONO tasks differed in the number of SFONO indications they elicited. It was hypothesized that these differences may reflect varying salience of numerical order across the tasks. In the Find the Mother task, numerical order was likely more salient because in order to spontaneously focus on numerical order, children needed to reproduce the numerical order presented at the beginning of the task. In contrast, in the Tower Building task and the Duck Family task, the children needed to notice themselves that the sets can be arranged in numerical order. In addition, the Tower Building task included blocks with fixed sets of stickers, while the Duck Family task, allowed children to rearrange the families (even though the task instruction stated that the families should stay together) and included more options for actions compared to the Tower Building task, which may have made numerical order less salient. Therefore, Study III introduced tasks that were more consistent in their answer formats and task features, with the aim of creating a set of tasks in which numerical order was consistently salient. It included the Tower Building task from Study II and developed three new tasks modelled on its structure. Thus, all the tasks included four elements (e.g., blocks, ladybugs, t-shirts, boxes) in two different colours, each including fixed sets of items (e.g., 1-4 stickers). In all tasks, children needed to notice themselves that the sets can be arranged in numerical order. To investigate the effect of task context to SFONO, the task context varied across the tasks, including building a tower using blocks, helping ladybugs, hanging laundry, and building a tower from boxes (for pictures of the task material and the initial setup, see Fig. 4). Otherwise, similar guidelines to Study II in the development of the SFONO measures were followed.

Figure 4. The task material and the initial setup of a) the Tower Building task, b) the Ladybug task, c) the Laundry task, and d) the Boxes task.

In Studies II and III, children's SFONO was scored based on children's actions and utterances in the task situations. The children were scored as spontaneously focusing on numerical order, if they arranged sets of items in numerical order (e.g., built a tower in ascending 1-2-3-4, or descending 4-3-2-1 numerical order). Thus, in order to arrange the sets of items in numerical order in the tasks, the children must have taken into account that the task included numerically different sets, to notice the order relations between the sets, and to finally produce a numerical order. In addition, if the children showed any behavioral indications of SFONO, such as a) verbal and/or behavioral indications of numerical order (e.g., showing to the researcher that there are "*One, two, and three*" while pointing to the sets in numerical order), b) referring to the number sequence (e.g., "*Two comes after three.*"), or c) interpreting the task's goal as numerical ordering (e.g., *"These should be in the same order as numbers."*), they were also regarded to spontaneously focus on numerical order, even if they did not arrange the sets of items in numerical order in the task, or made a mistake in numerical ordering. The measure of children's SFONO tendency was a sum score of SFONO scores from the tasks. The sum score of SFONO was regarded as an indicator of the amount of children's self-initiated practice of numerical order in their everyday environments (Hannula et al., 2005; Hannula-Sormunen, 2015).

In Study II, the three SFONO tasks had three trials each. The first and the last trial included sets of items with consecutive numbers starting from 1 (e.g., 1, 2, 3, and 4 stickers in the blocks), and the second trial included sets of items with consecutive numbers starting from 2 (e.g., 2, 3, 4, and 5 stickers in the blocks). However, the first trial of the Duck Family task showed weak inter-item correlations with the other SFONO items ($r < .19$). Notably, an unusually large number of children ordered the duck families in descending numerical order (i.e., 3-2-1), which may have been influenced by the closest proximity of the family with three ducklings to the child. Thus, the first item of the Duck family task was not further used, leading to a SFONO sum score of 8 with a good internal consistency (*Cronbach's alpha* = .75).

In Study III, there were four SFONO tasks with four trials each. In all tasks, similar trials as in Study II were used (i.e., consecutive numbers starting with 1, and consecutive numbers starting with 2) to compare the frequencies of SFONO responses between the two samples. In addition, as ordinality studies often include also numerical sequences beyond small consecutive numbers (e.g., Hutchison et al., 2022), Study III expanded the measurement of SFONO by including sets of items with small non-consecutive numbers (e.g., 1, 2, 4, and 5 stickers in the blocks), and larger numbers that were all beyond the subitizing range (e.g., 8, 9, 11, and 12 stickers in the blocks) in the task trials (see Figure 4 for examples of the different types of trials in the task material). This way, it was possible to investigate whether SFONO occurs mainly in small consecutive numbers (possibly due to high familiarity, see Devlin et al., 2024), or whether children exhibit SFONO in other types of numerical sequences as well. Thus, all four tasks had four trials, one with each numerical sequence (i.e., trial type), resulting in a maximum SFONO score of 16. Notably, using this many tasks and trials is uncommon in studies of spontaneous mathematical focusing tendencies. The SFONO measure indicated a good internal consistency, with Cronbach's alpha of .81.

3.4.2 Guided focusing on numerical order

A central consideration in developing tasks to measure spontaneous mathematical focusing tendencies is distinguishing these tendencies from the cognitive and mathematical skills required to complete the tasks (e.g., Hannula & Lehtinen, 2005). In other words, if children do not show any signs of spontaneously focusing on the mathematical aspect in the tasks, this should reflect a lack of spontaneous recognition and use of that aspect — not a lack of the necessary cognitive or mathematical skills. Although some considerations in task design (e.g., using small enough numbers for all children to enumerate, maintaining engagement, etc.) aim to keep the tasks within children's capabilities, they primarily serve to minimize the likelihood that performance differences are due to general competency limitations rather than differences in spontaneous mathematical focusing.

One method for distinguishing spontaneous mathematical focusing tendencies from the requisite skills in the previous studies on SFON and SFOR has been to directly measure children's abilities to solve the spontaneous mathematical focusing tasks by using the mathematical aspect of interest when they are explicitly guided to do so (Hannula & Lehtinen, 2005; McMullen et al., 2014). Study III aimed to distinguish whether the individual differences in SFONO tendency were due to a lack of spontaneous focusing on numerical order in the SFONO tasks, or due to differences in the requisite skills needed to solve the tasks. Therefore, all children completed guided versions of each trial in the Tower Building task (administered after the SFONO tasks), in which they were explicitly shown that the blocks contained different numbers of stickers and were instructed to build the tower in numerical order, from the fewest to the most stickers. This allowed to identify the group of children who had the skills to produce a numerical order in the SFONO tasks, and investigate whether there were differences in the SFONO scores within this group of children. A similar approach was previously described in Hannula & Lehtinen (2005), where all children who got a score of 0 from the SFON tendency measure (i.e., did not show indications of SFON in any of the tasks), completed guided measures of the SFON tasks to ensure it was the lack of SFON and not the requisite skills needed in the tasks to spontaneously focus on numerosity.

3.4.3 Measures of SFON tendency and mathematical skills

Children's *SFON tendency* was assessed in Studies I and II with measures previously reported in Hannula & Lehtinen (2005). In Study I, children's SFON tendency was indicated by a sum score of three SFON tasks (Imitation, Finding, and Model tasks), by two SFON tasks (Imitation tasks) in Study II. These tasks assessed how frequently the children noticed and used exact number of items spontaneously. In the Imitation task, the researcher asked the children to watch carefully as a toy animal was fed treats or as letters were inserted into a postbox, and then to repeat what the researcher did. If the children spontaneously focused on numerosity, they either fed the same number of treats to the toy animal (or posted the same number of letters), or showed other signs of noticing the numerosity aspect in the task, such as showed signs of counting or said *"I know you fed three."*. In the *Model* task, children observed the experimenter constructing a model—for example, stamping a specific number of spikes on a dinosaur's back—and were then asked to replicate the model (while the original model was not visible) (Hannula & Lehtinen, 2005). In the *Finding* task, children watched as the experimenter hid a treasure under one of a series of 27 hats placed in a semicircle, and were then asked to indicate where the treasure had been hidden (Hannula & Lehtinen, 2005). Each task included three trials with small numerosities. The children were assigned a SFON score of 1 from each trial if they produced the correct numerosity in the trial, or showed any quantifying acts such as a) verbal expressions involving number words (e.g., "I'll give him two berries"), b) using fingers to indicate quantities, c) performing counting-like actions (such as whispering number words or pointing), d) saying comments that refer to quantities or counting (e.g., "Oh, I miscounted them"), or e) interpreting the goal of the task in numerical terms (e.g., "I gave exactly the right number") (Hannula & Lehtinen, 2005). Each task included three trials with small numerosities, leading to a SFON score of 9 in Study I, and SFON score of 6 in Study II.

Children's *numerical ordering skills* were assessed in Studies II and IV with a modified version of a previously used "Ordering task" (Spaepen et al., 2018). In this task, the child was presented with cards each with a unique number of dots, and asked to put them in order from least to most dots. In total, the task included four trials with increasing difficulty as the number of cards and dots in them increased. The trials in the task were ordering 1-3, 1-5, 1-7, and 2-5 dots. As numerical ordering may require also understanding of mathematical language such as the words "least" and "most" (Chan et al., 2022; Hornburg et al., 2018), several things were done to ensure that the individual differences found in the task were not due to individual differences in understanding the task instruction. First, the "least to most" - instruction was further explained by the researcher elaborating that *"The card with the least dots goes here, then the next card that has a little bit more dots, then the next card with a little bit more dots… And here goes the card that has the most dots."*, while pointing to the corresponding places on the table and using the tone of the voice to be indicative of the increasing number of dots. Second, if the child did not succeed to start with the correct first card (e.g., if the child put the card of 2 dots first instead of the card with 1 dot in the trial of 1-3), the trial was repeated. In this second attempt, the researcher showed which card is the first one, and put it in the correct place. Then, the child was asked to put the rest of the cards in order from least to most dots. This was done, as it was hypothesized that especially the card with one dot would be a strong hint towards the numerical order of the cards. Third, one inclusion criteria to participate in the study was to have Finnish as first language (which was also important as the SFONO measures were used for the first time in a larger group of children). The child's better performance of the (possible) two attempts in the trial was scored, and no minus points were given for the researcher's help[2]. The task was discontinued if the child was unable to correctly order cards displaying 1–3 or 1–5 dots, or if the only correct response in these two trials occurred during the first trial with assistance from the experimenter. The score of in each trial was based on the highest number up to which the children could arrange the cards in numerical order. For example, if the produced order in the trial of 1-5 was 1-2-3-5-4, a score of three points was given from the trial. However, if all the cards were arranged correctly, the score in the task was the highest number minus one, as to correctly order all the cards necessitates that all the cards up to the second last card are in the correct places (Spaepen et al., 2018). The maximum scores in the trials were 2, 4, 6, and 3 respectively, leading to a maximum sum score of 15 in the task.

