YOUNG CHILDREN'S SPONTANEOUS FOCUSING ON QUANTITATIVE ASPECTS AND VERBALIZATIONS OF THEIR QUANTITATIVE REASONING

McMullen, Jake A., Hannula-Sormunen, Minna M., & Lehtinen, Erno

Centre for Learning Research & Dept. of Teacher Education, University of Turku

This paper presents a cross-sectional study of young children's Spontaneous Focusing on quantitative Relations (SFOR), Spontaneous Focusing on Numerosity (SFON) and verbalizations of their quantitative reasoning. Two tasks were presented to 4.5 to 8 year old children (N=86) during two separate sessions, first in order to measure their SFOR and SFON tendencies and second in order to measure their verbalizations of their quantitative reasoning through stimulated response questions. Children were found to differ in their SFOR and SFON tendencies. Children's SFOR tendency increased with age. Children were more likely to perform the task based on SFOR or SFON than they were likely to verbalize about their quantitative reasoning during the stimulated recall session.

INTRODUCTION

Young children have a variety of skills for reasoning about quantitative relations already before school age (Boyer, Levine, & Huttenlocher, 2008; Sophian, 2000). Reasoning about quantitative relations, such as proportional relations or relations between numerosities, can be seen as the informal understanding of ratios and fractions and has been described as a "quantitative form of analogous reasoning" (Boyer et al., 2008, pg. 1479). Young children have been found to have well-developed reasoning abilities about a concept or skill long before they are able to verbalize these abilities (Becker, 1993; Sophian, 2000).Children's verbalizations about quantitative relations may provide a key link in the development from these informal concepts to formal mathematical concepts such as fractions.

Sophian (2000) examined young children's ability to verbalize their reasoning about proportional relations and found that this ability seemed to appear after they were already able to reason proportionally. In her study, 4- and 5-year-old children were asked to compare a model pair of circles and find the matching proportional pair of circles from two pairs of choices. After a set of 48 test trials, the children were asked to explain their choices on 6 post-test trials, being asked, "Why did you pick that one?" The 4-year-olds were unable to verbalize their reasoning, while still performing above chance on the task. 5-year-olds proved somewhat more able to verbalize their reasoning, while still performing above chance; though only 4 children (out of 20 5-year-olds) were able to explicitly relate size of the circles to each other.

Similarly, Becker (1993) found that 4- to 5.5-year-old children were unable to verbalize the correct answer to a many-to-one correspondence problem, but were able to distribute the correct amounts. A majority of the participants gave incorrect answers when asked how many items would be needed in total if x number of dolls needed 2 or 3 items each. However, when asked to distribute the number of items they had verbalized, almost all of the participants who gave incorrect answers were able to parcel out the items in correct amounts (e.g. 2 items for 1 doll), even though they would run out of items before giving all dolls their share.

There are substantial individual differences in how often young children focus their attention on exact numbers of items or incidents in their natural surroundings (Hannula & Lehtinen, 2001; 2005). A child's frequent Spontaneous Focusing On Numerosity (SFON) leads to increased practice with enumeration skills, which is crucial for the development of numerical skills and number concept (Hannula & Lehtinen, 2001; 2005; Hannula, Lepola, & Lehtinen, 2010). Focusing attention on more complex mathematical aspects of a task may be needed in order to use more advanced numerical skills beyond basic number recognition. Based on this notion, in the current study, we propose that there may exist differences in how often children spontaneously focus their attention on quantitative relations. Thus, Spontaneous Focusing On quantitative Relations (SFOR) is defined as the spontaneous (i.e. undirected) focusing of attention on quantitative relations and the use of these relations in reasoning (McMullen, Hannula-Sormunen & Lehtinen, in preparation).

By combining stimulated recall methods (cf. Sophian, 2000) with methods used to measure spontaneous focusing tendencies, the present study can provide insight into the development of young children's quantitative reasoning. Two tasks were presented to children during two separate sessions, first in order to measure their SFOR and SFON tendencies and second in order to measure verbalizations of their quantitative reasoning. In the first session, children were undirected to the mathematical nature of the task and completed the imitation task based on the aspect of the activity which they found most relevant, be it quantitative relations, numerosity, or non-mathematical aspects, without feedback. The tasks were reintroduced to the child in a second session with stimulated recall questions.

