Solar Physics (2026) 301:6
https://doi.org/10.1007/s11207-025-02600-2

RESEARCH

Long Duration Gamma-Ray Flares: The Number Problem

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Received: 31 March 2025 / Accepted: 16 December 2025

Abstract

Long-duration gamma-ray flare (LDGRF) events observed by the Fermi/ Large Area Telescope have presented a puzzle for modelers. One of the ideas to account for their long duration is to assume that particles accelerated at shocks driven by fast coronal mass ejections (CMEs) would be the able to be trapped in the space between the shock and the Sun and would be able to precipitate from this volume over an extended amount of time. We present a simple leaky-box type model for the precipitating > 500 MeV proton flux in a system, where a coronal shock feeds accelerated protons into the volume between the shock and the solar surface and a relatively small amount of scattering keeps the distribution isotropic and homogeneous inside the volume. We demonstrate that by choosing fully realistic shock parameters the total number of precipitating protons can be brought to an agreement with observations. We also demonstrate that durations of several hours for these events are fully within reach of the modeling without using unreasonable choices for parameters. Thus, CME-driven shocks in the coronal and inner solar wind plasma are a plausible candidate to account for the LDGRF events.

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Keywords Energetic particles, acceleration · Energetic particles, propagation · Energetic particles, protons · Waves, shock

1. Introduction

It is well known (e.g. Desai and Giacalone 2016) that solar energetic particle (SEP) events – temporary intensity enhancements of ions and electrons in a range of energies from suprathermal to relativistic, observed in the interplanetary (IP) space by a spacecraft – are associated with solar eruptive processes, flares and coronal mass ejections (CMEs). However, how SEPs gain their energy is not fully understood. One of the long-standing problems of the SEP physics is the acceleration of protons (and heavier ions) to the highest, relativistic energies (Desai and Giacalone 2016; Klein and Dalla 2017). Such particles can be detected not only from space but also on the ground via registration of enhancements of secondary particles caused by nuclear interactions with the atmosphere (e.g., registration of neutrons with neutron monitors). Such secondary particle enhancements are known as ground level enhancements (GLEs) (e.g. Shea and Smart 2012). In spite of a wealth of observational data, even the site(s) of acceleration of GLE producing particles within the flare-CME system is still an open question. A flare, a CME-driven shock, and the current sheet environment forming in the wake of an erupting CME, all these are possible acceleration sites.

A related problem is the one of the so-called long-duration gamma-ray flare (LDGRF) events. These are temporary enhancements of solar γ-ray emission with energies $E > 100$ MeV associated with flare-CME eruptions, but lasting significantly longer (sometimes for many hours) than other electromagnetic emissions related to the flare impulsive phase. Being observed quite rarely in the past, such events appear to be more frequent as observations with the Fermi/Large Area Telescope (LAT) reveal (Ajello et al. 2021). The γ-ray emission itself occurs due to the decay of pions produced in interactions of relativistic protons and α-particles with the solar chromosphere and photosphere (Share et al. 2018). However, the presence of the high-energy, pion-producing particles in the corona for a long time, while the other energetic particles (electrons) responsible for the X-ray and radio (microwave) emissions are absent, remains a puzzle. Several hypotheses exist in this regard. According to the flare-loop hypothesis, the high-energy particles are accelerated by a flare-related process, fill up large-scale coronal loops and stay trapped and possibly reaccelerated there (Ryan and Lee 1991). Another, shock, hypothesis ascribes the high-energy particle source to the propagating CME-driven shock, from where the accelerated particles have to travel back to the Sun in order to produce γ-rays (Cliver, Kahler, and Vestrand 1993). Recently, the current sheet hypothesis has received some attention, according to which the particle source is associated with the current sheet forming in the wake of an erupting CME (Kocharov et al. 2020). Kocharov et al. (2021) argued based on detailed observational analysis that multiple sources of relativistic particles, associated with the flare, the CME shock, and the CME current sheet, can contribute to the September 10, 2017, LDGRF event.