A collection of children's early mathematical skills was assessed in all three studies. Studies I, II, and III all included measures of children's *cardinality recognition* and *number sequence production*. In Study I, *cardinality recognition* was measured by asking the children to bring a toy caterpillar as many socks as it needed (ages 3 and 4), or to count out loud how many objects there were on the table (ages 5 and 6) (see, Hannula & Lehtinen, 2005). In Studies II and III, children were administered a Give-a-Number (Wynn, 1990) task, where the children were asked to give a certain number of items (e.g., three apples) from a collection of items. Importantly, in all these tasks, if the children failed to produce a correct answer in a trial, a second attempt was given. The task was discontinued after two failed attempts in the same number. A score of 1 was given from a correctly produced number of items in either of the attempts. In Studies I, II, and III, *number sequence production* was assessed by asking children to count as far as they could. The children's counting was stopped at 50, and better out of two attempts of counting was scored. The score was based on the highest number up to which the children correctly produced the number sequence. For example, if a child correctly counted from 1 to 20, and continued 22, 23, 25… in the better attempt, a score of 20 was given from the task.

Additionally, Study I included an assessment of children's *subitizing-based enumeration* (see Hannula et al., 2007), in which children were quickly shown first a collection of objects (for 120 ms), followed by four groups of dwarves, and asked to identify which of the groups had stolen the collection of objects that had been shown previously. If the children had correctly enumerated the number of objects in

[2] See exception in Study IV, where only the child's first response in the trial of 1-3 was scored.

the collection by subitizing, they could choose the group that had the same number of dwarves as there were stolen objects. The children were carefully guided to understand in a trial-period that the number of objects would reveal the correct group of dwarves. The upper limit of each child's subitizing-based enumeration range was defined as the highest number for which they correctly matched sets of items and dwarves in at least three out of four trials (Hannula et al., 2007).

In addition to early mathematical skills, Study I included a follow-up measure of children's *mathematical achievement* at the age of 12. The children were administered the RMAT-test, which is a Finnish 10-minute time-limited standardized test for children aged 9 to 12 years (Räsänen, 2004). The test included 56 tasks, ranging from single-digit arithmetic to basic algebra. The majority of tasks (80%) focused on calculations involving the four basic arithmetic operations, requiring rapid retrieval of arithmetic facts and procedures. The remaining tasks evaluated curriculum-based content, such as converting measurement units and performing basic operations with decimals and fractions. The score was determined by the total number of correct responses. (Hannula-Sormunen et al., 2015).

3.4.4 Statistical analyses

The statistical analyses used in the different studies are presented in Table 3. The studies consisted of quantitative data that was analyzed using IBM SPSS Statistics (version 27 or higher). The Latent variable modelling used in Study IV was conducted using Mplus version 8.9 (Múthen & Múthen, 1998-2023).

Table 3. Overview of the statistical analyses used in the studies.

Statistical test	**Study**	**Purpose**
Cronbach's alpha	II, III, IV	Determine the internal consistency of the measures: SFONO tendency (Studies II and III), SFON tendency (Study II), numerical ordering skills (Studies II and IV), cardinality recognition (Studies II, III, and IV) Investigate the internal consistency of SFONO within different task contexts and numerical sequences (Study III)
Inter-item correlations	III	Investigate the internal consistency of SFONO items within different task contexts and numerical sequences
Pearson correlation	I, II	Examine the relation between SFONO and SFON tendencies and early mathematical skills (Studies I, II), and their relation to mathematical achievement at 12 years (Study I)
Regression analysis	I, II	Examine the predictive relation of focusing on numerical order and SFON tendency to mathematical achievement after controlling for subitizing-based enumeration (Study I). Examine whether SFONO tendency explains unique variance in numerical ordering performance after controlling for age, cardinality skills, number sequence production, and SFON tendency (Study II).
Repeated measures ANOVA	III	Compare differences in the mean SFONO scores between different task contexts and numerical sequences
Growth Mixture Modeling	IV	Examine different developmental trajectories in numerical ordering skills, and classification by the development in numerical ordering skills (by Trajectory group).
Latent Growth Curve Modeling	IV	Examine the development in cardinality recognition and number sequence production.
BHC procedure (chi-square distributed Wald tests)	IV	Compare group differences in initial level and growth in cardinality recognition and number sequence production between Low, Gentle, Steep, and High Trajectory groups of numerical ordering development.
Cohen's d	III, IV	Determine effect sizes for SFONO sum score (Study III) and group (Study IV) differences

4 Overview of Studies

4.1 Study I

Harju, H., Lehtinen, E., & Hannula-Sormunen, M. (2022). Focusing on numerical order in preschool predicts mathematical achievement six years later. In C. Fernández, S. Llinares, A. Gutiérrez, & N. Planas (Eds.), *Proceedings of the 45th conference of the International Group for the Psychology of Mathematics Education* (vol. 2, pp. 347–354).

This article reports results from a former longitudinal study following children's numerical development over a 9-year period, from the age of 3 years to the age of 12 years. The study aimed to investigate children's development in focusing on numerical order, which included noticing numerical order in a play-based task that was not explicitly mathematical in nature, and using numerical ordering skills in this task situation. In addition, Study I explored the predictive role of focusing on numerical order to later mathematical skills.

The participants were 36 Finnish-speaking children (18 girls and 18 boys, $M_{age, T1}$ = 3.0 years, *SD* = 1.5 months) with no developmental delays. The study took place in ECEC centers. Children were tested for their focusing on numerical order, cardinality recognition, and number sequence production at the ages of 3, 4, 5, and 6 years. In addition, subitizing-based enumeration was assessed at the age of 5, and SFON tendency at the age of 6. At 12 years of age, children's curriculum-based math achievement was measured. Focusing on numerical order was assessed individually with a video-recorded task, where the children were first presented with boxes that contained caterpillars with a unique number of legs and matching number of socks (e.g., a two-legged caterpillar with two socks). Then, the experimenter re-organized the boxes in numerical order based on the number of socks, while the caterpillars were away from the boxes. After that, the children needed to help the caterpillars to find their own sock boxes. However, the numerical order of the socks was not pointed out for the children, but was rather left for them to notice. Thus, to succeed in the task, the children were required to notice the numerical order of the socks, and to use their numerical ordering skills to arrange the caterpillars in corresponding numerical order based on the number of their legs.

The results showed that there were individual differences between children in focusing on numerical order within each time point. In addition, focusing on numerical order developed greatly: many children scored low in the task at the age of 3 years but a majority of them showed mastery in the task at the age of 6 years. An investigation into the co-development of children's focusing on numerical order, cardinality recognition, and number sequence production between ages 3 and 6 years revealed that their highest skills were in number sequence production, followed by cardinality recognition, and finally numerical ordering. Next, a multiple linear regression was conducted to investigate whether preschool numerical skills, especially focusing on numerical order, predicted mathematical achievement at the age of 12 years. The results showed, that only focusing on numerical order at 5 years ($\beta = .36, p < .05$) and SFON tendency at 6 years ($\beta = .40, p < .05$) were significant predictors of mathematical achievement at 12 years, when children's number sequence production ($\beta = .14$) and subitizing-based enumeration at the age of 5 ($\beta = -.27$) were also included in the regression. The model explained 44 % of the variance in mathematical achievement ($R^2 = .44, p < .001$).

Together, the findings of this study indicated that children had individual differences in 1) noticing numerical order in a novel task, and 2) their numerical ordering skills. Furthermore, these individual differences explained unique variance in children's mathematical achievement at the end of primary school, suggesting that the recognition of numerical order and better numerical ordering skills may be important aspects to support mathematical development. In this study, the instruction of focusing on numerical order -task included some hints towards the numerical nature of the task (e.g., towards focusing on the number of socks and legs of the caterpillars), and thus it did not fully capture children's *spontaneous* focusing on numerical order. The results were limited to only one task context and were not indicative of children's more general *tendency* to focus on numerical order in varying situations. In addition, as the task was scored based on the children's numerical ordering performance, the measure may have been more indicative of their numerical ordering skills. Thus, three new measures of spontaneous focusing on numerical order were developed, and administered on a larger sample of 150 children (Study II). Since children's performance in numerical ordering predicted later mathematics achievement, the developement of formal numerical ordering skills was examined in a longitudinal setting (Study III).

4.2 Study II

Harju, H., Van Hoof, J., Nanu, C. E., McMullen, J., & Hannula-Sormunen, M. (2024). Spontaneous focusing on numerical order and numerical skills of 3- to 4-year-old children. *Educational Studies in Mathematics*, *117*(1), 43–65. https://doi.org/10.1007/s10649-024-10327-3

In this article, we report the first timepoint from a longitudinal research project following 150 children's numerical development. The aim of this study was to identify individual differences in SFONO tendency in 3-4 -year-old children, and to determine whether SFONO tendency was related to children's early mathematical skills, especially numerical ordering skills. The tasks used in this study were the first to specifically measure SFONO in different situations.

Participants were 150 Finnish-speaking children (*M* = 4 years 1 month, *SD* = 4 months) from 13 ECEC centres. The children were assessed individually in video-recorded sessions that included measures of SFONO tendency, SFON tendency, numerical ordering skills, cardinality recognition, and number sequence production. SFONO tendency was assessed with three non-explicitly mathematical tasks with open-ended instructions, where numerical ordering was one possible aspect to focus on. The tasks included two production tasks, where one possible answer was to arrange sets of items in numerical order. The third task (a modification from the task used in Study I) was a reproduction task, where children had to notice themselves that sets of items were arranged in numerical order to reproduce it later in the task. The children were not given any indications of the mathematical nature of the tasks before or during their participation. The children were regarded to have spontaneously focused on numerical order in the tasks, if they arranged sets of items in numerical order, or showed any other verbal or behavioral signs of noticing the numerical order aspect in the tasks. SFONO tendency was a sum score of the tasks, indicating children's general tendency to spontaneously focus on numerical order across various situations.