Research Questions

In the present cross-sectional study of three age groups, we aim to investigate children's SFOR and SFON tendencies and their abilities to verbalize about their quantitative reasoning. To this end, we ask two questions: 1) Do children's SFOR and SFON tendencies differ from their verbalizations of their mathematical reasoning during stimulated recall interviews? 2) Are there differences in children's verbalizations of their reasoning about quantitative relations, numerosities, or non-mathematical aspects?

METHODS

Video-recordings of 86 Finnish-speaking children (50% female), with no diagnosed learning impairments, were collected during two 30-minute sessions in a quiet room at the child's day-care or school by a trained male researcher. Children were between the ages of 4y; 5m and 8y; 4m (M= 6y; 8m; SD=1.0 years) at the time of their testing. Children were separated into three age-groups based on their placement in kindergarten (n=31; M_{AGE}=5y; 6m), pre-school (n=27; M_{AGE}=6y; 9m), or first grade (n=28; M_{AGE} =7y; 9m). Two tasks from the first session and stimulated recall interviews of the two tasks from the second session will be reported in this paper.

Bread Task

Two identical stuffed-dogs named "Nassu and Tassu", 20 cm in height, were used as characters that were fed bread. The breads were circular foam pieces 6.5cm in diameter, which were a different color for each trial. The whole breads, originally the same size, were cut into different proportions (halves, thirds, quarters or sixths), and disarranged on the plate in order to prevent the direct mapping of the area of the two sets of bread (Figure 1). The child was told that the bread "*have broken…but, Nassu and Tassu don't mind.*"

In the first session, Nassu and Tassu were introduced as being two friends who always do the same (e.g. run or jump the same, eat the same), and want the same for a snack (cf., Imitation task, Hannula & Lehtinen, 2005). Two plates of the breads were then placed on the table in front the child and in front of the researcher. The child was told "*Watch carefully what I give Nassu, and then you give Tassu exactly the same.*" The experimenter gave the bread from his plate to Nassu, one at a time, turned over his plate, and said, "*Now you give Tassu exactly the same.*" The child then gave from the plate in front of them to Tassu. The child got no feedback about his or her task performance. There were altogether 4 trials in the task (see Table 1).

Figure 1. Sets of bread, trials 1-4, child's plate on the left in each trial

For the stimulated recall task, the dogs were reintroduced and the child was told that they would be giving bread again and that they would be asked some questions. The instructions to "*watch carefully*" were repeated and the experimenter gave the same amounts as in the first session. However, the child was stopped before giving their bread and asked "*How do you know, from this bread (points to child's bread) what to give Tassu?*" To check their memory, the child was then asked, "*How many pieces of bread did I give Nassu?*" Finally, the child was allowed to give the bread to Tassu. Again no feedback was given to the child about his or her task performance.

Rice Task

Two stuffed-monkeys "Miina" and "Pate" were fed using two pairs of spoons (Figure 2), each pair proportional in height. Set A were plastic cylinders, the small spoon 3cm in diameter and 3cm high and the larger spoon (twice the size) 3cm in diameter and 6cm high. Set B were metal rectangular prisms, the small spoon was 2.5cm x 2.5cm x 2.66cm, and the big spoon (three times the size) was 2.5cm x 2.5cm x 8cm.

Trial	Researcher	Participant	Response Type		
Bread			Relation	Numerosity	Non-math
1	2 Halves- Give 1	4 Fourths	2 pieces	1 piece	3-4 pieces
2	6 Sixths- Give 2	3 Thirds	1	2	3-6
3	6 Sixths- Give 3	4 Fourths	2	3	1, 4-6
4	3 Thirds- Give 2	6 Sixths	4	2	1, 3, 5-6
Rice		Size of spoon			
1	Set A Big- Give 1	Small A	2 spoons	1 spoon	3+ spoons
2	Set B Big- Give 1	Small B	3	1	2, 4+
3	Set B Small- Give 3	Big B	1	3	2, 4+
4	Set A Big- Give 2	Small A	4	2	1, 3, 5+

Table 1. Task trials. Researcher and Participants materials and possible responses.