Each hypothesis has its pros and cons. In particular, the shock hypothesis naturally provides the high-energy particle source that in principle could be active on the time scale of a LDGRF event. However, the main issue here is that particles escaping from the shock vicinity and propagating toward the Sun should experience the effect of magnetic mirroring, which reduces the number of particles able to precipitate in the chromosphere. The effect must become more severe with the shock moving away from the Sun. At the same time, particle scattering on magnetic turbulence is expected to facilitate populating the distribution loss cone, increasing the chance for precipitation. This was investigated in a number of modeling studies (Afanasiev et al. 2018; Hutchinson et al. 2022).

In particular, Hutchinson et al. (2022) modeled the back-propagation of 300 MeV protons that were injected from a moving source mimicking a small portion of a CME shock in the Parker-type interplanetary magnetic field. The modeling was done using full-orbit particle transport simulations in which pitch-angle scattering was included. It was found that, although scattering indeed enhances the particle precipitation compared to the scatter-free case, the total (simulation-integrated) fraction of precipitated particles was always below a few percent. Moreover, the simulated precipitation rate as a function of the source distance from the Sun was found to be inconsistent (too fast decrease) with the observed time profile of the γ-ray flux.

The fractions of precipitated particles obtained by Hutchinson et al. (2022) for a number of LDGRFs are much smaller than those derived from the observations by Bruno et al. (2023) for most of the events. Based on this disagreement, Bruno et al. (2023) disqualified the shock hypothesis of the LDGRF origin. It should be noted, however, that the precipitation fractions obtained by Bruno et al. (2023) are based on the SEP measurements at the energy of 80 MeV at 1 AU. Therefore, there is still a possibility that their precipitation fractions are overestimated, if the (event-integrated) number of 80 MeV protons actually ejected from the shock is larger than the number of particles observed at 1 AU. This would be the case, if the acceleration process is controlled by self-generated waves that trap a significant amount of accelerated particles close to the shock, in the foreshock region (see, e.g., Vainio et al. 2014). How relevant this acceleration scenario, at so high energies, is, has been unfortunately poorly explored so far (see Ng and Reames 2008, for an example). It should also be noted that the calculations of Hutchinson et al. (2022) are made under the assumption that the shock is completely transparent for particles mirrored from the corona and propagating away from the Sun. This assumption does not in fact agree with the diffusive mechanism of particle acceleration, according to which the turbulent region downstream of the shock has to have a large extent to allow particles to cross the shock multiple times (e.g. Ostrowski and Schlickeiser 1996). This also leads to it being very difficult for particles to get back to the shock from far downstream. Therefore, for a transparent shock a different mechanism should be assumed to accelerate particles to hundreds of MeV energies at the shock.

The idea of this study is to compute the number of precipitated high-energy (> 500 MeV) protons applying the theory of diffusive particle acceleration at the shock and considering a generic model of the particle transport from the downstream vicinity of the shock to the Sun, which is free of the details of the particle scattering process. The goal is to investigate whether the characteristics (the total number of precipitating > 500-MeV protons and the event duration) of the observed LDGRFs can be reproduced.

The article is organized as follows. The model is presented in Section 2, calculation results and discussion are included in Section 3, and conclusions in Section 4.

2. The Model

We use a simple model to evaluate whether a propagating coronal shock is able to produce enough protons at high energies to account for the pion-decay γ-ray emission from the Sun at time scales relevant to the LDGRFs. We present a leaky-box type model that considers particle injection from the shock to the region between the shock and the Sun, the precipitation of particles from this region to the solar surface and adiabatic cooling of particles in the expanding reservoir (Figure 1). The equation we solve is

$$\frac{\partial N_{\mathrm{p}}}{\partial t} + \frac{\partial}{\partial y}\left(\dot{y} N_{\mathrm{p}}\right) = Q_{\mathrm{p}} - \gamma N_{\mathrm{p}}, \tag{1}$$