The results revealed individual differences in children's SFONO tendency. Overall, 67 % of the children were found to have spontaneously focused on numerical order at least in one trial (out of eight trials) in the SFONO tasks. Importantly, it was observed that not all children arranged the sets of items in numerical order in the SFONO tasks, but showed other indications that revealed their SFONO, such as pointing towards the sets and pointing out the numerical order aspect for the examiner (36 % in the Duck Family task, 26 % in the Tower Building task, and 11 % in the Find the Mother task). To examine the association between SFONO tendency and numerical skills, analyses were conducted on a subsample of children (*n* = 103) who were considered to have sufficient numerical ordering skills to plausibly focus on numerical order spontaneously in the SFONO tasks. Children were included in this subsample if they demonstrated the ability to correctly arrange sets of 1–3 dots in the numerical ordering task, indicating adequate numerical ordering competence. The results showed significant associations between SFONO tendency and numerical skills, with partial correlations controlling for age varying from .39 to .42. Furthermore, SFONO tendency explained 2 % unique variance in children's numerical ordering skills after age, number sequence production, cardinality recognition, and SFON tendency were controlled for in the analyses. The model overall explained 48 % of the variance in numerical ordering skills ($R^2 = .48$, $F(5, 97) = 17.73$, $p < .001$).

This study suggested that the newly developed SFONO tasks were able to capture individual differences in children's SFONO tendency. As many children were observed using counting or cardinal number words in the SFONO tasks, it was plausible that the children were using exact numbers to produce a numerical order in the tasks, instead of approximation of quantities (least, more, most), thus providing further validation for the novel SFONO tasks. Importantly, individual differences in SFONO tendency were observed even among a group of children who had the skills to numerically order sets from one to three items, indicating that not all children who had the skills to numerically order the sets of items in the SFONO tasks, did so. Together with the significant associations between SFONO and early numerical skills, especially numerical ordering skills, the results suggest that SFONO tendency is a relevant factor in early numeracy.

4.3 Study III

Harju, H., Van Belle, L., Van Dooren, W., McMullen, J., & Van Hoof, J. (Under review). "I need to follow the numbers" - developing and validating a more comprehensive measure of Spontaneous Focusing On Numerical Order.

Based on Study II, it was still unclear whether the individual differences in SFONO tendency were due to children's requisite skills needed to solve the SFONO tasks, or varying task characteristics (i.e., differing answer formats in the production and reproduction tasks). In addition, the previous SFONO measure included only numerical sequences with small, consecutive numbers, which may be more salient in terms of numerical order. Thus, it was not known whether SFONO tendency occurs also in numerical sequences with gaps or larger numbers. Study III aimed to answer these gaps by investigating SFONO tendency with an improved measure in a different sample of children. More specifically, this cross-sectional study aimed to determine whether the individual differences in SFONO tendency were due to differences in the ability to numerically order sets or in SFONO tendency. In addition, the effect of task context and numerical sequence to SFONO tendency was examined.

The participants were 5- to 6-year-old children ($N = 51$, $M_{age} = 5.75$ years) from the region of Flanders, Belgium. The improved SFONO measure included four tasks, the Tower Building task used in Study II, and three novel tasks with a similar answer format and salience of the numerical aspects (see Fig. 4), with four trials each. The four trials in each task included the same trial types (i.e., numerical sequences) used in Study II (e.g., sets with 1-2-3-4, and 2-3-4-5 items), as well as two novel numerical sequences (e.g., sets with 1-2-4-5, and 8-9-11-12 items). Concurrently, all children completed measurements of Guided Focusing On Numerical Order (GFONO), where the children were presented with the four trials of the Tower Building task again, but this time they were explicitly guided to focus on the aspect of numerical order by asking them to build the tower in numerical order based on the stickers on the blocks. In addition, the children completed measures of cardinality recognition and number sequence production.

The results revealed that there were substantial individual differences in children's SFONO tendency in the improved SFONO measure covering a broader range of numerical sequences. Importantly, these differences could not be entirely explained by children's requisite skills needed in the SFONO tasks, as there were individual differences in SFONO tendency even among those children who successfully completed the GFONO tasks, had the skills to recognize all the cardinal values and recite the number sequence above the highest number included in the trials (see Figure 5 for the frequencies of SFONO scores). For example, while 43 children (84 % of the sample) successfully built the tower in numerical order in the GFONO task, recognized cardinal values to a minimum of 5, and produced the number sequence to 5 or above in the trials of 1-2-3-4 and 2-3-4-5 (i.e., the same as in Study II), still 21 % of these children did not spontaneously focus on numerical order in any of these trials across the four tasks. Therefore, this study was the first to show that that the individual differences found in the SFONO measure were not solely due to differences in the numerical skills needed to use numerical order in the tasks, but due to differences in SFONO tendency. Furthermore, children's SFONO responses appeared more affected by the numerical sequences used in the task than by the task context. While the SFONO scores did not differ significantly across tasks, they differed significantly across the different numerical sequences. In particular, trials with small consecutive numbers, especially those beginning like the number sequence (e.g., 1, 2…), elicited more spontaneous focusing on numerical order than the other types of trials.

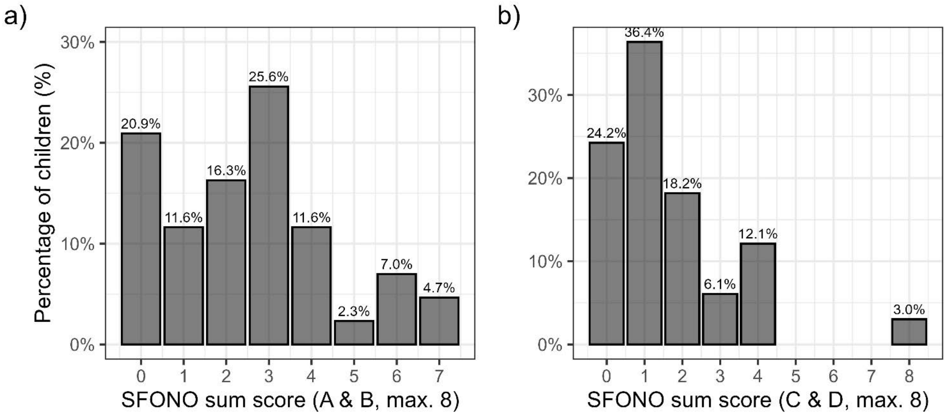

Figure 5. Percentages of SFONO sum scores among the children demonstrating adequate requisite skills to spontaneously focus on numerical order in (a) trial types 1–2–3–4 and 2–3–4–5, and (b) trial types 1–2–4–5 and 8–9–11–12.

4.4 Study IV

Harju, H., Van Hoof, J., McMullen, J., Nanu, C. E., Laakkonen, E., & Hannula-Sormunen, M. (Under review). When "one, two, three" gains ordinal meaning: Investigating the development of numerical ordering skills in 3- to 5-year-old children.

Study IV investigated the development of numerical ordering skills in relation to other early numerical skills, particularly cardinality recognition and number sequence production. The study aimed to capture differences in children's numerical ordering development by using Growth Mixture Modeling (GMM), to determine if children exhibited distinct developmental trajectories. This allowed to explore how cardinality recognition and number sequence production were related to the different developmental trajectories. In addition, video-recordings of children’s performance in the numerical ordering task were analyzed to identify the mathematical skills they were utilizing, and to observe how the use of these skills was changing along with their development.

For this longitudinal study, the same sample of children to Study II were followed for one year. Children completed measures of numerical ordering skills, cardinality recognition, and number sequence production three times, with 6-month time intervals. In the numerical ordering task, children were asked to order a set of cards with different numbers of dots (e.g., cards with 1, 2, and 3 dots) from least to most. Analyzing the video-recordings of children’s numerical ordering included observing any signs of comparison, use of approximate quantities (e.g., little, lots), cardinality recognition, use of the number sequence to order the cards, or use of ordinal number words. Each of these signs of numerical behavior was coded as 1 if it occurred, and 0 if it did not occur, to calculate how frequently they were appearing.

Model fit results of the Growth Mixture Modeling revealed that a 4-group solution was the most appropriate for modeling the different developmental trajectories in numerical ordering skills. Thus, numerical ordering development in the sample was best characterized by four different developmental trajectories: Low, Gentle, Steep, and High (see Fig. 6). The children were assigned into Trajectory Groups based on their most likely class membership. The Low Trajectory group (n = 14) performed low in the numerical ordering task throughout the follow-up. The Gentle Trajectory group (n = 23) began with the majority of children lacking numerical ordering skills, and showed minor development in these skills during the follow-up. The Steep Trajectory group (n = 67) developed from having very little numerical ordering skills to the most of the children mastering the task by the end of the follow-up. This group included almost half of the sample of children. The High Trajectory group (n = 46) had high scores in the numerical ordering task throughout the follow-up.

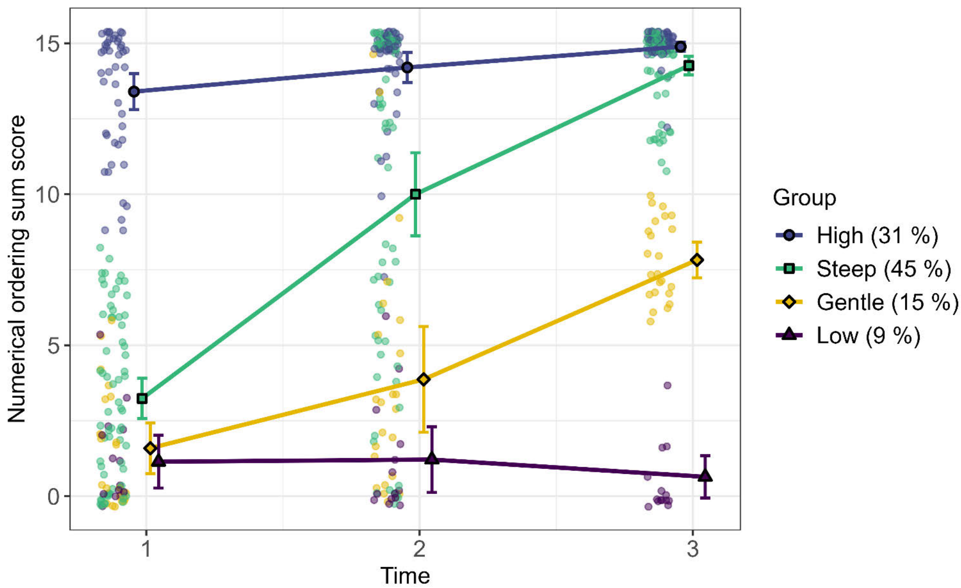

Figure 6. The identified developmental trajectories in numerical ordering skills over a one-year follow-up period. *Note.* Lines depict the mean numerical ordering skill progression for each group; error bars represent 95% confidence intervals.