In the first session, the monkeys were said to be brother and sister who like to have the same for lunch. A bowl of rice was placed on the table in front of each the monkeys and an empty bowl was placed in front of each these bowls. The plastic spoons were held up for possible comparison and the child was told that "*we will use these spoons*" and that "*Pate and Miina always want full spoonfuls*". For the first trial, the smaller spoon was placed in the bowl in front of the child, the experimenter held the larger spoon, and then said "*watch carefully what I give to Pate, and you give Miina exactly the same*." The experimenter placed one spoon of rice in the bowl and then asked the child to "*give exactly the same*." No feedback was given to the child. In total there were four trials (see Table 1).

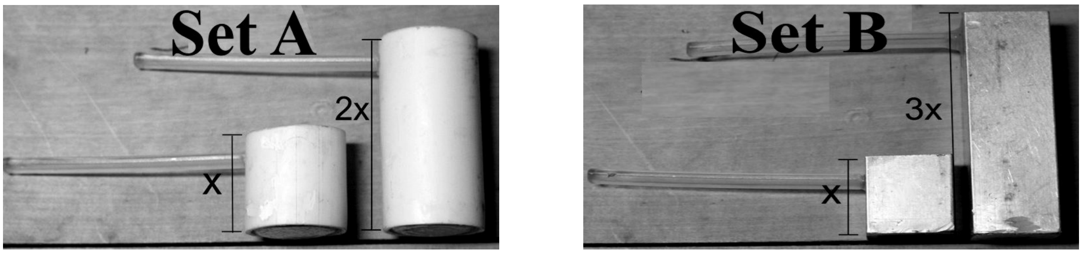

Figure 2. Rice Task spoons. See Table 1 for more detail.

For the stimulated recall task, a method similar to the bread interview was used. The child was told that they would "*give Pate and Miina rice again,*" and the child would be asked a couple of questions. The child was told to "*watch carefully,*" and the experimenter gave the same amounts as in the first session. The child was asked to wait before giving rice to Miina and asked, "*How do you know what to give Miina now?*" and "*How many spoonfuls did I give Pate?*" The child was then allowed to give rice to Miina. No feedback was given about the child's task performance.

Analysis

For the tasks in the first session, task performance was analyzed for each trial based on the most complex aspect on which the child focused, with quantitative relations (SFOR) being the most complex, then numerosity (SFON), and finally non-mathematical aspects. A child's scores were based on, a) the number of pieces or spoonfuls given (see Table 1) or b) any utterances, classified as relational (e.g., "You put 2 of those big ones, so I should put 4 small ones"), numerical ("Now just one!"), or non-mathematical. The maximum total score for each task was 4. Two independent raters analyzed 21% of the participants' responses and agreement was found on 98% of the trials for the bread task, and 97% of the trials on the rice task.

For the stimulated recall tasks, the child's response to the first question, "*How do you know…what to give…?*" was analyzed to determine if the child's answer was based on either a) quantitative relations (e.g. "*because two pieces like this are formed by one of those*"), b) numerosity (e.g., "*One bread, because you gave one as well.*"), or c) non-mathematical aspects, including no explanation (e.g., "*This piece, because you gave from the same location.", "I don't know"*) Again, the response was coded based on the highest level that the child discussed in this answer. 21% of the trials were analyzed by two independent raters and 94% agreement was found on both tasks.

RESULTS

Table 2 and Figure 2 display evidence that children differ in their SFOR and SFON tendencies during their performance on these tasks, as well as in their verbalizations of their reasoning during the stimulated recall tasks. Intraclass correlations calculated for all trials across the bread and rice tasks for the responses to the task performance, ICC=0.85, and for the stimulated recall answers, ICC=0.88, were found to be sufficiently high enough to warrant combining both the bread and rice tasks in analysis.