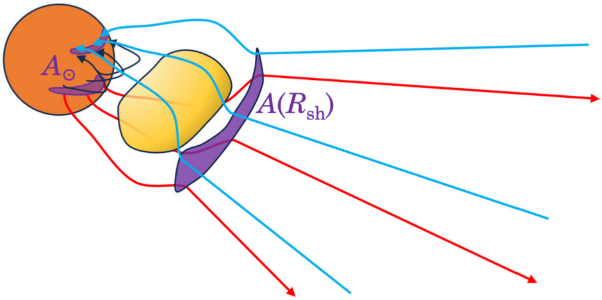

Figure 1 Sketch of the model geometry. The volume hosting the accelerated particles extends from the solar surface to the shock and the cross-sectional area of the volume is $A(R_{\mathrm{sh}})$ at the location of the shock; the corresponding area at the solar surface is $A_\odot$ (here split between two regions with different magnetic polarities, but the model does not depend on this feature). The yellow cloud represents the CME "piston" that is driving the shock wave.

where N_{p} is the number of protons per unit logarithmic momentum interval, $y = \ln(p/mc)$ is the logarithm of momentum p measured in natural units mc with m and c the proton mass and speed of light, respectively, $\dot{y} = -\frac{1}{3}\mathrm{d}\ln V/\mathrm{d}t$ is the adiabatic momentum change rate related to the expansion of the volume V hosting the particles, Q_{p} is the source function (i.e., rate of protons per unit logarithmic momentum interval entering the volume), and

$$\gamma N_{\mathrm{p}} = \frac{v A_\odot}{4V} N_{\mathrm{p}} \tag{2}$$

is the rate of precipitation of protons of speed v from the volume to the surface of the Sun. Here, we have assumed that the density of protons at the inner boundary can be approximated as their total number divided by the volume, and that the particle distribution is also isotropic inside the volume. Finally, $A_\odot$ is the cross-sectional area of the volume at the solar surface. The model does not allow the particles to escape the volume in any other way than precipitation so it only assesses the upper limit of the precipitating particles.

Since $\dot{y}$ is independent of y, Equation 1 can be written as a quasi-linear partial differential equation,

$$\frac{\partial N_{\mathrm{p}}}{\partial t} - \frac{1}{3V}\frac{\mathrm{d}V}{\mathrm{d}t}\frac{\partial N_{\mathrm{p}}}{\partial y} = Q_{\mathrm{p}} - \frac{v A_\odot}{4V} N_{\mathrm{p}}, \tag{3}$$

which can be solved with the method of characteristics. The characteristic curves in the (t, y) plane are given as

$$y(t') = y - \frac{1}{3}\ln\frac{V(t')}{V(t)}, \quad 0 \le t' \le t, \tag{4}$$

and the solution can be obtained by integrating the Equation 3 along the characteristics from $(0, y + \frac{1}{3}\ln\frac{V(t)}{V(0)})$ to (t, y):

$$N_{\mathrm{p}}(t, p) = \exp\left\{-\int_0^t \frac{v' A_\odot}{4V(t')}\mathrm{d}t'\right\}\Bigg[N_{\mathrm{p}}(0, p_0) + \int_0^t Q_{\mathrm{p}}(t', p')\exp\left\{\int_0^{t'} \frac{v'' A_\odot}{4V(t'')}\mathrm{d}t''\right\}\mathrm{d}t'\Bigg], \tag{5}$$

where $p_0 = p[V(t)/V(0)]^{1/3}$ and

$$v' = p'c/\sqrt{m^2c^2 + p'^2} \tag{6}$$

$$p' = mc\,\mathrm{e}^{y(t')} = p[V(t)/V(t')]^{1/3} \tag{7}$$

$$v'' = p''c/\sqrt{m^2c^2 + p''^2} \tag{8}$$

$$p'' = p'[V(t')/V(t'')]^{1/3} = p[V(t)/V(t'')]^{1/3}. \tag{9}$$

Thus, we can write

$$N_\mathrm{p}(t,p) = \mathrm{e}^{-G(t)} N_\mathrm{p}(0,p_0) + \int_0^t \mathrm{e}^{-G(t)+G(t')} Q_\mathrm{p}(t',p')\,\mathrm{d}t' \tag{10}$$