The results showed, that the Steep and the High Trajectory groups had significantly higher initial cardinality recognition and number sequence skills compared to the Low and the Gentle Trajectory groups. These results were in line with the results of Study I, indicating that children may first need adequate cardinality recognition and number sequence skills to start to learn numerical ordering. However, merely having the skills to tell how many dots each card had did not seem to be sufficient for fully mastering the numerical order task, as the mean scores of cardinality recognition task in both the Low and the Gentle Trajectory groups indicated that the children had acquired the necessary cardinality skills to succeed in the numerical ordering task by the end of the follow-up, however, not all children were able to order the cards numerically. Results of the video-analysis showed, that cardinality recognition was used quite consistently in the numerical ordering task throughout the follow-up in all groups. Interestingly, in the Steep Trajectory group the use of the number sequence increased over time, as the children started to develop in their numerical ordering skills. This may be an indication that the children needed to integrate their knowledge of cardinality and the number sequence in order to learn numerical ordering.

This study showed that children developed differently in their numerical ordering skills between the ages of 3 to 5 years, with majority of the children developing at least some numerical ordering skills during this age. This highlighted the crucial age range when numerical ordering skills are starting to develop and could be supported. Importantly, the findings suggest that children's numerical ordering skills may not develop gradually at a uniform pace but might require a conceptual shift that involves integrating their existing knowledge of cardinal values and the number sequence. This study provided a valuable starting point for further research on numerical ordering development, which is currently lacking in the research field of early numeracy.

5 Main Findings and Discussion

The aim of the present dissertation was to shed light on the role of numerical ordering skills and SFONO tendency in mathematical development. Specifically, the four studies in this dissertation aimed to 1) investigate whether it was theoretically and methodologically possible to distinguish a tendency to spontaneously focus on numerical order as a partially distinct aspect of task performance from the ability to numerically order sets of items, 2) examine the development of numerical ordering skills in 3- to 6-year-old children in relation to early numerical skills and later mathematical performance, and 3) explore the associations between SFONO tendency, numerical ordering skills, and early numerical skills. The theoretical framework was based on first, the definition and operationalization of numerical ordering skills as part of ordinality knowledge, second, the theoretical models and empirical findings on the development of early numerical skills, and third, the role of spontaneous mathematical focusing tendencies in the development of mathematical skills. To achieve the specific aims, a number of novel tasks were developed for the assessment of children's SFONO, that successfully captured individual differences in 3- to 5-year-old children's SFONO tendency. Furthermore, children's development in numerical ordering skills was modeled over a one-year period. Most importantly, the studies in the current dissertation explored the contribution of early numerical skills and SFONO tendency for numerical ordering skills, providing insights that advance our understanding of early numerical development.

As one of the aims of this dissertation was to investigate a separate attentional process of Spontaneous Focusing on Numerical Order, it was important to develop and validate appropriate measures. Previously, one study had investigated SFONO as part of a broader concept of recognition of mathematical structures and used a few task items for its measurement (Sharir & Mevarech, 2022). Studies I, II, and III collectively form a set of studies developing and validating novel tasks to specifically assess children's SFONO tendency. These tasks operationalize children's SFONO tendency as the amount of children's self-initiated recognition and use of numerical order across different, non-explicitly mathematical contexts. First, the Caterpillar Order task used in Study I together with the previous studies on spontaneous mathematical focusing tendencies acted as a starting point for developing a broader set of SFONO tasks in Study II. Second, Study II developed three novel SFONO tasks that together formed a measure of SFONO tendency, and developed a scoring framework for SFONO indications based on video-recordings of 150 children's performance in the SFONO tasks. The results of Study II found individual differences in children's SFONO across and within the three tasks and a good internal consistency of the measure. However, the results showed differing amounts of SFONO between the tasks, which may have been due to varying salience of numerical order in the tasks (e.g., Chan & Mazzocco, 2017; Mazzocco et al., 2020).

To address the limitations in the SFONO measures in Study II, Study III developed a set of SFONO tasks with similar task features, but different contexts to the Tower building task used in Study II, to account for the varying level of salience in numerical order. The new set of tasks showed good internal consistency and similar levels of SFONO across the four tasks, indicating that the individual differences in SFONO could not be explained by varying salience levels of numerical order. In addition, Study III showed that the trials with small numbers, especially those with sets of one and two items, elicited significantly more SFONO than trials with larger sets of items (e.g., 8-12). Furthermore, the internal consistency was higher within the trials of the same task context than within the trials using the same sets of items (e.g., between the trials that included sets of 1, 2, 3, and 4 items). Together, the results indicated that the developed measures were able to capture individual differences in SFONO tendency, and highlighted important considerations in the measurement of spontaneous mathematical focusing.

Overall, the results of Studies II and III support the hypotheses regarding the presence of SFONO tendency and its positive association with numerical skills, particularly numerical ordering skills. These results suggest that it is possible to distinguish a separate attentional process of Spontaneous Focusing on Numerical Order, that triggers the use of numerical order in situations where a child's attention is not explicitly guided towards the numerical order aspect. This conclusion is supported by several results. Studies II and III showed that there are substantial individual differences in children's SFONO tendency, indicating that some children are more inclined than others to spontaneously recognize and use numerical order in situations that are not explicitly mathematical. Furthermore, results of Study II showed that higher SFONO tendency is positively associated with aspects of early numeracy, such as number sequence production, cardinality recognition, and SFON tendency, and especially with numerical ordering skills. Importantly, the association between SFONO tendency and numerical ordering skills remained significant even after controlling for other important factors for mathematical skills such as age, early numerical skills, and SFON tendency. Furthermore, Study III showed that the individual differences in SFONO tendency could not be fully explained by the numerical or cognitive skills required to spontaneously focus on numerical order during the tasks. Instead, the findings suggest that these differences stemmed primarily from variation in the tendency to spontaneously focus on numerical order itself. Together, these results from three different samples and two different countries suggest that SFONO tendency may be a relevant, distinct factor within children's mathematical competencies, especially in numerical ordering skills.

Next, Studies I and IV extend previous research on the development of young children's ordinality knowledge by examining how non-symbolic numerical ordering skills develop. First, results of Studies I and IV indicated that 3- to 6-year-old children had substantial individual differences in their numerical ordering skills. Furthermore, Study IV showed that children developed in their numerical ordering skills between the ages of 4 and 5 years in different rates. Specifically, the results of the growth mixture modeling identified four distinct developmental trajectories in numerical ordering skills: 1) those who demonstrated mastery of the numerical ordering task from the beginning of the follow-up, 2) those who showed rapid development from minimal or no skills to full mastery, 3) those who progressed more slowly and reached the ability to order only the smallest sets (i.e., 1–3 dots), and 4) those who showed little to no development over time. Importantly, these distinct developmental patterns could not be fully explained by age differences among the children, suggesting that other factors contributed to the variability in skill development.

Previous studies have suggested that children's cardinality skills and number sequence may play a role in the acquisition of ordinality skills (e.g., Spaepen et al., 2018; Hutchison et al., 2022). In this light, Studies I and IV examined the development of numerical ordering skills in relation to these early numerical skills. Investigation of the co-development of numerical ordering (indicated by the measure for focusing on numerical order), cardinality recognition, and number sequence production in Study I showed that children's number range in numerical ordering was the lowest of all three skills measured in four timepoints, suggesting that numerical ordering skills may start to develop later than number sequence production and cardinality recognition. Consistent with this finding, Study IV showed that children who exhibited rapid development in numerical ordering skills during the follow-up had higher initial abilities in number sequence production and cardinality recognition compared to those who showed slower or no progress. In addition, Study IV investigated children's numerical behavior during numerical ordering to see the what numerical skills the children were utilizing in the task. The behavioral analyses indicated that cardinality recognition was used in the tasks consistently throughout the follow-up across all children in the sample, while mainly children in the groups showing full mastery or rapid development in the numerical ordering task, used number sequence as a tool to order the sets in numerical order. Taken together, the quantitative and qualitative results of Study IV indicated that merely having adequate cardinality skills (assessed through a Give-N -task and children's verbal references to set sizes in the numerical ordering task) may not be sufficient for successful numerical ordering, but may rather require a knowledge integration of how cardinal values and the number sequence are connected.

5.1 Theoretical implications

The present set of studies contributes to the theories of early numerical development by examining how three foundational skills—number sequence production, cardinality recognition, and numerical ordering—develop in relation to one another, and how spontaneous focusing on numerical order is associated with these skills. This work proposes a theoretical model outlining the subskills involved in numerical ordering and their interrelations. It also provides evidence suggesting that the integration of cardinality and number sequence knowledge, which may be necessary for the development of ordinal understanding, does not necessarily occur automatically after children acquire the cardinality principle. Moreover, it broadens the scope of research on ordinality by incorporating non-symbolic numerical ordering and exploring the factors that contribute to the development and individual differences in these abilities.

5.1.1 Disentangling the development of numerical ordering skills in early childhood

Previous studies have suggested that children are able to learn numerical ordering only after acquiring knowledge of the cardinality principle, possibly because only after this acquisition children start to understand the relations between numbers (Knudsen et al., 2015; Spaepen et al., 2018). Investigating the developmental relations between number sequence production, cardinality recognition, and numerical ordering skills in Studies I and IV indicated a similar relationship, where numerical ordering skills develop after cardinality recognition. In addition, Study IV confirmed that children who showed mastery or rapid development in numerical ordering skills during the follow-up, had the skills to recognize cardinal values of five or more, which is often considered as a sign of knowing the cardinality principle (e.g., Le Corre & Carey, 2007), at the beginning of the follow-up. Study IV extended the results of Spaepen et al. (2018) by qualitatively investigating children's numerical behavior during the numerical ordering task. This way it was possible to specifically investigate children's skills in numerical ordering task. The behavioral data showed that although children across all developmental trajectories used cardinal number words during the task, not all were able to order even the smallest sets correctly. This suggests that the ability to recognize the cardinal values does not, on its own, guarantee an understanding of numerical order. Consistent with this, children's performance on a separate measure of cardinality recognition indicated that most children had acquired the ability to identify set sizes up to the level required by the numerical ordering task by the last timepoint the latest. Therefore, differences in numerical ordering development cannot be fully explained by limitations in cardinality skills.