Task	Relational	Numerosity	Non-Math
Task Performance			
Bread	17.2 %	61.9 %	20.9 %
Rice	14.8 %	66.3 %	18.9 %
Total	16.0 %	64.1 %	19.9 %
Stimulated Recall			
Bread	18.3 %	20.9 %	60.8 %
Rice	15.4 %	23.8 %	60.8 %
Total	16.9 %	22.3 %	60.8 %

Table 2. Percentage of responses by reasoning level. ($N_{bread\ \&\ rice}$=344; N_{total}=688)

A 2 x 2 x 3 ANOVA [(Task Performance, Stimulated Recall) x Reasoning Level (Relational, Numerosity) x Age Group (Kindergarten, Pre-school, First Grade)] was run for children's responses. Main effects of task ($F(1, 83)=103.37$, $p<0.001$), reasoning level ($F(1, 83)=38.01$, $p<0.001$), and age group ($F(2, 83)=6.58$, $p<0.01$) were significant. Children were a) more likely to spontaneously focus on mathematical aspects in their task performance than verbalize about their quantitative reasoning, b) more likely to spontaneously focus on or verbalize about numerosity than quantitative relations, and c) more likely to perform the task based on SFOR and SFON and verbalize about their mathematical reasoning the older they were. Interaction effects of Reasoning Level x Age Group ($F(2, 83)=6.73$, $p<0.01$) and Task x Reasoning Level ($F(2, 83)=63.45$, $p<0.001$) were also significant. First graders were more likely to reason about quantitative relations than younger children and the difference between children's relational and numerical reasoning was larger in their task performance than in their verbalizations.

Planned pair-wise comparisons were run in order to compare children's SFOR and SFON scores with their verbalizations of their mathematical reasoning. Children displayed SFOR in their task performance with the same frequency as they verbalized about quantitative relations, $F(1, 83)=0.16$, p=ns. However, children displayed SFON in their task performance significantly more than they verbalized about their numerical reasoning, $F(1, 83)=98.22$, $p<0.001$. This difference was seen in all age groups separately (all $p<0.001$).

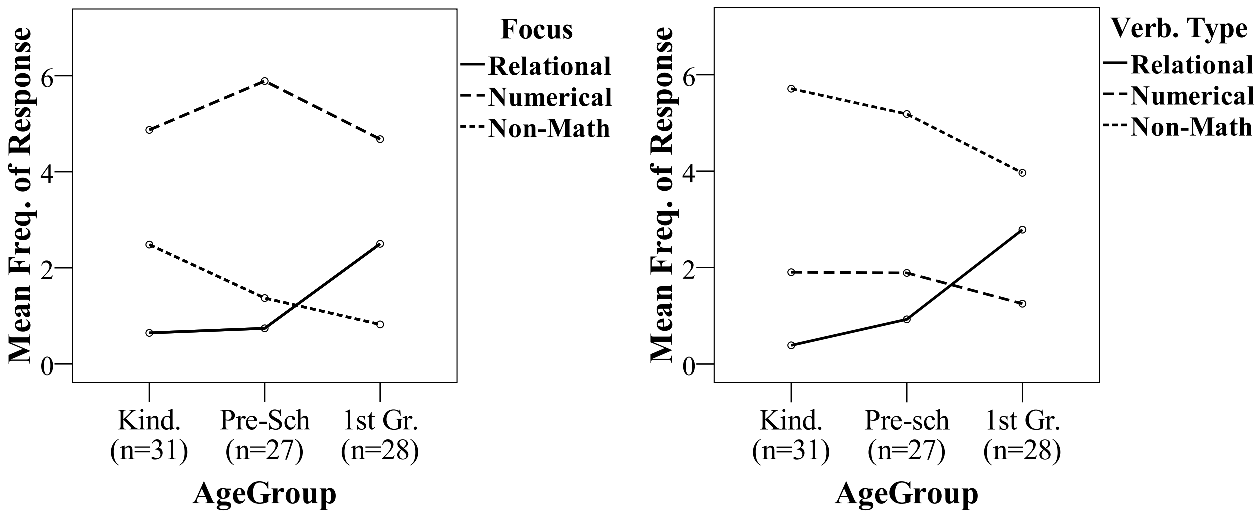

Figure 2 - Mean frequencies of spontaneous focus in the task performance (left), verbalizations (right) by Age

Children's non-mathematical verbalizations were categorized (Table 3). For the non-mathematical responses children verbalized about a) the act or way of giving ("*I have to give this rice because you gave that rice.*"), b) location ("*This one, because you gave from the same spot.*"), c) shape or size ("*That one, because it's shaped like a party hat…*"), d) the nature of the bread or rice (*"Bread", "Rice", "Breakfast"*), or e) no explanation ("*I don't know*").