$$G(t) = \int_0^t \frac{v'' A_\odot}{4V(t'')}\,\mathrm{d}t''. \tag{11}$$

We model the source function as being due to protons emitted downstream from a coronal shock wave propagating at speed U_sh outwards along the magnetic field. We take the solar wind speed along the magnetic field as u_sw and, thus, the upstream flow speed in the de Hoffmann–Teller frame (HTF) as $u_1 = U_\mathrm{sh} - u_\mathrm{sw}$. The downstream flow speed in HTF is $u_2 = u_1 r_B / r_\mathrm{gas}$, where r_B and r_gas are the magnetic and gas compression ratios of the shock. Thus, we write

$$Q_\mathrm{p}(t',p') = u_2(t') A_\mathrm{sh}(t')\, 4\pi p'^3 f_\mathrm{sh}(t',p') \tag{12}$$

where $A_\mathrm{sh}(t')$ is the cross-sectional area of the considered volume at the shock and $f_\mathrm{sh}(t',p')$ is the distribution function of protons at the shock. Here, we have assumed that there is a co-moving strongly turbulent region just downstream of the shock where protons are advected to a region further downstream of the shock where the transport is no longer advective but dominated by diffusion or even free streaming.

The distribution function at the shock can be given as (e.g. Vainio et al. 2014)

$$f_\mathrm{sh}(t',p') = \frac{\sigma \epsilon_\mathrm{inj} n_\mathrm{p}(R'_\mathrm{sh})}{4\pi p_\mathrm{inj}^3} \left(\frac{p'}{p_\mathrm{inj}}\right)^{-\sigma} g(t',p') \tag{13}$$

where $R'_\mathrm{sh} = R_\mathrm{sh}(t')$ is the shock location, p_inj is the injection momentum, ϵ_inj is the injection efficiency of the shock, $n_\mathrm{p}(R)$ is the plasma proton density as a function of radial distance,

$$\sigma = \frac{3r_\mathrm{sc}}{r_\mathrm{sc} - 1} \tag{14}$$

is the spectral index of the accelerated protons depending on the scattering center compression ratio of the shock, r_sc, and $g(t',p')$ is a function that implements the softening of the spectrum at high energies, often taken as an exponential roll-over. Motivated by the results presented by Raukunen et al. (2018) for a large sample of > 300 MeV proton events observed at Earth, we take the high-momentum form of the spectrum to be a steep power law instead

$$g(p') = \frac{1}{1 + (p'/p_\mathrm{c})^{\delta-\sigma}}, \tag{15}$$

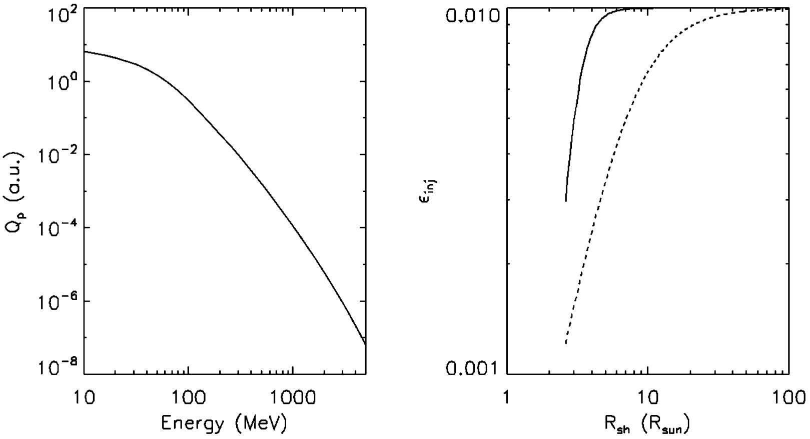

Figure 2 Left: spectral form of the injected proton spectrum from the shock with arbitrary normalization, with the cutoff momentum corresponding to a kinetic energy of 50 MeV. Right: spatial dependence of the injection efficiency, $\epsilon_{\rm inj}$, for $\kappa = 6$, $R_{\rm inj} = 3\, R_\odot$ (solid curve) and $\kappa = 2$, $R_{\rm inj} = 7\, R_\odot$ (dashed curve).