The behavioral data of numerical ordering in Study IV indicated that some children showing rapid development or full mastery in the numerical ordering task used the number sequence to order the sets in numerical order, while children exhibiting slower progress or no development did not. Importantly, the use of number sequence to order the sets numerically increased around the same time the children started to successfully order even the largest sets in the numerical ordering task. This might have indicated that these children had integrated their knowledge of how the cardinal values and number sequence are connected, and thus understood that the sets of items needed to be arranged based on their cardinal value. This finding contrasts with earlier theories (Carey, 2009; Krajewski, 2008 as cited in Krajewski & Schneider 2009) and empirical studies (Marchand & Barner, 2018; Sarnecka & Carey, 2008), which suggested that learning the cardinality principle involves connecting the ordinal structure of the count list with the corresponding cardinal values—for example, recognizing that each successive number word refers to a quantity that increases by one. Instead, the result supports the knowledge integration theory, which proposes that children initially possess separate, potentially interrelated pieces of numerical knowledge that must be integrated through learning (e.g., Karmiloff-Smith, 1992; M. Schneider & Stern, 2009). It may be that, at first, the procedural ability to recite the number sequence to determine "how many," and the conceptual understanding that the last number word represents the total quantity in a set, are two separate concepts. This may be due to children's initial experiences of using the number sequence, which is recited first separately on its own or mainly used as a tool for counting (Fuson, 1988; Gelman & Gallistel, 1978). Only later do children come to understand how these distinct components—procedural counting and conceptual understanding of cardinality—are integrated, allowing for their use in numerical ordering. This is an important finding for the research field of numerical cognition.

The different developmental trajectories in numerical ordering skills found in Study IV indicate that the process of developing these skills is not uniform for all children. Based on the findings of Study IV, two potential developmental pathways for numerical ordering skills and the proposed knowledge integration may be hypothesized. First, the qualitative analyses of children's behavior during the numerical ordering task suggested that those children who showed gentle development of being able to order approximately the smallest sets (i.e., 1-3 dots) by the end of the follow-up, also labeled the sets with approximate quantities (e.g., lots of dots) most frequently. Therefore, these children might have been able to order the smallest sets with approximation (e.g., a bit, a bit more, most), but were unsuccessful with the larger sets. Such approximate ordering may align with Krajewski's (2008, as cited in Krajewski & Schneider, 2009) model, which proposes that before children can order quantities based on exact cardinal values, they may initially classify and compare quantities using imprecise quantifiers (e.g., "a bit," "much," "very much"), and also Resnick's (1991) theory of layers of mathematical knowledge which suggests that on the earliest level, mathematics of protoquantities, children may be initially able to order sets based on salient size differences.

Second, findings by Hutchison et al. (2022) indicate that small, subitizable quantities (i.e., 1–4) tend to be processed more similarly in non-symbolic and symbolic forms, likely due to their more precise nature in contrast to larger, more approximate non-symbolic quantities. When combined with recent findings on ordinality, which highlight the high familiarity of the beginning of the number sequence (e.g., Devlin et al., 2024; Dubinkina et al., 2021; Sella, Sasanguie, et al., 2020), this suggests that children may initially be able to order small sets (e.g., 1, 2, and 3 items) from rote memory, even before fully understanding the conceptual idea of numerical ordering. It should be noted that the familiarity effect in ordinality has been observed in studies with adult participants, and therefore the familiarity effect might stem from highly practiced patterns (e.g., the beginning of the number sequence, multiplication patterns) (Devlin et al., 2024). In addition, these findings are based on reaction time data from order processing -tasks, and may not reflect the cognitive processes of young children in ordinality tasks, such as non-symbolic numerical ordering, based on children's ordering accuracy. Being able to order the smallest sets in Study IV could be explained by the results of Sella and Lucangeli (2020), who observed that inside the subitizing range, nearly all 5- to 6-year-old children knew the directional property of the number sequence, that adding one item to a set, results in the next number in the number sequence (i.e., successor function, n+1) and removing one item results in the previous number (i.e., predecessor function, n-1). However, even children who understood the cardinality principle showed a lack of understanding the directional property of the number sequence, especially the predecessor function, outside the subitizing range. Therefore, to fully understand numerical ordering, children may need to grasp that each subsequent set represents a larger quantity and corresponds to the next number in the sequence and vice versa. Such insight could mark a conceptual shift that allows children to order any sets within their numerical range. Evidence for this kind of developmental leap was observed in Study IV, where children tended to score either quite low or near the maximum in the numerical ordering measure, indicating a discontinuity in performance that may reflect this emerging conceptual understanding.

5.1.2 The association between SFONO tendency and numerical ordering skills

Previous studies showed that already young children spontaneously focus on mathematical structures such as numerical order (Sharir & Mevarech, 2022), and engage in math talk with their caregivers concerning the order of numbers (Ramani et al., 2015; Trickett et al., 2022) in situations that are not explicitly mathematical. The present set of studies was successful to specifically show, similarly to other studies on spontaneous mathematical focusing tendencies (Hannula & Lehtinen, 2005; McMullen et al., 2019; Verschaffel et al., 2020) that children have individual differences in their SFONO tendency. In other words, the present set of studies showed that some children are more likely to notice and use numerical order without guidance in situations that are not explicitly mathematical. Furthermore, the fact that individual differences in SFONO tendency were found in two different countries with different schooling systems, partly using different measures and numerical sequences, further supports the robustness of SFONO tendency. Similar to previous studies on spontaneous mathematical focusing tendencies (e.g., Gloor et al., 2017; Hannula & Lehtinen, 2005; McMullen et al., 2016; Poltz et al., 2022; Rathé et al., 2022a; Wijns et al., 2020), results of Study II showed that next to the positive associations between SFONO tendency and early numerical skills, SFONO tendency explained unique variance in children's numerical ordering skills after controlling for age, concurrent numerical skills, and SFON tendency, which are all important contributors to mathematical skills. Therefore, SFONO may be an important factor contributing to the individual differences in numerical ordering skills.

There are a few possible explanations for the unique, positive association between SFONO tendency and numerical ordering skills. First, previous studies have suggested that children with a higher tendency to spontaneously focus on a mathematical aspect, may be more inclined to notice and use this numerical aspect in their surroundings, and therefore gain more self-initiated practice in the relevant mathematical skills needed in this process (e.g., Hannula & Lehtinen, 2005; McMullen et al., 2014; Rathé et al., 2019). For example, children with a higher SFON tendency may pay more attention to the aspect of exact number in their everyday lives (e.g., there are three flowers in a vase), and gain more practice in their cardinality and counting skills that are needed to determine the exact number of flowers. Similarly, children who show a higher SFONO tendency, may be more inclined to recognize and use numerical order, and thereby gain more self-initiated practice in their numerical ordering skills. This proposed mechanism draws on

Ericsson and Lehman's (1996) theory of expertise, which proposes that increased deliberate practice leads to greater expertise in a given domain (Lehtinen et al., 2017). However, the current set of studies did not address the underlying mechanisms behind SFONO tendency. Therefore, it remains unclear why children with a higher SFONO tendency tend to demonstrate better numerical ordering skills. One promising direction for future research is to explore the hypothesized mechanism of self-initiated practice—for instance, by examining how children's performance on SFONO tasks relates to their spontaneous focusing on numerical order in everyday environments (e.g., Batchelor et al., 2024).

Alternatively, it has been previously suggested that spontaneous mathematical focusing tendencies may be an indication of children's ability to flexibly use their existing mathematical skills in non-routine tasks (Halme, 2024). Supporting this view, a recent meta-analysis by Li et al. (2025) found that mathematical skills more strongly predicted SFON tendency than the reverse, suggesting that children's attention to exact number may be driven by their mathematical skills. This interpretation is particularly relevant in the context of the SFONO tasks used in this dissertation, where numerical order was deliberately embedded in the different task contexts. In real-life settings, opportunities to encounter and practice numerical ordering skills with consecutive sets of items may be relatively rare. Therefore, it may be plausible that children with stronger numerical ordering skills are better equipped to detect and utilize numerical order even in less structured, everyday situations. Therefore, given the cross-sectional design of Study II, it is also possible that children's more advanced numerical ordering skills enabled them to recognize and use numerical order spontaneously, rather than the other way around.

5.2 Methodological implications

The present set of studies has methodological implications for measuring spontaneous focusing on mathematical aspects, as well as numerical skills in young children. Prior to the current dissertation, SFONO had been only measured as part of spontaneous focusing on mathematical structures (Sharir & Mevarech, 2020), with tasks that had limitations considering the guidelines on measuring spontaneous mathematical focusing tendencies. The current dissertation details six novel tasks that were created to measure children's SFONO, thus broadening the methodology on spontaneous mathematical focusing tendencies. Several considerations support the view that these tasks were able to successfully measure SFONO and distinguish it from the requisite skills needed to demonstrate one's spontaneous focusing on numerical order. First, the tasks in Studies II and III were designed so, that they did not have any indications towards the numerical aspects, allowing for multiple numerical and non-numerical interpretations. Second, the tasks mostly included such small sets of items that they were within the children's numerical ordering abilities. Third, all the sets of items were visible for the children during the whole task, in comparison to previously used Imitation tasks (Hannula & Lehtinen, 2005) where reproducing the correct number of items requires noticing it and keeping it in memory (Bathelor, 2014), therefore minimizing the working memory demand of the SFONO tasks. Fourth, children could demonstrate SFONO also through their actions during the SFONO tasks, not only by producing a correct numerical order in them. This way, it was possible to detect SFONO even if the children ended up making a mistake in the order produced, or focused on multiple aspects in the tasks and decided to do something else than numerical ordering. Fifth, and most importantly, Study III showed individual differences in SFONO even within the group of children who showed to have the necessary skills to order sets of items numerically in the guided version of the SFONO tasks, where the children were explicitly guided to use numerical order in the task by ordering the sets of items from least to most similarly to Hannula & Lehtinen (2005) and McMullen et al. (2014). Therefore, the current set of studies seem to be successful in developing tasks separately assessing children's attentional process of SFONO.