Task	Giving	Location	Size/Shape	Bread/Rice	No Response
Bread	20.6 %	10.5 %	7.2%	18.6%	43.1%
Rice	33.5%	3.8%	3.8%	32.5%	26.4%
Total	27.0%	7.1%	5.5%	25.6%	34.8%

Table 3. Non-mathematical verbalizations by category. (N=418 responses)

DISCUSSION

Previous studies have found that there are substantial individual differences in children's SFON tendency (Hannula & Lehtinen, 2001; 2005; Hannula et al., 2010). The present study indicates that there are also differences in children's SFOR tendency in tasks that can be attended to based on both mathematical and non-mathematical aspects. Children were also found to increase in their SFOR tendency with age. The novel methods of this study seem promising for the isolation of SFOR tendency in children's task performance. These findings suggest that spontaneous attentional processes may be relevant for more areas of mathematical development, including the recognition and reasoning about quantitative relations.

Prior studies have found that children's ability to verbalize about their mathematical reasoning lags behind their ability to display this reasoning in their task performance (Becker, 1993; Sophian, 2000). The results reported in this study show a similar pattern, with children being less likely to verbalize about their quantitative reasoning than they display spontaneous focusing on mathematical aspects in their task performance. The majority of children displayed SFON in the completion of these tasks, but a majority were then unable or unwilling to verbalize about their quantitative reasoning when asked to do so on a subsequent occasion. However, this discrepancy between spontaneous focusing and verbalizations of reasoning does not hold true for all levels of mathematical reasoning. The frequency of children's task performance based on SFOR did not significantly differ from the frequency of verbalizations based on quantitative relations. This discrepancy can be a result of several reasons.

One interpretation of these findings is that the wording of the stimulated recall question may have influenced children's responses, as linguistic cues have been found to influence children's answers (Wagner & Carey, 2003). The stimulated recall question specifically directed the child toward his/her own bread or rice, asking "*How do you know, from this bread, what to give…"* Those who were more likely to spontaneously focus on numerosity in these tasks may have found this question confusing, as they may have focused only on the researcher's portion of the task (e.g. the number of breads the researcher gave), whereas those children who were more likely to spontaneously focus on quantitative relations during these tasks would have needed to focus on both sets of bread or spoons of rice to determine what to give. This confusion among those who were more likely to focus on number may have lead to the relatively low frequency of numerical verbal responses during the stimulated recall session. The sizable number of responses not providing any substantive reasoning (categories d and e, of non-mathematical responses) could be a result of this confusion.

One alternative explanation for the large disparity between children's SFON tendency in the task performance and numerical verbalizations in the stimulated recall is that numerical aspects of a situation may be more automatically perceived than quantitative relational aspects, which may require more active processing in order to be perceived. SFOR requires both the recognition of numerosity in the two sets of items and relating of the sizes and numerosities involved with respect to each other. It is plausible that this requires more deliberate reasoning than pure recognition of numerosity. More studies are needed to fully understand these preliminary findings. It is also possible that those children who have a well-developed understanding of quantitative relations may be more likely spontaneously focus on quantitative relations in completing these tasks and, as well, be more able to verbalize their reasoning about relations. This could be a consequence of stronger general cognitive abilities in children with high SFOR tendency or possibly the influence of schooling, as the first graders are more likely to have had to develop the ability to verbalize their reasoning in the classroom.

The findings of this cross-sectional study provide a better understanding of the role of spontaneous attentional processes and verbalizations in children's mathematical reasoning. Children are found to have differed in their SFOR tendency in these tasks and this SFOR tendency seems to be related to the ability to verbalize about quantitative relations. However, conclusions regarding true developmental patterns require a longitudinal study of these skills. Nonetheless, unguided imitation tasks seem to be potential measures for the isolation of spontaneous attentional processes, such as SFON and SFOR. Finally, the combination of these tasks with carefully constructed, open-ended, stimulated recall tasks may provide an even more useful tool for the investigation of children's mathematical reasoning.

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