where $p_{\rm c}$ is the cutoff momentum in the spectrum and δ is a parameter fixing the asymptotic behavior of the spectrum. Thus,

$$Q_{\rm p}(R'_{\rm sh}, p') = u_2(R'_{\rm sh}) A(R'_{\rm sh})\, \sigma \epsilon_{\rm inj} n_{\rm p}(R'_{\rm sh}) \frac{(p'/p_{\rm inj})^{3-\sigma}}{1 + (p'/p_{\rm c})^{9-\sigma}}, \tag{16}$$

where we fix δ so that the source has a p^{-6} behavior at high momenta (Raukunen et al. 2018). The spectral form of the injected particle spectrum $Q_{\rm p}$ as a function of kinetic energy is plotted in Figure 2 (left panel) for $\sigma = 4$ and a cutoff momentum corresponding to a kinetic energy of 50 MeV.

In principle, all parameters of the shock can be functions of shock position or time explicitly, but we take a simplified approach and consider $U_{\rm sh}$ and $r_{\rm sc} = r_{\rm gas}$ as constants and take $r_B = 1$ (a parallel shock). For the plasma proton density, we consider a simple model

$$n_{\rm p}(R) = n_\odot (R_\odot/R)^6 + n_\oplus (R_\oplus/R)^2, \tag{17}$$

where $R_\odot = 6.96\ 10^5$ km is the solar radius, $R_\oplus = 1$ AU, $n_\oplus = 7\ {\rm cm}^{-3}$ is the density at 1 AU and $n_\odot = 3\ 10^8\ {\rm cm}^{-3}$ is the density at the coronal base. We take the magnetic field lines to be radial, and consider a solar wind speed derived from the conservation of mass as

$$R^2 u_{\rm sw}(R) n_{\rm p}(R) = R_\oplus^2 u_\oplus n_\oplus, \tag{18}$$

where $u_\oplus = 400\ {\rm km\ s}^{-1}$.

For the cross-sectional area of the volume, we employ $A(R) = R^2$ for distances $R > R_0$, where $R_0 = 2.5\, R_\odot$ is the initial distance of the shock, i.e., assuming that the cross-sectional area above $R > R_0$ corresponds to a solid angle of 1 sr in coordinate space. The volume, therefore, is

$$V(R) = V_0 + \tfrac{1}{3}(R^3 - R_0^3), \tag{19}$$

where V_0 is the initial volume. We assume that the cross-sectional area of the volume is proportional to R^3 below the initial distance and, hence, take

$$V_0 = \tfrac{1}{4}(R_0^4 - R_\odot^4)R_0^{-1} \tag{20}$$

$$A_\odot = R_\odot^3 R_0^{-1}. \tag{21}$$

These assumptions are roughly consistent with coronal conditions outside active regions with modest over-expansion of the magnetic field relative to radial, and they correspond to a solid angle of 0.4 sr of the precipitation region at the surface of the Sun, i.e., about 3.2% of the total area of the surface. Note that the cross-sectional area of the global coronal/interplanetary shock may well scale differently from the R^2 scaling assumed here and even reach a circumsolar extent in extreme cases (e.g. Wijsen et al. 2025). However, if we consider only the strongest regions of the shock to be able to accelerate particles to the highest energies, the area is most likely limited to parts of the shock with most favorable conditions, presumably close to the nose of the shock where it is strongest (e.g. Desai and Giacalone 2016). Since CMEs tend to expand in a self-similar way after the initial phase of lateral expansion (e.g. Balmaceda et al. 2020), this scaling law seems reasonable. Note also that if the spatial extent of the flux tube has a different value Ω_0 in solid angle at $R > R_0$, the results can be obtained by scaling the fluxes with $\Omega_0/(1\ \mathrm{sr})$.