In addition to the creation of the SFONO measures, the studies in the dissertation highlighted some factors influencing the measurement of SFONO and other spontaneous mathematical focusing tendencies. Similar to previous studies (Batchelor et al., 2015; Chan & Mazzocco, 2017; Mazzocco et al., 2020; McMullen et al., 2019), Studies II and III showed that task features such as the salience of the numerical information, task demands, and context may influence the measurement of spontaneous mathematical focusing. Study II included three tasks that had differing answer formats (i.e., order production vs. order reproduction), and possibly differing salience of numerical order aspect, and the results showed differing amounts of SFONO between the tasks. Notably, in Study III, these aspects were kept as similar as possible only varying the task context, which led to approximately the same amount of SFONO between the four tasks.

Additionally, Study III showed that there were significantly more SFONO in trials that included sets of items with small numbers, especially with sets of 1 and 2 items. This may indicate that the salience of numerical order is higher with small subitizable sets than with larger sets that require counting, and especially with such numbers of items that correspond to the start of the number sequence. This finding is in line with Savelkouls et al. (2020), who suggested that small sets of items overall elicit more spontaneous focusing on mathematical aspects in the measures. Furthermore, Study III showed that the task context may also influence the measurement of spontaneous mathematical focusing tendencies, as the trials within the same task context had better internal consistency compared to the trials that used the same sets of items (e.g., trials with sets of 1, 2, 3, and 4 items), which indicated that children answered more consistently within a task. As the aim of measuring spontaneous mathematical focusing tendencies is to get an indicator of children's general tendency to recognize and use the mathematical aspect of interest across different situations, it may be that several different tasks capture this better than using fewer tasks with more trials. This is in line with the guidelines of designing tasks assessing spontaneous mathematical focusing tendencies (Hannula & Lehtinen, 2005; Hannula-Sormunen, 2015; McMullen et al., 2019). In general, the present dissertation underscores the importance of thoughtful task design, as measuring spontaneous mathematical focusing tendencies is sensitive to task-related factors that may impact the validity of the results.

Next to the methodological implications for spontaneous mathematical focusing tendencies, the studies of this dissertation also emphasized the added value of observing children's behavior during task engagement. All of the studies included video-analysis of children's actions during the tasks, which revealed valuable details about the children's skills that contributed on the results and their interpretation. For example, Studies II and III revealed a group of children who showed signs of spontaneously focusing on numerical order in the SFONO tasks without ordering the sets of items in numerical order. If the SFONO score would have been based on merely children's final answer (e.g., the order produced) in the SFONO tasks, it would have led to an underrepresentation of SFONO answers in the results. Furthermore, by investigating children's numerical behavior in the numerical ordering task in Study IV, it was possible to observe that success in the numerical ordering task was not merely explained by having adequate cardinality and number sequence skills. Thus, the video-analyses added a rich set of data that affected the conclusions over the results, leading to more detailed picture of children's numerical ordering skills. Importantly, in combination of carefully designed tasks and task materials, the video-analyses contributed to both validity and reliability of the measures. Together, these considerations highlight that focusing only on children's task performance may lead to a limited view and underestimation of their skills. This is important, as currently research in the field is more often based only on the correctness of the solution than actions leading up to it.

Additionally, the current dissertation broadened the current research on ordinality skills by investigating them from the aspect of non-symbolic numerical ordering and the development of ordinality understanding in younger ages than previously. The results of Study IV showed, that even with the non-symbolic stimuli, the children were observed using number words and the number sequence to order the cards therefore using symbolic elements in the task, which may indicate that the children reason about ordinality similarly with non-symbolic as to symbolic stimuli. Furthermore, numerical ordering skills may be especially important at the early ages, as they may contribute to the conceptual understanding of the next number being always more than the previous one. This understanding may be relevant for children to understand that also non-consecutive numbers can be in order, which has been suggested to be a leap in children's ordinality development (Hutchison et al., 2022).

In addition, studies have shown that also numerical ordering with non-symbolic stimuli seem to be related to concurrent and future mathematical achievement (e.g., Berteletti et al., 2010; Purpura & Lonigan, 2013; Stock et al., 2009), which was supported by results of Study I indicating that focusing on numerical order was predictive of mathematical achievement 9 years later. Therefore, the research on ordinality could consider also using measures with non-symbolic stimuli, as the field is so far heavily focused on only using number symbols as stimuli. Moreover, as especially the much-used order processing measure has been suggested to rather measure fact retrieval of familiar sequences than ordinality knowledge (e.g., Devlin et al., 2024; Sella, Sasanguie, et al., 2020), it is important to investigate ordinality knowledge from different aspects. To deepen our understanding of ordinality, future research could investigate the structural and developmental connections among different components of ordinality knowledge, clarifying its underlying structure and the extent to which different task types tap into shared or distinct constructs.

5.3 Practical implications

The present dissertation has practical implications for young children's education in early numerical skills. Understanding ordinality is a crucial step in numerical development, and therefore it should be considered as an explicit goal in early education (cf. Hutchison et al., 2022). While the development of cardinality skills has received considerable attention in research (e.g., Bermejo et al., 2004; P. Cheung et al., 2022; Fuson, 1988; Mix et al., 2012; Wynn, 1992) and in the Finnish national core curriculum for ECEC (National Agency of Education, 2022), the development of ordinality has been comparatively overlooked. As the results from Study IV showed that numerical ordering skills are actively developing between the ages of 3 and 5 years, this period may be particularly important for educational support. Notably, as the findings also indicated that understanding how cardinal values and the number sequence are connected may play a role in the development of numerical ordering skills, children may benefit from educational activities that help children to see the relation between the two pieces of knowledge and thus support this knowledge integration (M. Schneider & Stern, 2009). Therefore, educational activities drawing connections between quantities and the number sequence, especially how quantities change as the number sequence is recited forward and backward, may be beneficial. In addition, the results showed substantial individual differences in children's numerical ordering skills. This suggests that incorporating numerical ordering activities into early education may not only support children's understanding of ordinality but also serve as a valuable tool in identifying children who may need additional support. Such activities could complement existing assessments and provide a more comprehensive picture of children's early numerical competencies.

In addition to supporting the development of ordinality through structured educational activities, fostering children's spontaneous focusing on mathematical aspects may offer another powerful avenue for enhancing early mathematical development. Previous studies have shown that spontaneous mathematical focusing tendencies can be supported through targeted interventions (Braham et al., 2018; Hannula et al., 2005; Hannula-Sormunen et al., 2020; McMullen, Hannula-Sormunen et al., 2019; Määttä et al., 2022, 2024). Moreover, children who received training in focusing on mathematical aspects, such as exact number, showed significantly greater improvements in their mathematical skills compared to control groups (Hannula et al., 2005; Hannula-Sormunen et al., 2020). These findings suggest that fostering spontaneous mathematical focusing tendencies may positively influence children's mathematical development. Furthermore, it has been suggested that students face difficulties in considering real world situations in school mathematics (Verschaffel et al., 2000). Given this, enhancing spontaneous mathematical focusing from early childhood may be especially important. In particular, encouraging children to recognize opportunities to apply numerical ordering skills in everyday situations may not only strengthen their SFONO tendency, but also support a deeper understanding of ordinality. The individual differences in SFONO tendency identified in the present set of studies suggest that some children may already benefit from the use of numerical order in their everyday environments, which could facilitate more successful mathematical development. Supporting the development of SFONO tendency in all children, particularly those with a lower initial tendency, may increase their chances of developing a stronger base for mathematical development, which is important for later mathematical development (e.g., Jordan et al., 2009; Liu et al., 2025; Nguyen et al., 2016).

Building on these findings, practical implications emerge for how early childhood educators can support children's mathematical development through everyday interactions. By intentionally promoting the use of mathematical skills in everyday activities, educators may not only nurture children's spontaneous focusing on mathematical aspects but also contribute meaningfully to their broader mathematical development (Hannula-Sormunen et al., 2020). Recognizing and encouraging the use of concepts such as numerical order during routine interactions—whether during play, transitions, or daily routines—can provide rich and meaningful opportunities for learning that are both accessible and engaging for young children. This perspective offers a powerful and practical tool for early education. Rather than requiring separate, formal instruction, educators can integrate mathematical thinking into the natural flow of daily life (Hannula et al., 2005). Even in the fast-paced and often unpredictable environment of early education and care, there are countless opportunities to highlight and discuss mathematical ideas in playful and meaningful ways (Clements & Sarama, 2007). Making these connections explicit for children may help close the gap between everyday experiences and formal mathematical understanding, setting a strong foundation for later learning.

5.4 Limitations and future studies

The studies presented in this dissertation represent some of the first systematic efforts to conceptualize and measure the tendency to spontaneously focus on numerical order (SFONO), and should be interpreted with that pioneering context in mind. While the findings offer promising initial insights, further validation of the SFONO measures, additional empirical studies, and continued refinement of the assessment tools are necessary to more fully understand and establish the role of SFONO tendency in children's mathematical development. Moreover, this dissertation examined ordinality knowledge through the lens of non-symbolic numerical ordering skills, an area that has received limited attention in previous research. As such, the current findings open up new directions for future inquiry. However, these contributions must also be considered in light of several limitations, which can inform and guide subsequent studies. Specifically, both methodological and conceptual limitations constrain the generalizability and interpretation of the findings.

Importantly, the present set of studies focused on examining associations between variables, and therefore no causal conclusions can be drawn from the results. While these findings offer a valuable starting point for understanding how different aspects of early numerical development may relate to one another, the interpretations are based on hypothesized relationships rather than demonstrated causal effects. This opens several promising directions for future research using intervention-based designs. First, future research could examine whether strengthening numerical ordering skills promotes broader mathematical development. Study I suggested that numerical ordering ability at the age of 5 years predicted mathematical achievement at the age of 12 years. Supporting this, Morsanyi et al. (2024) found that training order processing—an aspect of ordinality—improved mathematical performance in 4- to 5-year-old children. Conversely, James-Brabham et al. (2024) observed that training number ordering via a board game produced no learning gains beyond those attributable to regular classroom instruction in 4- to 5-year-old children. These findings indicate that further research is needed to determine whether, and which aspects of, ordinality knowledge benefit broader numerical development when strengthened through training. Second, as the results of this dissertation suggest, both the integration of cardinality knowledge with the number sequence and a stronger tendency to focus on numerical order may support numerical ordering skill development. In addition, it has been argued that randomized controlled trials aimed at stimulating spontaneous mathematical focusing tendencies (such as SFONO) are still needed to make strong claims about their causal role in mathematical learning (Inglis, 2020). Therefore, future intervention studies could explore effective ways to foster these skills—for example, by designing training that enhances children's SFONO tendency—and test whether this leads to improvements in numerical ordering.

Next, there are certain limitations regarding the developed SFONO measures. As the aim of having multiple SFONO tasks in different contexts is to gain an indicator of children's general tendency to focus on numerical order across different situations (Hannula & Lehtinen, 2005; Hannula-Sormunen, 2015; McMullen et al., 2019), having only three or four measures of SFONO limits the conclusions that can be drawn. On the other hand, many previous studies in the field of spontaneous mathematical focusing tendencies have used mainly one or two tasks (e.g., Nanu et al., 2018; McMullen et al., 2014; Rathé et al., 2019; Wijns et al., 2020), and including even more SFONO tasks might have been unfeasible. The limited number of SFONO tasks may risk the possibility that there may not be a general SFONO tendency, but instead the individual differences found in these measures could be a result of the particular features of these tasks (McMullen, 2014). However, the fact that individual differences in SFONO tendency were found in Studies II and III conducted in different countries with partly different set of tasks, indicates that the individual differences in SFONO tendency appear across different contexts. On the other hand, the repetitiveness of the SFONO tasks across trials may, to some extent, have contributed to children's demonstration of SFONO. The children were given three or four trials of the same task that shared a common theme. The SFONO tasks had sets of similar items that could be counted and ordered numerically, and they were open for multiple interpretations. Even though the individual trials included different colors or animals to reduce repetition, it is possible that SFONO may have increased along with the testing. Therefore, using multiple tasks with several trials may hinder the measurement of children's spontaneous behavior, and rather reflect an ability to recognize consistency across tasks instead. Thus, future studies should investigate SFONO tendency with other samples and different tasks, and investigate whether SFONO increases in the latter tasks. In addition, it would be interesting to investigate whether children show SFONO also in Picture description tasks that requires a verbal answer instead of a behavioral one, or whether SFONO is mainly observed in behavioral measures like those used in the current studies.

There are also certain factors regarding task design of the SFONO tasks that may have affected children's SFONO responses, such as task characteristics, like the task context and salience of the numerical features (e.g., Batchelor et al., 2015; Mazzocco et al., 2020). First, it is possible that the different task contexts used in Studies II and III contributed to the emergence of the individual differences found in SFONO, as contextual familiarity may have raised or lowered the salience of numerical order. For example, as block play is quite a common activity for young children to engage in where tower building may not usually be based on numbers of stickers on the blocks, the context may have reduced the salience of numerical order. In contrast, in a less familiar play activity of hanging laundry, it may be easier for the children to focus on the number of stickers on the items, leading to an increased perceived salience of numerical order as no prior experiences effect their task performance. On the other hand, for some children block play may be inherently more mathematical situation compared to a daily activity such as hanging laundry, and therefore the effect of task context likely varies by child. This aligns with the results of Study III, where no significant differences in SFONO were observed between the task contexts. Second, the numerical size of sets used in the tasks may have contributed to the salience of numerical order especially in Study III. As the numerical ratio is larger in small sets of items (e.g., 1-2-3-4) compared to large sets of items (e.g., 8-9-11-12), it was probably possible for the children to recognize without counting that the set sizes differed in the trials with small numbers, whereas with large numbers this recognition required explicit counting. Therefore, the perceived salience of numerical order was possibly higher in the trials with using small numbers. Third, it should be considered that other task characteristics such as the size, color, and contrast of the items, as well as task format (order production vs. order reproduction) may have contributed to the salience of numerical order. This may have been reflected in the results of Study II, where we observed the least SFONO in the Duck family task, followed by the Tower Building task and the most in the Find the Mother task.

Next, the studies in this dissertation did not investigate the test-retest reliability of the SFONO measures. Such an approach would be valuable in establishing the temporal stability of SFONO tendency as it is hypothesized to be a fairly stable trait, in other words, children who spontaneously focus on numerical order more across the measures, should do so at other points in time as well. The method of investigating test-retest reliability includes administering the same measure to the same sample of children in two different times (Bryman, 2004, p. 70). A high positive correlation between the results at the two different times would show good test-retest reliability, indicating consistency in children’s responses in the measure. However, the spontaneous nature of the SFONO measures complicates investigating test-retest reliability. According to the criteria for measuring spontaneous focusing on a mathematical aspect, the tasks should be novel for children to enable capturing their spontaneous behavior (Hannula-Sormunen, 2015). Therefore, administering the same tasks repeatedly in a sufficiently short interval for test-retest purpose may hinder capturing spontaneous mathematical focusing, as the children's prior answers or experiences in the task may affect their responses. One option for assessing test-retest reliability of the SFONO measures would be to develop parallel versions of the measures that differ slightly in their material (e.g., changing the blocks and stickers in the Tower Building task) that are administered by a different tester after a longer time period, which could be done for example in a longitudinal setting in future studies. This would be valuable in further validating the construct of SFONO tendency and ensuring the replicability of the results.

Another limitation concerning the measurement of SFONO tendency relates to the ecological validity of the tasks. It remains unclear whether children's performance in the SFONO tasks accurately reflects their spontaneous tendency to notice and use numerical order in real-world settings. In everyday life, situations featuring neatly arranged, consecutive sets of items are likely rare. As a result, SFONO may occur infrequently in naturalistic contexts—unless children themselves create opportunities to engage with numerical ordering using non-symbolic stimuli. Therefore, future research should explore if, how, and under what circumstances children demonstrate SFONO in their daily environments. There are additional ways in which SFONO might manifest in everyday life that were not captured in the current tasks. First, children might apply numerical ordering to non-consecutive sets (e.g., 2, 5, and 8 items) through repeated comparison—for example, identifying who has the fewest, the second most, and the most candies. Second, as symbolic numerical order is commonly embedded in everyday contexts (e.g., house numbers, page numbers, floors in elevators), children may spontaneously engage with symbolic numerical order more frequently than with non-symbolic forms. Third, as the current SFONO measures only investigated ordinality from the aspect of numerical ordering, it remains unclear whether SFONO also includes spontaneous focusing on ordinal numbers. This aspect of SFONO might be more prominent in children's everyday experiences, for instance, when waiting for their turn (e.g., "*I am third in the line.*").

These considerations highlight the need for future investigations of SFONO. To fully understand the construct, it is essential to clarify what types of numerical content and representations it encompasses. Future studies should examine whether SFONO is best understood as a general tendency to attend to numerical order, as a set of distinct, partially independent focusing tendencies involving different aspects of numerical order (e.g., symbolic vs. non-symbolic, consecutive vs. non-consecutive), or even as a broader tendency to focus on numerical relations in general. Investigating whether certain types of numerical order are more readily recognized, or whether certain aspects of ordinality are more strongly related to mathematical skills than others, could offer valuable insights for instructional design.

Such findings could guide educators in intentionally supporting children's attention to numerical order across diverse learning environments.

Additionally, there are a number of issues concerning the measurement of numerical ordering skills in studies II and IV worth considering. First, a subset of children appeared unsure of how to approach the task. While this may reflect underdeveloped numerical ordering skills, it is also possible that some children did not fully comprehend the task instructions. However, this explanation seems less likely given that for children unable to place even the first card correctly, the experimenter gave a second attempt with the help of showing which card to start with and placed it in its correct place. This was hypothesized to serve as a strong cue towards the number sequence. In addition, as the task instruction included concepts such as least and most, many children who failed in ordering the cards numerically were able to correctly recognize the cards that had the least and the most dots.

Beyond task comprehension, other design-related factors may also have influenced the measurement of numerical ordering skills. One such factor is the use of only consecutive numbers in the task design. Although this design choice allowed for the investigation of early emerging numerical ordering abilities, it may not capture more advanced forms of ordinality understanding. For instance, the ability to recognize non-consecutive sequences (e.g., 2–6–8) as being in numerical order may represent a more sophisticated conceptual understanding (Hutchison et al., 2022). This may limit the construct validity of the numerical ordering measure. However, it is important to note that the difficulty of non-consecutive sequences may depend on the task: children might correctly order stimuli in tasks that require concrete ordering but label non-consecutive sequences as unordered in order verification tasks (i.e., "Are these numbers in order?"). Furthermore, including non-consecutive sets may have allowed children to rely more heavily on perceptual cues of the stimuli, as the quantitative differences between sets would be more pronounced (Spaepen et al., 2018).

In addition to task design, the scoring of the numerical ordering task may have resulted in an underestimation of some children's numerical ordering skills. However, these limitations were mitigated by the use of partial credit (based on the longest correctly ordered sequence) and multiple trials, instead of using a correct - incorrect scoring at the whole trial -level. As the scoring was based on the highest number up to which the cards were correctly ordered, a mistake of placing only two cards the wrong way (e.g., 1-2-*4-3*-5) could result in a loss of many points, and were scored equal to those responses that had only the first few cards in order and the rest of the cards placed randomly (e.g., 1-2-5-4-3). Furthermore, because the number of cards varied across trials, the maximum achievable score also differed for each trial (i.e., 2, 4, 6, and 3 points). As a result, the total score may have disproportionately reflected performance on the more difficult trials, particularly the one with a maximum of 6 points.

Next to the methodological limitations, there are also important considerations regarding the interpretations of the findings. Specifically, certain limitations affect the conclusions that can be drawn from the results concerning the development of numerical ordering skills in Study IV and their predictive relation to later mathematical skills in Study I. First, both studies had at least partly limited sample size for the analyses conducted. Study IV used Growth Mixture Modeling to examine children's development in numerical ordering skills, and identified four distinct developmental trajectories. Although qualitative investigations of children's numerical ordering skills provided further support for these trajectories, the sample size of 150 was relatively small for using Growth Mixture Modeling. In addition, Study I included a sample of 36 children, thus limiting the generalizability of the results. Therefore, further research is needed to confirm if these results hold for different and larger samples of children as well.