We take the injection momentum to be $p_{\mathrm{inj}} = m u_1$. We employ a model where the shock source switches on gradually, i.e.,

$$\epsilon_{\mathrm{inj}} = \frac{\epsilon_\infty}{1 + (R_{\mathrm{inj}}/R)^\kappa}, \tag{22}$$

where R_{inj} is the distance of half of the maximum injection, ϵ_∞, and κ determines the steepness of the rise of the injection as a function of time (see Figure 2, right panel). We also couple the injection efficiency and the cutoff momentum by (Vainio et al. 2014)

$$p_{\mathrm{c}}(R) = p_{\mathrm{c},\infty}\left(\frac{\epsilon_{\mathrm{inj}}}{\epsilon_\infty}\right)^{1/(\sigma-3)}, \tag{23}$$

where $p_{\mathrm{c},\infty}$ is the cutoff momentum of the spectrum. These choices allow us to mimic the finite acceleration time of relativistic protons in the coronal shock. For the initial proton spectrum, related to earlier phases of acceleration (e.g., the flare) we use

$$N_{\mathrm{p}}(0, p_0) = N_0 \frac{(p_{\mathrm{inj}}/p_0)^{3-\sigma}}{1 + (p_0/p_{\mathrm{c},\infty})^{9-\sigma}}, \tag{24}$$

where N_0 fixes the total number of protons at $p_0 > p_{\mathrm{inj}}$.

It should be emphasized that R_{inj} is not the distance of initial particle injection, rather it characterizes the height at which the injection efficiency achieves half of its asymptotic value and, together the κ parameter, determines the scale of increase of the shock acceleration efficiency. Therefore, it should not be directly compared with, e.g., particle injection heights derived from GLE observations (e.g. Reames 2009), corresponding to impulsive (prompt) components of GLEs. Prompt injections into the interplanetary medium could well be connected to the impulsive phase of the gamma-ray flare. In the present model, particles generating the impulsive component of the gamma-ray flare are modeled as an initial particle distribution in the volume hosting the accelerated particles, i.e., the model does not account for their acceleration mechanism or time evolution.

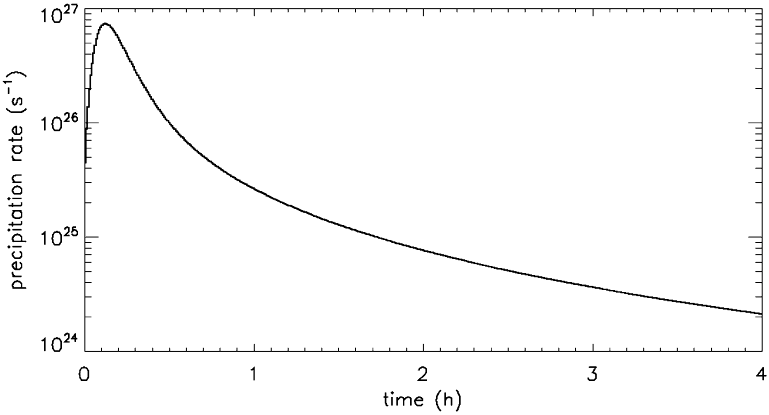

Figure 3 Rate of precipitation of > 500 MeV protons at the solar surface behind a shock propagating through the corona at 2300 km s^{-1}. See text for the details of the simulation.

We emphasize that the model is purely phenomenological and does not correspond to a detailed self-consistent calculation of shock-accelerated protons. It, however, allows us to estimate the maximum number and precipitation rate of protons that can produce pion-decay γ-rays for an event of maximal duration.

3. Results and Discussion

We consider the model with the following parameters: shock speed $V_{\mathrm{sh}} = 2300$ km s^{-1} and a compression ratio $r_{\mathrm{sc}} = 3$, corresponding to a fast CME driving a moderately strong shock; injection distance $R_{\mathrm{inj}} = 3\,R_\odot$, $\kappa = 6$ and injection efficiency $\epsilon_\infty = 0.01$; cutoff momentum $p_{\mathrm{c},\infty} = c^{-1}\sqrt{E_{\mathrm{c}}^2 + 2mc^2 E_{\mathrm{c}}}$, corresponding to kinetic energy of $E_{\mathrm{c}} = 50$ MeV. The initial number of protons in the system is set to $N_0 = 10^{31}$.