Next, the knowledge integration hypothesis proposed in Study IV should also be interpreted with caution. The suggestion was primarily based on qualitative observations of children's numerical behavior during the numerical ordering task, that were compared across groups of children with different developmental trajectories in numerical ordering skills. However, the group sizes varied considerably (14, 23, 67, and 46 children), which may limit the comparability over the frequencies of numerical behavior, even though these were reported as percentages within each group. Moreover, the majority of children did not seem to display overt numerical behavior in the video recordings. While the qualitative observations were supported by quantitative findings—indicating that neither adequate cardinality skills nor number sequence production alone guarantees successful numerical ordering—further research is needed to substantiate the knowledge integration hypothesis.

Despite these limitations, the findings offer a promising starting point for future investigations. These insights suggest that there may be considerable value in examining how young children begin to integrate basic numerical skills towards an integrated framework of early numerical concepts. This opens several fruitful avenues for future research. First, studies could explore in greater depth how children integrate their knowledge of cardinal and ordinal aspects of number. Although this integration may be difficult to assess directly, one promising approach could be to investigate the factor structure of these skills (and potentially their subcomponents) across development, or by cross-sectionally looking at the invariance across age groups. This would allow to examine whether and how these aspects of numerical knowledge become more interconnected over time towards a more integrated knowledge structure, similar to approaches used to study relations among conceptual and procedural knowledge (e.g., M. Schneider et al., 2011). Second, while the present dissertation focused on three key factors of numerical ordering skills, namely cardinality skills, number sequence production, and SFONO tendency, future research could examine the role of additional early numerical competencies suggested in the theoretical framework of the current dissertation, such as comparison skills and mathematical language, in contributing to individual differences in numerical ordering skills.

An additional consideration is that the relationship between SFONO tendency and numerical ordering skills was explored solely through cross-sectional data in Study II. Although the results of Study II indicated that SFONO tendency was positively associated with numerical ordering skills after controlling for age and numerical skills, SFONO explained additional 2 % of the variance in numerical ordering skills. This finding is in line with a study of Rathé et al. (2022a) which observed only weak (but significant) relations between spontaneous mathematical focusing tendencies (i.e., SFON and SFONS) and mathematical skills with a larger sample of children (N = 128), contradictory to the current theoretical expectations (McMullen et al., 2019; Verchaffel et al., 2020). The weak association in Study II may be explained by limited variance in children's numerical ordering skills and SFONO tendency. The children in the sample were between 3.5 and 4.5 years of age, and the results indicated that majority of the children in the sample scored very low in the measures of numerical ordering skills and SFONO tendency, likely because these skills may have just started to develop. This may limit the conclusions that can be drawn about the relation between numerical ordering skills and SFONO tendency. It is hypothesized that SFONO tendency may increase with age (e.g., Hannula & Lehtinen, 2005; Hannula-Sormunen et al., 2023), and thus it's relation with numerical skills may strengthen in time. Given that previous studies have suggested spontaneous mathematical focusing tendencies to be important factors in children's mathematical development and to develop reciprocally with the related mathematical skills possibly due to the self-initiated practice -hypothesis (e.g., Hannula & Lehtinen, 2005; McMullen et al., 2017, 2019), future studies should investigate the longitudinal bidirectional relations between SFONO tendency and numerical ordering skills, as well their contribution to later mathematical skills such as arithmetic. Additionally, if both SFONO tendency and numerical ordering skills would predict later mathematical skills, the added value of SFONO tendency over numerical ordering skills should be examined. This would further clarify whether SFONO tendency plays a distinct role in mathematical development, or whether it would be better characterized by a measure of implicit numerical ordering skills or applying one's skills in real-life situations.

Beyond examining the reciprocal relations between SFONO tendency and related mathematical skills that may be due to the increased self-initiated practice, further research is needed regarding its origins and mechanics. More specifically, is SFONO tendency, or are the different spontaneous mathematical focusing tendencies in general, driven by bottom-up processes (e.g., perceptual salience of mathematical features), or top-down influences (e.g., social goals, children's beliefs about the importance of mathematics)? According to McMullen et al. (2019), both of these mechanisms are present in any situation where a person recognizes and uses mathematical aspects and are therefore related to spontaneous mathematical focusing tendencies. Consistent with the bottom-up processes, SFON interventions using "SFON baits" that enhance the salience of exact number in a set of everyday materials in daycare settings led to significant improvements in SFON tendency (and related numerical skills) (Hannula et al., 2005; Hannula-Sormunen et al., 2020). At the same time, these interventions also utilized top-down influences like social interaction between the teachers and children (e.g., teachers asking questions to direct focus towards or raising motivational interest on exact numbers) (e.g., Hannula-Sormunen et al., 2020) that may make the use of exact number a valued goal. However, further work is needed to understand the mechanisms behind spontaneous focusing on mathematical aspects. A better understanding of why some children spontaneously focus on different mathematical features while others do not, could have valuable theoretical and practical educational implications.

Another open question for future research concerns the domain-specificity of SFONO tendency. First, as numerical order can be viewed as a distinct type of increasing pattern, where the rule of the pattern is always adding one item to the previous set, SFONO tendency may be closely related to SFOP tendency. More specifically, it is possible that children who are more inclined to recognize and use patterns (e.g., red-blue-red-blue) may also be more likely to recognize and use numerical order. Second, the SFONO tasks did not control for the overall surface area of the numerical sets. It is therefore possible that the children may have ordered the sets by relying on this continuous cue rather than on exact number. Even though the results of Study II showed that many children who spontaneously focused on numerical order also displayed numerical behaviour, such as counting, which suggests they ordered sets by exact number rather than by continuous cues like total surface area, this should be further explored. Therefore, although the developed SFONO tasks included numerical stimuli, it is still possible that they were reflective of children's spontaneous recognition and use of patterns or ordinality (i.e., increasing / decreasing sequence) in general. If this is the case, the association between SFONO tendency and numerical skills could possibly be explained by a combination of numerical ordering skills and general intelligence (i.e., pattern recognition). Thus, future studies should investigate the relation between SFONO and SFOP tendencies in same children, as well as control for the continuous ordinality cues in the SFONO tasks.

While Study II investigated the associations between SFONO and SFON tendency, and suggested them to be related but distinct constructs due to observing only a weak association between the two constructs and differing relations with numerical ordering skills, the present dissertation did not investigate the associations between SFONO tendency and other spontaneous mathematical focusing tendencies. Especially, as SFONO, SFOR, and SFOP tendencies all require relational thinking between sets of items, it could be assumed that the three tendencies are related. Thus, it is still an open question whether all these three tendencies are distinct constructs. For the present dissertation, this is an important open question, as it suggested SFONO tendency to be a distinct factor in numerical development. This highlights an important avenue for future research concerning spontaneous mathematical focusing tendencies in general. While there are only a few studies investigating the relations between two spontaneous mathematical focusing tendencies, mostly between SFON and another tendency (Verschaffel et al., 2020), there seems to be no studies investigating the relations between multiple different spontaneous mathematical focusing tendencies. Therefore, further empirical research is needed to determine whether, and to what extent, the different spontaneous mathematical focusing tendencies can be distinguished (Inglis, 2020; Verschaffel et al., 2020).

Another possible avenue for future research would be to investigate the construct of spontaneous mathematical focusing tendencies and their development. In the current literature, the different tendencies are suggested to be distinct attentional processes (e.g., Hannula & Lehtinen, 2005; McMullen et al., 2019), for which the joint term of *spontaneous mathematical focusing tendencies* is used as a broader umbrella concept. However, it is possible that some children are generally more inclined to spontaneously focus on mathematical aspects in their surroundings, depending on which mathematical information is relevant in situations of different contexts. From this perspective, all the suggested spontaneous mathematical focusing tendencies could stem from a broader tendency to focus on mathematical aspects in everyday lives in general.

Alternatively, it may be that the spontaneous mathematical focusing tendencies indeed are distinct attentional processes that trigger the use of the specific mathematical skills. This view aligns with theories of skill acquisition, such as Ericsson's (2016) work on expertise, which suggests that the nature of practice evolves as individuals progress in their abilities. Similarly, spontaneous mathematical focusing tendencies may emerge in response to the developmental relevance of particular skills (McMullen, 2014). For example, at the time when children are in the process of developing their cardinality skills, SFON may support this development. However, when more complex mathematical skills, such as numerical ordering skills, start to develop, SFONO may come into play. In addition, as the more complex spontaneous mathematical focusing tendencies may require

SFON (e.g., to notice a numerical order in sets of items, one may first need to spontaneously focus on how many items each set includes), SFON tendency may form the necessary basis for other spontaneous mathematical focusing tendencies (Li et al., 2025; Rathé et al., 2018; Verschaffel et al., 2020). Eventually, all the different spontaneous mathematical focusing tendencies may become joint features of flexible mathematical skills (see McMullen et al., 2022).

6 Conclusion

Overall, the findings of this dissertation shed light on the development of numerical ordering skills in early numeracy and provide novel insights into the role of spontaneous focusing on numerical order as a distinct attentional process. The results suggest that developing numerical ordering skills is a complex developmental process, closely tied to foundational numerical competencies, such as number sequence production and cardinality recognition. Furthermore, SFONO tendency was found to be positively associated with early numerical skills, particularly numerical ordering, even after accounting for other key predictors. This suggests that SFONO may represent a distinct factor in early numerical development and could support the development of numerical ordering skills.

These findings contribute to theoretical models of early numerical development by emphasizing the potential role of knowledge integration and spontaneous mathematical focusing tendencies in shaping individual differences in mathematical development. In sum, this work highlights the value of examining both spontaneous mathematical focusing tendencies and core numerical skills to gain a more comprehensive understanding of early numeracy. In particular, individual differences in numerical ordering skills, alongside spontaneous mathematical focusing tendencies such as SFONO, may provide valuable insights into the varied developmental pathways children follow. This knowledge can inform the design of assessment tools that capture these individual differences and support timely educational interventions before the start of formal schooling.

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