The rate of precipitation of > 500 MeV protons is plotted in Figure 3. The total number of precipitated protons is $8.2\ 10^{29}$, when integrated over a 50 $R_\odot$ propagation distance of the shock from the source surface. The model produces time scales of the order of hours and the long-time decay of the precipitation rate is similar to $-\gamma N_{\mathrm{p}} \propto t^{-2}$.

Comparing the result with values given by Bruno et al. (2023) for LDGRF events related to large SEP events, the computed value of the total number of > 500 MeV protons precipitated at the solar surface ranks among the largest, with only one event with a greater value obtained with the methodology of Ajello et al. (2021). That event, i.e., the LDGRF of March 7, 2012, had an extremely high speed of the CME (3146 km s^{-1}). Taking the shock speed as 3200 km s^{-1} and a somewhat larger value of the compression ratio, $r_{\mathrm{c}} = 3.3$, gives us $1.9\ 10^{30} > 500$ MeV protons precipitating, matching the value derived for the event by Ajello et al. (2021). This, of course, is but one combination of parameters that reproduces the number and should not be taken as anything else than a demonstration that even the largest LDGRF events can in principle be reproduced by the shock model, at least when it comes to the number of precipitating protons. However, it would be difficult for the model, at least with the present parameterization, to account for the extremely long duration of the event (~ 20 hours).

To investigate the possibility to obtain slower time evolution of the events, we use more gradual rise of the injection efficiency with $R_{\mathrm{inj}} = 7\,R_\odot$ and $\kappa = 2$, a strong shock with

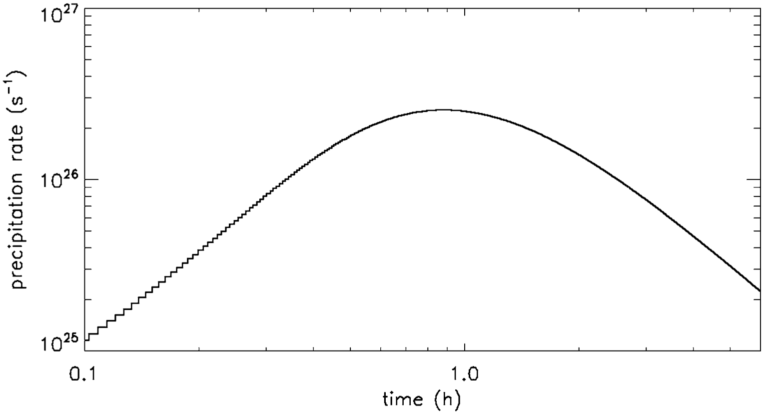

Figure 4 Rate of precipitation of > 500 MeV protons at the solar surface behind a shock propagating through the corona at 3200 km s^{-1}. See text for the details of the model parameters. Note the logarithmic time axis.

$r_c = 4$ and $E_c = 100$ MeV, and $N_0 = 10^{32}$, while keeping the shock speed at 3200 km s^{-1}. The resulting precipitation profile is plotted in Figure 4. The total number of > 500 MeV protons precipitating during the simulated shock propagation distance of 100 $R_\odot$ is 2.1 10^{30}, but now decay of the event is significantly slower. Extrapolating the decay to about 20 hours indicates that the event could be detectable at the longest time scales of the LDGRF events.

The number of precipitating > 500 MeV protons quoted by Bruno et al. (2023) varies by three orders of magnitude, between $\sim$2 10^{27} and $\sim$2 10^{30}. Possible reasons for lower numbers of protons than those calculated above are numerous, among others: (1) the shock can be less efficient in accelerating high-energy protons (lower ϵ_{inj} or E_c); (2) the protons may experience strong advection away from the solar surface if the turbulent region behind the shock fills the volume behind the shock and the solar surface, which could be particularly important when the shock is still close to the Sun; (3) there may be other losses than precipitation of protons from the volume between the shock and the Sun (e.g., via perpendicular diffusion); or (4) the cross-sectional area of the acceleration volume at the solar surface is significantly smaller than the value used in our study.

The presented model employs a distribution function at the shock, so in principle it is possible to evaluate the flux of particles escaping in the upstream region (to be detected by spacecraft observations) provided that a magnetic connection to the shock wave exists and that the upstream scattering mean free path is known. If the proton acceleration to energies exceeding 100 MeV occurs mainly due to self-generated waves in the upstream region, the upstream mean free path can be expected to be a rather complicated function of plasma and shock magnetohydrodynamic (MHD) parameters as well as of the shock acceleration efficiency (see, e.g., Afanasiev, Wijsen, and Vainio 2025). Furthermore, self-generated waves can lead to streaming-limited particle fluxes at 1 AU (Reames and Ng 1998), which prevents a direct comparison of the particle intensity at the shock and in the far upstream region. Other processes like, e.g., the loss processes due to perpendicular transport in the downstream region may contribute to the SEP population measured at 1 AU (see also Kocharov et al. 2015). We feel it is beyond the reach of the simple model presented here to address the particles escaping from the upstream of the shock in addition to the particles precipitating at the Sun for their detailed comparison (e.g., to calculate precipitation ratios, see Vainio, Kocharov, and Laitinen 2000) without introducing an unjustifiably large number of free parameters.

To assess the number (and time profile) of precipitating shock accelerated protons more thoroughly, detailed MHD modeling of the erupting plasma and accelerated protons should be conducted. The same applies to the case-by-case comparison of in situ SEP observations to the pion-decay γ-ray flux, as the structure of the ambient coronal and interplanetary magnetic field and plasma properties will affect the local acceleration efficiency of a shock wave and the resulting evolution of fluxes in the solar wind and those precipitating at the Sun.

It is important to note, however, that even though the protons precipitating at the Sun and those escaping from the upstream of the shock may originate from the same source at the shock, all those processes taking place in the upstream and downstream regions can blur the existing relationship between these two proton populations. In fact, the observed number of high-energy protons in SEP events (measured at 1 AU) corresponding to LDGRFs and the number of precipitated protons are only poorly/moderately correlated (Bruno et al. 2023).

4. Conclusions

We have modeled the number of shock accelerated > 500 MeV protons that are able to precipitate to the solar surface to produce pion-decay γ-rays. We have demonstrated that the total number of particles consistent with the γ-ray fluence spectrum can be produced by a shock with reasonable parameterizations of the shock-accelerated particle spectrum injected in the volume between the shock and the solar surface. The model is able to reproduce gamma-ray event durations measured in hours, although the longest observed durations need quite extreme choices of model parameters. While our modeling results give confidence for the shock origin of the LDGRF events, more detailed modeling should be performed, including a more realistic evolution of the CME-driven shock, transport conditions, and accelerated particle spectrum produced at the shock. Such modeling is already within reach of modern simulation tools (Husidic et al. 2024; Afanasiev, Wijsen, and Vainio 2025; Jin et al. 2018, 2024).

Author Contributions RV and AA constructed the model and wrote the manuscript based on discussions of all co-authors during the ISEE Workshop on Solar Gamma-Ray Events on 16-20 October, 2023, in Nagoya, Japan. All co-authors read and commented the final manuscript.

Funding Information RV and AA acknowledge the funding from the Research Council of Finland (project N. 352847). MJ acknowledges support from NASA SWxC grant 80NSSC23M0191 (CLEAR), NASA LWS grant 80NSSC21K1327, and NASA HSR grant 80NSSC24K0136.

Data Availability No datasets were generated or analyzed during the current study.

Materials Availability Not applicable.

Code Availability Not applicable.

Declarations

Competing Interests The authors declare no competing interests.

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